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Step-by-step solutions for the 2026-27 NCERT (Latest Edition), Class 12 Mathematics

Chapter 7: Integrals

About this Chapter

Exercise 7.8 applies the **Second Fundamental Theorem of Calculus**: if $F'(x) = f(x)$ on $[a, b]$, then $\int_a^b f(x)\,dx = F(b) - F(a)$. Find an antiderivative, evaluate it at the two limits, and subtract.

Topics covered: Second FTC • Direct evaluation $[F(x)]_a^b$ • Trig identities • Definite integrals across breakpoints

Quick Formula Sheet

Second FTC: $\displaystyle\int_a^b f(x)\,dx = F(b) - F(a)$, where $F' = f$ on $[a, b]$.

Useful:
$\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ (for $n \neq -1$).
$\displaystyle\int \sin x\,dx = -\cos x$; $\displaystyle\int \cos x\,dx = \sin x$.
$\displaystyle\int \frac{dx}{1+x^2} = \tan^{-1} x$; $\displaystyle\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x$.
$\displaystyle\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|$.

Exercise 7.8

Q 7.1 Evaluate $\displaystyle\int_{-1}^{1} (x+1)\,dx$.

SOLUTION

Concept used. **Second Fundamental Theorem of Calculus**. If $F'(x) = f(x)$ on $[a, b]$ then $\int_a^b f\,dx = F(b) - F(a)$. Integrate term-by-term using $\int x^n\,dx = \dfrac{x^{n+1}}{n+1}$ for $n \neq -1$, then plug the limits.

Step 1. Antiderivative: $F(x) = \dfrac{x^2}{2} + x$.

Step 2. Evaluate at $x = 1$: $F(1) = \frac{1}{2} + 1 = \frac{3}{2}$.

Step 3. Evaluate at $x = -1$: $F(-1) = \frac{1}{2} - 1 = -\frac{1}{2}$.

Step 4. Subtract: $\int_{-1}^{1}(x+1)\,dx = F(1) - F(-1) = \frac{3}{2} - (-\frac{1}{2}) = 2$.

Final Answer: 2

☞ **Power rule**

$\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$. The -1 case becomes $\log|x| + C$.

EXPERT'S SOLUTION *: Anaya Nair, M.Sc Mathematics, IIT Madras*

Sanity check by area. The line $y = x + 1$ between -1 and 1 traces a right triangle of base 2 and height 2, area $\frac{1}{2} \cdot 2 \cdot 2 = 2$.

Step 1. Antiderivative $F(x) = x^2/2 + x$.

Step 2. $F(1) - F(-1) = 3/2 - (-1/2) = 2$.

Step 3. Matches the geometric area. ✓

Final Answer: 2

Q 7.2 **Evaluate** $\displaystyle\int_2^3 \frac{1}{x}\,dx$.

SOLUTION

Concept used. The antiderivative of $1/x$ is $\log|x|$ (natural log, base e).

Step 1. Antiderivative $F(x) = \log|x|$.

Step 2. $F(3) = \log 3$, $F(2) = \log 2$.

Step 3. $\int_2^3 \dfrac{dx}{x} = \log 3 - \log 2 = \log\dfrac{3}{2}$.

Final Answer: $\log(3/2)$

EXPERT'S SOLUTION : *Ishaan Bhandari, M.Sc Mathematics, Delhi University*

Property of log. $\log a - \log b = \log(a/b)$ — a one-line move.

Step 1. $F = \log|x|$.

Step 2. Difference: $\log 3 - \log 2 = \log(3/2)$.

Final Answer: $\log(3/2)$

Q 7.3 **Evaluate** $\int_1^2 (4x^3 - 5x^2 + 6x + 9)\,dx$.

SOLUTION

Concept used. Linearity of integration plus power rule.

Step 1. Antiderivative term-by-term:

$$F(x) = 4\cdot\frac{x^4}{4} - 5\cdot\frac{x^3}{3} + 6\cdot\frac{x^2}{2} + 9x = x^4 - \frac{5x^3}{3} + 3x^2 + 9x.$$

Step 2. $F(2) = 16 - \frac{40}{3} + 12 + 18 = 46 - \frac{40}{3} = \frac{138-40}{3} = \frac{98}{3}$.

Step 3. $F(1) = 1 - \frac{5}{3} + 3 + 9 = 13 - \frac{5}{3} = \frac{39-5}{3} = \frac{34}{3}$.

Step 4. $I = F(2) - F(1) = \frac{98}{3} - \frac{34}{3} = \frac{64}{3}$.

Final Answer: $\frac{64}{3}$

EXPERT'S SOLUTION : *Pratyush Hegde, M.Sc Mathematics, IIT Hyderabad*

Tidy. Integrate each piece independently and add.

Step 1. Antiderivative $= x^4 - \frac{5x^3}{3} + 3x^2 + 9x$.

Step 2. $F(2) = 98/3$, $F(1) = 34/3$.

Step 3. $F(2) - F(1) = 64/3$.

Final Answer: $64/3$

Q 7.4 **Evaluate** $\int_0^{\pi/4} \sin 2x \, dx$.

SOLUTION

Concept used. $\int \sin(ax)\,dx = -\frac{\cos(ax)}{a}$, with $a = 2$.

Step 1. Antiderivative: $F(x) = -\frac{\cos 2x}{2}$.

Step 2. $F(\pi/4) = -\frac{\cos(\pi/2)}{2} = -\frac{0}{2} = 0$.

Step 3. $F(0) = -\frac{\cos 0}{2} = -\frac{1}{2}$.

Step 4. $I = F(\pi/4) - F(0) = 0 - (-\frac{1}{2}) = \frac{1}{2}$.

Final Answer: $1/2$

EXPERT'S SOLUTION *: Ayan Mukherjee, M.Sc Mathematics, IIT Kharagpur*

Reading. The chain rule pulls a $1/a$ inside; $a = 2$ here.

Step 1. $F = -\cos(2x)/2$.

Step 2. Values: $F(\pi/4) = 0$, $F(0) = -1/2$.

Step 3. Difference: $1/2$.

Final Answer: $1/2$

Q 7.5 **Evaluate** $\int_0^{\pi/2} \cos 2x \, dx$.

SOLUTION

Concept used. $\int \cos(ax)\,dx = \frac{\sin(ax)}{a}$ with $a = 2$.

Step 1. Antiderivative $F(x) = \frac{\sin 2x}{2}$.

Step 2. $F(\pi/2) = \frac{\sin \pi}{2} = 0$.

Step 3. $F(0) = 0$.

Step 4. $I = 0 - 0 = 0$.

Final Answer: 0

EXPERT'S SOLUTION *: Riddhi Sharma, M.Sc Mathematics, IIT BHU*

Why zero. $\cos 2x$ on $[0, \pi/2]$ has equal positive and negative areas about the x-axis (sign flips at $\pi/4$), so they cancel exactly.

Step 1. Antiderivative $\sin(2x)/2$.

Step 2. Both endpoint values equal 0.

Final Answer: 0

Q 7.6 **Evaluate** $\int_4^5 e^x\,dx$**.**

SOLUTION

Concept used. $\int e^x\,dx = e^x$.

Step 1. $F(x) = e^x$.

Step 2. $I = e^5 - e^4 = e^4(e-1)$.

Final Answer: $e^5 - e^4$

EXPERT'S SOLUTION *: Sara Iyer, M.Sc Mathematics, Anna University*

Factor. $e^5 - e^4 = e^4(e-1) \approx 54.6 \times 1.72 \approx 93.8$.

Step 1. Direct FTC: $e^5 - e^4$.

Final Answer: $e^5 - e^4$

Q 7.7 **Evaluate** $\int_0^{\pi/4} \tan x\,dx$**.**

SOLUTION

Concept used. $\int \tan x\,dx = \log|\sec x| + C$, derived from $\tan x = \dfrac{\sin x}{\cos x}$ with the substitution $u = \cos x$.

Step 1. $F(x) = \log|\sec x|$.

Step 2. $F(\pi/4) = \log|\sec(\pi/4)| = \log\sqrt{2} = \frac{1}{2}\log 2$.

Step 3. $F(0) = \log 1 = 0$.

Step 4. $I = \frac{1}{2}\log 2 - 0 = \frac{1}{2}\log 2$.

> **Final Answer:** $\frac{1}{2}\log 2$

EXPERT'S SOLUTION *: Naman Solanki, M.Sc Mathematics, IIT Roorkee*

Alternative form. $\log\sqrt{2} = \log 2^{1/2} = \frac{1}{2}\log 2$. Same value.

Step 1. Apply $F(x) = \log|\sec x|$.

Step 2. Evaluate at $\pi/4$ and 0.

Step 3. Difference: $\frac{1}{2}\log 2$.

> **Final Answer:** $\frac{1}{2}\log 2$

Q 7.8 **Evaluate** $\displaystyle\int_{\pi/6}^{\pi/4} \operatorname{cosec} x\,dx$.

SOLUTION

Concept used. $\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + C$ (one of the four "standard logs").

Step 1. $F(x) = \log|\operatorname{cosec} x - \cot x|$.

Step 2. At $x = \pi/4$: $\operatorname{cosec}(\pi/4) = \sqrt{2}$, $\cot(\pi/4) = 1$. So $F(\pi/4) = \log|\sqrt{2} - 1|$.

Step 3. At $x = \pi/6$: $\operatorname{cosec}(\pi/6) = 2$, $\cot(\pi/6) = \sqrt{3}$. So $F(\pi/6) = \log|2 - \sqrt{3}|$.

Step 4. Subtract: $I = \log|\sqrt{2} - 1| - \log|2 - \sqrt{3}| = \log\left|\dfrac{\sqrt{2} - 1}{2 - \sqrt{3}}\right|$.

Final Answer: $\log\left|\dfrac{\sqrt{2}-1}{2-\sqrt{3}}\right|$

EXPERT'S SOLUTION : *Yashika Pandey, M.Sc Mathematics, BHU Varanasi*

Alternate form. $\log|\operatorname{cosec} x - \cot x| = \log|\tan(x/2)|$ (half-angle identity).

Step 1. $F = \log|\tan(x/2)|$. $F(\pi/4) = \log\tan(\pi/8)$; $F(\pi/6) = \log\tan(\pi/12)$.

Step 2. Difference: $\log[\tan(\pi/8)/\tan(\pi/12)]$. Equivalent to the boxed answer.

Final Answer: $\log\left|\dfrac{\sqrt{2}-1}{2-\sqrt{3}}\right|$

Q 7.9 **Evaluate** $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^2}}$.

SOLUTION

Concept used. $\int \dfrac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$.

Step 1. $F(x) = \sin^{-1} x$.

Step 2. $F(1) = \sin^{-1}(1) = \dfrac{\pi}{2}$.

Step 3. $F(0) = \sin^{-1}(0) = 0$.

Step 4. $I = \dfrac{\pi}{2}$.

Final Answer: $\pi/2$

EXPERT'S SOLUTION : *Bhavya Goyal, M.Sc Mathematics, IIT Bombay*

Read it off. The standard antiderivative does the job.

Step 1. $\sin^{-1} 1 - \sin^{-1} 0 = \pi/2$.

Final Answer: $\pi/2$

Q 7.10 **Evaluate** $\int_0^1 \frac{dx}{1+x^2}$.

SOLUTION

Concept used. $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$.

Step 1. $F(x) = \tan^{-1} x$.

Step 2. $F(1) = \tan^{-1}(1) = \frac{\pi}{4}$.

Step 3. $F(0) = 0$.

Step 4. $I = \frac{\pi}{4}$.

Final Answer: $\pi/4$

EXPERT'S SOLUTION *: Sahil Khanna, M.Sc Mathematics, IIT Delhi*

Reading. The fundamental arctan integral.

Step 1. $\tan^{-1}(1) - \tan^{-1}(0) = \pi/4$.

Final Answer: $\pi/4$

Q 7.11 **Evaluate** $\int_2^3 \frac{dx}{x^2-1}$.

SOLUTION

Concept used. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + C$ with $a = 1$.

Step 1. Antiderivative: $F(x) = \frac{1}{2} \log\left|\frac{x-1}{x+1}\right|$.

Step 2. $F(3) = \frac{1}{2} \log\frac{2}{4} = \frac{1}{2} \log\frac{1}{2}$.

Step 3. $F(2) = \frac{1}{2} \log\frac{1}{3}$.

Step 4. Subtract:

$$I = \frac{1}{2} \log\frac{1}{2} - \frac{1}{2} \log\frac{1}{3} = \frac{1}{2} \log\frac{1/2}{1/3} = \frac{1}{2} \log\frac{3}{2}.$$

Final Answer: $\frac{1}{2}\log(3/2)$

EXPERT'S SOLUTION *: Manvi Sehgal, M.Sc Mathematics, IIT Kanpur*

Property used. $\log(a/b) - \log(c/d) = \log(ad/(bc))$. Apply on $\frac{1}{2}$ and $\frac{1}{3}$ ratios.

Step 1. Apply standard antiderivative.

Step 2. Combine logs to $\frac{1}{2}\log(3/2)$.

Final Answer: $\frac{1}{2}\log(3/2)$

Q 7.12 **Evaluate** $\int_0^{\pi/2} \cos^2 x \, dx$.

SOLUTION

Concept used. Power-reducing identity $\cos^2 x = \dfrac{1+\cos 2x}{2}$.

Step 1. Rewrite: $\cos^2 x = \dfrac{1+\cos 2x}{2}$.

Step 2. Antiderivative:

$$F(x) = \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) = \frac{x}{2} + \frac{\sin 2x}{4}.$$

Step 3. $F(\pi/2) = \dfrac{\pi}{4} + \dfrac{\sin \pi}{4} = \dfrac{\pi}{4} + 0 = \dfrac{\pi}{4}$.

Step 4. $F(0) = 0$.

Step 5. $I = \dfrac{\pi}{4}$.

Final Answer: $\pi/4$

EXPERT'S SOLUTION *: Karthik Iyer, M.Sc Mathematics, IIT Madras*

Average-value shortcut. Over $[0, \pi/2]$, average of $\cos^2$ is $1/2$, times length $\pi/2$, gives $\pi/4$.

Step 1. Recognise $\cos^2 x$ has period π, average $1/2$.

Step 2. Multiply: $\frac{1}{2} \cdot \frac{\pi}{2} = \pi/4$.

Final Answer: $\pi/4$

Q 7.13 **Evaluate** $\displaystyle\int_2^3 \frac{x\,dx}{x^2+1}$.

SOLUTION

Concept used. Substitution $t = x^2 + 1 \Rightarrow dt = 2x\,dx$.

Step 1. Antiderivative: $\int \dfrac{x\,dx}{x^2+1} = \dfrac{1}{2}\log|x^2+1| = \dfrac{1}{2}\log(x^2+1)$.

Step 2. $F(3) = \frac{1}{2}\log 10$, $F(2) = \frac{1}{2}\log 5$.

Step 3. $I = \frac{1}{2}(\log 10 - \log 5) = \frac{1}{2}\log 2$.

Final Answer: $\frac{1}{2}\log 2$

EXPERT'S SOLUTION *: Tanmay Suri, M.Sc Mathematics, IIT Bombay*

Pattern. Numerator is, up to a factor of 2, the derivative of the denominator. That always gives a log.

Step 1. $\frac{1}{2}\log(x^2+1)$ at $x = 3$ vs $x = 2$.

Step 2. Difference: $\frac{1}{2}\log 2$.

Final Answer: $\frac{1}{2}\log 2$

Q 7.14 **Evaluate** $\displaystyle\int_0^1 \frac{2x+3}{5x^2+1}\,dx$.

SOLUTION

Concept used. Split the numerator: write $2x + 3 = (2x) + 3$ so the first piece matches the derivative of $5x^2 + 1$ (which is $10x$), and the second piece becomes a $\tan^{-1}$ integral.

Step 1. Split:

$$I = \int_0^1 \frac{2x}{5x^2+1}\,dx + \int_0^1 \frac{3}{5x^2+1}\,dx = I_1 + I_2.$$

Step 2. I_1: let $t = 5x^2 + 1$, $dt = 10x\,dx$, so $2x\,dx = dt/5$:

$$I_1 = \int_1^6 \frac{dt/5}{t} = \frac{1}{5}\log\frac{6}{1} = \frac{\log 6}{5}.$$

Step 3. I_2: factor 5 out:

$$I_2 = 3\int_0^1 \frac{dx}{5x^2+1} = \frac{3}{5}\int_0^1 \frac{dx}{x^2+1/5}.$$

Use $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}(x/a)$ with $a = 1/\sqrt{5}$:

$$I_2 = \frac{3}{5}\cdot\sqrt{5}\left[\tan^{-1}\sqrt{5}\,x\right]_0^1 = \frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}.$$

Step 4. Sum:

$$I = \frac{\log 6}{5} + \frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}.$$

Final Answer: $\frac{\log 6}{5} + \frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$

EXPERT'S SOLUTION : *Pari Sharma, M.Sc Mathematics, IIT Delhi*

Two-piece plan. Whenever the numerator is "α(derivative of denominator) $+ \beta$," break into a log piece and a $\tan^{-1}$ piece.

Step 1. $2x\,dx \to$ log piece $\frac{\log 6}{5}$.

Step 2. Constant $3 \to \tan^{-1}$ piece $\frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$.

Step 3. Sum.

Final Answer: $\frac{1}{5}\log 6 + \frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$

Q 7.15 **Evaluate** $\int_0^1 x\,e^{x^2}\,dx$.

SOLUTION

Concept used. Substitution $t = x^2 \Rightarrow dt = 2x\,dx$.

Step 1. Antiderivative: $\int xe^{x^2}\,dx = \frac{1}{2}e^{x^2}$ (since $\frac{d}{dx}e^{x^2} = 2xe^{x^2}$).

Step 2. $F(1) = \frac{e}{2}$, $F(0) = \frac{1}{2}$.

Step 3. $I = \frac{e}{2} - \frac{1}{2} = \dfrac{e-1}{2}$.

Final Answer: $\dfrac{e-1}{2}$

EXPERT'S SOLUTION *: Aakash Kohli, M.Sc Mathematics, IIT Bombay*

Pattern. x in front of e^{x^2} is exactly what's needed for the inner derivative.

Step 1. Antiderivative $\frac{1}{2}e^{x^2}$.

Step 2. $(e-1)/2$.

Final Answer: $(e-1)/2$

Q 7.16 **Evaluate** $\displaystyle\int_1^2 \frac{5x^2}{x^2+4x+3}\,dx$.

SOLUTION

Concept used. Degree of numerator equals degree of denominator ⇒ do polynomial long division first; then partial fractions on the proper remainder.

Step 1. Divide $5x^2$ by x^2+4x+3: quotient 5, remainder $5x^2 - 5(x^2+4x+3) = -20x - 15$. So

$$\frac{5x^2}{x^2+4x+3} = 5 - \frac{20x+15}{x^2+4x+3}.$$

Step 2. Factor denominator: $x^2+4x+3 = (x+1)(x+3)$. Partial fractions:

$$\frac{20x+15}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}.$$

Multiply: $20x+15 = A(x+3) + B(x+1)$. Set $x=-1$: $-5 = 2A \Rightarrow A = -5/2$. Set $x=-3$: $-45 = -2B \Rightarrow B = 45/2$.

Step 3. Integrand becomes:

$$5 + \frac{5/2}{x+1} - \frac{45/2}{x+3}.$$

Antiderivative:

$$F(x) = 5x + \tfrac{5}{2}\log|x+1| - \tfrac{45}{2}\log|x+3|.$$

Step 4. $F(2) = 10 + \frac{5}{2}\log 3 - \frac{45}{2}\log 5.$
$F(1) = 5 + \frac{5}{2}\log 2 - \frac{45}{2}\log 4.$

Step 5. Subtract:

$$I = 5 + \tfrac{5}{2}\log\frac{3}{2} - \tfrac{45}{2}\log\frac{5}{4}.$$

> **Final Answer:** $5 + \frac{5}{2}\log\frac{3}{2} - \frac{45}{2}\log\frac{5}{4}$

EXPERT'S SOLUTION *: Vivek Roy, M.Sc Mathematics, IIT Kharagpur*

Bookkeeping. The constant "5" from the quotient contributes $5 \cdot (2-1) = 5$. The logs come from each pole separately.

Step 1. Quotient 5, remainder $-(20x + 15)$.

Step 2. Partial fractions: $-5/2$ and $45/2$ for poles at -1 and -3.

Step 3. Integrate and evaluate at endpoints.

> **Final Answer:** $5 + \frac{5}{2}\log\frac{3}{2} - \frac{45}{2}\log\frac{5}{4}$

Q 7.17 **Evaluate** $\displaystyle\int_0^{\pi/4} (2\sec^2 x + x^3 + 2)\,dx$.

SOLUTION

Concept used. Integrate term-by-term: $\int \sec^2 x\,dx = \tan x$; $\int x^3 = x^4/4$; $\int 2\,dx = 2x$.

Step 1. Antiderivative:

$$F(x) = 2\tan x + \frac{x^4}{4} + 2x.$$

Step 2. $F(\pi/4) = 2\tan(\pi/4) + \dfrac{(\pi/4)^4}{4} + 2\cdot\dfrac{\pi}{4} = 2 + \dfrac{\pi^4}{4\cdot 256} + \dfrac{\pi}{2} = 2 + \dfrac{\pi^4}{1024} + \dfrac{\pi}{2}.$

Step 3. $F(0) = 0.$

Step 4. Therefore $I = 2 + \dfrac{\pi}{2} + \dfrac{\pi^4}{1024}.$

> **Final Answer:** $2 + \dfrac{\pi}{2} + \dfrac{\pi^4}{1024}$

EXPERT'S SOLUTION : *Devyani Hooda, M.Sc Mathematics, IIT Roorkee*

Per-piece read. $2\tan x$ contributes 2; $x^4/4$ contributes $(\pi/4)^4/4 = \pi^4/1024$; $2x$ contributes $\pi/2$.

Step 1. Sum the three pieces.

> **Final Answer:** $2 + \dfrac{\pi}{2} + \dfrac{\pi^4}{1024}$

Q 7.18 **Evaluate** $\displaystyle\int_0^{\pi} \left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right) dx$.

SOLUTION

Concept used. Identity $\cos x = \cos^2(x/2) - \sin^2(x/2)$, equivalently $\sin^2(x/2) - \cos^2(x/2) = -\cos x$.

Step 1. Simplify: integrand $= -\cos x$.

Step 2. Antiderivative: $F(x) = -\sin x$.

Step 3. $F(\pi) - F(0) = -\sin\pi - (-\sin 0) = 0 - 0 = 0$.

> **Final Answer:** 0

☞ **Double-angle**

$\cos x = 1 - 2\sin^2(x/2) = 2\cos^2(x/2) - 1$. Subtracting gives the simplification used above.

EXPERT'S SOLUTION : *Aarna Mishra, M.Sc Mathematics, IIT BHU*

Symmetry argument. $\cos x$ is symmetric about $x = \pi/2$ with equal positive area on $[0, \pi/2]$ and equal negative area on $[\pi/2, \pi]$, summing to 0.

Step 1. Identity reduces integrand to $-\cos x$.

Step 2. Definite integral 0 over $[0, \pi]$.

> **Final Answer:** 0

Q 7.19 **Evaluate** $\displaystyle\int_0^{2} \frac{6x+3}{x^2+4}\, dx$.

SOLUTION

Concept used. Split into a derivative-of-denominator piece and a $\tan^{-1}$ piece.

Step 1. Split:

$$\frac{6x+3}{x^2+4} = \frac{6x}{x^2+4} + \frac{3}{x^2+4}.$$

Step 2. For the first piece, $t = x^2+4 \Rightarrow dt = 2x\,dx$: $\int \frac{6x}{x^2+4}\,dx = 3\log(x^2+4)$.

Step 3. For the second piece, $\int \frac{3}{x^2+4}\,dx = \frac{3}{2}\tan^{-1}(x/2)$ (using $a=2$).

Step 4. Antiderivative: $F(x) = 3\log(x^2+4) + \frac{3}{2}\tan^{-1}(x/2)$.

Step 5. $F(2) = 3\log 8 + \frac{3}{2}\tan^{-1}(1) = 3\log 8 + \frac{3\pi}{8}$.

Step 6. $F(0) = 3\log 4 + 0$.

Step 7. $I = 3\log 8 - 3\log 4 + \frac{3\pi}{8} = 3\log 2 + \frac{3\pi}{8}$.

Final Answer: $3\log 2 + \frac{3\pi}{8}$

EXPERT'S SOLUTION *: Aniruddha Mohanty, M.Sc Mathematics, IIT Bhubaneswar*

Reading. $8/4 = 2$, $\tan^{-1}(1) = \pi/4$, $\frac{3}{2}\cdot\pi/4 = 3\pi/8$.

Step 1. Split and integrate.

Step 2. Evaluate at 2 and 0.

Step 3. Result $3\log 2 + 3\pi/8$.

Final Answer: $3\log 2 + \frac{3\pi}{8}$

Q 7.20 **Evaluate** $\int_0^1 \left(xe^x + \sin\frac{\pi x}{4}\right)dx$.

SOLUTION

Concept used. $\int xe^x\,dx = (x-1)e^x$ by parts; $\int \sin(\pi x/4)\,dx = -\frac{4}{\pi}\cos(\pi x/4)$.

Step 1. Antiderivative:

$$F(x) = (x-1)e^x - \frac{4}{\pi}\cos\frac{\pi x}{4}.$$

Step 2. $F(1) = 0 \cdot e - \frac{4}{\pi}\cos(\pi/4) = -\frac{4}{\pi} \cdot \frac{1}{\sqrt{2}} = -\frac{2\sqrt{2}}{\pi}.$

Step 3. $F(0) = (-1)(1) - \frac{4}{\pi}\cos 0 = -1 - \frac{4}{\pi}.$

Step 4. Subtract:

$$I = -\frac{2\sqrt{2}}{\pi} - \left(-1 - \frac{4}{\pi}\right) = -\frac{2\sqrt{2}}{\pi} + 1 + \frac{4}{\pi} = 1 + \frac{4-2\sqrt{2}}{\pi}.$$

Final Answer: $1 + \frac{4-2\sqrt{2}}{\pi}$

EXPERT'S SOLUTION : *Mira Bhalla, M.Sc Mathematics, IIT Guwahati*

Two-piece. The exponential piece uses parts; the trig piece uses the chain rule.

Step 1. Compute each antiderivative.

Step 2. Plug limits.

Step 3. Combine into $1 + (4 - 2\sqrt{2})/\pi$.

Final Answer: $1 + \frac{4-2\sqrt{2}}{\pi}$

Q 7.21 Choose the correct answer.

The value of the integral $\int dx/(1+x^2)$ from 1 to $\sqrt{3}$ is

(A) $\pi/3$ **(B)** $2\pi/3$ **(C)** $\pi/6$ **(D)** $\pi/12$.

SOLUTION

Concept used. $\int \frac{dx}{1+x^2} = \tan^{-1} x.$

Step 1. $F(\sqrt{3}) - F(1) = \tan^{-1}(\sqrt{3}) - \tan^{-1}(1) = \frac{\pi}{3} - \frac{\pi}{4} = \frac{4\pi - 3\pi}{12} = \frac{\pi}{12}.$

Final Answer: Option (D)

EXPERT'S SOLUTION : *Sneha Kulkarni, M.Sc Mathematics, IIT Bombay*

Quick. $\pi/3 - \pi/4 = \pi/12$.

Step 1. Convert to common denominator 12.

Step 2. $4\pi/12 - 3\pi/12 = \pi/12$.

Final Answer: Option (D)

Q 7.22 **Choose the correct answer.**
The value of the integral $\int dx/(4+9x^2)$ from 0 to $2/3$ is
(A) $\pi/6$ **(B)** $\pi/12$ **(C)** $\pi/24$ **(D)** $\pi/4$.

SOLUTION

Concept used. Bring into the form $\int \frac{du}{u^2+a^2}$ via substitution.

Step 1. Rewrite: $4 + 9x^2 = 9(x^2 + 4/9) = 9\big(x^2 + (2/3)^2\big)$. So

$$I = \frac{1}{9}\int_0^{2/3} \frac{dx}{x^2 + (2/3)^2}.$$

Step 2. Apply $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}(x/a)$ with $a = 2/3$:

$$I = \frac{1}{9}\cdot\frac{3}{2}\left[\tan^{-1}\frac{3x}{2}\right]_0^{2/3} = \frac{1}{6}\big[\tan^{-1}(1) - \tan^{-1}(0)\big].$$

Step 3. Evaluate: $\tan^{-1}(1) = \pi/4$.

Step 4. $I = \frac{1}{6}\cdot\frac{\pi}{4} = \frac{\pi}{24}$. Matches (C).

Final Answer: Option (C)

EXPERT'S SOLUTION : *Ravi Kant, M.Sc Mathematics, IIT Hyderabad*

Substitution. $u = 3x$ gives $du = 3\,dx$, $4 + 9x^2 = 4 + u^2$: $I = \frac{1}{3}\int_0^2 \frac{du}{u^2+4}$.

Step 1. $= \frac{1}{3}\cdot\frac{1}{2}\tan^{-1}(u/2)\Big|_0^2 = \frac{1}{6}\tan^{-1}1 = \frac{\pi}{24}$.

Final Answer: Option (C)

Key Takeaways

- **FTC-II:** $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F' = f$. Constant of integration drops out.
- Always check whether the integrand is improper (rational with $\deg$num $\geq \deg$den); long-divide first.
- Trig powers reduce via $\sin^2 x = (1 - \cos 2x)/2$, $\cos^2 x = (1 + \cos 2x)/2$.
- Linear combination split: a $(\alpha \cdot D'(x) + \beta)/D(x)$ integrand splits into a $\log$ piece and a $\tan^{-1}$ / $\sin^{-1}$ piece.
- For $\int dx/(x^2 + a^2)$ on definite limits, the answer is always $\frac{1}{a}[\tan^{-1}(b/a) - \tan^{-1}(a_0/a)]$.

End of Exercise 7.8