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Collegedunia NCERT Solutions

Step-by-step solutions for the 2026-27 NCERT (Latest Edition), Class 12 Mathematics

Chapter 7: Integrals

About this Chapter

Exercise 7.7 applies the **three special integrals** of the form $\int\sqrt{x^2 \pm a^2}\,dx$ and $\int\sqrt{a^2 - x^2}\,dx$. We complete the square inside the radical, substitute to push the integral into one of the three standard forms, then read the answer.

Topics covered: $\int\sqrt{a^2 - x^2}\,dx$ • $\int\sqrt{x^2 + a^2}\,dx$ • $\int\sqrt{x^2 - a^2}\,dx$ • Completing the square

Quick Formula Sheet

$$\int\sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

$$\int\sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + C$$

$$\int\sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$$

Exercise 7.7

Q 7.1 **Integrate $\sqrt{4 - x^2}$.**

SOLUTION

Concept used. The standard integral $\int\sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}(x/a) + C$. Identify a by matching the constant inside the radical.

Step 1. Compare $\sqrt{4 - x^2} = \sqrt{2^2 - x^2}$, so $a = 2$.

Step 2. Apply the formula:

$$\int\sqrt{4 - x^2}\,dx = \frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2} + C.$$

Step 3. Simplify $4/2 = 2$:

$$I = \frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2} + C.$$

Final Answer: $\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}+C$

☞ **Three special forms**

$\sqrt{a^2-x^2}$: $\sin^{-1}$ piece. $\sqrt{x^2+a^2}$ or $\sqrt{x^2-a^2}$: log piece.

EXPERT'S SOLUTION *: Vihaan Agarwal, M.Sc Mathematics, IIT Delhi*

Direct read-off. The integrand matches the a^2-x^2 template with $a=2$.

Step 1. Identify $a=2$.

Step 2. Plug into template.

Step 3. Result: $\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}(x/2)+C$.

Final Answer: $\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}+C$

Q 7.2 Integrate $\sqrt{1-4x^2}$.

SOLUTION

Concept used. Factor the 4 out so the coefficient of x^2 becomes 1: $1-4x^2=4\left(\frac{1}{4}-x^2\right)=4\left((1/2)^2-x^2\right)$. Then apply $\int\sqrt{a^2-x^2}\,dx$ with $a=1/2$ and multiply by $\sqrt{4}=2$.

Step 1. Rewrite: $\sqrt{1-4x^2}=2\sqrt{(1/2)^2-x^2}$.

Step 2. Apply the formula with $a=1/2$:

$$\int\sqrt{(1/2)^2-x^2}\,dx=\frac{x}{2}\sqrt{(1/2)^2-x^2}+\frac{(1/2)^2}{2}\sin^{-1}(2x)+C.$$

Step 3. Multiply by 2:

$$I=2\left[\frac{x}{2}\sqrt{\tfrac{1}{4}-x^2}+\frac{1}{8}\sin^{-1}(2x)\right]+C=x\sqrt{\tfrac{1}{4}-x^2}+\frac{1}{4}\sin^{-1}(2x)+C.$$

Step 4. Convert $\sqrt{\frac{1}{4}-x^2}$ back to $\frac{1}{2}\sqrt{1-4x^2}$:

$$I=\frac{x}{2}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}(2x)+C.$$

Final Answer: $\frac{x}{2}\sqrt{1-4x^2} + \frac{1}{4}\sin^{-1}(2x) + C$

EXPERT'S SOLUTION *: Anvita Rao, M.Sc Mathematics, IIT Madras*

Substitution route. Let $u = 2x$ so $du = 2\,dx$ and $1 - 4x^2 = 1 - u^2$. $I = \frac{1}{2}\int \sqrt{1-u^2}\,du$.

Step 1. Use $\int \sqrt{1-u^2}\,du = \frac{u}{2}\sqrt{1-u^2} + \frac{1}{2}\sin^{-1} u$.

Step 2. Multiply by $\frac{1}{2}$: $I = \frac{u}{4}\sqrt{1-u^2} + \frac{1}{4}\sin^{-1} u + C$.

Step 3. Restore $u = 2x$: $I = \frac{x}{2}\sqrt{1-4x^2} + \frac{1}{4}\sin^{-1}(2x) + C$.

Final Answer: $\frac{x}{2}\sqrt{1-4x^2} + \frac{1}{4}\sin^{-1}(2x) + C$

Q 7.3 Integrate $\sqrt{x^2+4x+6}$.

SOLUTION

Concept used. Complete the square inside the radical. $x^2 + 4x + 6 = (x+2)^2 + 2$. Substitute $y = x + 2$ to bring the integral into the $\int \sqrt{y^2+a^2}\,dy$ form with $a^2 = 2$.

Step 1. Complete the square: $x^2 + 4x + 6 = (x^2 + 4x + 4) + 2 = (x+2)^2 + 2$.

Step 2. Put $y = x + 2 \Rightarrow dy = dx$, so $I = \int \sqrt{y^2 + (\sqrt{2})^2}\,dy$.

Step 3. Apply $\int \sqrt{y^2+a^2}\,dy = \frac{y}{2}\sqrt{y^2+a^2} + \frac{a^2}{2}\log\left|y + \sqrt{y^2+a^2}\right|$ with $a^2 = 2$:

$$I = \frac{y}{2}\sqrt{y^2+2} + \log\left|y + \sqrt{y^2+2}\right| + C.$$

Step 4. Back-substitute $y = x + 2$:

$$I = \frac{x+2}{2}\sqrt{x^2+4x+6} + \log\left|(x+2) + \sqrt{x^2+4x+6}\right| + C.$$

Final Answer: $\frac{(x+2)}{2}\sqrt{x^2+4x+6} + \log\left|(x+2) + \sqrt{x^2+4x+6}\right| + C$

EXPERT'S SOLUTION : *Aditi Banerjee, M.Sc Mathematics, IIT Kharagpur*

Two-step plan. Complete the square first; recognise the matching standard form.

Step 1. $(x+2)^2 + (\sqrt{2})^2$; substitute $y = x+2$.

Step 2. Apply $\int \sqrt{y^2+a^2}\,dy$ template with $a = \sqrt{2}$.

Step 3. Restore $y \to x+2$.

Final Answer: $\frac{(x+2)}{2}\sqrt{x^2+4x+6} + \log\left|(x+2) + \sqrt{x^2+4x+6}\right| + C$

Q 7.4 **Integrate $\sqrt{x^2+4x+1}$.**

SOLUTION

Concept used. Completing the square gives $x^2+4x+1 = (x+2)^2 - 3$. Now the form is $\sqrt{y^2-a^2}$ with $a^2 = 3$.

Step 1. Complete the square: $x^2+4x+1 = (x+2)^2 + 1 - 4 = (x+2)^2 - 3$.

Step 2. Substitute $y = x+2$: $I = \int \sqrt{y^2-3}\,dy$.

Step 3. Apply $\int \sqrt{y^2-a^2}\,dy = \frac{y}{2}\sqrt{y^2-a^2} - \frac{a^2}{2}\log\left|y + \sqrt{y^2-a^2}\right|$ with $a^2 = 3$:

$$I = \frac{y}{2}\sqrt{y^2-3} - \frac{3}{2}\log\left|y + \sqrt{y^2-3}\right| + C.$$

Step 4. Back-substitute $y = x+2$:

$$I = \frac{x+2}{2}\sqrt{x^2+4x+1} - \frac{3}{2}\log\left|(x+2) + \sqrt{x^2+4x+1}\right| + C.$$

Final Answer: $\frac{x+2}{2}\sqrt{x^2+4x+1} - \frac{3}{2}\log\left|(x+2) + \sqrt{x^2+4x+1}\right| + C$

EXPERT'S SOLUTION : *Kabir Bansal, M.Sc Mathematics, IIT Bombay*

Pattern. Sign of the constant after squaring decides the family: positive $\to \sqrt{y^2+a^2}$ (log + plus), negative $\to \sqrt{y^2-a^2}$ (log + minus).

Step 1. $(x+2)^2 - 3$: negative constant, use $\sqrt{y^2-a^2}$ template.

Step 2. $a = \sqrt{3}$.

Step 3. Apply and restore.

Final Answer: $\frac{x+2}{2}\sqrt{x^2+4x+1} - \frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right| + C$

Q 7.5 Integrate $\sqrt{1-4x-x^2}$.

SOLUTION

Concept used. Complete the square on the negative-leading quadratic. $1-4x-x^2 = -(x^2+4x-1) = -[(x+2)^2-5] = 5-(x+2)^2$. Form is $\sqrt{a^2-y^2}$ with $a^2=5$.

Step 1. Rewrite: $1-4x-x^2 = 5-(x+2)^2$.

Step 2. Substitute $y=x+2$: $I = \int \sqrt{5-y^2}\,dy$.

Step 3. Apply $\int \sqrt{a^2-y^2}\,dy = \frac{y}{2}\sqrt{a^2-y^2} + \frac{a^2}{2}\sin^{-1}(y/a)$ with $a=\sqrt{5}$:

$$I = \frac{y}{2}\sqrt{5-y^2} + \frac{5}{2}\sin^{-1}\frac{y}{\sqrt{5}} + C.$$

Step 4. Back-substitute $y=x+2$:

$$I = \frac{x+2}{2}\sqrt{1-4x-x^2} + \frac{5}{2}\sin^{-1}\frac{x+2}{\sqrt{5}} + C.$$

Final Answer: $\frac{x+2}{2}\sqrt{1-4x-x^2} + \frac{5}{2}\sin^{-1}\frac{x+2}{\sqrt{5}} + C$

✗ Common Mistake

When the coefficient of x^2 is negative, factor a minus sign *out* of the whole quadratic before completing the square. Skipping this step flips signs and breaks the template match.

EXPERT'S SOLUTION *: Sneha Bose, M.Sc Mathematics, IIT Guwahati*

Three-line plan. Negative leading → pull out -1; complete the square; spot $\sqrt{a^2-y^2}$.

Step 1. $1-4x-x^2 = 5-(x+2)^2$, $a=\sqrt{5}$.

Step 2. Apply template.

Step 3. Restore.

Final Answer: $\dfrac{x+2}{2}\sqrt{1-4x-x^2}+\dfrac{5}{2}\sin^{-1}\dfrac{x+2}{\sqrt{5}}+C$

Q 7.6 **Integrate $\sqrt{x^2+4x-5}$.**

SOLUTION

Concept used. Complete the square. $x^2+4x-5=(x+2)^2-9$. The form is $\sqrt{y^2-a^2}$ with $a^2=9$, i.e. $a=3$.

Step 1. $(x+2)^2-9$; substitute $y=x+2$.

Step 2. Apply $\int\sqrt{y^2-9}\,dy=\frac{y}{2}\sqrt{y^2-9}-\frac{9}{2}\log\left|y+\sqrt{y^2-9}\right|$.

Step 3. Restore $y=x+2$:

$$I=\frac{x+2}{2}\sqrt{x^2+4x-5}-\frac{9}{2}\log\left|(x+2)+\sqrt{x^2+4x-5}\right|+C.$$

Final Answer: $\dfrac{x+2}{2}\sqrt{x^2+4x-5}-\dfrac{9}{2}\log\left|(x+2)+\sqrt{x^2+4x-5}\right|+C$

EXPERT'S SOLUTION *: Manav Joshi, M.Sc Mathematics, IIT Kanpur*

One-pass. Same family as Q4. $a=3$ this time.

Step 1. Complete square.

Step 2. Apply template.

Step 3. Restore.

Final Answer: $\dfrac{x+2}{2}\sqrt{x^2+4x-5}-\dfrac{9}{2}\log\left|(x+2)+\sqrt{x^2+4x-5}\right|+C$

Q 7.7 **Integrate $\sqrt{1+3x-x^2}$.**

SOLUTION

Concept used. Complete the square on a negative-leading quadratic. $1+3x-x^2 = -(x^2-3x-1) = -\big((x-3/2)^2 - 9/4 - 1\big) = \frac{13}{4} - (x-3/2)^2$. The form is $\sqrt{a^2-y^2}$ with $a^2 = 13/4$.

Step 1. Algebra: $1+3x-x^2 = \frac{13}{4} - (x-3/2)^2$.

Step 2. Substitute $y = x - 3/2$: $I = \int \sqrt{\frac{13}{4} - y^2}\, dy$.

Step 3. Apply $\int \sqrt{a^2-y^2}\, dy$ with $a = \sqrt{13}/2$:

$$I = \frac{y}{2}\sqrt{\frac{13}{4} - y^2} + \frac{13/4}{2}\sin^{-1}\frac{2y}{\sqrt{13}} + C.$$

Step 4. Simplify the second term and back-substitute $y = x - 3/2$:

$$I = \frac{(2x-3)}{4}\sqrt{1+3x-x^2} + \frac{13}{8}\sin^{-1}\frac{2x-3}{\sqrt{13}} + C.$$

Final Answer: $\frac{(2x-3)}{4}\sqrt{1+3x-x^2} + \frac{13}{8}\sin^{-1}\frac{2x-3}{\sqrt{13}} + C$

EXPERT'S SOLUTION : *Suhana Kapoor, M.Sc Mathematics, IIT Roorkee*

Halved coefficient trick. When the x-coefficient is odd-over-even, the half is a fraction; the substitution $y = x - 3/2$ pulls the half out cleanly.

Step 1. Complete square $\to 13/4 - (x-3/2)^2$.

Step 2. Apply $\sqrt{a^2-y^2}$ template with $a^2 = 13/4$.

Step 3. Restore; tidy fractions to $(2x-3)/4$ and $13/8$.

Final Answer: $\frac{(2x-3)}{4}\sqrt{1+3x-x^2} + \frac{13}{8}\sin^{-1}\frac{2x-3}{\sqrt{13}} + C$

Q 7.8 **Integrate $\sqrt{x^2+3x}$.**

SOLUTION

Concept used. Complete the square. $x^2+3x = (x+3/2)^2 - 9/4$. Form is $\sqrt{y^2-a^2}$ with $a^2 = 9/4$.

Step 1. $(x+3/2)^2-(3/2)^2$; substitute $y=x+3/2$.

Step 2. Apply $\int\sqrt{y^2-a^2}\,dy$ template with $a=3/2$:

$$I=\frac{y}{2}\sqrt{y^2-9/4}-\frac{9/4}{2}\log\left|y+\sqrt{y^2-9/4}\right|+C.$$

Step 3. Back-substitute and tidy:

$$I=\frac{2x+3}{4}\sqrt{x^2+3x}-\frac{9}{8}\log\left|(x+3/2)+\sqrt{x^2+3x}\right|+C.$$

Final Answer: $\frac{2x+3}{4}\sqrt{x^2+3x}-\frac{9}{8}\log\left|(x+3/2)+\sqrt{x^2+3x}\right|+C$

EXPERT'S SOLUTION *: Aryaman Saluja, M.Sc Mathematics, IIT Bombay*

Quick. No constant term, but the procedure is unchanged.

Step 1. Complete: $(x+3/2)^2-9/4$.

Step 2. Template $\sqrt{y^2-a^2}$.

Step 3. Restore.

Final Answer: $\frac{2x+3}{4}\sqrt{x^2+3x}-\frac{9}{8}\log\left|(x+3/2)+\sqrt{x^2+3x}\right|+C$

Q 7.9 Integrate $\sqrt{1+\frac{x^2}{9}}$.

SOLUTION

Concept used. Factor $1/9$ from the radical so the form becomes $\frac{1}{3}\sqrt{x^2+9}$, then apply the $\sqrt{x^2+a^2}$ template with $a=3$.

Step 1. $\sqrt{1+x^2/9}=\sqrt{(9+x^2)/9}=\frac{1}{3}\sqrt{x^2+9}$.

Step 2. So $I=\frac{1}{3}\int\sqrt{x^2+9}\,dx$.

Step 3. Apply the template with $a=3$:

$$\int\sqrt{x^2+9}\,dx=\frac{x}{2}\sqrt{x^2+9}+\frac{9}{2}\log\left|x+\sqrt{x^2+9}\right|+C.$$

Step 4. Multiply by $1/3$:

$$I = \frac{x}{6}\sqrt{x^2+9} + \frac{3}{2}\log\left|x + \sqrt{x^2+9}\right| + C.$$

Final Answer: $\frac{x}{6}\sqrt{x^2+9} + \frac{3}{2}\log\left|x + \sqrt{x^2+9}\right| + C$

EXPERT'S SOLUTION : *Rhea Banerjee, M.Sc Mathematics, IISER Kolkata*

Bring out the constant. $\sqrt{1 + x^2/9} = \sqrt{x^2+9}/3$. Then template.

Step 1. Pull factor of $1/3$ outside.

Step 2. Apply $\sqrt{x^2+a^2}$ formula with $a = 3$.

Step 3. Multiply through.

Final Answer: $\frac{x}{6}\sqrt{x^2+9} + \frac{3}{2}\log\left|x + \sqrt{x^2+9}\right| + C$

Q 7.10 $\int \sqrt{1+x^2}\, dx$ **is equal to**

(A) $\frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\log\left(x + \sqrt{1+x^2}\right) + C$

(B) $\frac{2}{3}(1+x^2)^{3/2} + C$

(C) $\frac{2}{3}x(1+x^2)^{3/2} + C$

(D) $\frac{x^2}{2}\sqrt{1+x^2} + \frac{1}{2}x^2\log\left|x + \sqrt{1+x^2}\right| + C$

SOLUTION

Concept used. Standard form $\int \sqrt{x^2+a^2}\, dx$ with $a = 1$.

Step 1. Identify $a = 1$ from $1 + x^2 = x^2 + 1$.

Step 2. Apply the formula:

$$\int \sqrt{1+x^2}\, dx = \frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\log\left|x + \sqrt{1+x^2}\right| + C.$$

Step 3. Compare with the options: matches (A).

Final Answer: Option (A)

EXPERT'S SOLUTION : *Maitri Desai, M.Sc Mathematics, IIT Madras*

Eliminator check. (B) and (C) have the wrong functional shape (no log); (D) has an x^2 multiplying the log term which is dimensionally wrong. Only (A) survives.

Step 1. Apply $\sqrt{x^2+a^2}$ template with $a=1$.

Step 2. Match.

Final Answer: Option (A)

Q 7.11 $\int \sqrt{x^2-8x+7}\,dx$ **is equal to**

(A) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+9\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$
(B) $\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log\left|x+4+\sqrt{x^2-8x+7}\right|+C$
(C) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$
(D) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

SOLUTION

Concept used. Complete the square inside the radical: $x^2-8x+7=(x-4)^2-9$. Apply $\int\sqrt{y^2-a^2}\,dy$ template with $a^2=9$.

Step 1. Complete the square: $x^2-8x+7=x^2-8x+16-9=(x-4)^2-9$.

Step 2. Substitute $y=x-4$: $I=\int\sqrt{y^2-9}\,dy$.

Step 3. Apply $\int\sqrt{y^2-a^2}\,dy=\frac{y}{2}\sqrt{y^2-a^2}-\frac{a^2}{2}\log\left|y+\sqrt{y^2-a^2}\right|$ with $a^2=9$:

$$I=\frac{y}{2}\sqrt{y^2-9}-\frac{9}{2}\log\left|y+\sqrt{y^2-9}\right|+C.$$

Step 4. Back-substitute $y=x-4$:

$$I=\frac{x-4}{2}\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|(x-4)+\sqrt{x^2-8x+7}\right|+C.$$

Step 5. Matches option (D).

Final Answer: Option (D)

EXPERT'S SOLUTION : *Tarun Gowda, M.Sc Mathematics, IIT Hyderabad*

Sign-of-log check. The $\sqrt{y^2 - a^2}$ template carries a *negative* sign in front of the log, so (A) and (B) (both positive) are out; (C) has $-3\sqrt{2}$ instead of $-9/2$, which is wrong; (D) matches.

Step 1. Complete square $\to (x-4)^2 - 9$.

Step 2. Apply template; result $-\frac{9}{2}\log$.

Step 3. Pick (D).

Final Answer: Option (D)

Key Takeaways

- Three special integrals: $\sqrt{a^2 - x^2} \to \sin^{-1}$; $\sqrt{x^2 + a^2} \to +\log$; $\sqrt{x^2 - a^2} \to -\log$.
- Always complete the square inside the radical before applying the template.
- For a negative-leading quadratic (like $1 - 4x - x^2$), pull -1 out so the leading coefficient becomes $+1$; the form is then $\sqrt{a^2 - y^2}$.
- Sign of the constant after completing the square decides whether you are in the $+a^2$ family or the $-a^2$ family.
- For coefficients like $4x^2$ or $x^2/9$, factor the scalar out first so the leading coefficient is ± 1; the integral picks up an extra constant multiplier.

End of Exercise 7.7