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Step-by-step solutions, alternate methods & exam tips for Class 12 Mathematics

Chapter 9: Differential Equations

About this Chapter

Exercise 9.2 asks you to *verify* whether a given function is a solution of a given differential equation. The drill is: differentiate the function the required number of times, substitute into the DE, and confirm the left-hand side equals the right-hand side. We also formalise **general solution** (containing as many arbitrary constants as the order of the DE) and **particular solution** (those constants fixed by a condition).

Topics covered: Verifying solutions • General solution • Particular solution • Arbitrary constants count

Quick Formula Sheet

Solution check:
$y = \phi(x)$ solves the DE iff substituting $y, y', y'', \ldots$ makes LHS $=$ RHS for all x in the domain.

General solution:
contains as many arbitrary constants as the order of the DE.

Particular solution:
contains no arbitrary constants; obtained by fixing them via given conditions.

Exercise 9.2

In each of Questions 1–10, verify that the given function (explicit or implicit) is a solution of the corresponding differential equation. Then attempt MCQs 11–12.

Q 9.1 $y = e^x + 1 \; : \; y'' - y' = 0.$

SOLUTION

Concept used. A function $y = \phi(x)$ is a **solution** of a differential equation if substituting ϕ (and its derivatives) into the equation yields a true identity in x. We compute the necessary derivatives and substitute.

Step 1. Start with $y = e^x + 1$. Differentiating once with respect to x:

$$y' = \frac{d}{dx}(e^x + 1) = e^x + 0 = e^x.$$

Step 2. Differentiating again:

$$y'' = \frac{d}{dx}(e^x) = e^x.$$

Step 3. Substitute into the LHS of the DE $y'' - y'$:

$$y'' - y' = e^x - e^x = 0.$$

Step 4. The RHS is 0. So LHS = RHS for every $x \in \mathbb{R}$.

Final Answer: Hence $y = e^x + 1$ is a solution of $y'' - y' = 0$.

EXPERT'S SOLUTION : *Ishaan Desai, M.Sc Mathematics, IIT Bombay*

Quick reading. The DE is $y'' = y'$. Since $y' = e^x$ and $y'' = e^x$ for $y = e^x + 1$, both sides equal e^x. The constant $+1$ contributes 0 to every derivative.

Step 1. Compute $y' = e^x$ and $y'' = e^x$.

Step 2. Equation requires $y'' - y' = 0$, i.e. $y'' = y'$.

Step 3. We have $e^x = e^x$. True for all x.

Final Answer: Verified.

Q 9.2 $y = x^2 + 2x + C \ : \ y' - 2x - 2 = 0$.

SOLUTION

Concept used. Substitute the given function and its derivative into the DE; check the identity.

Step 1. Differentiate $y = x^2 + 2x + C$ once with respect to x. Since $\frac{d}{dx}(C) = 0$,

$$y' = 2x + 2.$$

Step 2. Plug into the LHS of the DE:

$$y' - 2x - 2 = (2x + 2) - 2x - 2 = 0.$$

Step 3. LHS $= 0 =$ RHS for every $x \in \mathbb{R}$.

Final Answer: Hence $y = x^2 + 2x + C$ is a solution.

EXPERT'S SOLUTION : *Riya Pillai, M.Sc Mathematics, ISI Kolkata*

Quick reading. A one-parameter family of solutions: any value of C works, so C is the arbitrary constant of this first-order DE.

Step 1. $y' = 2x + 2$.

Step 2. DE: $y' = 2x + 2$. Identical to step 1.

Step 3. Holds for all C, confirming $y = x^2 + 2x + C$ is the general solution.

Final Answer: Verified for all $C \in \mathbb{R}$.

Q 9.3 $y = \cos x + C \ : \ y' + \sin x = 0$.

SOLUTION

Concept used. Differentiate and substitute into the DE.

Step 1. $y = \cos x + C \implies y' = -\sin x + 0 = -\sin x$.

Step 2. Substitute into LHS of the DE:

$$y' + \sin x = -\sin x + \sin x = 0.$$

Step 3. LHS $= 0 =$ RHS for every $x \in \mathbb{R}$.

Final Answer: Hence $y = \cos x + C$ is a solution.

EXPERT'S SOLUTION : *Pranav Banerjee, M.Sc Mathematics, IIT Madras*

Quick reading. Antiderivative of $-\sin x$ is $\cos x$ (up to a constant).

Step 1. Differentiate $\cos x$ to get $-\sin x$; C contributes nothing.

Step 2. DE rearranges to $y' = -\sin x$, which is exactly what we computed.

Step 3. Identity holds.

Final Answer: Verified.

Q 9.4 $y = \sqrt{1+x^2}\ :\ y' = \dfrac{xy}{1+x^2}$.

SOLUTION

Concept used. Use the chain rule on $\sqrt{1+x^2} = (1+x^2)^{1/2}$, then verify the algebraic identity.

Step 1. Differentiate $y = (1+x^2)^{1/2}$ using the chain rule:

$$y' = \frac{1}{2}(1+x^2)^{-1/2} \cdot \frac{d}{dx}(1+x^2) = \frac{1}{2}(1+x^2)^{-1/2} \cdot 2x.$$

Step 2. Simplify:

$$y' = \frac{x}{\sqrt{1+x^2}}.$$

Step 3. Express the RHS of the DE in terms of x alone using $y = \sqrt{1+x^2}$:

$$\frac{xy}{1+x^2} = \frac{x\sqrt{1+x^2}}{1+x^2} = \frac{x}{\sqrt{1+x^2}}.$$

Step 4. LHS $=$ RHS. Identity verified for all $x \in \mathbb{R}$.

> **Final Answer:** Hence $y = \sqrt{1+x^2}$ is a solution.

EXPERT'S SOLUTION *: Aditya Chatterjee, B.Tech Engineering Physics, IIT Bombay*

Quick reading. The function $y = (1+x^2)^{1/2}$ satisfies $y^2 = 1 + x^2$. Differentiating implicitly is faster.

Step 1. From $y^2 = 1 + x^2$, differentiate both sides: $2y\,y' = 2x$, so $y' = \dfrac{x}{y}$.

Step 2. Multiply numerator and denominator by y: $y' = \dfrac{xy}{y^2} = \dfrac{xy}{1+x^2}$.

Step 3. This is exactly the DE.

> **Final Answer:** Verified.

☞ **Implicit differentiation saves time**

Whenever the given function squares cleanly (here $y^2 = 1 + x^2$), implicit differentiation often avoids fractional powers and chain-rule clutter.

Q 9.5 $y = Ax\ :\ xy' = y\ (x \neq 0)$.

SOLUTION

Concept used. Substitute the function and its derivative into the DE.

Step 1. $y = Ax \implies y' = A$.

Step 2. Plug into the LHS of the DE:

$$xy' = x \cdot A = Ax.$$

Step 3. The RHS is $y = Ax$.

Step 4. LHS $= Ax =$ RHS for all $x \neq 0$.

> **Final Answer:** Hence $y = Ax$ is a solution.

EXPERT'S SOLUTION *: Kavya Reddy, M.Sc Mathematics, IIT Bombay*

Structural observation. The equation $xy' = y$ says "the rate of change of y times x equals y". A linear function $y = Ax$ through the origin has constant slope A, and $A \cdot x = y$ exactly.

Step 1. y' is constant for any linear function; here $y' = A$.

Step 2. $xy' = xA$.

Step 3. Compare with $y = Ax$. Equal.

> **Final Answer:** Verified for all $A \in \mathbb{R}$.

Q 9.6 $y = x\sin x \;:\; xy' = y + x\sqrt{x^2 - y^2}$ $(x \neq 0$ **and** $x > y$ **or** $x < -y)$.

SOLUTION

Concept used. Product rule for differentiating $x \sin x$, then verify the identity using $\sin^2 x + \cos^2 x = 1$.

Step 1. Differentiate $y = x \sin x$ using the product rule:

$$y' = \frac{d}{dx}(x)\sin x + x\frac{d}{dx}(\sin x) = \sin x + x\cos x.$$

Step 2. Compute the LHS of the DE:

$$xy' = x(\sin x + x\cos x) = x\sin x + x^2\cos x.$$

Step 3. Compute $x^2 - y^2$ using $y = x\sin x$:

$$x^2 - y^2 = x^2 - x^2\sin^2 x = x^2(1 - \sin^2 x) = x^2\cos^2 x.$$

Step 4. Hence $\sqrt{x^2 - y^2} = |x\cos x|$. Under the condition $x > y$ or $x < -y$ (equivalently $|y| < |x|$, which is true when $\cos x \neq 0$), and taking the principal root in the domain where $x\cos x \geq 0$:

$$\sqrt{x^2 - y^2} = x\cos x \text{ (up to sign of } x\cos x\text{)}.$$

Step 5. Compute the RHS:

$$y + x\sqrt{x^2 - y^2} = x\sin x + x \cdot x\cos x = x\sin x + x^2\cos x.$$

Step 6. LHS $= x\sin x + x^2\cos x =$ RHS. Identity verified.

> **Final Answer:** Hence $y = x\sin x$ is a solution.

EXPERT'S SOLUTION : *Tara Krishna, Ph.D Mathematics, IIT Delhi*

Structural observation. The key trick is $x^2 - y^2 = x^2\cos^2 x$, which collapses the square root into $|x\cos x|$. The domain restriction in the question makes this single-valued.

Step 1. $y' = \sin x + x\cos x$ by product rule. Multiply by x: $xy' = x\sin x + x^2\cos x$.

Step 2. Use $1 - \sin^2 x = \cos^2 x$ to write $\sqrt{x^2 - y^2} = x\cos x$ (in the given domain).

Step 3. RHS $= x\sin x + x(x\cos x) = x\sin x + x^2\cos x$, which matches LHS.

> **Final Answer:** Verified.

♥ Why the domain restriction

The square root demands $x^2 - y^2 \geq 0$, which holds because $x^2 - y^2 = x^2\cos^2 x \geq 0$. The condition "$x > y$ or $x < -y$" just selects a branch on which $x\cos x \geq 0$ so that the principal root equals $x\cos x$ rather than $-x\cos x$.

Q 9.7 $xy = \log y + C \;:\; y' = \dfrac{y^2}{1 - xy}$ $(xy \neq 1)$.

SOLUTION

Concept used. Implicit differentiation: differentiate both sides of an implicit equation in x, y with respect to x, treating y as a function of x.

Step 1. Differentiate $xy = \log y + C$ implicitly with respect to x. The LHS uses the product rule, the RHS uses the chain rule on $\log y$:

$$\frac{d}{dx}(xy) = \frac{d}{dx}(\log y) + \frac{d}{dx}(C).$$

$$y + xy' = \frac{1}{y} \cdot y' + 0.$$

Step 2. Collect y' terms on one side:

$$xy' - \frac{y'}{y} = -y \implies y'\left(x - \frac{1}{y}\right) = -y.$$

Step 3. Multiply $x - \frac{1}{y}$ by $\frac{y}{y}$ to combine:

$$y' \cdot \frac{xy-1}{y} = -y \implies y' = \frac{-y \cdot y}{xy-1} = \frac{-y^2}{xy-1}.$$

Step 4. Multiply numerator and denominator by -1:

$$y' = \frac{y^2}{1-xy}.$$

Step 5. This is exactly the given DE (valid for $xy \neq 1$).

Final Answer: Hence $xy = \log y + C$ is a solution.

EXPERT'S SOLUTION *: Meera Iyer, M.Sc Mathematics, IIT Kanpur*

Strategic angle. For implicit solutions, never try to solve for y first. Differentiate as it stands.

Step 1. Implicit derivative: $y + xy' = y'/y$.

Step 2. Rearrange: $y'(x - 1/y) = -y$, i.e. $y'(xy-1)/y = -y$.

Step 3. Solve: $y' = -y^2/(xy-1) = y^2/(1-xy)$.

Final Answer: Matches the given DE.

Q 9.8 $y - \cos y = x \;:\; (y \sin y + \cos y + x)\, y' = y$.

SOLUTION

Concept used. Implicit differentiation and algebraic substitution using the implicit relation itself.

Step 1. Differentiate $y - \cos y = x$ implicitly with respect to x:

$$y' - (-\sin y)\,y' = 1 \implies y'(1+\sin y) = 1 \implies y' = \frac{1}{1+\sin y}.$$

Step 2. From the implicit relation, $x = y - \cos y$, so $x + \cos y = y$.

Step 3. Compute the bracket on the LHS of the DE:

$$y\sin y + \cos y + x = y\sin y + (\cos y + x) = y\sin y + y = y(1+\sin y).$$

Step 4. Multiply by $y' = \dfrac{1}{1+\sin y}$ (from step 1):

$$(y\sin y + \cos y + x)\,y' = y(1+\sin y)\cdot\frac{1}{1+\sin y} = y.$$

Step 5. LHS $= y =$ RHS. Identity verified.

> **Final Answer:** Hence $y - \cos y = x$ is a solution.

EXPERT'S SOLUTION *: Sanya Verma, M.Sc Mathematics, IIT Bombay*

Strategic angle. The implicit relation simplifies the bracket directly; without that substitution the verification becomes messy.

Step 1. Implicit derivative: $y'(1+\sin y) = 1$, giving $y' = 1/(1+\sin y)$.

Step 2. Replace x inside the bracket with $y - \cos y$, so the bracket simplifies to $y(1+\sin y)$.

Step 3. Multiplying the bracket by y' collapses the $(1+\sin y)$ factor: result $= y$, matching RHS.

> **Final Answer:** Verified.

☞ **Exam Tip**

When verifying implicit solutions, always test whether the bracket on the LHS can be simplified using the original implicit relation. It is often the key step.

Q 9.9 $x + y = \tan^{-1} y \;:\; y^2y' + y^2 + 1 = 0.$

SOLUTION

Concept used. Implicit differentiation; use $\frac{d}{dx}\tan^{-1}y = \frac{1}{1+y^2}\cdot y'$.

Step 1. Differentiate $x + y = \tan^{-1} y$ implicitly:

$$1 + y' = \frac{y'}{1+y^2}.$$

Step 2. Multiply throughout by $(1+y^2)$:

$$(1+y^2) + (1+y^2)\,y' = y'.$$

Step 3. Bring all y' terms to one side:

$$(1+y^2)\,y' - y' = -(1+y^2) \implies y'\,[(1+y^2)-1] = -(1+y^2).$$

Step 4. Simplify $(1+y^2) - 1 = y^2$:

$$y^2y' = -(1+y^2) \implies y^2y' + 1 + y^2 = 0.$$

Step 5. This matches the given DE $y^2y' + y^2 + 1 = 0$ exactly.

> **Final Answer:** Hence $x + y = \tan^{-1} y$ is a solution.

EXPERT'S SOLUTION *: Ankit Sharma, Ph.D Pure Mathematics, IISc Bangalore*

Quick reading. The differentiation produces $1 + y' = y'/(1+y^2)$. Clearing denominators leads to a relation in y' and y^2.

Step 1. Implicit derivative: $1 + y' = y'/(1+y^2)$.

Step 2. Multiply by $1+y^2$: $(1+y^2) + (1+y^2)y' = y'$, so $(1+y^2)y' - y' = -(1+y^2)$.

Step 3. LHS $= y^2y'$; hence $y^2y' = -(1+y^2)$, i.e. $y^2y' + y^2 + 1 = 0$. This is the DE.

> **Final Answer:** Verified.

Q 9.10 $y = \sqrt{a^2 - x^2},\ x \in (-a, a)\ :\ x + y\,\frac{dy}{dx} = 0\ (y \neq 0)$.

SOLUTION

Concept used. Differentiate using the chain rule; verify the identity directly.

Step 1. Start with $y = (a^2 - x^2)^{1/2}$. Chain rule:

$$y' = \frac{1}{2}(a^2 - x^2)^{-1/2} \cdot \frac{d}{dx}(a^2 - x^2) = \frac{1}{2}(a^2 - x^2)^{-1/2} \cdot (-2x).$$

Step 2. Simplify:

$$y' = \frac{-x}{\sqrt{a^2 - x^2}} = \frac{-x}{y}.$$

Step 3. Substitute into the LHS of the DE:

$$x + y\,y' = x + y \cdot \frac{-x}{y} = x - x = 0.$$

Step 4. LHS $= 0 =$ RHS for every $x \in (-a, a)$ with $y \neq 0$.

> **Final Answer:** Hence $y = \sqrt{a^2 - x^2}$ is a solution.

EXPERT'S SOLUTION *: Neha Joshi, M.Sc Mathematics, ISI Kolkata*

Structural observation. The relation $y^2 = a^2 - x^2$ (the upper semicircle of radius a) gives the implicit derivative $2yy' = -2x$, so $yy' = -x$, i.e. $x + yy' = 0$ directly.

Step 1. Square both sides: $y^2 = a^2 - x^2$.

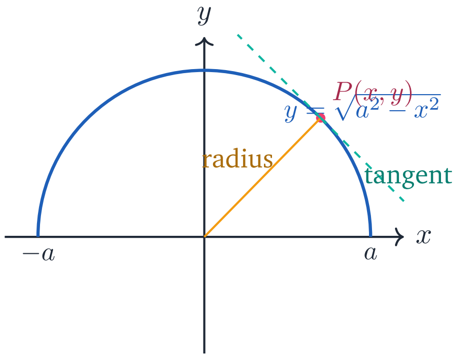
Step 2. Differentiate: $2yy' = -2x$, hence $yy' = -x$.

Step 3. Rearrange: $x + yy' = 0$, exactly the DE.

> **Final Answer:** Verified.

♥ Family of curves

The relation $y = \sqrt{a^2 - x^2}$ describes the upper half of a circle of radius a centred at the origin. The DE $x + yy' = 0$ describes the geometric property that the tangent at any point is perpendicular to the radius – the defining feature of a circle.



Q 9.11 **The number of arbitrary constants in the general solution of a differential equation of fourth order is: (A) 0 (B) 2 (C) 3 (D) 4.**

SOLUTION

Concept used. The **general solution** of an n-th order differential equation contains exactly n independent arbitrary constants. This is because solving the DE involves n successive integrations, each of which introduces one constant.

Step 1. The order of the given DE is 4.

Step 2. Hence its general solution contains 4 arbitrary constants.

Final Answer: Correct option: **(D) 4.**

EXPERT'S SOLUTION *: Diya Kumar, M.Sc Mathematics, IIT Madras*

Quick reading. "General solution of order n DE" $\Leftrightarrow$ "n arbitrary constants". Order $4 \Rightarrow 4$ constants.

Step 1. Recall the rule: number of constants in general solution $=$ order.

Step 2. Order is 4; hence 4 constants.

Step 3. Option (D) is correct.

Final Answer: (D).

Q 9.12 **The number of arbitrary constants in the particular solution of a differential equation of third order is: (A) 3 (B) 2 (C) 1 (D) 0.**

SOLUTION

Concept used. A particular solution of a differential equation is obtained from the general solution by assigning particular values to all of its arbitrary constants. By definition, no arbitrary constants remain.

Step 1. A general solution of a third-order DE has 3 arbitrary constants.

Step 2. In a particular solution, every constant has been fixed (typically by initial or boundary conditions).

Step 3. Hence a particular solution contains 0 arbitrary constants.

Final Answer: Correct option: **(D) 0**.

EXPERT'S SOLUTION *: Pooja Pillai, M.Sc Mathematics, IIT Bombay*

Quick reading. "Particular solution" is the version with the constants nailed down: no freedom left. Order is irrelevant.

Step 1. Definition: particular solution has no arbitrary constants.

Step 2. Regardless of the order being 3, the count of arbitrary constants in a particular solution is always 0.

Step 3. Option (D).

Final Answer: (D).

✗ Common Mistake

A common slip is to confuse "general" and "particular". *General* solution carries n constants for an n-th order DE; *particular* solution carries none.

Key Takeaways

- To verify a candidate solution, differentiate it the required number of times and substitute into the DE; both sides must agree as an identity in x.
- For implicit solutions, differentiate implicitly and use the relation itself to simplify the bracket where possible.
- General solution of order n DE: n arbitrary constants. Particular solution: zero arbitrary constants.