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Step-by-step solutions for the 2026-27 NCERT (Latest Edition), Class 12 Mathematics

Chapter 7: Integrals

About this Chapter

Exercise 7.9 evaluates definite integrals by **substitution**. After choosing the new variable $t = g(x)$, change the limits to $t = g(a)$ and $t = g(b)$. Once the integrand is written purely in t with new limits, the Second FTC finishes it without back-substituting.

Topics covered: Definite integrals by substitution • Limit-changing rule • $\int_0^{\pi/2}$ trig substitutions • Property check (odd / even integrand)

Quick Formula Sheet

Substitution (definite):
if $t = g(x)$, $dt = g'(x)\,dx$, $\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(t)\,dt$.

Useful integrals:
$\int \frac{dx}{(x-h)^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x-h}{a}$.

First FTC: $\frac{d}{dx}\int_0^x f(t)\,dt = f(x)$.

Exercise 7.9

Q 7.1 **Evaluate by substitution:** $\displaystyle\int_0^1 \frac{x}{x^2+1}\,dx$.

SOLUTION

Concept used. **Substitution** with $t = x^2 + 1$, so $dt = 2x\,dx$ and $x\,dx = dt/2$. Change the limits: when $x = 0$, $t = 1$; when $x = 1$, $t = 2$.

Step 1. Substitute:

$$\int_0^1 \frac{x\,dx}{x^2+1} = \int_1^2 \frac{dt/2}{t} = \frac{1}{2}\int_1^2 \frac{dt}{t}.$$

Step 2. Integrate: $\frac{1}{2}[\log t]_1^2 = \frac{1}{2}(\log 2 - \log 1) = \frac{1}{2}\log 2.$

Final Answer: $\frac{1}{2}\log 2$

☞ **Change-of-limits trick**

After substituting, the new integrand is in t *with new limits*. No need to revert to x — the FTC works directly with the new variable.

EXPERT'S SOLUTION *: Aanya Saxena, M.Sc Mathematics, IIT Delhi*

Alternative. Keep antiderivative in x: $\int \frac{x\,dx}{x^2+1} = \frac{1}{2}\log(x^2+1)$.

Step 1. Evaluate at 1 and 0: $\frac{1}{2}(\log 2 - \log 1) = \frac{1}{2}\log 2$.

Final Answer: $\frac{1}{2}\log 2$

Q 7.2 **Evaluate** $\int_0^{\pi/2} \sqrt{\sin\phi}\,\cos^5\phi\,d\phi$.

SOLUTION

Concept used. Write $\cos^5\phi = \cos^4\phi\cdot\cos\phi = (1-\sin^2\phi)^2\cos\phi$. Substitute $t=\sin\phi$, $dt = \cos\phi\,d\phi$. Limits: $\phi = 0 \to t = 0$; $\phi = \pi/2 \to t = 1$.

Step 1. Transform:

$$I = \int_0^{\pi/2} \sqrt{\sin\phi}\,(1-\sin^2\phi)^2\cos\phi\,d\phi = \int_0^1 \sqrt{t}\,(1-t^2)^2\,dt.$$

Step 2. Expand $(1-t^2)^2 = 1-2t^2+t^4$. Multiply by $\sqrt{t} = t^{1/2}$:

$$\sqrt{t}(1-t^2)^2 = t^{1/2} - 2t^{5/2} + t^{9/2}.$$

Step 3. Integrate term-by-term:

$$\int_0^1 (t^{1/2} - 2t^{5/2} + t^{9/2})\,dt = \frac{2}{3} - 2\cdot\frac{2}{7} + \frac{2}{11} = \frac{2}{3} - \frac{4}{7} + \frac{2}{11}.$$

Step 4. Common denominator 231:

$$\frac{2}{3} = \frac{154}{231}, \quad \frac{4}{7} = \frac{132}{231}, \quad \frac{2}{11} = \frac{42}{231}.$$

Sum: $\dfrac{154-132+42}{231} = \dfrac{64}{231}$.

Final Answer: $\dfrac{64}{231}$

EXPERT'S SOLUTION *: Reet Khanna, M.Sc Mathematics, IIT Bombay*

Plan. Keep one $\cos\phi$ aside for the dt, rewrite the rest in $\sin\phi = t$.

Step 1. Transform integrand to $t^{1/2}(1-t^2)^2$ on $[0,1]$.

Step 2. Expand and integrate.

Step 3. Combine fractions to $64/231$.

Final Answer: $64/231$

Q 7.3 **Evaluate** $\displaystyle\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$.

SOLUTION

Concept used. From Ex 7.6 Q22, $\sin^{-1}(\frac{2x}{1+x^2}) = 2\tan^{-1} x$ for $|x| \le 1$. So the integral becomes $2\int_0^1 \tan^{-1} x\, dx$.

Step 1. Identity reduces integrand: $I = 2\int_0^1 \tan^{-1} x\, dx$.

Step 2. $\int \tan^{-1} x\, dx = x\tan^{-1} x - \frac{1}{2}\log(1+x^2)$ (from Ex 7.6 Q13).

Step 3. Evaluate at $x = 1$: $\tan^{-1} 1 - \frac{1}{2}\log 2 = \frac{\pi}{4} - \frac{\log 2}{2}$.

Step 4. Evaluate at $x = 0$: $0 - 0 = 0$.

Step 5. Multiply by 2: $I = 2\left[\frac{\pi}{4} - \frac{\log 2}{2}\right] = \frac{\pi}{2} - \log 2$.

Final Answer: $\dfrac{\pi}{2} - \log 2$

☞ **Exam Tip**

The identity $\sin^{-1}(\frac{2x}{1+x^2}) = 2\tan^{-1} x$ is a constant-time simplification — drilling it saves a full page of substitution work.

EXPERT'S SOLUTION : *Madhav Tripathi, M.Sc Mathematics, IIT Kanpur*

Reading. Identity, then known antiderivative, then plug.

Step 1. Simplify integrand to $2\tan^{-1}x$.

Step 2. Apply $\int \tan^{-1}x\,dx$ formula.

Step 3. Boxed: $\pi/2 - \log 2$.

Final Answer: $\pi/2 - \log 2$

Q 7.4 **Evaluate $\displaystyle\int_0^2 x\sqrt{x+2}\,dx$ (put $x+2=t^2$).**

SOLUTION

Concept used. Substitute $x+2=t^2$, so $x=t^2-2$ and $dx=2t\,dt$. Change limits: $x=0\to t=\sqrt{2}$; $x=2\to t=2$.

Step 1. Transform:

$$I=\int_{\sqrt{2}}^{2}(t^2-2)\cdot t\cdot 2t\,dt = 2\int_{\sqrt{2}}^{2}(t^4-2t^2)\,dt.$$

Step 2. Distribute the leading 2: antiderivative $G(t)=\dfrac{2t^5}{5}-\dfrac{4t^3}{3}$.

Step 3. Evaluate at $t=2$: $G(2)=\dfrac{2\cdot 32}{5}-\dfrac{4\cdot 8}{3}=\dfrac{64}{5}-\dfrac{32}{3}=\dfrac{192-160}{15}=\dfrac{32}{15}$.

Step 4. $G(\sqrt{2})=\dfrac{2(\sqrt{2})^5}{5}-\dfrac{4(\sqrt{2})^3}{3}=\dfrac{2\cdot 4\sqrt{2}}{5}-\dfrac{4\cdot 2\sqrt{2}}{3}=\dfrac{8\sqrt{2}}{5}-\dfrac{8\sqrt{2}}{3}=$
$8\sqrt{2}\left(\dfrac{1}{5}-\dfrac{1}{3}\right)=8\sqrt{2}\cdot\left(-\dfrac{2}{15}\right)=-\dfrac{16\sqrt{2}}{15}$.

Step 5. Difference: $I=G(2)-G(\sqrt{2})=\dfrac{32}{15}+\dfrac{16\sqrt{2}}{15}=\dfrac{32+16\sqrt{2}}{15}=\dfrac{16(2+\sqrt{2})}{15}$.

Final Answer: $\dfrac{16(2+\sqrt{2})}{15}$

EXPERT'S SOLUTION : *Aman Khanna, M.Sc Mathematics, IIT Roorkee*

Why $x+2=t^2$. The radical disappears entirely, leaving a polynomial.

Step 1. Sub gives $\int_{\sqrt{2}}^{2} 2t^2(t^2-2)\,dt$.

Step 2. Expand to $2t^4-4t^2$, integrate.

Step 3. Plug limits; result $16(2+\sqrt{2})/15$.

Final Answer: $\dfrac{16(2+\sqrt{2})}{15}$

Q 7.5 Evaluate $\displaystyle\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,dx$.

SOLUTION

Concept used. Substitute $t = \cos x$, $dt = -\sin x\,dx$, so $\sin x\,dx = -dt$. Limits: $x = 0 \to t = 1$; $x = \pi/2 \to t = 0$.

Step 1. Transform:

$$I = \int_1^0 \frac{-dt}{1+t^2} = \int_0^1 \frac{dt}{1+t^2}.$$

Step 2. Antiderivative $\tan^{-1} t$.

Step 3. Evaluate: $\tan^{-1} 1 - \tan^{-1} 0 = \dfrac{\pi}{4}$.

Final Answer: $\pi/4$

EXPERT'S SOLUTION : *Tara Mehta, M.Sc Mathematics, IIT Hyderabad*

Pattern. $\sin x$ on top of $1 + \cos^2 x$ is the textbook setup for $t = \cos x$.

Step 1. Sub flips the limits; the minus sign re-flips them.

Step 2. $\tan^{-1}$ on $[0, 1]$ gives $\pi/4$.

Final Answer: $\pi/4$

Q 7.6 Evaluate $\displaystyle\int_0^{2} \frac{dx}{x+4-x^2}$.

SOLUTION

Concept used. Complete the square in the denominator and recognise $\int \frac{dx}{a^2-(x-h)^2}$.

Then apply $\int \frac{dx}{a^2-u^2} = \frac{1}{2a}\log\left|\frac{a+u}{a-u}\right|$.

Step 1. Rewrite:

$$x+4-x^2 = -(x^2-x-4) = -\big((x-1/2)^2-1/4-4\big) = \frac{17}{4}-\big(x-1/2\big)^2.$$

Step 2. Substitute $u = x-1/2$: $\int_{-1/2}^{3/2} \frac{du}{(\sqrt{17}/2)^2-u^2}$.

Step 3. Apply formula with $a=\sqrt{17}/2$:

$$I = \frac{1}{2a}\left[\log\left|\frac{a+u}{a-u}\right|\right]_{-1/2}^{3/2} = \frac{1}{\sqrt{17}}\left[\log\left|\frac{(\sqrt{17}/2)+u}{(\sqrt{17}/2)-u}\right|\right]_{-1/2}^{3/2}.$$

Step 4. At $u=3/2$: $\log\left|\frac{\sqrt{17}+3}{\sqrt{17}-3}\right|$.

At $u=-1/2$: $\log\left|\frac{\sqrt{17}-1}{\sqrt{17}+1}\right|$.

Step 5. Subtract:

$$I = \frac{1}{\sqrt{17}}\log\left|\frac{(\sqrt{17}+3)(\sqrt{17}+1)}{(\sqrt{17}-3)(\sqrt{17}-1)}\right|.$$

Final Answer: $\frac{1}{\sqrt{17}}\log\left|\frac{(\sqrt{17}+3)(\sqrt{17}+1)}{(\sqrt{17}-3)(\sqrt{17}-1)}\right|$

EXPERT'S SOLUTION : *Sneha Patel, M.Sc Mathematics, IIT Gandhinagar*

Recognising a^2-u^2. Once the denominator is (constant) − (shifted square), the family is fixed: log of $|(a+u)/(a-u)|$.

Step 1. Complete square: $\frac{17}{4}-(x-\frac{1}{2})^2$.

Step 2. Apply log formula.

Step 3. Plug limits $0,2$ (equivalently $u=-1/2, 3/2$).

Final Answer: $\frac{1}{\sqrt{17}}\log\left|\frac{(\sqrt{17}+3)(\sqrt{17}+1)}{(\sqrt{17}-3)(\sqrt{17}-1)}\right|$

Q 7.7 **Evaluate** $\int_{-1}^{1} \frac{dx}{x^2+2x+5}$.

SOLUTION

Concept used. Complete the square: $x^2+2x+5=(x+1)^2+4$. Apply $\int \frac{du}{u^2+a^2} = \frac{1}{a}\tan^{-1}(u/a)$ with $a=2$.

Step 1. Substitute $u=x+1$: limits become $-1+1=0$ and $1+1=2$.

$$I = \int_0^2 \frac{du}{u^2+4} = \frac{1}{2}\left[\tan^{-1}(u/2)\right]_0^2.$$

Step 2. Evaluate: $\frac{1}{2}\left[\tan^{-1}(1)-\tan^{-1}(0)\right] = \frac{1}{2}\cdot\frac{\pi}{4} = \frac{\pi}{8}$.

Final Answer: $\pi/8$

EXPERT'S SOLUTION *: Veer Singh, M.Sc Mathematics, IIT Bombay*

Quick. $(x+1)^2+4$ instantly $\to \tan^{-1}$.

Step 1. Shift to $u=x+1$; new limits $0, 2$.

Step 2. $\frac{1}{2}\tan^{-1}(u/2)$ on $[0,2]$ gives $\pi/8$.

Final Answer: $\pi/8$

Q 7.8 **Evaluate** $\int_1^2 \left(\frac{1}{x}-\frac{1}{2x^2}\right) e^{2x}\,dx$.

SOLUTION

Concept used. Spot the $e^{ax}[f(x)+f'(x)]$ template with $a=2$. The general identity reads

$$\int e^{ax}[af(x)+f'(x)]\,dx = e^{ax}f(x)+C.$$

Try $f(x)=\frac{1}{2x}$. Then $f'(x) = -\frac{1}{2x^2}$, and $af(x)+f'(x) = \frac{1}{x}-\frac{1}{2x^2}$, exactly the bracket above.

Step 1. Identify $f(x)=\frac{1}{2x}$, so the antiderivative is $e^{2x}\cdot\frac{1}{2x} = \frac{e^{2x}}{2x}$.

Step 2. Evaluate at $x=2$: $\frac{e^4}{4}$.

Step 3. Evaluate at $x = 1$: $\dfrac{e^2}{2}$.

Step 4. Subtract: $I = \dfrac{e^4}{4} - \dfrac{e^2}{2}$.

Final Answer: $\dfrac{e^4}{4} - \dfrac{e^2}{2}$

☞ **Generalised template**

$\int e^{ax}[af + f']\,dx = e^{ax}f + C$. Set $a = 1$ to recover the usual $\int e^x[f + f']\,dx$.

EXPERT'S SOLUTION *: Aarush Garg, M.Sc Mathematics, IIT Madras*

Direct verify. $\frac{d}{dx}\left[\frac{e^{2x}}{2x}\right] = \frac{2e^{2x}}{2x} - \frac{e^{2x}}{2x^2} = \left(\frac{1}{x} - \frac{1}{2x^2}\right)e^{2x}$. ✓

Step 1. $F(x) = e^{2x}/(2x)$.

Step 2. Plug limits.

Final Answer: $\dfrac{e^4}{4} - \dfrac{e^2}{2}$

Q 7.9 **The value of** $\displaystyle\int_{1/3}^{1} \frac{(x - x^3)^{1/3}}{x^4}\,dx$ **is**

(A) 6 **(B)** 0 **(C)** 3 **(D)** 4

SOLUTION

Concept used. Factor x out from the cube root: $x - x^3 = x(1 - x^2)$, so $(x - x^3)^{1/3} = x^{1/3}(1 - x^2)^{1/3}$. Better: pull x^3 inside the radical, which leaves $(x - x^3)^{1/3} = x \cdot (\frac{1}{x^2} - 1)^{1/3}$ if $0 < x < 1$. The latter form pairs neatly with $1/x^4$.

Step 1. Algebra: for $x \in (0, 1)$, $x - x^3 = x^3(\frac{1}{x^2} - 1)$, so $(x - x^3)^{1/3} = x\left(\frac{1}{x^2} - 1\right)^{1/3}$.

Step 2. The integrand is

$$\frac{x\left(\frac{1}{x^2} - 1\right)^{1/3}}{x^4} = \frac{\left(\frac{1}{x^2} - 1\right)^{1/3}}{x^3}.$$

Step 3. Substitute $t = \dfrac{1}{x^2} - 1$. Then

$$dt = -\frac{2}{x^3}\,dx, \quad \text{so} \quad \frac{dx}{x^3} = -\frac{dt}{2}.$$

Change of limits: $x = 1/3 \to t = 9 - 1 = 8$; $x = 1 \to t = 0$.

Step 4. Transform:

$$I = \int_8^0 t^{1/3} \cdot \left(-\frac{dt}{2}\right) = \frac{1}{2}\int_0^8 t^{1/3}\,dt.$$

Step 5. Antiderivative: $\frac{1}{2} \cdot \frac{3t^{4/3}}{4} = \frac{3t^{4/3}}{8}$. At $t = 8$: $8^{4/3} = (2^3)^{4/3} = 2^4 = 16$. So $\dfrac{3 \cdot 16}{8} = 6$. At $t = 0$: 0.

Step 6. Therefore $I = 6$. Matches (A).

> **Final Answer:** Option (A): 6

EXPERT'S SOLUTION : *Vihaan Kapoor, M.Sc Mathematics, IIT Roorkee*

Clever factoring. Pulling x^3 inside the cube root looks unintuitive but it exposes $1/x^2 - 1$, whose derivative is $-2/x^3$ — matching the $1/x^4 \cdot x$ outside.

Step 1. Substitute $t = 1/x^2 - 1$.

Step 2. Convert limits.

Step 3. Integrand becomes $-\frac{1}{2}t^{1/3}$; integrate to $\frac{3t^{4/3}}{8}\Big|_8^0$ (with the sign flip in limits), giving 6.

> **Final Answer:** Option (A)

Q 7.10 **If** $f(x) = \displaystyle\int_0^x t\sin t\,dt$**, then** $f'(x)$ **is**

(A) $\cos x + x\sin x$ **(B)** $x\sin x$ **(C)** $x\cos x$ **(D)** $\sin x + x\cos x$

SOLUTION

Concept used. First Fundamental Theorem of Calculus: if $F(x) = \int_a^x g(t)\,dt$ then $F'(x) = g(x)$.

Step 1. Apply directly with $g(t) = t\sin t$.

Step 2. $f'(x) = x\sin x$. Matches (B).

> **Final Answer:** Option (B): $x\sin x$

♥ First FTC in one line

The First FTC says: differentiating the area-function $A(x) = \int_a^x g(t)\,dt$ recovers the integrand g evaluated at the upper limit. No integration needed!

EXPERT'S SOLUTION *: Akshat Verma, M.Sc Mathematics, IIT Bombay*

Trap. If you actually integrate by parts, you get $f(x) = -x\cos x + \sin x$. Differentiate: $f'(x) = -\cos x + x\sin x + \cos x = x\sin x$. ✓

Step 1. Apply First FTC: $f'(x) =$ (integrand at $t = x$) $= x\sin x$.

Final Answer: Option (B)

Key Takeaways

- Change of variable in a definite integral changes the limits, not just the integrand: if $t = g(x)$, the new limits are $g(a)$ and $g(b)$.
- No need to back-substitute when you change the limits — the FTC works directly with t.
- Spot patterns: $\sin x\,dx/(1+\cos^2 x) \to t = \cos x$; rational with D' in numerator $\to t = D(x)$; $\int(\ldots)^{1/2}$ next to that root's derivative $\to t = \ldots$
- First FTC: $\frac{d}{dx}\int_a^x g(t)\,dt = g(x)$. Use it whenever the question asks for the derivative of an area-defined function.

End of Exercise 7.9