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Step-by-step solutions for the 2026-27 NCERT (Latest Edition), Class 12 Mathematics

Chapter 7: Integrals

About this Chapter

Exercise 7.3 builds on substitution by adding the **trigonometric-identity toolbox**. You rewrite powers of $\sin$ and $\cos$ using the half-angle identities $\sin^2 x = \frac{1-\cos 2x}{2}$ and $\cos^2 x = \frac{1+\cos 2x}{2}$, and products of $\sin$ and $\cos$ using the product-to-sum identities $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$ etc. Once the integrand is a linear combination of $\sin(kx)$ and $\cos(kx)$, the rest is direct integration.

Topics covered: Half-angle identities • Product-to-sum • Power-reduction • $\tan^n x$ via $\sec^2 x = 1 + \tan^2 x$

Quick Formula Sheet

Half-angle:
$\sin^2 x = \dfrac{1-\cos 2x}{2}, \quad \cos^2 x = \dfrac{1+\cos 2x}{2}$

Product-to-sum:
$2\sin A\cos B = \sin(A+B) + \sin(A-B)$
$2\cos A\cos B = \cos(A-B) + \cos(A+B)$
$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Sine cube:
$\sin^3 x = \dfrac{3\sin x - \sin 3x}{4}$

Cosine cube:
$\cos^3 x = \dfrac{3\cos x + \cos 3x}{4}$

Exercise 7.3

Q 7.1 **Find** $\int \sin^2(2x+5)\,dx$.

SOLUTION

Concept used. The **half-angle identity** $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$ converts a squared sine into a linear combination of 1 and $\cos$, which integrates immediately.

Step 1. Apply the identity with $\theta = 2x + 5$:

$$\sin^2(2x+5) = \frac{1-\cos(4x+10)}{2} = \frac{1}{2} - \frac{1}{2}\cos(4x+10).$$

Step 2. Integrate term-by-term:

$$\int \sin^2(2x+5)\,dx = \tfrac{1}{2}\int 1\,dx - \tfrac{1}{2}\int \cos(4x+10)\,dx.$$

Step 3. Compute each: $\frac{1}{2}\int 1\,dx = x/2$. For the cosine, the inner derivative is 4, so $\int \cos(4x+10)\,dx = \frac{1}{4}\sin(4x+10)$. Multiply by $-1/2$:

$$-\tfrac{1}{2}\cdot\tfrac{1}{4}\sin(4x+10) = -\tfrac{1}{8}\sin(4x+10).$$

Step 4. Combine: $\dfrac{x}{2} - \dfrac{1}{8}\sin(4x+10) + C$.

> **Final Answer:** $\dfrac{x}{2} - \dfrac{1}{8}\sin(4x+10) + C$

EXPERT'S SOLUTION *: Aarav Sharma, M.Sc Mathematics, IIT Bombay*

Strategic angle. Any $\sin^2$ or $\cos^2$ comes down to its half-angle form before integrating.

Step 1. $\sin^2(2x+5) = \frac{1}{2} - \frac{1}{2}\cos(4x+10)$.

Step 2. Anti derivative term-by-term: $\frac{x}{2} - \frac{1}{8}\sin(4x+10)$.

Step 3. Add C.

> **Final Answer:** $\dfrac{x}{2} - \dfrac{\sin(4x+10)}{8} + C$

Q 7.2 **Find** $\displaystyle\int \sin 3x\ \cos 4x\,dx$.

SOLUTION

Concept used. The product-to-sum identity $\sin A\cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$. With $A = 3x$, $B = 4x$:

$$\sin 3x\cos 4x = \tfrac{1}{2}[\sin 7x + \sin(-x)] = \tfrac{1}{2}[\sin 7x - \sin x].$$

Step 1. Apply product-to-sum:

$$\sin 3x \cos 4x = \tfrac{1}{2}\sin 7x - \tfrac{1}{2}\sin x.$$

Step 2. Integrate each piece:

$$\tfrac{1}{2}\int \sin 7x\,dx = \tfrac{1}{2}\cdot\left(-\tfrac{1}{7}\cos 7x\right) = -\tfrac{1}{14}\cos 7x,$$
$$-\tfrac{1}{2}\int \sin x\,dx = -\tfrac{1}{2}\cdot(-\cos x) = \tfrac{1}{2}\cos x.$$

Step 3. Combine: $-\dfrac{1}{14}\cos 7x + \dfrac{1}{2}\cos x + C$.

Final Answer: $-\dfrac{\cos 7x}{14} + \dfrac{\cos x}{2} + C$

EXPERT'S SOLUTION : *Sneha Iyer, M.Sc Applied Mathematics, IIT Kanpur*

Quick reading. $\sin A\cos B$ splits cleanly via product-to-sum.

Step 1. $\sin 3x\cos 4x = \tfrac{1}{2}\sin 7x - \tfrac{1}{2}\sin x$.

Step 2. Anti derivatives: $-\frac{\cos 7x}{14}, \frac{\cos x}{2}$.

Final Answer: $\dfrac{\cos x}{2} - \dfrac{\cos 7x}{14} + C$

Q 7.3 **Find** $\displaystyle\int \cos 2x\ \cos 4x\ \cos 6x\,dx$.

SOLUTION

Concept used. Apply $2\cos A\cos B = \cos(A-B) + \cos(A+B)$ repeatedly. Combine two of the three factors first, then multiply by the third and apply product-to-sum again.

Step 1. Group $\cos 2x\cos 6x$ and use $2\cos A\cos B = \cos(A-B)+\cos(A+B)$:

$$2\cos 2x\cos 6x = \cos 4x + \cos 8x \Rightarrow \cos 2x\cos 6x = \tfrac{1}{2}(\cos 4x + \cos 8x).$$

Step 2. Multiply by $\cos 4x$:

$$\cos 2x\cos 4x\cos 6x = \tfrac{1}{2}\cos 4x(\cos 4x + \cos 8x) = \tfrac{1}{2}\cos^2 4x + \tfrac{1}{2}\cos 4x\cos 8x.$$

Step 3. Use $\cos^2 4x = \frac{1+\cos 8x}{2}$ and $2\cos 4x \cos 8x = \cos 4x + \cos 12x$:

$$\tfrac{1}{2}\cos^2 4x = \tfrac{1}{4}(1+\cos 8x), \qquad \tfrac{1}{2}\cos 4x\cos 8x = \tfrac{1}{4}(\cos 4x + \cos 12x).$$

Step 4. Add the two:

$$\cos 2x \cos 4x \cos 6x = \tfrac{1}{4} + \tfrac{1}{4}\cos 8x + \tfrac{1}{4}\cos 4x + \tfrac{1}{4}\cos 12x.$$

Step 5. Integrate term-by-term:

$$\int = \tfrac{x}{4} + \tfrac{1}{4}\cdot\tfrac{\sin 8x}{8} + \tfrac{1}{4}\cdot\tfrac{\sin 4x}{4} + \tfrac{1}{4}\cdot\tfrac{\sin 12x}{12}.$$

Step 6. Simplify:

$$\frac{x}{4} + \frac{\sin 4x}{16} + \frac{\sin 8x}{32} + \frac{\sin 12x}{48} + C.$$

Final Answer: $\dfrac{x}{4} + \dfrac{\sin 4x}{16} + \dfrac{\sin 8x}{32} + \dfrac{\sin 12x}{48} + C$

♥ Iterated product-to-sum

For products of three or more trig factors, group two at a time and reduce. A triple product of cosines becomes a sum of four (or fewer) cosines, all of which integrate directly.

EXPERT'S SOLUTION : *Karan Mehta, B.Tech Engineering Physics, IIT Bombay*

Strategic angle. Pair smallest and largest argument first to land on a nice intermediate.

Step 1. $\cos 2x \cos 6x = \frac{1}{2}(\cos 4x + \cos 8x)$.

Step 2. Multiply by $\cos 4x$: $\frac{1}{2}\cos^2 4x + \frac{1}{2}\cos 4x \cos 8x$.

Step 3. Expand each: $\frac{1}{4} + \frac{\cos 8x}{4} + \frac{\cos 4x}{4} + \frac{\cos 12x}{4}$.

Step 4. Integrate: $\frac{x}{4} + \frac{\sin 4x}{16} + \frac{\sin 8x}{32} + \frac{\sin 12x}{48} + C$.

Final Answer: $\dfrac{x}{4} + \dfrac{\sin 4x}{16} + \dfrac{\sin 8x}{32} + \dfrac{\sin 12x}{48} + C$

Q 7.4 **Find** $\displaystyle\int \sin^3(2x+1)\,dx$.

SOLUTION

Concept used. The identity $\sin^3\theta = \dfrac{3\sin\theta - \sin 3\theta}{4}$ (from $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$) reduces a cube of sine to a linear combination integrable directly.

Step 1. Apply with $\theta = 2x+1$:

$$\sin^3(2x+1) = \frac{3\sin(2x+1) - \sin(6x+3)}{4}.$$

Step 2. Integrate term-by-term:

$$\int \tfrac{3}{4}\sin(2x+1)\,dx = \tfrac{3}{4}\cdot\left(-\tfrac{1}{2}\cos(2x+1)\right) = -\tfrac{3}{8}\cos(2x+1),$$
$$\int -\tfrac{1}{4}\sin(6x+3)\,dx = -\tfrac{1}{4}\cdot\left(-\tfrac{1}{6}\cos(6x+3)\right) = \tfrac{1}{24}\cos(6x+3).$$

Step 3. Combine: $-\dfrac{3}{8}\cos(2x+1) + \dfrac{1}{24}\cos(6x+3) + C$.

Final Answer: $-\dfrac{3}{8}\cos(2x+1) + \dfrac{1}{24}\cos(6x+3) + C$

EXPERT'S SOLUTION *: Pranav Patel, Ph.D Mathematics, IIT Delhi*

Strategic angle. Use the triple-angle identity to flatten $\sin^3$.

Step 1. $\sin^3(2x+1) = \frac{3\sin(2x+1)-\sin(6x+3)}{4}$.

Step 2. Anti derivatives: $-\frac{3\cos(2x+1)}{8}, +\frac{\cos(6x+3)}{24}$.

Final Answer: $-\dfrac{3\cos(2x+1)}{8} + \dfrac{\cos(6x+3)}{24} + C$

Q 7.5 **Find** $\displaystyle\int \sin^3 x\ \cos^3 x\,dx$.

SOLUTION

Concept used. The double-angle identity $\sin x\cos x = \frac{1}{2}\sin 2x$ turns $\sin^3 x\cos^3 x = (\sin x\cos x)^3$ into $\frac{1}{8}\sin^3 2x$. Then apply the $\sin^3$ reduction identity.

Step 1. Rewrite: $\sin^3 x\cos^3 x = (\sin x\cos x)^3 = \left(\frac{1}{2}\sin 2x\right)^3 = \frac{1}{8}\sin^3 2x$.

Step 2. Use $\sin^3 2x = \frac{3\sin 2x - \sin 6x}{4}$:

$$\frac{1}{8}\sin^3 2x = \frac{1}{32}(3\sin 2x - \sin 6x).$$

Step 3. Integrate term-by-term:

$$\int \frac{3}{32}\sin 2x\,dx = \frac{3}{32}\cdot\left(-\frac{1}{2}\cos 2x\right) = -\frac{3}{64}\cos 2x,$$
$$\int -\frac{1}{32}\sin 6x\,dx = -\frac{1}{32}\cdot\left(-\frac{1}{6}\cos 6x\right) = \frac{1}{192}\cos 6x.$$

Step 4. Combine: $-\dfrac{3}{64}\cos 2x + \dfrac{1}{192}\cos 6x + C$.

Final Answer: $-\dfrac{3\cos 2x}{64} + \dfrac{\cos 6x}{192} + C$

EXPERT'S SOLUTION : *Riya Gupta, M.Sc Mathematics, ISI Kolkata*

Alternative. Substitute $t = \sin x$, $dt = \cos x\,dx$. Then $\sin^3 x\cos^3 x\,dx = t^3(1-t^2)\,dt$, integrate, back-substitute.

Step 1. $t = \sin x$; $\cos^2 x = 1 - t^2$.

Step 2. Integrand: $t^3(1-t^2) = t^3 - t^5$.

Step 3. Anti derivative: $\frac{t^4}{4} - \frac{t^6}{6} = \frac{\sin^4 x}{4} - \frac{\sin^6 x}{6}$.

Step 4. Equivalent to the main solution (differs by a constant).

Final Answer: $\dfrac{\sin^4 x}{4} - \dfrac{\sin^6 x}{6} + C$

Q 7.6 **Find** $\displaystyle\int \sin x\ \sin 2x\ \sin 3x\,dx$.

SOLUTION

Concept used. Apply $2\sin A\sin B = \cos(A-B) - \cos(A+B)$ on two factors, then multiply.

Step 1. Group $\sin x\sin 3x$:

$$2\sin x\sin 3x = \cos 2x - \cos 4x \Rightarrow \sin x\sin 3x = \frac{1}{2}(\cos 2x - \cos 4x).$$

Step 2. Multiply by $\sin 2x$:

$$\sin x \sin 2x \sin 3x = \tfrac{1}{2}(\sin 2x \cos 2x - \sin 2x \cos 4x).$$

Step 3. Use $\sin 2x \cos 2x = \tfrac{1}{2}\sin 4x$ and $2\sin 2x \cos 4x = \sin(2x + 4x) + \sin(2x - 4x) = \sin 6x - \sin 2x$:

$$\sin x \sin 2x \sin 3x = \tfrac{1}{4}\sin 4x - \tfrac{1}{4}(\sin 6x - \sin 2x) = \tfrac{1}{4}\sin 2x + \tfrac{1}{4}\sin 4x - \tfrac{1}{4}\sin 6x.$$

Step 4. Integrate:

$$\int = \tfrac{1}{4}\cdot\left(-\tfrac{\cos 2x}{2}\right) + \tfrac{1}{4}\cdot\left(-\tfrac{\cos 4x}{4}\right) - \tfrac{1}{4}\cdot\left(-\tfrac{\cos 6x}{6}\right).$$

Step 5. Simplify:

$$-\frac{\cos 2x}{8} - \frac{\cos 4x}{16} + \frac{\cos 6x}{24} + C.$$

Final Answer: $\dfrac{\cos 6x}{24} - \dfrac{\cos 4x}{16} - \dfrac{\cos 2x}{8} + C$

EXPERT'S SOLUTION *: Yash Joshi, M.Sc Mathematics, IIT Bombay*

Strategic angle. Three-factor product: pair the two non-adjacent ones first.

Step 1. $\sin x \sin 3x = \tfrac{1}{2}(\cos 2x - \cos 4x)$.

Step 2. Multiply by $\sin 2x$ and apply product-to-sum once more.

Step 3. Net integrand: $\tfrac{1}{4}\sin 2x + \tfrac{1}{4}\sin 4x - \tfrac{1}{4}\sin 6x$.

Step 4. Integrate to get $-\tfrac{\cos 2x}{8} - \tfrac{\cos 4x}{16} + \tfrac{\cos 6x}{24} + C$.

Final Answer: $\dfrac{\cos 6x}{24} - \dfrac{\cos 4x}{16} - \dfrac{\cos 2x}{8} + C$

Q 7.7 Find $\int \sin 4x \, \sin 8x \, dx$.

SOLUTION

Concept used. $2\sin A \sin B = \cos(A - B) - \cos(A + B)$.

Step 1. $2\sin 4x \sin 8x = \cos 4x - \cos 12x$, so $\sin 4x \sin 8x = \tfrac{1}{2}(\cos 4x - \cos 12x)$.

Step 2. Integrate:

$$\frac{1}{2}\int \cos 4x\,dx - \frac{1}{2}\int \cos 12x\,dx = \frac{1}{2}\cdot\frac{\sin 4x}{4} - \frac{1}{2}\cdot\frac{\sin 12x}{12}.$$

Step 3. Simplify: $\dfrac{\sin 4x}{8} - \dfrac{\sin 12x}{24} + C.$

Final Answer: $\dfrac{\sin 4x}{8} - \dfrac{\sin 12x}{24} + C$

EXPERT'S SOLUTION *: Aanya Singh, M.Sc Mathematics, IIT Bombay*

Quick reading. Product-to-sum, then integrate.

Step 1. Convert: $\sin 4x \sin 8x = \frac{1}{2}(\cos 4x - \cos 12x)$.

Step 2. Anti derivative: $\frac{\sin 4x}{8} - \frac{\sin 12x}{24}$.

Final Answer: $\dfrac{\sin 4x}{8} - \dfrac{\sin 12x}{24} + C$

Q 7.8 **Find** $\displaystyle\int \frac{1-\cos x}{1+\cos x}\,dx$.

SOLUTION

Concept used. Use the half-angle identities $1-\cos x = 2\sin^2(x/2)$ and $1+\cos x = 2\cos^2(x/2)$ to reduce the integrand to $\tan^2(x/2)$.

Step 1. Apply the identities:

$$\frac{1-\cos x}{1+\cos x} = \frac{2\sin^2(x/2)}{2\cos^2(x/2)} = \tan^2(x/2).$$

Step 2. Use $\tan^2\theta = \sec^2\theta - 1$:

$$\int \tan^2(x/2)\,dx = \int \sec^2(x/2)\,dx - \int 1\,dx.$$

Step 3. For the first, substitute $t = x/2$: $\int \sec^2(x/2)\,dx = 2\tan(x/2)$. For the second, $\int 1\,dx = x$.

Step 4. Combine: $2\tan(x/2) - x + C$.

Final Answer: $2\tan(x/2) - x + C$

☞ **Half-angle identities**

$1 - \cos\theta = 2\sin^2(\theta/2)$ and $1 + \cos\theta = 2\cos^2(\theta/2)$. The formulas are derived from $\cos 2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$ with $\alpha = \theta/2$.

EXPERT'S SOLUTION : *Diya Kapoor, M.Sc Mathematics, IIT Bombay*

Strategic angle. Whenever you see $1 \pm \cos x$, half-angle. Reduces to a tan form.

Step 1. $\dfrac{1-\cos x}{1+\cos x} = \tan^2(x/2)$.

Step 2. $\int \tan^2(x/2)\,dx = 2\tan(x/2) - x + C$.

Final Answer: $2\tan(x/2) - x + C$

Q 7.9 Find $\displaystyle\int \frac{\cos x}{1+\cos x}\,dx$.

SOLUTION

Concept used. Rewrite $\cos x = (1+\cos x) - 1$ to split the fraction, then apply the half-angle identity $1+\cos x = 2\cos^2(x/2)$.

Step 1. Write $\cos x = (1+\cos x) - 1$:

$$\frac{\cos x}{1+\cos x} = \frac{(1+\cos x)-1}{1+\cos x} = 1 - \frac{1}{1+\cos x}.$$

Step 2. Use $1+\cos x = 2\cos^2(x/2)$, so $\dfrac{1}{1+\cos x} = \dfrac{1}{2}\sec^2(x/2)$.

Step 3. Integrate:

$$\int 1\,dx - \tfrac{1}{2}\int \sec^2(x/2)\,dx = x - \tfrac{1}{2}\cdot 2\tan(x/2) = x - \tan(x/2).$$

Final Answer: $x - \tan(x/2) + C$

EXPERT'S SOLUTION : *Vivaan Nair, Ph.D Pure Mathematics, IISc Bangalore*

Quick reading. Subtract-and-add trick: $\cos x = (1+\cos x) - 1$.

Step 1. $\dfrac{\cos x}{1+\cos x} = 1 - \dfrac{1}{1+\cos x} = 1 - \tfrac{1}{2}\sec^2(x/2)$.

Step 2. Integrate: $x - \tan(x/2) + C$.

Final Answer: $x - \tan(x/2) + C$

Q 7.10 **Find** $\int \sin^4 x\, dx$.

SOLUTION

Concept used. Square the half-angle identity $\sin^2 x = \frac{1-\cos 2x}{2}$.

Step 1. $\sin^4 x = (\sin^2 x)^2 = \left(\dfrac{1-\cos 2x}{2}\right)^2 = \dfrac{1 - 2\cos 2x + \cos^2 2x}{4}$.

Step 2. Use $\cos^2 2x = \dfrac{1+\cos 4x}{2}$:

$$\sin^4 x = \frac{1 - 2\cos 2x + \frac{1+\cos 4x}{2}}{4} = \frac{2 - 4\cos 2x + 1 + \cos 4x}{8} = \frac{3 - 4\cos 2x + \cos 4x}{8}.$$

Step 3. Integrate term-by-term:

$$\int \sin^4 x\, dx = \tfrac{1}{8}\int (3 - 4\cos 2x + \cos 4x)\, dx = \tfrac{1}{8}\left(3x - 4\cdot\tfrac{\sin 2x}{2} + \tfrac{\sin 4x}{4}\right).$$

Step 4. Simplify:

$$\tfrac{1}{8}\left(3x - 2\sin 2x + \tfrac{\sin 4x}{4}\right) = \frac{3x}{8} - \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C.$$

Final Answer: $\dfrac{3x}{8} - \dfrac{\sin 2x}{4} + \dfrac{\sin 4x}{32} + C$

EXPERT'S SOLUTION *: Tara Bhat, M.Sc Mathematics, ISI Kolkata*

Strategic angle. For even powers of $\sin$ or $\cos$, square the half-angle identity and reduce repeatedly until only $\cos(2^k x)$ terms remain.

Step 1. $\sin^4 x = \dfrac{3 - 4\cos 2x + \cos 4x}{8}$.

Step 2. Integrate term-by-term; absorb constants.

Final Answer: $\dfrac{3x}{8} - \dfrac{\sin 2x}{4} + \dfrac{\sin 4x}{32} + C$

Q 7.11 **Find** $\int \cos^4 2x\, dx$.

SOLUTION

Concept used. Square the half-angle identity $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ with $\theta = 2x$.

Step 1. $\cos^2 2x = \dfrac{1+\cos 4x}{2}$.

Step 2. Square:

$$\cos^4 2x = \left(\frac{1+\cos 4x}{2}\right)^2 = \frac{1+2\cos 4x+\cos^2 4x}{4}.$$

Step 3. Use $\cos^2 4x = \dfrac{1+\cos 8x}{2}$:

$$\cos^4 2x = \frac{1+2\cos 4x+\frac{1+\cos 8x}{2}}{4} = \frac{2+4\cos 4x+1+\cos 8x}{8} = \frac{3+4\cos 4x+\cos 8x}{8}.$$

Step 4. Integrate:

$$\int \cos^4 2x\, dx = \tfrac{1}{8}\left(3x + 4\cdot\tfrac{\sin 4x}{4} + \tfrac{\sin 8x}{8}\right) = \frac{3x}{8} + \frac{\sin 4x}{8} + \frac{\sin 8x}{64} + C.$$

Final Answer: $\dfrac{3x}{8} + \dfrac{\sin 4x}{8} + \dfrac{\sin 8x}{64} + C$

EXPERT'S SOLUTION *: Ishaan Reddy, B.Tech CSE, IIT Roorkee*

Quick reading. Symmetric template to Q10.

Step 1. $\cos^4 2x = \dfrac{3+4\cos 4x+\cos 8x}{8}$.

Step 2. Anti derivative: $\dfrac{3x}{8} + \dfrac{\sin 4x}{8} + \dfrac{\sin 8x}{64} + C$.

Final Answer: $\dfrac{3x}{8} + \dfrac{\sin 4x}{8} + \dfrac{\sin 8x}{64} + C$

Q 7.12 **Find** $\displaystyle\int \frac{\sin^2 x}{1+\cos x}\, dx$.

SOLUTION

Concept used. $\sin^2 x = 1 - \cos^2 x = (1 - \cos x)(1 + \cos x)$. The factor $(1 + \cos x)$ cancels.

Step 1. Factor: $\sin^2 x = (1 - \cos x)(1 + \cos x)$.

Step 2. Cancel:

$$\frac{\sin^2 x}{1 + \cos x} = 1 - \cos x.$$

Step 3. Integrate:

$$\int (1 - \cos x)\, dx = x - \sin x + C.$$

Final Answer: $x - \sin x + C$

EXPERT'S SOLUTION *: Dev Chatterjee, B.Tech CSE, IIT Roorkee*

Structural observation. Difference of squares: $1 - \cos^2 x = \sin^2 x$, which factors as $(1 - \cos x)(1 + \cos x)$.

Step 1. Cancel $(1 + \cos x)$.

Step 2. Integrate $1 - \cos x$.

Final Answer: $x - \sin x + C$

Q 7.13 **Find** $\displaystyle\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}\, dx$.

SOLUTION

Concept used. Use $\cos 2\theta = 2\cos^2\theta - 1$ in both numerator terms, then factor.

Step 1. Apply the identity:

$$\cos 2x - \cos 2\alpha = (2\cos^2 x - 1) - (2\cos^2 \alpha - 1) = 2(\cos^2 x - \cos^2 \alpha).$$

Step 2. Factor as a difference of squares:

$$\cos^2 x - \cos^2 \alpha = (\cos x - \cos \alpha)(\cos x + \cos \alpha).$$

Step 3. Substitute and cancel $(\cos x - \cos \alpha)$:

$$\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} = 2(\cos x + \cos \alpha).$$

Step 4. Integrate:

$$\int 2(\cos x + \cos\alpha)\,dx = 2\sin x + 2x\cos\alpha + C.$$

(Here $\cos\alpha$ is a constant.)

Final Answer: $2(\sin x + x\cos\alpha) + C$

EXPERT'S SOLUTION *: Krishna Rao, M.Sc Mathematics, IIT Bombay*

Strategic angle. Spot the algebraic identity that cancels the denominator before trying anything else.

Step 1. $\cos 2x - \cos 2\alpha = 2(\cos^2 x - \cos^2\alpha) = 2(\cos x - \cos\alpha)(\cos x + \cos\alpha)$.

Step 2. Cancel to get $2(\cos x + \cos\alpha)$.

Step 3. Integrate: $2\sin x + 2x\cos\alpha + C$.

Final Answer: $2\sin x + 2x\cos\alpha + C$

Q 7.14 **Find** $\int \dfrac{\cos x - \sin x}{1 + \sin 2x}\,dx$.

SOLUTION

Concept used. $1 + \sin 2x = \sin^2 x + \cos^2 x + 2\sin x\cos x = (\sin x + \cos x)^2$. Substitute $t = \sin x + \cos x$.

Step 1. Rewrite the denominator:

$$1 + \sin 2x = (\sin x + \cos x)^2.$$

Step 2. Put $t = \sin x + \cos x$. Then $dt = (\cos x - \sin x)\,dx$ — exactly the numerator times dx.

Step 3. Substitute:

$$\int \frac{\cos x - \sin x}{(\sin x + \cos x)^2}\,dx = \int \frac{dt}{t^2} = -\frac{1}{t}.$$

Step 4. Back-substitute: $-\dfrac{1}{\sin x + \cos x} + C$.

Final Answer: $-\dfrac{1}{\sin x + \cos x} + C$

EXPERT'S SOLUTION : *Siddharth Pillai, M.Sc Mathematics, IIT Bombay*

Structural observation. $1+\sin 2x$ is a perfect square in $\sin x+\cos x$.

Step 1. $1+\sin 2x=(\sin x+\cos x)^2$.

Step 2. $t=\sin x+\cos x$; $dt=(\cos x-\sin x)\,dx$.

Step 3. $\int dt/t^2=-1/t$.

Final Answer: $-\dfrac{1}{\sin x+\cos x}+C$

Q 7.15 **Find** $\displaystyle\int \tan^3(2x)\sec(2x)\,dx$.

SOLUTION

Concept used. Split $\tan^3=\tan^2\cdot\tan$, use $\tan^2=\sec^2-1$, and substitute $t=\sec(2x)$ since $\dfrac{d}{dx}\sec(2x)=2\sec(2x)\tan(2x)$.

Step 1. Split:

$$\tan^3(2x)\sec(2x)=\tan^2(2x)\cdot\tan(2x)\sec(2x)=[\sec^2(2x)-1]\cdot\tan(2x)\sec(2x).$$

Step 2. Put $t=\sec(2x)$. Then $dt=2\sec(2x)\tan(2x)\,dx$, so $\sec(2x)\tan(2x)\,dx=dt/2$.

Step 3. Substitute:

$$\int(t^2-1)\cdot\frac{dt}{2}=\tfrac{1}{2}\int(t^2-1)\,dt=\tfrac{1}{2}\left(\frac{t^3}{3}-t\right)=\frac{t^3}{6}-\frac{t}{2}.$$

Step 4. Back-substitute:

$$\frac{\sec^3(2x)}{6}-\frac{\sec(2x)}{2}+C.$$

Final Answer: $\dfrac{\sec^3(2x)}{6}-\dfrac{\sec(2x)}{2}+C$

EXPERT'S SOLUTION : *Aditi Verma, M.Sc Mathematics, IIT Bombay*

Strategic angle. $\tan^3\sec$ converts to a polynomial in $\sec$ via $\tan^2=\sec^2-1$.

Step 1. Split: $\tan^3(2x)\sec(2x)=(\sec^2(2x)-1)\tan(2x)\sec(2x)$.

Step 2. $t=\sec(2x)$; $\sec(2x)\tan(2x)\,dx=dt/2$.

Step 3. Integrate $\frac{1}{2}(t^2-1)\,dt = \frac{t^3}{6} - \frac{t}{2}$.

Final Answer: $\dfrac{\sec^3(2x)}{6} - \dfrac{\sec(2x)}{2} + C$

Q 7.16 Find $\displaystyle\int \tan^4 x\,dx$.

SOLUTION

Concept used. Reduce powers of $\tan$ via $\tan^2 x = \sec^2 x - 1$ recursively.

Step 1. Split $\tan^4 = \tan^2 \cdot \tan^2 = \tan^2 x(\sec^2 x - 1) = \tan^2 x \sec^2 x - \tan^2 x$.

Step 2. Substitute $\tan^2 x = \sec^2 x - 1$ in the second term:

$$\tan^4 x = \tan^2 x \sec^2 x - (\sec^2 x - 1) = \tan^2 x \sec^2 x - \sec^2 x + 1.$$

Step 3. Integrate term-by-term. For $\int \tan^2 x \sec^2 x\,dx$, substitute $t = \tan x$, $dt = \sec^2 x\,dx$:

$$\int t^2\,dt = \frac{t^3}{3} = \frac{\tan^3 x}{3}.$$

Step 4. $\int \sec^2 x\,dx = \tan x$; $\int 1\,dx = x$.

Step 5. Combine:

$$\frac{\tan^3 x}{3} - \tan x + x + C.$$

Final Answer: $\dfrac{\tan^3 x}{3} - \tan x + x + C$

EXPERT'S SOLUTION *: Ananya Bhat, M.Sc Mathematics, IIT Bombay*

Strategic angle. Repeatedly replace $\tan^2$ with $\sec^2 - 1$ until you have a combination of $\tan^k \sec^2$ and powers of $\sec$.

Step 1. $\tan^4 x = \tan^2 x \sec^2 x - \sec^2 x + 1$.

Step 2. Anti derivatives: $\tan^3 x/3$, $-\tan x$, x.

Step 3. Combine.

Final Answer: $\dfrac{\tan^3 x}{3} - \tan x + x + C$

Q 7.17 **Find** $\displaystyle\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}\,dx$.

SOLUTION

Concept used. Split the fraction term-by-term, simplify each, and use standard anti derivatives of $\sec x \tan x$ and $\csc x \cot x$.

Step 1. Split:

$$\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} = \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x} = \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x}.$$

Step 2. Recognise the standard forms: $\sin x/\cos^2 x = \tan x \sec x$ and $\cos x/\sin^2 x = \cot x \csc x$.

Step 3. Integrate:

$$\int \tan x \sec x\,dx = \sec x, \qquad \int \cot x \csc x\,dx = -\csc x.$$

Step 4. Combine: $\sec x - \csc x + C$.

Final Answer: $\sec x - \csc x + C$

EXPERT'S SOLUTION *: Rohit Desai, M.Sc Mathematics, IIT Bombay*

Quick reading. Term-by-term division produces standard derivatives of sec and csc.

Step 1. Split into $\tan x \sec x + \cot x \csc x$.

Step 2. Anti derivatives: $\sec x$ and $-\csc x$.

Final Answer: $\sec x - \csc x + C$

Q 7.18 **Find** $\displaystyle\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}\,dx$.

SOLUTION

Concept used. Simplify the numerator using $\cos 2x = 1 - 2\sin^2 x$.

Step 1. Numerator: $\cos 2x + 2\sin^2 x = (1 - 2\sin^2 x) + 2\sin^2 x = 1$.

Step 2. Hence the integrand is $\dfrac{1}{\cos^2 x} = \sec^2 x$.

Step 3. Integrate: $\int \sec^2 x\, dx = \tan x$.

Final Answer: $\tan x + C$

♥ Look for identities first

A nasty-looking integrand often collapses to one or two terms after a single identity. Always simplify the integrand before integrating.

EXPERT'S SOLUTION *: Aditya Kumar, Ph.D Mathematics, IIT Delhi*

Structural observation. The numerator collapses to 1 using $\cos 2x = 1 - 2\sin^2 x$.

Step 1. Numerator $= 1$; integrand $= \sec^2 x$.

Step 2. Anti derivative: $\tan x + C$.

Final Answer: $\tan x + C$

Q 7.19 **Find** $\displaystyle\int \frac{1}{\sin x\ \cos^3 x}\, dx$.

SOLUTION

Concept used. Use $1 = \sin^2 x + \cos^2 x$ in the numerator to split into manageable pieces.

Step 1. Write the numerator as $\sin^2 x + \cos^2 x$:

$$\frac{1}{\sin x \cos^3 x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos^3 x} = \frac{\sin x}{\cos^3 x} + \frac{1}{\sin x \cos x}.$$

Step 2. Simplify the second fraction using $\sin x \cos x = \frac{1}{2}\sin 2x$, so $\dfrac{1}{\sin x \cos x} = \dfrac{2}{\sin 2x}$.

Alternatively, leave it as $\dfrac{\cos^2 x + \sin^2 x}{\sin x \cos x} \cdot \dfrac{1}{1} = \cot x + \tan x$ (using $\cos x/\sin x + \sin x/\cos x$).

Step 3. Better: rewrite as $\tan x + \cot x$: $\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} = \dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x}$.

Step 4. Anti derivative of first term ($\sin x/\cos^3 x$): substitute $t = \cos x$, $dt = -\sin x\,dx$:

$$\int \frac{\sin x}{\cos^3 x}\,dx = -\int \frac{dt}{t^3} = -\cdot\frac{t^{-2}}{-2} = \frac{1}{2t^2} = \frac{1}{2\cos^2 x} = \tfrac{1}{2}\sec^2 x.$$

(We can equivalently write this as $\frac{1}{2}\tan^2 x + \text{const}$, since $\sec^2 x = 1 + \tan^2 x$.)

Step 5. Anti derivatives of the second pair: $\int \tan x\,dx = -\log|\cos x|$ and $\int \cot x\,dx = \log|\sin x|$. Sum: $\log|\sin x| - \log|\cos x| = \log|\tan x|$.

Step 6. Combine all pieces:

$$\int \frac{dx}{\sin x\cos^3 x} = \tfrac{1}{2}\sec^2 x + \log|\tan x| + C.$$

(Equivalently $\frac{1}{2}\tan^2 x + \log|\tan x| + C$, absorbing the $\frac{1}{2}$ constant.)

Final Answer: $\dfrac{\tan^2 x}{2} + \log|\tan x| + C$

EXPERT'S SOLUTION *: Meera Banerjee, Ph.D Pure Mathematics, IISc Bangalore*

Alternative. Multiply top and bottom by $\sec^2 x/\cos x$ to get $\sec^4 x \cdot \tan x \cdot 0$... Actually the cleanest path is dividing by $\cos^4 x$: $\dfrac{1}{\sin x\cos^3 x} = \dfrac{\sec^4 x}{\tan x}\cdot\cos x = \dfrac{\sec^4 x}{\tan x}$. Wait, let us redo: $\dfrac{1}{\sin x\cos^3 x}\cdot\dfrac{1/\cos x}{1/\cos x} = \dfrac{\sec^4 x}{\tan x}$. Substitute $t = \tan x$, $dt = \sec^2 x\,dx$:

$$\int \frac{\sec^4 x\,dx}{\tan x} = \int \frac{(1+\tan^2 x)\sec^2 x\,dx}{\tan x} = \int \frac{1+t^2}{t}\,dt = \log|t| + \frac{t^2}{2}.$$

Step 1. Divide by $\cos^4 x$: integrand becomes $\sec^4 x/\tan x = (1+\tan^2 x)\sec^2 x/\tan x$.

Step 2. $t = \tan x$; $dt = \sec^2 x\,dx$.

Step 3. $\int (1+t^2)/t\,dt = \log|t| + t^2/2$.

Step 4. Back-substitute: $\log|\tan x| + \frac{1}{2}\tan^2 x + C$.

Final Answer: $\dfrac{\tan^2 x}{2} + \log|\tan x| + C$

Q 7.20 **Find** $\displaystyle\int \frac{\cos 2x}{(\cos x + \sin x)^2}\,dx$.

SOLUTION

Concept used. $\cos 2x = \cos^2 x - \sin^2 x = (\cos x - \sin x)(\cos x + \sin x)$.

Step 1. Factor: $\cos 2x = (\cos x - \sin x)(\cos x + \sin x)$.

Step 2. Cancel one factor of $(\cos x + \sin x)$:

$$\frac{\cos 2x}{(\cos x + \sin x)^2} = \frac{\cos x - \sin x}{\cos x + \sin x}.$$

Step 3. Compute the derivative of $\cos x + \sin x$:

$$\frac{d}{dx}(\cos x + \sin x) = -\sin x + \cos x = \cos x - \sin x,$$

which is exactly the numerator. Substitute $t = \cos x + \sin x$, so $dt = (\cos x - \sin x)\,dx$:

$$\int \frac{dt}{t} = \log|t|.$$

Step 4. Back-substitute: $\log|\cos x + \sin x| + C$.

Final Answer: $\log|\cos x + \sin x| + C$

EXPERT'S SOLUTION *: Neha Gupta, M.Sc Mathematics, ISI Kolkata*

Structural observation. $\cos 2x$ factors as a difference of squares, cancelling one factor of the denominator.

Step 1. $\cos 2x = (\cos x - \sin x)(\cos x + \sin x)$.

Step 2. Cancel; the result is in log-derivative form.

Step 3. Anti derivative: $\log|\cos x + \sin x| + C$.

Final Answer: $\log|\cos x + \sin x| + C$

Q 7.21 **Find** $\int \sin^{-1}(\cos x)\,dx$.

SOLUTION

Concept used. Use the co-function identity $\sin^{-1}(\cos x) = \sin^{-1}\big(\sin(\pi/2 - x)\big)$. Where the principal branch applies (i.e. $\pi/2 - x \in [-\pi/2, \pi/2]$, equivalently $x \in [0, \pi]$), we have $\sin^{-1}(\sin\theta) = \theta$. Thus $\sin^{-1}(\cos x) = \pi/2 - x$ on this branch.

Step 1. Use the identity $\cos x = \sin(\pi/2 - x)$:

$$\sin^{-1}(\cos x) = \sin^{-1}\big(\sin(\pi/2 - x)\big) = \pi/2 - x \quad (x \in [0, \pi]).$$

Step 2. Integrate:

$$\int (\pi/2 - x)\, dx = \frac{\pi x}{2} - \frac{x^2}{2}.$$

Step 3. Add the constant.

Final Answer: $\frac{\pi x}{2} - \frac{x^2}{2} + C$

EXPERT'S SOLUTION *: Priya Sharma, Ph.D Mathematics, IIT Delhi*

Strategic angle. Inverse-trig of a co-function collapses via $\pi/2 - x$.

Step 1. $\sin^{-1}(\cos x) = \pi/2 - x$.

Step 2. $\int (\pi/2 - x)\, dx = \pi x/2 - x^2/2$.

Final Answer: $\frac{\pi x - x^2}{2} + C$

Q 7.22 **Find** $\int \frac{1}{\cos(x-a)\cos(x-b)}\, dx$.

SOLUTION

Concept used. Multiply numerator by $\frac{\sin(a-b)}{\sin(a-b)}$ (a constant), then split using $\sin(a-b) = \sin\big[(x-b) - (x-a)\big]$ expanded via $\sin(A-B) = \sin A\cos B - \cos A \sin B$.

Step 1. Multiply and divide by $\sin(a-b)$:

$$I = \int \frac{dx}{\cos(x-a)\cos(x-b)} = \frac{1}{\sin(a-b)} \int \frac{\sin(a-b)\, dx}{\cos(x-a)\cos(x-b)}.$$

Step 2. Use

$$\sin(a-b) = \sin\big[(x-b) - (x-a)\big] = \sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a).$$

Step 3. Split:

$$\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)} = \frac{\sin(x-b)}{\cos(x-b)} - \frac{\sin(x-a)}{\cos(x-a)} = \tan(x-b) - \tan(x-a).$$

Step 4. Integrate each: $\int \tan(x-c)\,dx = -\log|\cos(x-c)|$. Hence

$$I = \frac{1}{\sin(a-b)}\big[-\log|\cos(x-b)| + \log|\cos(x-a)|\big] = \frac{1}{\sin(a-b)}\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right| + C.$$

Final Answer: $\frac{1}{\sin(a-b)}\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right| + C$

EXPERT'S SOLUTION *: Aanya Singh, M.Sc Mathematics, IIT Bombay*

Strategic angle. Whenever an integrand has $\cos(x-a)\cos(x-b)$ or $\sin(x-a)\sin(x-b)$ in the denominator, multiply by the constant $\sin(a-b)$ rewritten as a sine of a difference of the linear factors.

Step 1. Multiply top and bottom by $\sin(a-b)$.

Step 2. Expand $\sin(a-b) = \sin\big[(x-b)-(x-a)\big]$ and split.

Step 3. Integrate $\tan(x-b) - \tan(x-a)$ and divide by $\sin(a-b)$.

Final Answer: $\frac{1}{\sin(a-b)}\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right| + C$

Q 7.23 $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}\,dx$ **is equal to**

(A) $\tan x + \cot x + C$ **(B)** $\tan x + \csc x + C$

(C) $-\tan x + \cot x + C$ **(D)** $\tan x + \sec x + C$

SOLUTION

Concept used. Split the numerator term-by-term.

Step 1. Split:

$$\frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} = \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} = \sec^2 x - \csc^2 x.$$

Step 2. Integrate: $\int \sec^2 x\,dx = \tan x$ and $\int \csc^2 x\,dx = -\cot x$, so $\int -\csc^2 x\,dx = \cot x$.

Step 3. Sum: $\tan x + \cot x + C$. Matches option (A).

Final Answer: Option (A): $\tan x + \cot x + C$

EXPERT'S SOLUTION : *Aarav Sharma, M.Sc Mathematics, IIT Bombay*

Quick reading. Split and recognise $\sec^2 x - \csc^2 x$.

Step 1. $\sec^2 x - \csc^2 x$.

Step 2. Anti derivatives: $\tan x + \cot x$.

Final Answer: Option (A)

Q 7.24 $\int \frac{e^x(1+x)}{\cos^2(e^x x)}\,dx$ **equals**

(A) $-\cot(e^x x) + C$ **(B)** $\tan(xe^x) + C$

(C) $\tan(e^x) + C$ **(D)** $\cot(e^x) + C$

SOLUTION

Concept used. Differentiate $e^x x = xe^x$ by the product rule:

$$\frac{d}{dx}(xe^x) = e^x + xe^x = e^x(1+x).$$

Substitute $t = xe^x$.

Step 1. Put $t = xe^x$. Then $dt = e^x(1+x)\,dx$.

Step 2. Substitute:

$$\int \frac{e^x(1+x)\,dx}{\cos^2(xe^x)} = \int \frac{dt}{\cos^2 t} = \int \sec^2 t\,dt = \tan t.$$

Step 3. Back-substitute: $\tan(xe^x) + C$. Matches option (B).

Final Answer: Option (B): $\tan(xe^x) + C$

♥ **Recognising product-rule derivatives**

$e^x(1+x)$ is the derivative of xe^x. Whenever an integrand contains a sum like $f(x) + xf'(x)$ multiplied by $f'(x)/f'(x)$ structure, suspect a product-rule reversal.

EXPERT'S SOLUTION : *Sneha Iyer, M.Sc Applied Mathematics, IIT Kanpur*

Quick reading. $e^x(1+x) = (xe^x)'$.

Step 1. Substitute $t = xe^x$, $dt = e^x(1+x)\,dx$.

Step 2. $\int \sec^2 t\,dt = \tan t$.

Final Answer: Option (B)

Key Takeaways

- Half-angle identities reduce $\sin^2 x$ and $\cos^2 x$ to a linear combination of 1 and $\cos 2x$, integrable directly.
- Product-to-sum identities reduce products of trig functions to sums; apply once for two factors, iterate for three.
- $\sin^3\theta = (3\sin\theta - \sin 3\theta)/4$ and $\cos^3\theta = (3\cos\theta + \cos 3\theta)/4$ for cubes.
- For $\tan^n x$: split as $\tan^{n-2} x \cdot \tan^2 x = \tan^{n-2} x(\sec^2 x - 1)$ and reduce.
- Always simplify the integrand using identities before reaching for substitution.

End of Exercise 7.3