JELET-2019

Subject : B.Sc.

(Booklet Number)

Duration : 2 Hours **Full Marks : 100**

INSTRUCTIONS

1. All questions are of objective type having four answer options for each. Only one option is correct. Correct answer will carry full marks 1. In case of incorrect answer or any combination of more than one answer. ¼ marks will be deducted.
2. Questions must be answered on OMR sheet by darkening the appropriate bubble marked
   A, B, C or D.
3. Use only **Black/Blue ball point pen** to mark the answer by complete filling up of the respective bubbles.
4. Mark the answers only in the space provided. Do not make any stray mark on the OMR.
5. Write question booklet number and your roll number carefully in the specified locations of the **OMR**. Also fill appropriate bubbles.
6. Write your name (in block letter), name of the examination centre and put your full signature in appropriate boxes in the OMR.
7. The OMR is liable to become invalid if there is any mistake in filling the correct bubbles for question booklet number roll number or if there is any discrepancy in the name/ signature of the candidate, name of the examination centre. The OMR may also become invalid due to folding or putting stray marks on it or any damage to it. The consequence of such invalidation due to incorrect marking or careless handling by the candidate will be sole responsibility of candidate.
8. Candidates are not allowed to carry any written or printed material, calculator, pen, docu-pen, log table, wristwatch, any communication device like mobile phones etc. inside the examination hall. Any candidate found with such items will be **reported against** & his/her candidature will be summarily cancelled.
9. Rough work must be done on the question paper itself. Additional blank pages are given in the question paper for rough work.
10. Hand over the OMR to the invigilator before leaving the Examination Hall.

1. If α and β be the roots of $x^2 - 2x + 2 = 0$, then for any positive integer n, $\alpha^n + \beta^n =$

(A) $2^{\frac{n}{2}} \cos\frac{n\pi}{4}$ (B) $2^{\frac{n}{2}+1} \cos\frac{n\pi}{4}$

(C) $2^{\frac{n}{2}} \sin\frac{n\pi}{4}$ (D) $2^{\frac{n}{2}-1} \sin\frac{n\pi}{4}$

2. Roots of the equation $(1+x)^{2n} + (1-x)^{2n} = 0$, where n is a positive integer, are all on

(A) the real axis (B) the imaginary axis

(C) the line $y = x$ (D) the line $y = -x$

3. Let the points A, B and C represent complex numbers Z_1, Z_2 and Z_3 respectively and there exists scalars p, q, r such that $pZ_1 + qZ_2 + rZ_3 = 0$. Then

(A) A, B and C are never collinear

(B) A, B and C are collinear if $p + q + r = 0$

(C) A, B and C are vertices of a right-angled triangle

(D) A, B and C are vertices of an isosceles triangle

4. Let Z be any complex number. Then

(A) $|Z| < \frac{|Re(Z)| + |Im(Z)|}{\sqrt{2}}$ (B) $|Z| \geq \frac{|Re(Z)| + |Im(Z)|}{\sqrt{2}}$

(C) $|Z| = \frac{|Re(Z)| - |Im(Z)|}{\sqrt{2}}$ (D) $|Z| > \frac{|Re(Z)| - |Im(Z)|}{\sqrt{2}}$

5. Let $\Delta = \begin{vmatrix} 1 & a & b^2 + c^2 + bc \\ 1 & b & c^2 + a^2 + ac \\ 1 & c & a^2 + b^2 + ab \end{vmatrix}$. Then

(A) $a - b$ is the only linear factor of Δ

(B) $b - c$ is the only linear factor of Δ

(C) $c - a$ is the only linear factor of Δ

(D) $a - b$, $b - c$, $c - a$ are all factors of Δ

6. If $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = k\,abc(a+b+c)^3$, then $k =$

(A) 1 (B) 2

(C) 3 (D) 4

7. Consider the system of equations

$$x + y + z = 0$$

$$2x - y + 4z = 0$$

$$x + 5y - 7z = 0$$

The system has

(A) infinitely many solutions (B) finite number of non-zero solutions

(C) only trivial solution (D) unique non-trivial solution

8. Let $\Delta = \begin{vmatrix} a+b & 2a+b & 3a+b \\ 2a+b & 3a+b & 4a+b \\ 4a+b & 5a+b & 6a+b \end{vmatrix}$ Then

(A) $\Delta = 0$ (B) $\Delta > 0$

(C) $\Delta < 0$ (D) $\Delta = a^2 b^2$

9. M and G are two matrices such that

$$M = \begin{vmatrix} m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \\ p_1 & p_2 & p_3 \end{vmatrix} \text{ and } M - G = \begin{vmatrix} q_1 & q_2 & q_3 \\ r_1 & r_2 & r_3 \\ p_1 & p_2 & p_3 \end{vmatrix}$$

Then G is

(A) a null matrix (B) a scalar matrix

(C) a lower triangular matrix (D) an upper triangular matrix

10. If $k > 1$ and the determinant of a matrix A^2 is k^2.

where $A = \begin{vmatrix} k & k\alpha & \beta \\ 0 & \alpha & k\alpha \\ 0 & 0 & k \end{vmatrix}$

Then α is

(A) 1 (B) $\frac{1}{k}$

(C) $-\frac{1}{k}$ (D) k

11. Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$. Then

(A) A is a null matrix (B) A is not an orthogonal matrix

(C) A^{-1} does not exist (D) $A^2 = I$, the identity matrix of order 3

12. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ then, the value of the determinant $|A^{2009} - 5A^{2008}|$ is

(A) –6 (B) –5

(C) 1 (D) –1

13. If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $(2I + 3E)^2 = mI + nE$, m and n are two real numbers, then (m, n) =

(A) (4, 12) (B) (12, 4)

(C) (2, 4) (D) (4, 2)

14. The matrix $\begin{bmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is invertible if

(A) $\lambda \neq -15$ (B) $\lambda \neq -17$

(C) $\lambda \neq -16$ (D) $\lambda \neq -18$

15. If A and B are two square matrices of same order, then the matrix AB – BA is

(A) a symmetric matrix (B) skew symmetric matrix

(C) a null matrix (D) the identity matrix

16. Rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ is

(A) 3 (B) 2

(C) 1 (D) 0

17. Let $A = \begin{bmatrix} 1 & -3 \\ 3 & -5 \end{bmatrix}$.

Then the roots of $\det(A - \lambda I_2) = 0$ (I_2 be the identity matrix of order 2) are

(A) – 2, – 2 (B) 2, 2

(C) 2, – 2 (D) 1, 3

18. If A is a 3×3 non-singular matrix such that $A^T A = AA^T$, and $B = A^{-1}A^T$, then BB^T is

(A) $A^T A^{-1}$ (B) $(B^{-1})^T$

(C) B^{-1} (D) I, identity matrix of order 3

19. The equation $x^4 + 3x^2 + x - 1 = 0$ has

(A) exactly two real roots (B) at least two real roots

(C) four real roots (D) at least two imaginary roots

20. To remove the second term of the equation $x^3 + 6x^2 - 12x + 32 = 0$, we have to increase the roots by

(A) 2 (B) -2

(C) 1 (D) -1

21. Let α, β, γ be the roots of the equation $x^3 - 3px^2 + 3(p-1)x + 1 = 0$.

Then the equation whose roots are $1-\alpha$, $1-\beta$, $1-\gamma$ is given by

(A) $y^3 + 3p^2y^2 - 6py + 2 = 0$ (B) $y^3 - 3py^2 + 3(p^2-1)y - 1 = 0$

(C) $3y^3 - (p^2-1)y^2 + 3py - 1 = 0$ (D) $y^3 + 3(p-1)y^2 - 3py + 1 = 0$

22. On $\mathbb{R} \times \mathbb{R}$, the binary operation * be defined by $(a, b)*(c, d) = (a-c, b-d+2bd)$.

Then

(A) (S, *) has no identity element.

(B) (S, *) has identity element but $\left(a, -\frac{1}{2}\right)$ has no inverse.

(C) Every element of (S, *) has inverse

(D) (S, *) is a group

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23. Let $S = \left\{\begin{pmatrix} a & 0 \\ b & 1 \end{pmatrix} : a, b \in \mathbb{R}, a \neq 0\right\}$. Then

(A) S forms a group w. r. t. matrix multiplication (*)

(B) S is not a group w. r. t. matrix multiplication

(C) S is only a semi group

(D) (S, *) has no identity element

24. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x(x-1)(x+1)$. Then

(A) f is one-one and onto (B) f is neither one-one nor onto

(C) f is one-one but not onto (D) f is onto but not one-one

25. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are defined by $f(x) = |x|$ and $g(x) = [x - 3]$ for all $x \in \mathbb{R}$, then $\left\{g(f(x)) : -\frac{8}{5} < x < \frac{8}{5}\right\}$ is equal to

(A) $\{0, 1\}$ (B) $\{1, 2\}$

(C) $\{-3, -2\}$ (D) $\{2, 3\}$

26. In a group $(Q - \{-1\}, *)$, where * is defined by $a*b = a + b + ab$, the inverse of 3 is

(A) -3 (B) $\frac{3}{4}$

(C) $\frac{1}{3}$ (D) $-\frac{3}{4}$

27. Let a binary operation '*' on $\mathbb{Q}$ be defined by $a * b = a + 2b$ for all $a, b \in \mathbb{Q}$. Then

(A) the operation is commutative.

(B) the operation obeys the associative property.

(C) there does not exist any identity element with respect to the operation defined.

(D) there exists left identity element with respect to the operation defined.

28. Which is the simplified representation of $(A' \cap B' \cap C) \cup (B \cap C) \cup (A \cap C)$ where A, B, C are non-void subsets of X ?

(A) A (B) B

(C) C (D) X

29. Which of the following statements is true ?

(A) The mapping $f : \mathbb{N} \to \mathbb{N}$ given by $f(x) = x + 2$ is bijective.

(B) The mapping $f : \mathbb{Q} \to \mathbb{Q}$ given by $f(x) = 3x + 7$ is bijective.

(C) The mapping $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^3 - x$ is injective.

(D) For the mapping $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2 - 3$, $f^{-1}(\sqrt{2}) = \phi$

30. For three non-void subsets A, B, C of a set X, if $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then

(A) $A = B = C$ (B) $B = C$

(C) $A = C$ (D) $B = C = X$

31. On $\mathbb{R}^+$, the set of positive reals, the operator * is defined by

$a * b = a \log_q b \; \forall a, b \in \mathbb{R}^+$ and $q \in \mathbb{R}^+$, $q \neq 1$. Then

(A) $(a * b) * c = a * (b * c)$ holds $\forall$ a, b, c $\in \mathbb{R}^+$

(B) $a * b = b * a \; \forall$ a, b $\in \mathbb{R}^+$

(C) $(a * b) * c \neq a * (b * c) \; \forall$ a, b, c $\in \mathbb{R}^+$

(D) $\exists$ no $b \in \mathbb{R}^+$ for which $a * b = a$ holds

32. If ρ be a relation on Z defined by $a \rho b$ iff $a^2 - b^2$ is divisible by 5, then ρ is

(A) only reflexive (B) only symmetric

(C) only reflexive and transitive (D) an equivalence relation

33. With reference to a universal set, the inclusion of a set in another, is a relation, which is

(A) symmetric only (B) an equivalence relation

(C) reflexive only (D) reflexive and transitive

34. In a college of 300 students, every student needs 5 newspapers and every newspaper is read by 60 students. The number of newspapers are

(A) at least 30 (B) at most 20

(C) exactly 25 (D) exactly 15

35. If $f : A \to B$ is surjective, then

(A) $n(A) \leq n(B)$

(B) $n(A) = n(B)$

(C) $n(A) \geq n(B)$

(D) no specific order relation can be ascertained

[n(p) denotes the number of elements in p]

36. If the expression ax + by changes to a′x′ + b′y′ by a rotation of rectangular axes about origin, then

(A) $ab = a'b'$ (B) $a + b = a' + b'$

(C) $\frac{a}{b} = \frac{a'}{b'}$ (D) $a^2 + b^2 = a'^2 + b'^2$

37. For the new origin (h, k) without changing the directions of the axes, if the equation $5x^2 - 2y^2 - 30x + 8y = 0$ changes to the form $Ax'^2 + By'^2 = 1$, then (h, k) =

(A) (–3, 2) (B) (3, 2)

(C) (3, –2) (D) (–3, –2)

38. If the centroid of the triangle formed by the points (0, 0), (cos θ, sin θ) and (sin θ, – cos θ) lies on the line $y = x$ then θ is equal to

(A) 60° (B) 30°

(C) 45° (D) 90°

39. The ratio in which the join of (1, 2, 3) and (4, 6, –5) is divided by the xy-plane is

(A) 1 : 4 (B) 1 : 3

(C) 3 : 5 (D) 5 : 3

40. If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then

(A) p = q (B) p + q = 0

(C) pq + 1 = 0 (D) p + q = 1

41. The angle between the lines joining the origin to the intersection of the line $y = 3x + 2$ with the curve $x^2 + 2xy + 3y^2 + 4x + 8y - 11 = 0$ is

(A) $\frac{\pi}{3}$ (B) $\tan^{-1}\frac{2\sqrt{2}}{3}$

(C) $\tan^{-1}\frac{2\sqrt{3}}{5}$ (D) $\tan^{-1}\frac{3\sqrt{2}}{4}$

42. If one of the lines of $ax^2 + 2hxy + by^2 = 0$ makes the same angle with the x-axis as the other makes with y-axis, then

(A) $a = b$ (B) $a + b = 0$

(C) $a + b = 1$ (D) $a = b + 1$

43. The equation $xy - ax - by + ab = 0$ represents

(A) an ellipse (B) a circle

(C) a hyperbola (D) a pair of straight lines

44. Which one of the following is correct ?

(A) $y^2 = 4ax$ is a central conic but $\frac{x^2}{3} + \frac{y^2}{9} = 1$ is not so.

(B) $x^2 = 4ay$ is a non-central conic but $\frac{x^2}{9} - \frac{y^2}{2} = 1$ is a central conic.

(C) $y = 2x^2 + 3x + 7$ is a central conic but $x^2 + \frac{y^2}{2} = 1$ is non-central conic.

(D) $x^2 + 3xy + 2y^2 = 0$ is a central conic but $y^2 + 5xy + 6x^2 = 0$ is not so.

45. Consider the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. The conjugate axis of the hyperbola is less or greater than the transverse axis according as

(A) $e > \sqrt{2}$ or $e < \sqrt{2}$ respectively

(B) $e < \sqrt{2}$ or $e > \sqrt{2}$ respectively

(C) $e > \sqrt{3}$ or $e < \sqrt{2}$ respectively

(D) $e < \sqrt{3}$ or $e > \sqrt{2}$ respectively

46. The distance of a point on the conic $\frac{x^2}{6} + \frac{y^2}{2} = 1$ from the centre is 2. Then the eccentric angle is/are given by

(A) only $\frac{\pi}{4}$ (B) only $\frac{3\pi}{4}, \frac{5\pi}{4}$

(C) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ (D) only $\frac{3\pi}{4}, \frac{7\pi}{4}$

47. Chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ touch the circle $4x^2+4y^2=c^2$. Then locus of their poles is

(A) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{c^2}$

(B) $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{c^4}$

(C) $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{2}{c^4}$

(D) $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{4}{c^2}$

48. The polar of a point P with respect to the parabola $y^2=4ax$ is parallel to the line $lx+my=1$. The locus of P is

(A) $lx+2am=0$

(B) $ly+2am=0$

(C) $lx-2am=0$

(D) $ly-2am=0$

49. The equation $r^2\sin 2\theta=2a^2$ represents

(A) an ellipse

(B) a circle

(C) a hyperbola of eccentricity 2

(D) a hyperbola of eccentricity √2

50. An ellipse of axes 2a and 2b slides in a plane always touching the co-ordinate axes. The locus of the centre is

(A) a circle

(B) an ellipse

(C) a parabola

(D) a straight line

51. If the line $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane $lx+my-z=9$, then l^2+m^2 will be

(A) 26

(B) 18

(C) 6

(D) 2

52. A variable plane is at a distance 3 units from the origin and meets the axes at A, B and C. The locus of the centroid of the triangle ABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\alpha$. Then α =

(A) 1

(B) – 2

(C) 1/3

(D) 3

53. The equation of the plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to x-axis is

(A) $y - 3z + 6 = 0$ (B) $3y - z + 6 = 0$

(C) $y + 3z + 6 = 0$ (D) $3y - 2z + 6 = 0$

54. The shortest distance from the plane $12x + 4y + 3z = 327$ to the centre of the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is

(A) 26 unit (B) 13 unit

(C) 12 unit (D) 39 unit

55. If the planes $x = cy + bz$, $y = az + cx$ and $z = bx + ay$ have a common line of intersection, then $a^2 + b^2 + c^2 = 1 + kabc$, where $k =$

(A) 1 (B) -1

(C) 2 (D) -2

56. The angle between the vectors $\vec{\alpha} = 2\vec{p} + 4\vec{q}$ and $\vec{\beta} = \vec{p} - \vec{q}$, where $\vec{p}$ and $\vec{q}$ are unit vectors inclined at an angle 120°, is

(A) 60° (B) 30°

(C) 120° (D) 45°

57. The value of $[\vec{a}\ \vec{b} + \vec{c}\ \vec{a} + \vec{b} + \vec{c}]$ is

(A) $[\vec{a}\ \vec{b}\ \vec{c}]$ (B) 0

(C) $2[\vec{a}, \vec{b}\ \vec{c}]$ (D) $\vec{a} \times (\vec{b} \times \vec{c})$

58. ABCD is a quadrilateral with $\overrightarrow{AB} = \vec{a}$, $\overrightarrow{AD} = \vec{b}$ and $\overrightarrow{AC} = 2\vec{a} + 3\vec{b}$. If area of the quadrilateral is α times the area of the parallelogram with AB, AD as adjacent sides, then α is

(A) 5 (B) $\frac{5}{2}$

(C) 1 (D) $\frac{1}{2}$

59. For non-zero vectors $\vec{a}, \vec{b}, \vec{c}$, $|(\vec{a} \times \vec{b}) \cdot \vec{c}| = |\vec{a}||\vec{b}||\vec{c}|$ holds iff

(A) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = 0$

(B) $\vec{a}\cdot\vec{c} = \vec{b}\cdot\vec{c} = 0$

(C) $\vec{c}\cdot\vec{a} = \vec{a}\cdot\vec{b} = 0$

(D) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$

60. If D, E, F be the mid points of the sides BC, CA and AB respectively of Δ ABC, then $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} =$

(A) $\vec{0}$

(B) $\overrightarrow{DE}$

(C) $\overrightarrow{EF}$

(D) $\overrightarrow{FD}$

61. Let $f(x) = \dfrac{\sin[x]}{[x]}$. Then

(A) $\lim\limits_{x \to 0} f(x)$ exists

(B) the limit does not exist

(C) $x = 0$ is point of infinite discontinuity

(D) if we take $f(0) = 1$, $f(x)$ will be continuous at $x = 0$

62. Let $f : \mathbb{R} \to \mathbb{R}$ be such that $f(x + y) = f(x)\, f(y)$ for all $x, y \in \mathbb{R}$.
Also let $f(x) = 1 + x\, g(x)$, where $\lim\limits_{x \to 0} g(x) = 1$. Then $f(x)$ is

(A) e^x

(B) 2^x

(C) a non-constant polynomial

(D) 1 for all x

63. Let $f : [-1, 1] \to \mathbb{R}$ be twice differentiable and $f(0) = f'(0) = 0$, $f''(0) = 4$. Then the value of $\lim\limits_{x \to 0} \dfrac{2f(x) - 3f(2x) + f(4x)}{x^2}$ is

(A) 11

(B) 2

(C) 12

(D) 13

64. If $y = \lim_{x \to \infty} (1+x)(1+x^2)\ldots(1+x^{2^n})$ and $x^2 < 1$, then $\frac{dy}{dx} =$

(A) 1 (B) $\frac{1}{1+x}$

(C) $-\frac{1}{(1+x)^2}$ (D) $\frac{1}{(1-x)^2}$

65. A body of mass 6 gm is in rectilinear motion according to the law

$s = -1 + \ln(t+1) + (t+1)^3$. The kinetic energy of the body one second after it begins to move is

(A) 300 unit (B) 468 ¾ unit

(C) 415 unit (D) 384 unit

66. Let $f(x) = \begin{cases} x^{\alpha} \sin \frac{1}{x^{\beta}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then

(A) if $\alpha > 1, \beta > 0$, $f'(0)$ exists

(B) if $0 < \alpha - 1 \leq \beta$, f' is continuous at $x = 0$

(C) if $0 < \beta < \alpha - 1$, f' is discontinuous at $x = 0$

(D) $f'(0)$ exists for all $\alpha, \beta > 0$

67. Let $\lim_{x \to \infty} \{f(x) + f'(x)\} = a \ (\in \mathbb{R})$. Then

(A) $\lim_{x \to \infty} f(x) = 0$, $\lim_{x \to \infty} f'(x) = a$ (B) $\lim_{x \to \infty} f(x) = a$, $\lim_{x \to \infty} f'(x) = 0$

(C) $\lim_{x \to \infty} f(x) = \frac{a}{2} = \lim_{x \to \infty} f'(x)$ (D) one of the limits does not exist

68. If for $y = (x + \sqrt{x^2 - 1})^n$, $(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + ky = 0$ then k =

(A) m^2 (B) $2m^2$

(C) $-m^2$ (D) $-2m^2$

69. Choose the correct one:

(A) Rolle's Theorem is applicable to $|x|$ in $[a, b] (\subset \mathbb{R})$

(B) Rolle's Theorem is never applicable to $|x|$ in $[a, b] (\subset \mathbb{R})$

(C) Rolle's Theorem is applicable to $|x|$ in $[a, b] (\subset \mathbb{R})$, where $0 < a < b$

(D) Rolle's Theorem is applicable to $|x|$ in $[a, b] (\subset \mathbb{R})$, where $a < b < 0$

70. Let $f : [a, b] \to \mathbb{R}$ be continuous in $[a, b]$, differentiable in (a, b) and $f(a) = 0 = f(b)$. Then

(A) there exists at least one point $c \in (a, b)$ such that $f'(c) = f(c)$

(B) $f'(x) = f(x)$ does not hold at any point of (a, b)

(C) there exists a point $d \in (a, b)$ for which $f'(x) > f(x)$ holds

(D) there exists a point $p \in (a, b)$ for which $f'(p) < f(p)$ holds

71. Let $I = \tan\alpha - \tan\beta$, $J_1 = \frac{\alpha - \beta}{\cos^2\beta}$, $J_2 = \frac{\alpha - \beta}{\cos^2\alpha}$, $0 < \beta \le \alpha < \frac{\pi}{2}$. Then

(A) $J_1 \le I, J_2 \le I$ (B) $J_1 \le I, I \le J_2$

(C) $J_1 \le I, I \ge J_2$ (D) $J_1 \le I \le J_2$

72. If f, g be differentiable functions on [0, 3] such that

$f(0) = 2$, $f(3) = 6$, $g(0) = 0$, $g(3) \neq 0$ and $f'(x) = g'(x)$ $(\neq 0)$ in $(0, 3)$, then

(A) $g(3)$ may have any real value (B) $g(3) = 4$

(C) $g(3) = -1$ (D) $g(3) = 2$

73. In the expansion of $\log(1+x)$, $-1 < x \leq 1$, the co-efficient of x^5 is

(A) 1 (B) $\frac{1}{\lfloor 5}$

(C) $\frac{1}{5}$ (D) $-\frac{1}{5}$

74. If $\lim_{x \to 1} \frac{ax^2 + bx + c}{(x-1)^2} = 2$, then (a, b, c) is equal to

(A) (2, 4, 2) (B) (2, −4, 2)

(C) (2, 4, −2) (D) (2, −4, −2)

75. The slope of the tangents drawn from P to the parabola $y^2 = 4ax$ are m_1 and m_2. If $m_1 = km_2$ then locus of P is

(A) $y^2 = k^2ax$ (B) $y^2 = (k+1)^2ax$

(C) $y^2 = kax$ (D) $y^2 = \frac{(k+1)^2}{k}ax$

76. Let $f(x) = 2x^2 - \ln x$. Then

(A) f decreases for $x < \frac{1}{2}$ (B) f decreases for $0 < x < \frac{1}{2}$

(C) f decreases for $-\frac{1}{2} < x < \frac{1}{2}$ (D) f increases for $x < \frac{1}{4}$

77. Equation(s) of tangent(s) at the origin to the curve $(x^2 + y^2)^2 = a^2x^2 - b^2y^2$, $ab \neq 0$ is/are given by

(A) $x = 0$

(B) $y = 0$

(C) $y = \frac{b}{a}x$

(D) $y = \pm\frac{a}{b}x$

78. If the subnormal is of constant length, then the curve must be a

(A) circle

(B) parabola

(C) ellipse

(D) hyperbola

79. Let $z(x, y) = (x - 1)^2 - 2y^2$. Then

(A) z is minimum for $x = 1, y = 0$

(B) z is maximum for $x = 1, y = 0$

(C) there is no extrema at $(1, 0)$

(D) there is extrema at $(1, 0)$

80. Given that $1 + xy - \ln(e^{xy} + e^{-xy}) = 0$. Then $\frac{dy}{dx}$

(A) is $\frac{x}{y}$

(B) is $-\frac{y}{x}$

(C) is $\frac{2y}{x^2}$

(D) cannot be determined

81. Let $z = f(x + ay) + \varphi(x - ay)$ be twice differentiable function. Then

(A) $\frac{\partial^2 z}{\partial x^2} = a^2\frac{\partial^2 z}{\partial y^2}$

(B) $\frac{\partial^2 z}{\partial y^2} = a^2\frac{\partial^2 z}{\partial x^2}$

(C) $\frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial x^2}$

(D) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$

82. Let $f(x, y)$ and $g(x, y)$ be homogeneous functions of degree 0 and have continuous first order partial derivatives.

Also let $J(x, y) = \begin{vmatrix} f_x & f_y \\ g_x & g_y \end{vmatrix}$ Then for all x, y:

(A) $J > 0$

(B) $J < 0$

(C) $J = 0$

(D) J possesses both signs for various values of x and y

83. Let $f(x, y) = x^2 + y^2 + (x + y + 1)^2$. Then $f(x, y)$

(A) has no extrema

(B) has saddle point at $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

(C) has maxima at $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

(D) has maxima at $\left(-\frac{1}{3}, -\frac{1}{3}\right)$

84. Let $Z(x, y) = \frac{\cos y}{x}$ and $x = u^2 - v$, $y = e^v$. Then $\frac{\partial z}{\partial v}$ is

(A) $\frac{1}{x^2}\{\cos y - x^2 \sin y\}$

(B) $\frac{1}{x^2}\{\cos y - xy \sin y\}$

(C) $x^2 \sin y + \frac{1}{x} \cos y$

(D) $x^2 \cos y - y^2 \sin y$

85. If $u = \begin{vmatrix} x^2 & y^2 & z^2 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix}$ then, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} =$

(A) $2x$ (B) xy

(C) $2u$ (D) $3u$

86. Let $f(x, y) = \begin{cases} xy.\frac{x^2 - y^2}{x^2 + y^2}, & x^2 + y^2 \neq 0 \\ 0. & x^2 + y^2 = 0 \end{cases}$. Then

(A) f_{xy}, f_{yx} do not exist at (0, 0)

(B) $f_{xy}(0, 0) = f_{yx}(0, 0)$

(C) $f_{xy}(0, 0) \neq f_{yx}(0, 0)$

(D) $f_{xy}(0, 0)$ exists but f_{yx} does not exist

87. Let $u(x, y) = \log(y \sin x + x \sin y)$, $I = \frac{\partial^2 u}{\partial x \partial y}$, $J = \frac{\partial^2 u}{\partial y \partial x}$. Then

(A) $I = J$ (B) $I > J$

(C) $I < J$ (D) $I + J = 0$

88. If $I_1 = \int \sin^{-1} x \, dx$ and $I_2 = \int \sin^{-1}\sqrt{1-x^2} \, dx$, then

(A) $I_1 = I_2$ (B) $I_2 = \frac{\pi}{2} I_1$

(C) $I_1 + I_2 = \frac{\pi}{2} x$ (D) $I_1 \cdot I_2 = \frac{\pi}{2}$

89. $\lim_{n\to 0}\frac{3}{n}\left\{1+\sqrt{\frac{n}{n+3}}+\sqrt{\frac{n}{n+6}}+....+\sqrt{\frac{n}{n+3(n-1)}}\right\}$ is

(A) 2 (B) 3

(C) 1 (D) 15

90. Let $I=\int_a^b \frac{|x|}{x}dx$. Then

(A) $I = b - a$ (B) $I = a - b$

(C) $I = |b| - |a|$ (D) $I = |a| - |b|$

91. $I = \int \cos(\ln x)\,dx$. Then

(A) $I = \frac{1}{2}x^2 \sin(\ln x) + c$

(B) $I = \frac{1}{2}x^2 \cos(\ln x) + c$

(C) $I = \frac{1}{2}x\{\cos(\ln x) + \sin(\ln x)\} + c$

(D) $I = \frac{1}{2}x\{\sin(\ln x) - \cos(\ln x)\} + c$

(c is constant of integration)

92. $\int_0^\infty e^{-x^2}x^9\,dx =$

(A) 4 (B) 8

(C) 12 (D) does not exist

93. $\int_0^1 x^{m-1} (1 - x^2)^{n-1} \, dx$

(A) exists for all m, n

(B) exists for m > 0, n > 0 and is equal to $\frac{1}{2} \beta \left(\frac{m}{2}, n\right)$

(C) never exists

(D) exists for m > 1, n > 1 and is equal to $\frac{1}{2} \beta$ (m, n)

94. Consider $\Gamma(a)$ where the symbol has its usual meaning. Then

(A) $\Gamma(a) = a!$ for all $a \in \mathbb{R}$.

(B) $\Gamma(a) = (a + 1)!$ for all $a > 0$

(C) $\Gamma(a) = (a - 1)!$ for all positive integral values of a

(D) $\Gamma(a) = (a - 1)!$ for all rational positive values of a

95. Let $f(x) = \begin{cases} 2x^2 + 3x + 4, & x < 1 \\ 7, & x = 1 \\ 3x^2 + 6x + 1, & x > 1 \end{cases}$. Then

(A) f is continuous at all points $\in \mathbb{R}$

(B) $x = 1$ is a point of jump discontinuity

(C) $x = 1$ is a point of infinite discontinuity

(D) $x = 1$ is a point of removable discontinuity

96. The Integrating factor of the differential equation $\cos y + (x \sin y - 1) \frac{dy}{dx} = 0$ is

(A) $\sec x$

(B) $\sec y$

(C) $\tan x$

(D) $\tan y$

97. The general solution of $e^{dy/dx} = x$ is

(A) $y = x(\log x + 1) + k$

(B) $y = x(1 - \log x) + k$

(C) $y = (1 + \log x) + k$

(D) $y = x(\log x - 1) + k$

(where k is arbitrary constant)

98. $y = 2e^{2x} - e^{-x}$ is a solution of the differential equation

(A) $y_2 + y_1 - 2y = 0$

(B) $y_2 + y_1 + 2y = 0$

(C) $y_2 - y_1 - 2y = 0$

(D) $y_2 - y_1 + 2y = 0$

99. General solution of the differential equation $\frac{d^2y}{dx^2} + 3y = -2x$ is

(A) $y = c_1 \cos x + c_2 \sin x - \frac{2x}{3}$

(B) $y = c_1 \cos \sqrt{3}x + c_2 \sin x + \frac{x}{3}$

(C) $y = c_1 \cos \sqrt{3}x + c_2 \sin \sqrt{3}x - \frac{2x}{3}$

(D) $y = c_1 \cos x + c_2 \sin \sqrt{3}x + \frac{x}{3}$

(where c_1, c_2 are arbitrary constants)

100. The singular solution of the equation $y = px - p^2$ represents

(A) a circle

(B) a straight line

(C) a pair of straight lines

(D) a parabola