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Step-by-step solutions, alternate methods & exam tips for Class 12 Mathematics

Chapter 9: Differential Equations

About this Chapter

Differential equations relate a function to its derivatives. Exercise 9.1 trains you to identify two fundamental properties of any differential equation: its **order** (the highest derivative present) and its **degree** (the exponent of that highest derivative once the equation is a polynomial in derivatives). When the equation cannot be cast as a polynomial in derivatives, the degree is not defined.

Topics covered: Order of a DE • Degree of a DE • Polynomial-in-derivatives test • When degree is undefined

Quick Formula Sheet

Order:
highest derivative order in the equation.

Degree:
power of the highest-order derivative once the equation is a polynomial in derivatives; *not defined* if the equation contains sin, cos, log etc. of a derivative.

Polynomial test:
every derivative must appear with a non-negative integer exponent and no transcendental function around it.

Exercise 9.1

Determine the order and degree (if defined) of the differential equations in Questions 1–10. Then attempt MCQs 11–12.

Q 9.1 $\dfrac{d^4y}{dx^4} + \sin(y''') = 0.$

SOLUTION

Concept used. The **order** of a differential equation is the order of the highest derivative that appears in it. The **degree** is defined only when the equation can be expressed as a polynomial in all the derivatives that appear; it is then the power of the highest-order derivative. If any derivative is wrapped inside a transcendental function (such as $\sin$, $\cos$, $\log$, $e^{(\cdot)}$), the equation is not a polynomial in derivatives and the degree is *not defined*.

Step 1. Identify every derivative in the equation. We see $\frac{d^4y}{dx^4}$ and $y''' = \frac{d^3y}{dx^3}$.

Step 2. The highest-order derivative is $\frac{d^4y}{dx^4}$, so the order is 4.

Step 3. Test if the equation is a polynomial in derivatives. The third-order derivative y''' sits inside $\sin(\cdot)$. Since $\sin$ is a transcendental function of a derivative, the equation is *not* a polynomial in its derivatives.

Step 4. Therefore the degree is not defined.

> **Final Answer:** Order $= 4$; Degree: not defined.

☞ Polynomial-in-derivatives

"Polynomial in derivatives" means each derivative appears only raised to a non-negative integer power. Wrapping a derivative inside $\sin$, $\cos$, $\log$, $e^{(\cdot)}$, or $\sqrt{\ }$ that cannot be cleared by squaring breaks the polynomial form.

EXPERT'S SOLUTION *: Aarav Sharma, M.Sc Mathematics, IIT Bombay*

Quick reading. Two checks resolve every order/degree question: (a) what is the largest superscript on d/dx? (b) once you collect derivatives onto one side, is each one to a positive integer power, with no $\sin$ / $\cos$ / $\log$ etc. around them? If yes, degree is that integer; if no, degree is undefined.

Step 1. Largest derivative superscript: the 4 in $\frac{d^4y}{dx^4}$. Hence order $= 4$.

Step 2. Now scan the equation for transcendental wrappers around any derivative. The term $\sin(y''')$ is exactly such a wrapper: it is $\sin$ of the third derivative. A polynomial in the derivatives can have $(y''')^2$, $(y''')^3$, etc., but never $\sin(y''')$.

Step 3. Because the polynomial form fails, the degree cannot be assigned, so it is undefined.

> **Final Answer:** Order $= 4$; Degree: not defined.

Q 9.2 $y' + 5y = 0$.

SOLUTION

Concept used. Order is the order of the highest derivative; degree is the power of that highest-order derivative when the equation is a polynomial in all derivatives.

Step 1. The only derivative is $y' = \frac{dy}{dx}$, of order 1. So order $= 1$.

Step 2. Is the equation polynomial in y'? Rewrite it as $1 \cdot (y')^1 + 5y = 0$. The derivative y' appears with exponent 1 and is not inside any transcendental function. So yes, it is a polynomial in y'.

Step 3. The power of the highest-order derivative (y') is 1. Hence degree $= 1$.

> **Final Answer:** Order $= 1$; Degree $= 1$.

EXPERT'S SOLUTION : *Sneha Iyer, M.Sc Mathematics, ISI Kolkata*

Strategic angle. Equations linear in y and its first derivative, with constant coefficients, are the simplest possible: order $= 1$, degree $= 1$. Here $y' + 5y = 0$ matches that pattern exactly.

Step 1. Highest derivative: y'. Order $= 1$.

Step 2. It appears to the first power and is not inside any non-polynomial function. The equation *is* a polynomial (in fact linear) in y'. Degree $= 1$.

Step 3. Bonus check: this is a first-order linear DE; its general solution is $y = Ce^{-5x}$, which we will recognise in Exercise 9.5.

> **Final Answer:** Order $= 1$; Degree $= 1$.

☞ Exam Tip

First-order linear equations of the form $y' + ky = 0$ are the prototype for every exponential-decay and growth model in Class 12 (radioactive decay, Newton's law of cooling, RC circuits). Their order and degree are always 1.

Q 9.3 $\frac{ds}{dt} + 3s\frac{d^2s}{dt^2} = 0.$

SOLUTION

Concept used. Identify the highest-order derivative, then check that the equation is a polynomial in all derivatives.

Step 1. Derivatives present: $\dfrac{ds}{dt}$ (order 1) and $\dfrac{d^2s}{dt^2}$ (order 2). The highest order is 2.

Step 2. Rewriting: $\dfrac{ds}{dt} + 3s\,\dfrac{d^2s}{dt^2} = 0$. Each derivative is raised to the first power, and no derivative is inside a transcendental function. The equation is a polynomial in the derivatives.

Step 3. The power of the highest-order derivative $\dfrac{d^2s}{dt^2}$ is 1. Hence degree $= 1$.

Final Answer: Order $= 2$; Degree $= 1$.

EXPERT'S SOLUTION *: Arjun Patel, M.Tech CS, IIT Madras*

Quick reading. Multiplying a derivative by a function of s or t does *not* change the degree, because s is the dependent variable, not a derivative. Only powers and wrappers on derivatives matter.

Step 1. Highest derivative: $\dfrac{d^2s}{dt^2}$. Order $= 2$.

Step 2. Coefficient of the second derivative is $3s$. That makes the equation non-linear (because of $s \cdot s''$), but it is still polynomial in derivatives: s'' appears to power 1, s' to power 1. Degree $= 1$.

Step 3. Caution: "polynomial in derivatives" and "linear" are different. This DE is non-linear yet has degree 1.

Final Answer: Order $= 2$; Degree $= 1$.

✗ Common Mistake

Students sometimes mistake non-linearity for degree > 1. The factor s multiplying s'' makes the equation non-linear, but the degree is the power of the highest-order derivative, which is still 1.

Q 9.4 $\left(\dfrac{d^2y}{dx^2}\right)^2 + \cos\left(\dfrac{dy}{dx}\right) = 0.$

SOLUTION

Concept used. Degree is defined only if the equation is a polynomial in all derivatives. A cos of a derivative breaks the polynomial form.

Step 1. Derivatives present: $\frac{d^2y}{dx^2}$ (order 2) and $\frac{dy}{dx}$ (order 1). Highest order $= 2$.

Step 2. Although $\left(\frac{d^2y}{dx^2}\right)^2$ is polynomial in y'', the term $\cos\left(\frac{dy}{dx}\right)$ is a cosine of y'. The equation is therefore *not* a polynomial in its derivatives.

Step 3. Hence the degree is not defined.

> **Final Answer:** Order $= 2$; Degree: not defined.

EXPERT'S SOLUTION *: Priya Gupta, Ph.D Mathematics, IIT Delhi*

Structural observation. The presence of $\cos(y')$ alone is enough to kill the degree. You do not need to expand or simplify further.

Step 1. Order $= 2$ from the $\frac{d^2y}{dx^2}$ term.

Step 2. Test polynomial form: $\cos(y')$ has a Maclaurin expansion $1 - \frac{(y')^2}{2!} + \frac{(y')^4}{4!} - \cdots$, an *infinite* series in y'. An infinite series in a derivative is not a polynomial in that derivative.

Step 3. Degree therefore undefined.

> **Final Answer:** Order $= 2$; Degree: not defined.

Q 9.5 $\frac{d^2y}{dx^2} = \cos 3x + \sin 3x.$

SOLUTION

Concept used. The transcendental terms $\cos 3x$ and $\sin 3x$ involve only the independent variable x, not any derivative. They do *not* affect the polynomial-in-derivatives test.

Step 1. Rewrite as $\frac{d^2y}{dx^2} - \cos 3x - \sin 3x = 0$. The only derivative is $\frac{d^2y}{dx^2}$, so order $= 2$.

Step 2. The derivative appears to the first power; no derivative is inside a transcendental function. The terms $\cos 3x$, $\sin 3x$ are functions of x, not of any derivative, so the polynomial form in derivatives is intact.

Step 3. Hence degree $= 1$.

Final Answer: Order $= 2$; Degree $= 1$.

EXPERT'S SOLUTION *: Vivaan Mehta, M.Sc Applied Mathematics, IIT Kanpur*

Quick reading. "Polynomial in derivatives" is a test about the *derivatives only*. Whatever functions of x appear on the right-hand side are irrelevant to it.

Step 1. Highest derivative $= y''$. Order $= 2$.

Step 2. Power of y'' is 1, and y'' is not nested in $\sin / \cos / \log$. Degree $= 1$.

Step 3. This is in fact a linear DE; its general solution is found by integrating twice: $y = -\frac{1}{9}\cos 3x - \frac{1}{9}\sin 3x + C_1 x + C_2$.

Final Answer: Order $= 2$; Degree $= 1$.

Q 9.6 $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$.

SOLUTION

Concept used. Order is the highest derivative present; degree is the power of that highest-order derivative once polynomial form is confirmed.

Step 1. Derivatives present: y''', y'', y'. Highest order is 3.

Step 2. Each derivative is raised to a positive-integer power, and none sits inside a transcendental function. So the equation is polynomial in derivatives.

Step 3. The highest-order derivative y''' is raised to the power 2. Hence degree $= 2$.

Final Answer: Order $= 3$; Degree $= 2$.

EXPERT'S SOLUTION *: Aanya Bhat, M.Sc Mathematics, IIT Bombay*

Structural observation. Multiple derivatives all appearing to integer powers means the equation is polynomial in derivatives. Pick the highest-order derivative and read off its exponent.

Step 1. Largest derivative superscript is 3, so order $= 3$.

Step 2. Now find the exponent of y''' itself: it is $(y''')^2$. Exponent $= 2$, so degree $= 2$.

Step 3. The exponents on the lower-order derivatives (3 on y'', 4 on y', 5 on y) do not affect order or degree.

Final Answer: Order $= 3$; Degree $= 2$.

✗ Common Mistake

The degree is the power of the *highest-order* derivative, not the largest exponent in the equation. Here y has exponent 5, but y is not a derivative, so it is irrelevant.

Q 9.7 $y''' + 2y'' + y' = 0.$

SOLUTION

Concept used. Order = highest derivative's order; degree = its exponent under polynomial form.

Step 1. Derivatives: y''', y'', y'. Highest order is 3.

Step 2. Each derivative appears to the first power; no transcendental wrapping. The equation is polynomial in derivatives.

Step 3. Power of y''' is 1. Degree $= 1$.

Final Answer: Order $= 3$; Degree $= 1$.

EXPERT'S SOLUTION *: Rohit Singh, M.Sc Mathematics, IIT Madras*

Quick reading. A linear DE with constant coefficients always has degree 1. Here every term is a constant times a derivative, so the equation is linear.

Step 1. Order is the highest superscript, namely 3.

Step 2. All derivatives are raised to the first power, with no nesting inside $\sin / \cos / \log$. Polynomial form holds. Degree $= 1$.

Step 3. This is a third-order linear homogeneous DE; the characteristic equation $m^3 + 2m^2 + m = 0$ gives roots $0, -1, -1$, leading to $y = C_1 + (C_2 + C_3x)e^{-x}$. (Beyond Class 12 scope, but useful for context.)

Final Answer: Order $= 3$; Degree $= 1$.

Q 9.8 $y' + y = e^x$.

SOLUTION

Concept used. Order is the order of the highest derivative; degree is its power under the polynomial-in-derivatives test. Note that e^x is a function of x only, not of any derivative, so it does not affect the test.

Step 1. The only derivative is $y' = \dfrac{dy}{dx}$, of order 1. So order $= 1$.

Step 2. The derivative y' appears to the first power, with no transcendental wrapper. The term e^x depends on x only.

Step 3. Hence the equation is polynomial (in fact linear) in y', and degree $= 1$.

> **Final Answer:** Order $= 1$; Degree $= 1$.

EXPERT'S SOLUTION *: Karan Verma, B.Tech CSE, IIT Roorkee*

Quick reading. A first-order linear non-homogeneous equation: order 1, degree 1. The right side e^x is a forcing function in x, not in y'.

Step 1. Order: largest derivative is y', so order $= 1$.

Step 2. Degree: y' appears once, to the first power. Degree $= 1$.

Step 3. Solved by an integrating factor $e^{\int 1\,dx} = e^x$, giving $y = \frac{1}{2}e^x + Ce^{-x}$. (Exercise 9.5 territory.)

> **Final Answer:** Order $= 1$; Degree $= 1$.

Q 9.9 $y'' + (y')^2 + 2y = 0$.

SOLUTION

Concept used. Order = highest derivative's order; degree = its power once polynomial form is confirmed.

Step 1. Derivatives: y'' (order 2) and y' (order 1). Highest order $= 2$.

Step 2. Each derivative is raised to a positive-integer power: y'' to the first, y' to the second. No derivative is inside a transcendental function. Polynomial form holds.

Step 3. Power of y'' is 1. Hence degree $= 1$.

Final Answer: Order $= 2$; Degree $= 1$.

EXPERT'S SOLUTION *: Aditi Joshi, Ph.D Pure Mathematics, IISc Bangalore*

Structural observation. The square is on y', not on y''. Degree always reports the highest-order derivative's exponent.

Step 1. Order: y'' is the highest derivative. Order $= 2$.

Step 2. Look only at the exponent of y'': it is 1. Degree $= 1$.

Step 3. The $(y')^2$ term makes the DE non-linear, yet the degree is still 1 because the highest-order derivative is to the first power.

Final Answer: Order $= 2$; Degree $= 1$.

Q 9.10 $y'' + 2y' + \sin y = 0$.

SOLUTION

Concept used. The polynomial-in-derivatives test is only about the derivatives. The term $\sin y$ is a function of y, the dependent variable, not of any derivative.

Step 1. Derivatives present: y'' (order 2) and y' (order 1). Highest order $= 2$.

Step 2. Both y'' and y' appear to the first power, and neither is inside $\sin$, $\cos$, $\log$. The term $\sin y$ contains y, not a derivative, so it does not break the polynomial-in-derivatives form.

Step 3. Power of y'' is 1. Hence degree $= 1$.

Final Answer: Order $= 2$; Degree $= 1$.

EXPERT'S SOLUTION *: Yash Rao, M.Sc Mathematics, ISI Kolkata*

Quick reading. "Polynomial in derivatives" carefully excludes the dependent variable. Functions of y alone do not break the form.

Step 1. Order: y'' gives order $= 2$.

Step 2. Test: y'' and y' both linear, no $\sin / \cos$ around *them*. The $\sin y$ is on y alone, not on any derivative.

Step 3. Therefore the polynomial-in-derivatives test passes, and the degree is the power of y'', which is 1.

Final Answer: Order $= 2$; Degree $= 1$.

♥ Why $\sin y$ is allowed

A DE is studied as an equation in the unknown function $y(x)$ and its derivatives. The dependent variable y can appear inside any function whatsoever (sin, cos, exp, log); only when a *derivative* of y is wrapped in such a function does the polynomial test fail.

Q 9.11 **The degree of the differential equation**

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$$

is (A) 3 **(B)** 2 **(C)** 1 **(D) not defined.**

SOLUTION

Concept used. Degree is defined only if the DE is a polynomial in all the derivatives that appear in it. The presence of $\sin\left(\frac{dy}{dx}\right)$ violates this requirement.

Step 1. Check each term. $\left(\frac{d^2y}{dx^2}\right)^3$ is polynomial in y''; $\left(\frac{dy}{dx}\right)^2$ is polynomial in y'; the constant 1 is harmless. So far the form is polynomial.

Step 2. However, $\sin\left(\frac{dy}{dx}\right)$ wraps the derivative y' inside the transcendental function $\sin$. This is not a polynomial in y'.

Step 3. Since the equation is not polynomial in all the derivatives present, the degree is not defined.

Final Answer: Correct option: **(D) not defined**.

EXPERT'S SOLUTION *: Ananya Kapoor, M.Tech Applied Mathematics, IIT Delhi*

Strategic angle. For an MCQ on degree, do a fast scan for $\sin / \cos / \log / e^{(\cdot)}$ around any derivative. If found, the answer is "not defined".

Step 1. The exponent on $\frac{d^2y}{dx^2}$ is 3, which would suggest degree 3 if the equation were polynomial in all derivatives.

Step 2. But $\sin\left(\frac{dy}{dx}\right)$ is in the equation. This is $\sin$ of a derivative, so polynomial form fails.

Step 3. No degree can be assigned. Option (D) is correct.

> **Final Answer:** Correct option: **(D)**.

✗ Common Mistake

The high exponent 3 on y'' is a red herring. The polynomial test must hold for *every* derivative in the equation, not just the highest-order one.

Q 9.12 The order of the differential equation

$$2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$$

is (A) 2 **(B)** 1 **(C)** 0 **(D) not defined.**

SOLUTION

Concept used. Order is the highest order of a derivative that appears in the differential equation. Coefficients made of x, y, or constants do not affect order.

Step 1. List the derivatives: $\frac{d^2y}{dx^2}$ has order 2, and $\frac{dy}{dx}$ has order 1.

Step 2. The largest of these orders is 2.

Step 3. Therefore the order of the differential equation is 2.

> **Final Answer:** Correct option: **(A) 2**.

EXPERT'S SOLUTION *: Dev Nair, B.Tech CSE, IIT Roorkee*

Quick reading. Order questions reduce to "what's the largest superscript on d/dx?". Here it is 2.

Step 1. Scan the equation. The derivatives are $\frac{d^2y}{dx^2}$ (order 2) and $\frac{dy}{dx}$ (order 1).

Step 2. Pick the maximum: $\max\{2, 1\} = 2$.

Step 3. Order $= 2$, matching option (A).

Final Answer: Correct option: **(A)**.

Key Takeaways

- **Order** of a DE = order of the highest derivative present.
- **Degree** of a DE is defined only when the equation is a polynomial in all its derivatives, and is then the power of the highest-order derivative.
- Any $\sin$, $\cos$, $\log$, or $e^{(\cdot)}$ wrapped around a derivative makes the degree *not defined*.
- Functions of the dependent variable y alone (such as $\sin y$) do **not** affect the test.
- A non-linear DE may still have degree 1, e.g. $s' + 3ss'' = 0$.