GATE 2024 Aerospace Engineering Question Paper with Solutions

Time Allowed :3 Hour	Maximum Marks :100	Total Questions :65

General Instructions

Please read the following instructions carefully:

1. This question paper is divided into three sections:
   - **General Aptitude (GA):** 10 questions (5 questions × 1 mark + 5 questions × 2 marks) for a total of 15 marks.
   - **Environmental Science and Engineering + Engineering Mathematics:**
     - **Part A (Mandatory):** 36 questions (1 questions × 1 mark + 19 questions × 2 marks) for a total of 55 marks.
     - **Part B (Section 1):** Candidates can choose either Part B1 (Surveying and Mapping) or Part B2 (Section 2). Each part contains 16 questions (8 questions × 1 mark + 11 questions × 2 marks) for a total of 30 marks.
2. The total number of questions is **65**, carrying a maximum of **100 marks**.
3. The duration of the exam is **3 hours**.
4. Marking scheme:
   - For 1-mark MCQs, $\frac{1}{3}$ mark will be deducted for every incorrect response.
   - For 2-mark MCQs, $\frac{2}{3}$ mark will be deducted for every incorrect response.
   - No negative marking for numerical answer type (NAT) questions.
   - No marks will be awarded for unanswered questions.
5. Ensure you attempt questions only from the optional section (Part B1 or Part B2) you have selected.
6. Follow the instructions provided during the exam for submitting your answers.

1. If '→' denotes increasing order of intensity, then the meaning of the words [dry → arid → parched] is analogous to [diet → fast → ______]. Which one of the given options is appropriate to fill the blank?
(A) starve
(B) reject
(C) feast
(D) deny

Correct Answer: (A) starve

Solution: Concept: In analogy questions based on vocabulary, the relationship between words often depends on:

- Degree or intensity

- Progression or sequence
- Cause and effect

Here, the symbol '$\rightarrow$' explicitly denotes **increasing order of intensity**.
Step 1: Analyze the first sequence:

$$\text{dry} \rightarrow \text{arid} \rightarrow \text{parched}$$

This sequence shows a gradual increase in the severity of dryness:

- **Dry**: lacking moisture
- **Arid**: extremely dry
- **Parched**: severely dried, extremely thirsty

Step 2: Apply the same logic to the second sequence:

$$\text{diet} \rightarrow \text{fast} \rightarrow ?$$

- **Diet**: controlled or reduced intake of food
- **Fast**: complete avoidance of food for a period

The next word should indicate an even more intense or extreme condition related to lack of food.
Step 3: Evaluate the options:

- **Starve**: suffer or die due to extreme lack of food
- **Reject**: unrelated to food intensity
- **Feast**: opposite in meaning
- **Deny**: too general, does not imply intensity of hunger

Thus, the correct word that completes the analogy is **starve**.

Quick Tip

In intensity-based analogies:

- Identify the progression (mild $\rightarrow$ moderate $\rightarrow$ extreme)
- Ensure the same semantic relationship applies to the second pair

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2. If two distinct non-zero real variables x and y are such that $(x+y)$ is proportional to $(x-y)$, then the value of $\frac{x}{y}$
(A) depends on xy
(B) depends only on x and not on y
(C) depends only on y and not on x
(D) is a constant

Correct Answer: (D) is a constant

Solution: Concept: When one algebraic expression is said to be **proportional** to another, it means that the ratio of the two expressions is a constant. That is, if $A \propto B$, then:

$$\frac{A}{B} = k \quad \text{(constant)}$$

Step 1: Use the given proportionality condition.

$$(x + y) \propto (x - y)$$

This implies:

$$\frac{x+y}{x-y} = k \quad \text{where } k \text{ is a constant.}$$

Step 2: Rearrange the equation.

$$x + y = k(x - y)$$

$$x + y = kx - ky$$

Step 3: Collect like terms.

$$x - kx = -y - ky$$

$$x(1 - k) = -y(1 + k)$$

Step 4: Find the ratio $\frac{x}{y}$.

$$\frac{x}{y} = \frac{-(1+k)}{(1-k)}$$

Since k is a constant, the value of $\frac{x}{y}$ is also a **constant**.

Quick Tip

Whenever two expressions are proportional:

- Convert the proportionality into an equation using a constant.
- Check whether the required ratio depends on variables or only on constants.

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3. Consider the following sample of numbers:

$$9, \ 18, \ 11, \ 14, \ 15, \ 17, \ 10, \ 69, \ 11, \ 13$$

The median of the sample is
(A) 13.5
(B) 14
(C) 11
(D) 18.7

Correct Answer: (A) 13.5

Solution: Concept: The **median** of a data set is the middle value when the observations are arranged in ascending or descending order.

- If the number of observations n is **odd**, the median is the $\left(\frac{n+1}{2}\right)$th term.
- If n is **even**, the median is the average of the $\left(\frac{n}{2}\right)$th and $\left(\frac{n}{2}+1\right)$th terms.

Step 1: Arrange the given data in ascending order.

$$9,\ 10,\ 11,\ 11,\ 13,\ 14,\ 15,\ 17,\ 18,\ 69$$

Step 2: Count the number of observations.

$$n = 10 \quad \text{(even)}$$

Step 3: Identify the middle two terms.

$$5\text{th term} = 13, \quad 6\text{th term} = 14$$

Step 4: Compute the median.

$$\text{Median} = \frac{13 + 14}{2} = 13.5$$

Quick Tip

Always sort the data before finding the median. For an even number of observations, remember to take the **average of the two central values**.

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4. The number of coins of 1, 5, and 10 denominations that a person has are in the ratio $5 : 3 : 13$. Of the total amount, the percentage of money in 5 coins is

(A) 21%
(B) $14\frac{2}{7}$%
(C) 10%
(D) 30%

Correct Answer: (C) 10%

Solution: Concept: When the number of coins of different denominations are given in a ratio:

- Assume a common multiplying factor to find actual numbers.
- Multiply the number of coins by their respective denominations to get total values.
- Percentage is calculated as:

$$\text{Percentage} = \frac{\text{Part}}{\text{Whole}} \times 100$$

Step 1: Let the numbers of 1, 5, and 10 coins be:

$$5k,\ 3k,\ 13k \quad \text{respectively.}$$

Step 2: Find the total value of coins of each denomination.

$$\text{Value of 1 coins} = 1 \times 5k = 5k$$

$$\text{Value of 5 coins} = 5 \times 3k = 15k$$

$$\text{Value of 10 coins} = 10 \times 13k = 130k$$

Step 3: Find the total amount of money.

$$\text{Total amount} = 5k + 15k + 130k = 150k$$

Step 4: Calculate the percentage of money in 5 coins.

$$\text{Percentage} = \frac{15k}{150k} \times 100 = 10\%$$

Quick Tip

In coin-ratio problems:

- Always convert number ratios into value contributions.
- Cancel the common factor early to simplify percentage calculations.

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5. For positive non-zero real variables p and q, if

$$\log(p^2 + q^2) = \log p + \log q + 2\log 3,$$

then, the value of

$$\frac{p^4 + q^4}{p^2 q^2}$$

is
(A) 79
(B) 81
(C) 9
(D) 83

Correct Answer: (A) 79

Solution: Concept: Logarithmic identities allow us to combine expressions:

- $\log a + \log b = \log(ab)$
- $n \log a = \log(a^n)$

Also, useful algebraic identities:

$$p^4 + q^4 = (p^2 + q^2)^2 - 2p^2q^2$$

Step 1: Simplify the given logarithmic equation.

$$\log(p^2 + q^2) = \log p + \log q + 2\log 3$$

$$\log(p^2 + q^2) = \log(pq) + \log(3^2)$$
$$\log(p^2 + q^2) = \log(9pq)$$

Step 2: Remove logarithms.

$$p^2 + q^2 = 9pq \quad \cdots (1)$$

Step 3: Square equation (1).

$$(p^2 + q^2)^2 = (9pq)^2$$

$$p^4 + q^4 + 2p^2q^2 = 81p^2q^2$$

Step 4: Find $p^4 + q^4$.

$$p^4 + q^4 = 81p^2q^2 - 2p^2q^2$$

$$p^4 + q^4 = 79p^2q^2$$

Step 5: Compute the required value.

$$\frac{p^4 + q^4}{p^2q^2} = 79$$

Quick Tip

In logarithmic equations:

- First combine logs into a single logarithm.
- Convert to algebraic form before applying identities.

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6. In the given text, the blanks are numbered (i)–(iv). Select the best match for all the blanks.
Steve was advised to keep his head .... before heading .... to bat; for, while he had a head .... batting, he could only do so with a cool head .... his shoulders.
(A) (i) down (ii) down (iii) on (iv) for
(B) (i) on (ii) down (iii) for (iv) on
(C) (i) down (ii) out (iii) for (iv) on
(D) (i) on (ii) out (iii) on (iv) for

Correct Answer: (C)

Solution: Concept: This question tests knowledge of **idiomatic expressions** involving the word *head*. Correct usage depends on commonly accepted phrases in English.
Step 1: Analyze each blank independently.

- **(i) keep his head <u>down</u>** — means to stay calm or remain focused.
- **(ii) heading <u>out</u> to bat** — the correct phrasal verb used when a player goes to bat.
- **(iii) had a head <u>for</u> batting** — means having an aptitude or natural ability.
- **(iv) a cool head <u>on</u> his shoulders** — a standard idiom meaning being sensible and calm.

Step 2: Combine the correct idioms into the sentence.
All four expressions fit naturally and idiomatically only in option (C).

Quick Tip

In fill-in-the-blank questions based on idioms:

- Check for commonly used word pairs or fixed expressions.
- Read the sentence aloud mentally to test natural flow.

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7. A rectangular paper sheet of dimensions 54 cm $\times$ 4 cm is taken. The two longer edges of the sheet are joined together to create a cylindrical tube. A cube whose surface area is equal to the area of the sheet is also taken.
Then, the ratio of the volume of the cylindrical tube to the volume of the cube is

(A) $\frac{1}{\pi}$
(B) $\frac{2}{\pi}$
(C) $\frac{3}{\pi}$
(D) $\frac{4}{\pi}$

Correct Answer: (A) $\frac{1}{\pi}$

Solution: Concept:

- When a rectangular sheet is rolled into a cylinder, one side becomes the **circumference** and the other becomes the **height**.
- Volume of a cylinder: $V = \pi r^2 h$
- Surface area of a cube: $6a^2$

Step 1: Form the cylindrical tube.
The longer edges (54 cm) are joined, hence:

$$\text{Circumference of base} = 4 \text{ cm}$$

$$2\pi r = 4 \Rightarrow r = \frac{2}{\pi}$$

$$\text{Height } h = 54 \text{ cm}$$

Step 2: Find the volume of the cylinder.

$$V_{\text{cylinder}} = \pi r^2 h$$

$$= \pi \left(\frac{2}{\pi}\right)^2 \times 54$$

$$= \pi \cdot \frac{4}{\pi^2} \cdot 54$$

$$= \frac{216}{\pi}$$

Step 3: Find the volume of the cube.
Area of the rectangular sheet:

$$54 \times 4 = 216 \text{ cm}^2$$

This equals the surface area of the cube:

$$6a^2 = 216$$

$$a^2 = 36 \Rightarrow a = 6$$

$$V_{\text{cube}} = a^3 = 6^3 = 216$$

Step 4: Find the required ratio.

$$\text{Required ratio} = \frac{V_{\text{cylinder}}}{V_{\text{cube}}} = \frac{\frac{216}{\pi}}{216} = \frac{1}{\pi}$$

Quick Tip

Always identify correctly which dimension becomes the circumference and which becomes the height when a sheet is rolled into a cylinder.

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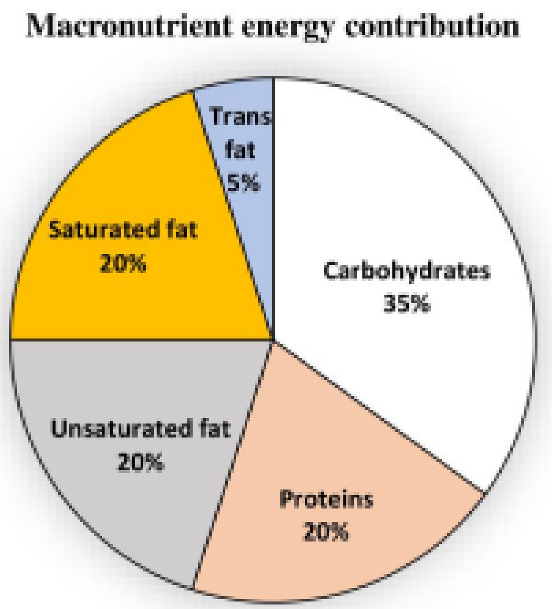
8. The pie chart presents the percentage contribution of different macronutrients to a typical 2000 kcal diet of a person.



The typical energy density (kcal/g) of these macronutrients is given in the table below.

Macronutrient	Energy density (kcal/g)
Carbohydrates	4
Proteins	4
Unsaturated fat	9
Saturated fat	9
Trans fat	9

The total fat (all three types), in grams, this person consumes is
(A) 44.4
(B) 77.8
(C) 100
(D) 3600

Correct Answer: (C) 100

Solution: Concept:

- Energy contribution from a macronutrient is calculated using its percentage share of total calories.
- Conversion from calories to grams is done using:

$$\text{Grams} = \frac{\text{Calories}}{\text{Energy density (kcal/g)}}$$

Step 1: Identify total percentage contribution from fats.
From the pie chart:

$$\text{Unsaturated fat} = 20\%, \quad \text{Saturated fat} = 20\%, \quad \text{Trans fat} = 5\%$$

$$\text{Total fat percentage} = 20 + 20 + 5 = 45\%$$

Step 2: Calculate total calories obtained from fats.

$$\text{Total daily intake} = 2000 \text{ kcal}$$

$$\text{Calories from fat} = 45\% \times 2000 = 900 \text{ kcal}$$

Step 3: Convert fat calories into grams.
Energy density of fat = 9 kcal/g

$$\text{Total fat consumed} = \frac{900}{9} = 100 \text{ g}$$

Quick Tip

When multiple categories contribute to the same nutrient (like different fats), always **add their percentages first** before converting calories into grams.

9. A rectangular paper of 20 cm $\times$ 8 cm is folded 3 times. Each fold is made along the line of symmetry, which is perpendicular to its long edge. The perimeter of the final folded sheet (in cm) is

(A) 18
(B) 24
(C) 20
(D) 21

Correct Answer: (B) 24

Solution: Concept:

- Folding along a line of symmetry perpendicular to the longer side halves the **length**.
- Width remains unchanged.
- Perimeter of a rectangle $= 2(l + b)$.

Step 1: Initial dimensions of the paper:

$$20 \text{ cm} \times 8 \text{ cm}$$

Step 2: Apply successive folds.
Each fold halves the longer side:

$$20 \to 10 \to 5 \to 2.5$$

Final dimensions:

$$2.5 \text{ cm} \times 8 \text{ cm}$$

Step 3: Calculate the perimeter of the final folded sheet.

$$\text{Perimeter} = 2(2.5 + 8) = 2 \times 10.5 = 21 \text{ cm}$$

Hence, the correct answer is $\boxed{21}$.

Quick Tip

When a sheet is folded repeatedly along the same orientation, only one dimension keeps changing while the other remains constant.

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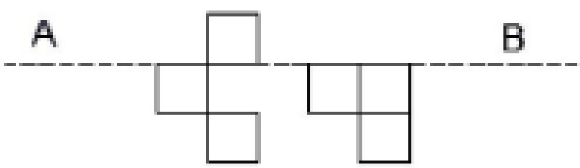
10. The least number of squares to be added in the figure to make AB a line of symmetry is



(A) 6
(B) 4
(C) 5
(D) 7

Correct Answer: (C) 5

Solution: Concept: A line of symmetry divides a figure into two mirror-image halves.

- For symmetry about line AB, each square on one side must have a corresponding square at the same distance on the opposite side.
- Missing mirror images must be added.

Step 1: Observe the given figure.
Squares on the left of line AB do not have corresponding mirror squares on the right side at matching positions.
Step 2: Count the missing mirror squares.
By matching each square on the left with its symmetric position on the right:

$$\text{Number of missing squares} = 5$$

Step 3: Add these squares to complete symmetry about line AB.
Thus, the **least number of squares required** is $\boxed{5}$.

Quick Tip

To check symmetry:

- Imagine folding the figure along the given line.
- Count how many shapes fail to overlap perfectly.

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11. The following system of linear equations

$$\begin{aligned} 7x - 3y + z &= 0 \\ 3x - y + z &= 0 \\ x - y - z &= 0 \end{aligned}$$

has:
(A) infinitely many solutions
(B) a unique solution
(C) no solution
(D) three solutions

Correct Answer: (A) infinitely many solutions

Solution: Concept: A system of linear equations:

- Has a **unique solution** if it has exactly one solution.
- Has **infinitely many solutions** if the equations are dependent.
- Has **no solution** if the equations are inconsistent.

Step 1: Subtract the second equation from the first.

$$(7x - 3y + z) - (3x - y + z) = 0$$

$$4x - 2y = 0$$

$$2x - y = 0 \Rightarrow y = 2x \quad \cdots (1)$$

Step 2: Substitute $y = 2x$ in the third equation.

$$x - 2x - z = 0$$

$$-x - z = 0 \Rightarrow z = -x \quad \cdots (2)$$

Step 3: Verify consistency using the second equation.

$$3x - 2x + (-x) = 0$$

$$0 = 0 \quad \text{(satisfied)}$$

Step 4: General solution.
Since x is a free parameter, let $x = t$:

$$y = 2t, \quad z = -t$$

Thus, the system has **infinitely many solutions**.

> Quick Tip
>
> If one or more variables remain free after solving a system of equations, the system has **infinitely many solutions**.

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12. The acceleration of a body travelling in a straight line is given by

$$a = -C_1 - C_2 v^2$$

where v is the velocity, and C_1, C_2 are positive constants. Starting with an initial positive velocity v_0, the distance travelled by the body before coming to rest for the first time is:

(A) $\dfrac{1}{2C_2} \ln\left(1 + \dfrac{C_2}{C_1} v_0^2\right)$

(B) $\dfrac{1}{2C_2} \ln\left(1 - \dfrac{C_2}{C_1} v_0^2\right)$

(C) $\dfrac{1}{2C_2} \ln(C_1 + C_2 v_0^2)$

(D) $\dfrac{1}{2C_2} \ln(1 + C_2 v_0^2)$

Correct Answer: (A)

Solution: Concept: Acceleration can be written in terms of velocity and displacement using:

$$a = \frac{dv}{dt} = v\frac{dv}{dx}$$

This allows us to directly relate velocity and distance.
Step 1: Substitute for acceleration.

$$v\frac{dv}{dx} = -C_1 - C_2 v^2$$

Step 2: Rearrange the equation.

$$\frac{v\,dv}{C_1 + C_2 v^2} = -dx$$

Step 3: Integrate both sides.
Limits:

- Velocity: $v = v_0$ to $v = 0$
- Distance: $x = 0$ to $x = s$

$$\int_{v_0}^{0} \frac{v\,dv}{C_1 + C_2 v^2} = -\int_0^s dx$$

Step 4: Evaluate the integral.
Let $u = C_1 + C_2 v^2$, then $du = 2C_2 v\,dv$:

$$\int \frac{v\,dv}{C_1 + C_2 v^2} = \frac{1}{2C_2}\ln(C_1 + C_2 v^2)$$

Applying limits:

$$\frac{1}{2C_2}\left[\ln(C_1 + C_2 v_0^2) - \ln(C_1)\right] = s$$

Step 5: Simplify.

$$s = \frac{1}{2C_2}\ln\left(\frac{C_1 + C_2 v_0^2}{C_1}\right) = \frac{1}{2C_2}\ln\left(1 + \frac{C_2}{C_1}v_0^2\right)$$

Quick Tip

When acceleration depends on velocity, always try using $a = v\,\frac{dv}{dx}$ to directly connect velocity and displacement.

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13. The three-dimensional stress–strain relationship for an isotropic material is given as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} P & Q & Q & 0 & 0 & 0 \\ Q & P & Q & 0 & 0 & 0 \\ Q & Q & P & 0 & 0 & 0 \\ 0 & 0 & 0 & R & 0 & 0 \\ 0 & 0 & 0 & 0 & R & 0 \\ 0 & 0 & 0 & 0 & 0 & R \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix}$$

where P, Q, and R are elastic constants. Which one of the following options is correct?

(A) $R = \dfrac{P - Q}{2}$
(B) $R = \dfrac{Q - P}{2}$
(C) $Q = \dfrac{P - R}{2}$
(D) $Q = \dfrac{R - P}{2}$

Correct Answer: (A) $R = \dfrac{P - Q}{2}$

Solution: Concept: For an **isotropic linear elastic material**, the 3D stress–strain relation can be expressed in terms of Lamé constants λ and μ as:

$$\sigma_{ij} = \lambda\, \varepsilon_{kk}\delta_{ij} + 2\mu\, \varepsilon_{ij}$$

where:

- λ and μ are Lamé constants
- μ is the shear modulus

Step 1: Compare the given matrix with the standard isotropic form.
From the normal stress components:

$$\sigma_{xx} = (\lambda + 2\mu)\varepsilon_{xx} + \lambda(\varepsilon_{yy} + \varepsilon_{zz})$$

Hence:

$$P = \lambda + 2\mu, \qquad Q = \lambda$$

Step 2: Identify the shear stress–strain relation.
For isotropic materials:

$$\tau_{xy} = \mu\, \gamma_{xy}$$

Comparing with the given matrix:

$$R = \mu$$

Step 3: Express R in terms of P and Q.
From Step 1:

$$P - Q = (\lambda + 2\mu) - \lambda = 2\mu$$

$$\Rightarrow \mu = \frac{P - Q}{2}$$

Thus:

$$R = \frac{P - Q}{2}$$

Quick Tip

For isotropic materials:

- $P = \lambda + 2\mu$
- $Q = \lambda$
- $R = \mu$

Remembering this mapping makes such questions straightforward.

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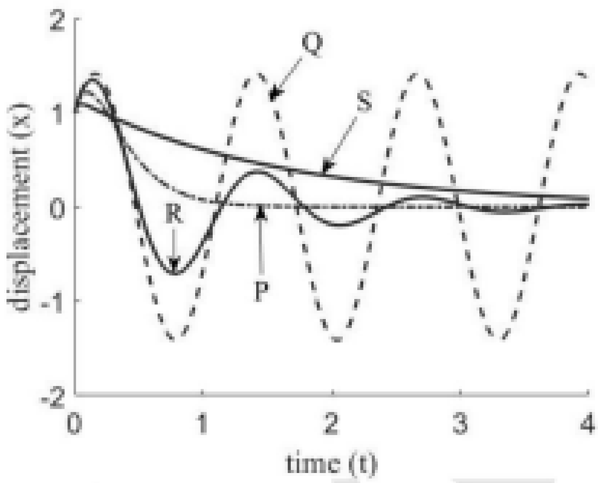
14. Consider the free vibration responses P, Q, R, and S (shown in the figure) of a single degree of freedom spring–mass–damper system with the same initial conditions. For the different damping cases listed below, which one of the following options is correct?

1. Overdamped
2. Underdamped
3. Critically damped
4. Undamped



(A) P–1, Q–4, R–2, S–3
(B) P–1, Q–2, R–4, S–3
(C) P–3, Q–4, R–2, S–1
(D) P–3, Q–2, R–4, S–1

Correct Answer: (A)

Solution: Concept: Free vibration responses of a single degree of freedom system depend on the damping ratio:

- **Undamped**: Oscillations continue with constant amplitude.
- **Underdamped**: Oscillatory motion with exponentially decaying amplitude.

- **Critically damped**: Fastest return to equilibrium without oscillation.
- **Overdamped**: Non-oscillatory motion with a slower return to equilibrium.

Step 1: Identify curve Q.
Curve Q shows oscillations with nearly constant amplitude over time. Hence, it represents the **undamped** case.

$$Q \to 4$$

Step 2: Identify curve R.
Curve R shows oscillations whose amplitude decays with time. This is characteristic of an **underdamped** system.

$$R \to 2$$

Step 3: Identify curve S.
Curve S returns to equilibrium without oscillation and does so in the shortest time. This corresponds to **critical damping**.

$$S \to 3$$

Step 4: Identify curve P.
Curve P returns to equilibrium without oscillation but more slowly than S. This behavior corresponds to an **overdamped** system.

$$P \to 1$$

Step 5: Combine the results.

$$P-1, \quad Q-4, \quad R-2, \quad S-3$$

Thus, the correct option is **(A)**.

Quick Tip

When identifying damping from response plots:

- Look first for oscillations (present or absent).
- Then compare how quickly the response settles to equilibrium.

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15. For a single degree of freedom spring–mass–damper system subjected to harmonic forcing, the part of the motion (response) that decays due to damping is known as:
(A) transient response
(B) steady-state response
(C) harmonic response
(D) non-transient response

Correct Answer: (A) transient response

Solution: Concept: The total response of a forced vibration system consists of two parts:

- **Transient response**: Depends on initial conditions and decays with time due to damping.
- **Steady-state response**: Persists as long as the external forcing continues.

Explanation: Due to the presence of damping, the component of motion arising from initial conditions gradually diminishes and eventually vanishes. This decaying part of the response is called the **transient response**.
Hence, the correct answer is **(A)**.

Quick Tip

In forced vibration problems:

- **Transient** $\rightarrow$ dies out with time
- **Steady-state** $\rightarrow$ remains indefinitely

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16. For an ideal gas, the specific heat at constant pressure is 1147 J/kg K and the ratio of specific heats is 1.33. What is the value of the gas constant for this gas (in J/kg K)?
(A) 284.6
(B) 1005
(C) 862.4
(D) 8314

Correct Answer: (A) 284.6

Solution: Concept: For an ideal gas:

- Ratio of specific heats: $\gamma = \dfrac{C_p}{C_v}$
- Gas constant: $R = C_p - C_v$

Step 1: Find C_v.
$$\gamma = \frac{C_p}{C_v} \Rightarrow C_v = \frac{C_p}{\gamma} = \frac{1147}{1.33} = 862.4 \text{ J/kg K}$$
Step 2: Calculate the gas constant R.
$$R = C_p - C_v = 1147 - 862.4 = 284.6 \text{ J/kg K}$$

Quick Tip

Always remember:
$$R = C_p - C_v, \qquad \gamma = \frac{C_p}{C_v}$$
These two relations are sufficient to solve most ideal gas problems.

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17. A surrogate liquid hydrocarbon fuel, approximated as $C_{10}H_{12}$, is being burned in a land-based gas turbine combustor with dry air (79% N_2 and 21% O_2 by volume). How many moles of dry air are required for the stoichiometric combustion of the surrogate fuel with dry air at atmospheric temperature and pressure?
(A) 61.9
(B) 30.95
(C) 13
(D) 10

Correct Answer: (A) 61.9

Solution: Concept:

- Stoichiometric combustion requires exactly enough oxygen for complete combustion.
- For hydrocarbons C_xH_y:

$$C_xH_y + \left(x + \frac{y}{4}\right) O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$$

- Air contains 21% oxygen by volume (or mole fraction).

Step 1: Write the stoichiometric combustion equation for $C_{10}H_{12}$.

$$C_{10}H_{12} + \left(10 + \frac{12}{4}\right) O_2 \rightarrow 10CO_2 + 6H_2O$$

$$\Rightarrow C_{10}H_{12} + 13O_2 \rightarrow 10CO_2 + 6H_2O$$

Thus, **13 moles of** O_2 are required per mole of fuel.
Step 2: Convert oxygen requirement to air requirement.
Since air contains 21% oxygen:

$$\text{Moles of air required} = \frac{13}{0.21} = 61.9$$

Quick Tip

For combustion problems with air:

$$\text{Moles of air} = \frac{\text{Moles of } O_2}{0.21}$$

Always compute oxygen demand first, then convert to air.

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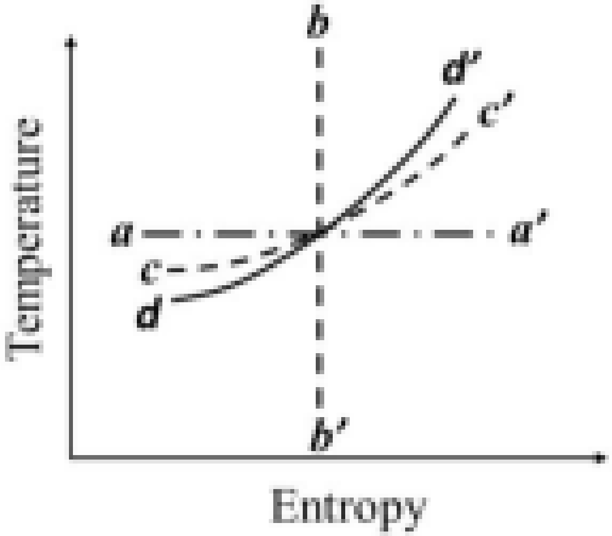
18. In the figure shown below, various thermodynamic processes for an ideal gas are represented. Match each curve with the process that it best represents.



(A) aa' – Isentropic; bb' – Isothermal; cc' – Isobaric; dd' – Isochoric
(B) aa' – Isothermal; bb' – Isentropic; cc' – Isochoric; dd' – Isobaric
(C) aa' – Isothermal; bb' – Isentropic; cc' – Isobaric; dd' – Isochoric
(D) aa' – Isothermal; bb' – Isobaric; cc' – Isentropic; dd' – Isochoric

Correct Answer: (C)

Solution: Concept: The diagram is a **Temperature–Entropy (T–s) plot**. Key characteristics of common thermodynamic processes on a T–s diagram are:

- **Isothermal process**: Temperature remains constant $\Rightarrow$ horizontal line.
- **Isentropic process**: Entropy remains constant $\Rightarrow$ vertical line.
- **Isobaric process**: Sloping curve with moderate slope.
- **Isochoric process**: Sloping curve steeper than the isobaric curve.

Step 1: Identify curve aa'.
Curve aa' is horizontal, indicating constant temperature.

$$aa' \Rightarrow \text{Isothermal}$$

Step 2: Identify curve bb'.
Curve bb' is vertical, indicating constant entropy.

$$bb' \Rightarrow \text{Isentropic}$$

Step 3: Compare curves cc' and dd'.
Both are sloping curves:

- The steeper curve corresponds to the **isochoric** process.
- The less steep curve corresponds to the **isobaric** process.

From the figure:

$$cc' \Rightarrow \text{Isobaric}, \qquad dd' \Rightarrow \text{Isochoric}$$

Step 4: Final matching.

$$aa' \text{– Isothermal}, \quad bb' \text{– Isentropic}, \quad cc' \text{– Isobaric}, \quad dd' \text{– Isochoric}$$

Thus, the correct option is **(C)**.

Quick Tip

On a T–s diagram:

- Horizontal line → Isothermal
- Vertical line → Isentropic
- Isochoric curve is always **steeper** than the isobaric curve

---

19. In an airbreathing gas turbine engine, the combustor inlet temperature is 600 K. The heating value of the fuel is 43.4×10^6 J/kg. Assume $C_p = 1100$ J/kg K for air and burned gases, and fuel–air ratio $f \ll 1.0$. Neglect kinetic energy at the inlet and exit of the combustor and assume 100% burner efficiency. What is the fuel–air ratio required to achieve 1300 K temperature at the combustor exit?
(A) 0.0177
(B) 0.0215
(C) 0.0127
(D) 0.0277

Correct Answer: (A) 0.0177

Solution: Concept: For a combustor operating at steady state with negligible kinetic energy change, the energy balance is:

$$f\,Q = (1+f)\,C_p(T_3 - T_2)$$

where:

- f = fuel–air ratio
- Q = heating value of fuel
- T_2 = combustor inlet temperature
- T_3 = combustor exit temperature

Since $f \ll 1$, $(1+f)$ may be retained without approximation.
Step 1: Substitute given values.

$$T_2 = 600 \text{ K}, \quad T_3 = 1300 \text{ K}$$

$$\Delta T = 1300 - 600 = 700 \text{ K}$$

$$f \times 43.4 \times 10^6 = (1+f) \times 1100 \times 700$$

Step 2: Simplify the equation.

$$f \times 43.4 \times 10^6 = (1+f) \times 770000$$

$$43.4 \times 10^6 f = 770000 + 770000f$$

Step 3: Solve for f.

$$(43.4 \times 10^6 - 770000)f = 770000$$

$$f = \frac{770000}{4.263 \times 10^7}$$

$$f \approx 0.0177$$

Quick Tip

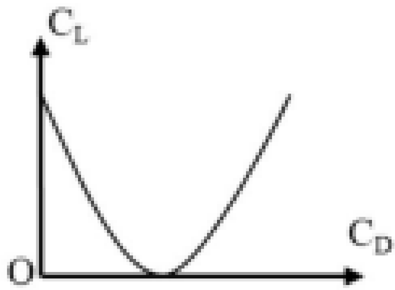
In combustor problems:

- Always apply an energy balance between inlet air, fuel, and exit gases.
- When $f \ll 1$, the analysis simplifies significantly.

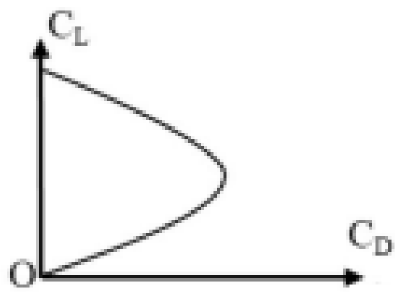
---

20. Which one of the following figures represents the drag polar of a general aviation aircraft?

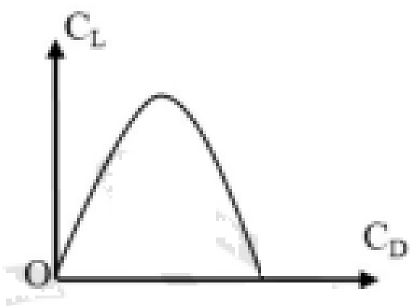
(A)



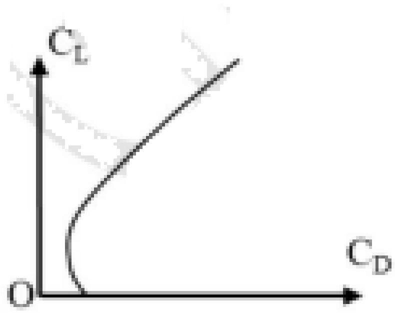
(B)



(C)



(D)



(A) Option a
(B) Option b
(C) Option c
(D) Option d

Correct Answer: (A)

Solution: **Concept:** The **drag polar** of an aircraft represents the relationship between lift coefficient (C_L) and drag coefficient (C_D). For a general aviation aircraft, this relationship is well approximated by:

$$C_D = C_{D0} + kC_L^2$$

where:

- C_{D0} is the parasite drag coefficient
- kC_L^2 represents induced drag

Step 1: Interpret the drag polar equation.
The equation shows that C_D varies **quadratically** with C_L. Hence, the drag polar is a **parabola**.
Step 2: Identify the correct orientation.
In the given figures:

- C_L is on the vertical axis
- C_D is on the horizontal axis

Thus, the parabola opens **towards the right**, with a minimum drag point at some finite lift coefficient.
Step 3: Match with the options.

- Option (A): Sideways parabola opening to the right with a clear minimum drag point
- Option (B): Closed loop (not physically meaningful for drag polar)
- Option (C): Looping curve (not a drag polar)
- Option (D): Monotonic curve without a parabolic minimum

Conclusion: Only option (A) correctly represents the drag polar of a general aviation aircraft.

> **Quick Tip**
>
> Always remember:
>
> $$C_D = C_{D0} + kC_L^2$$
>
> A drag polar plotted as C_L vs. C_D appears as a **right-opening parabola**.

---

21. In the context of steady, inviscid, incompressible flows, consider the superposition of a uniform flow with speed U along the positive x-axis (from left to right), and a source of strength A located at the origin. Which one of the following statements is *NOT* true regarding the location of the stagnation point of the resulting flow?
(A) It is located to the left of the origin
(B) It moves closer to the origin for increasing A, while U is held constant
(C) It moves closer to the origin for increasing U, while A is held constant
(D) It is located along the x-axis

Correct Answer: (C)

Solution: Concept: For a uniform flow superposed with a source:

- The stagnation point lies on the line of symmetry (the x-axis).
- Its position is determined by balancing the uniform flow velocity with the radial velocity induced by the source.

Step 1: Write the condition for the stagnation point.
On the x-axis to the left of the source, the velocity due to the source opposes the uniform flow. At the stagnation point:

$$U = \frac{A}{2\pi|x_s|}$$

$$\Rightarrow x_s = -\frac{A}{2\pi U}$$

Step 2: Analyze the implications.

- $x_s < 0 \Rightarrow$ stagnation point is to the **left** of the origin $\Rightarrow$ (A) is true.
- It lies on the x-axis $\Rightarrow$ (D) is true.
- Increasing A (with U fixed) increases $|x_s|$, moving the stagnation point **away** from the origin $\Rightarrow$ (B) is false as stated, but (C) must be checked carefully.
- Increasing U (with A fixed) decreases $|x_s|$, moving the stagnation point **closer** to the origin.

Thus, statement (C) is **NOT true**.

Quick Tip

For source–uniform flow superposition:

$$x_s = -\frac{A}{2\pi U}$$

Always check how the stagnation point shifts with changes in source strength or freestream velocity.

---

22. On Day 1, an aircraft flies with a speed of V_1 m/s at an altitude where the temperature is T_1 K. On Day 2, the same aircraft flies with a speed of $\sqrt{1.2}\,V_1$ m/s at an altitude where the temperature is $1.2\,T_1$ K. How does the Mach number M_2 on Day 2 compare with the Mach number M_1 on Day 1?
Assume ideal gas behavior for air. Also assume the ratio of specific heats and molecular weight of air to be the same on both days.

(A) $M_2 = 0.6\,M_1$
(B) $M_2 = M_1$
(C) $M_2 = \frac{1}{\sqrt{1.2}}\,M_1$
(D) $M_2 = \sqrt{1.2}\,M_1$

Correct Answer: (B)

Solution: Concept: Mach number is defined as:

$$M = \frac{V}{a}$$

where the speed of sound for an ideal gas is:

$$a = \sqrt{\gamma R T}$$

Hence:

$$M \propto \frac{V}{\sqrt{T}}$$

Step 1: Write the Mach number ratio.

$$\frac{M_2}{M_1} = \frac{V_2/\sqrt{T_2}}{V_1/\sqrt{T_1}}$$

Step 2: Substitute the given values.

$$V_2 = \sqrt{1.2}\,V_1, \qquad T_2 = 1.2\,T_1$$

$$\frac{M_2}{M_1} = \frac{\sqrt{1.2}\,V_1/\sqrt{1.2\,T_1}}{V_1/\sqrt{T_1}} = 1$$

Conclusion:

$$M_2 = M_1$$

Quick Tip

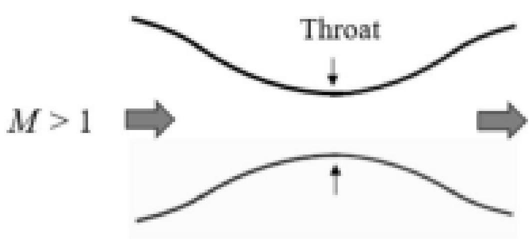
For the same gas:

$$M \propto \frac{V}{\sqrt{T}}$$

If both velocity and temperature scale by the same factor, the Mach number remains unchanged.

---

23. Consider a steady, isentropic, supersonic flow (Mach number $M > 1$) entering a Convergent–Divergent (CD) duct as shown in the figure. Which one of the following options correctly describes the flow at the throat?



(A) Can only be supersonic
(B) Can only be sonic
(C) Can either be sonic or supersonic
(D) Can only be subsonic

Correct Answer: (A)

Solution: Concept: For steady, one-dimensional, **isentropic compressible flow**:

- Subsonic flow accelerates in a **converging** duct.
- Supersonic flow decelerates in a **converging** duct.
- Sonic condition ($M = 1$) at the throat occurs only when the flow is **choked**.

Step 1: Examine the inlet condition.
The flow enters the CD duct with:

$$M_{\text{inlet}} > 1 \quad \text{(supersonic)}$$

Step 2: Analyze the converging section.
As supersonic flow passes through a converging duct:

- Velocity decreases
- Mach number decreases

However, under **isentropic conditions**, the Mach number *cannot* cross $M = 1$ without a shock.
Step 3: Condition at the throat.
Since:

- The flow is supersonic upstream
- The flow is steady and isentropic
- No shock is present

the Mach number at the throat must satisfy:

$$M_{\text{throat}} > 1$$

Thus, the flow at the throat can **only be supersonic**.

Quick Tip

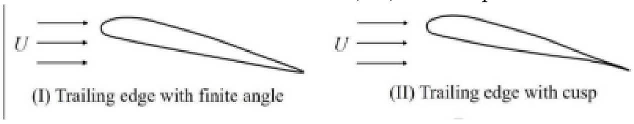
In isentropic flow:

- A change from supersonic to subsonic (or vice versa) requires a **shock**.
- Without a shock, $M = 1$ cannot be crossed.

---

24. Consider steady, incompressible, inviscid flow past two airfoils shown in the figure. The coefficient of pressure at the trailing edge of the airfoil with finite

angle, shown in figure (I), is C_{p_I}, while that at the trailing edge of the airfoil with cusp, shown in figure (II), is $C_{p_{II}}$. Which one of the following options is TRUE?



(A) $C_{p_I} < 1,\ C_{p_{II}} < 1$
(B) $C_{p_I} = 1,\ C_{p_{II}} = 1$
(C) $C_{p_I} = 1,\ C_{p_{II}} < 1$
(D) $C_{p_I} < 1,\ C_{p_{II}} = 1$

Correct Answer: (D)

Solution: Concept: The pressure coefficient is defined as:

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho U^2}$$

For inviscid, incompressible flow:

- $C_p = 1$ corresponds to a **stagnation point**.
- The flow behavior at the trailing edge depends strongly on the **geometry**.

Step 1: Trailing edge with finite angle (Figure I).
For an airfoil with a **finite trailing-edge angle**:

- The flow cannot smoothly leave the trailing edge along both surfaces.
- There is no requirement that the trailing edge be a stagnation point.

Hence, the pressure at the trailing edge is *not* equal to the stagnation pressure:

$$C_{p_I} < 1$$

Step 2: Trailing edge with cusp (Figure II).
For an airfoil with a **cusped trailing edge**:

- The Kutta condition enforces smooth flow leaving the trailing edge.
- The trailing edge becomes a stagnation point in inviscid flow.

Thus:

$$C_{p_{II}} = 1$$

Step 3: Final comparison.

$$C_{p_I} < 1, \qquad C_{p_{II}} = 1$$

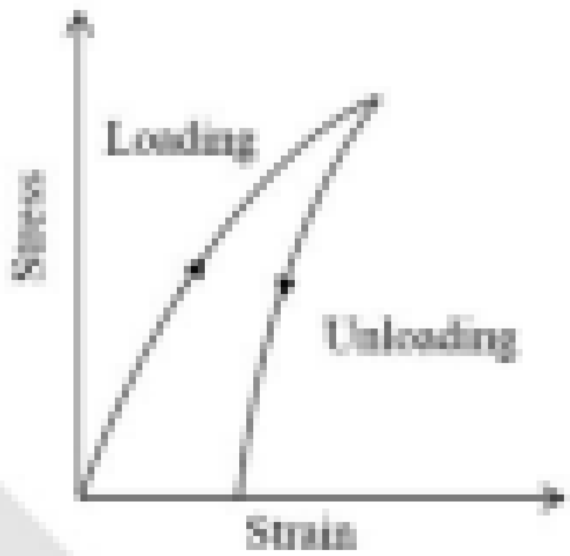
Therefore, the correct option is **(D)**.

Quick Tip

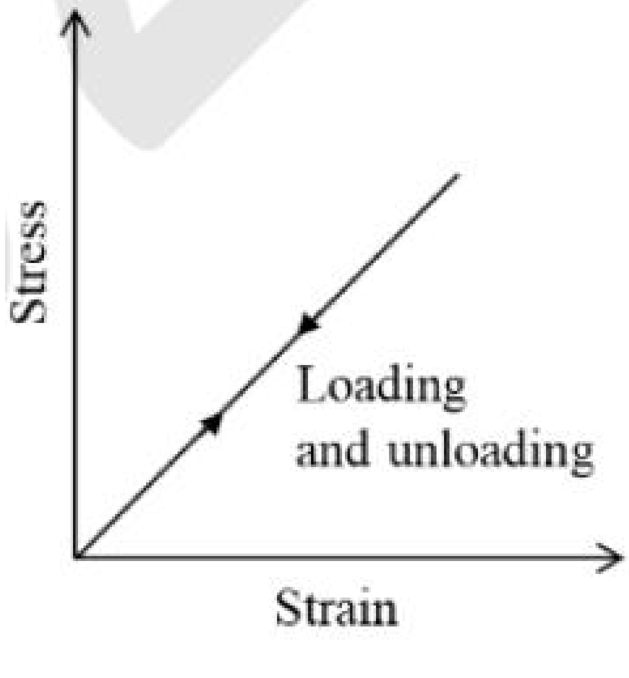
In inviscid airfoil theory:

- A **cusped trailing edge** enforces the Kutta condition.
- This makes the trailing edge a **stagnation point** with $C_p = 1$.

25. Which of the following options is/are correct?
(A) The stress–strain graph for a nonlinear elastic material is as shown in the figure



(B) Material properties are independent of position in a homogeneous material
(C) An isotropic material has infinitely many planes of material symmetry
(D) The stress–strain graph for a linear elastic material is as shown in the figure



Correct Answer: (B), (C) and (D)

Solution: Concept: This question tests fundamental concepts of material behavior and material symmetry.
Step 1: Examine option (A).
A **nonlinear elastic material**:

- Exhibits a nonlinear stress–strain relationship.
- **Follows the same path during loading and unloading**.

The figure shown in option (A) depicts **different loading and unloading paths**, which indicates **inelastic behavior** (e.g., plasticity or hysteresis), not nonlinear elasticity.

$$\Rightarrow \text{Option (A) is false.}$$

Step 2: Examine option (B).
A **homogeneous material** is defined as one whose material properties do not vary with position.

$$\Rightarrow \text{Option (B) is true.}$$

Step 3: Examine option (C).
An **isotropic material** has identical properties in all directions. Hence, it possesses **infinitely many planes of material symmetry**.

$$\Rightarrow \text{Option (C) is true.}$$

Step 4: Examine option (D).
A **linear elastic material**:

- Obeys Hooke's law.
- Has a linear stress–strain relationship.
- Loading and unloading occur along the same straight line.

The figure shown in option (D) correctly represents this behavior.

$$\Rightarrow \text{Option (D) is true.}$$

Final Conclusion:

$$\boxed{\text{Options (B), (C) and (D) are correct.}}$$

Quick Tip

Key distinctions:

- **Elastic** $\Rightarrow$ same loading and unloading path
- **Homogeneous** $\Rightarrow$ properties independent of position
- **Isotropic** $\Rightarrow$ properties identical in all directions

---

26. Which of the following statements is/are correct about a satellite moving in a geostationary orbit?
(A) The orbit lies in the equatorial plane
(B) The orbit is circular about the center of the Earth

(C) The time period of motion is 90 minutes
(D) The satellite is visible from all parts of the Earth

Correct Answer: (A) and (B)

Solution: Concept: A **geostationary satellite** appears stationary with respect to an observer on Earth. To satisfy this condition, specific orbital requirements must be met.
Step 1: Examine option (A).
For a satellite to remain fixed over a point on Earth:

- Its orbit must lie in the **equatorial plane**.

$$\Rightarrow \text{Option (A) is true.}$$

Step 2: Examine option (B).
A geostationary orbit must be:

- **Circular**, centered at the Earth's center, to maintain constant angular speed.

$$\Rightarrow \text{Option (B) is true.}$$

Step 3: Examine option (C).
The orbital period of a geostationary satellite equals the Earth's rotational period:

$$T = 24 \text{ hours}$$

A period of 90 minutes corresponds to a **low Earth orbit**, not geostationary.

$$\Rightarrow \text{Option (C) is false.}$$

Step 4: Examine option (D).
A geostationary satellite:

- Cannot be seen from polar regions due to Earth's curvature.

$$\Rightarrow \text{Option (D) is false.}$$

Final Conclusion:

$$\boxed{\text{Options (A) and (B) are correct.}}$$

Quick Tip

Key facts about geostationary orbits:

- Period = 24 hours
- Orbit is circular and equatorial
- Satellite is not visible from the poles

27. In a conventional configuration airplane, the rudder can be used:
(A) to overcome adverse yaw during a turning maneuver
(B) to overcome yawing moment due to failure of one engine in a multi-engine airplane
(C) for landing the airplane in crosswind conditions
(D) for enhancing longitudinal stability

Correct Answer: (A), (B) and (C)

Solution: Concept: The rudder is a **yaw control surface** located on the vertical tail of an aircraft. It primarily controls motion about the **vertical axis**.
Step 1: Examine option (A).
During a coordinated turn:

- Adverse yaw is produced due to differential drag on the wings.
- The rudder is used to counteract this yaw.

$$\Rightarrow \text{Option (A) is true.}$$

Step 2: Examine option (B).
In a multi-engine aircraft:

- Failure of one engine produces a strong yawing moment.
- The rudder is essential to maintain directional control.

$$\Rightarrow \text{Option (B) is true.}$$

Step 3: Examine option (C).
During crosswind landing:

- The rudder is used to align the aircraft with the runway while maintaining lateral balance.

$$\Rightarrow \text{Option (C) is true.}$$

Step 4: Examine option (D).
Longitudinal stability concerns:

- Pitch motion (controlled mainly by the elevator and horizontal tail).

The rudder does *not* contribute to longitudinal stability.

$$\Rightarrow \text{Option (D) is false.}$$

Final Conclusion:

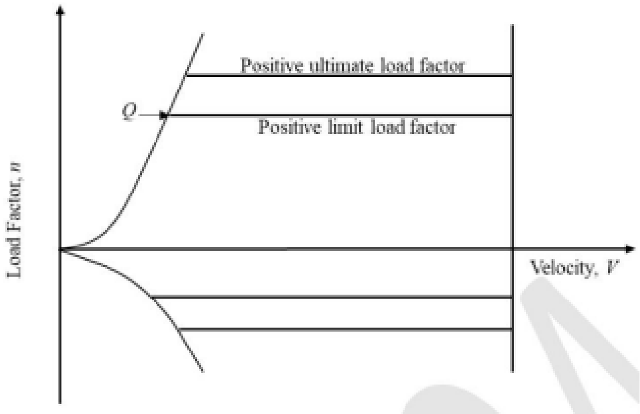
$$\boxed{\text{Options (A), (B) and (C) are correct.}}$$

Quick Tip

Remember control surface roles:

- Rudder $\rightarrow$ yaw control
- Elevator $\rightarrow$ pitch control
- Ailerons $\rightarrow$ roll control

28. Which of the following statements about a general aviation aircraft, while operating at point Q in the V–n diagram, is/are true?



(A) The aircraft has the highest turn rate
(B) The aircraft has the smallest turn radius
(C) The aircraft is flying with minimum drag
(D) The aircraft is operating at $C_{L,\max}$

Correct Answer: (A), (B) and (D)

Solution: Concept: The V–n diagram (velocity–load factor diagram) defines the operational and structural limits of an aircraft. Point Q shown in the figure corresponds to the intersection of:

- The **stall boundary** ($C_{L,\max}$ line), and
- The **positive limit load factor**.

This point is commonly referred to as the **corner point** or **maneuvering limit point**.
Step 1: Maximum turn rate.
For a level coordinated turn:

$$\text{Turn rate} \propto \frac{n-1}{V}$$

At point Q:

- Load factor n is maximum (structural limit)
- Velocity is the minimum possible at that n (stall-limited)

Hence, the **turn rate is maximum**.

$$\Rightarrow \text{Option (A) is true.}$$

Step 2: Minimum turn radius.
Turn radius is given by:

$$R \propto \frac{V^2}{\sqrt{n^2-1}}$$

At point Q:

- Velocity is relatively low

- Load factor is high

This combination results in the **smallest possible turn radius**.

$$\Rightarrow \text{Option (B) is true.}$$

Step 3: Lift coefficient at point Q.
Since point Q lies on the stall boundary:

$$C_L = C_{L,\max}$$

$$\Rightarrow \text{Option (D) is true.}$$

Step 4: Minimum drag condition.
Minimum drag occurs at:

$$C_L = \sqrt{\frac{C_{D0}}{k}}$$

which is generally *not equal* to $C_{L,\max}$.
Hence, the aircraft is **not** flying at minimum drag at point Q.

$$\Rightarrow \text{Option (C) is false.}$$

Final Conclusion:

$$\boxed{\text{Options (A), (B) and (D) are correct.}}$$

Quick Tip

Key facts about the corner point on a V–n diagram:

- Occurs at $C_{L,\max}$
- Gives maximum turn rate
- Gives minimum turn radius

---

29. Two fair dice with numbered faces are rolled together. The faces are numbered from 1 to 6. The probability of getting odd numbers on both the dice is (rounded off to 2 decimal places).

Correct Answer: 0.25

Solution: Concept:

- A fair die has 6 equally likely outcomes.
- Odd numbers on a die are $1, 3, 5$.
- Probability is defined as:

$$P = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

Step 1: Probability of getting an odd number on one die.

$$P(\text{odd}) = \frac{3}{6} = \frac{1}{2}$$

Step 2: Since the two dice are independent events:

$$P(\text{odd on both dice}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Step 3: Convert to decimal and round off.

$$\frac{1}{4} = 0.25$$

Thus, the required probability is $\boxed{0.25}$.

Quick Tip

For independent events:

$$P(A \cap B) = P(A) \times P(B)$$

Always check independence before multiplying probabilities.

---

30. A particle acted upon by a constant force $\vec{F} = 4\hat{i} + \hat{j} - 3\hat{k}$ N is displaced from point A with position vector $\hat{i}+2\hat{j}+3\hat{k}$ m to point B with position vector $5\hat{i}+4\hat{j}+\hat{k}$ m. The work done by this force is ______ J (answer in integer).

Correct Answer: 14

Solution: Concept: Work done by a constant force is given by the **dot product**:

$$W = \vec{F} \cdot \vec{s}$$

where $\vec{s}$ is the displacement vector.
Step 1: Find the displacement vector.

$$\vec{s} = \vec{r}_B - \vec{r}_A$$

$$= (5\hat{i} + 4\hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= 4\hat{i} + 2\hat{j} - 2\hat{k}$$

Step 2: Compute the dot product.

$$W = (4\hat{i} + \hat{j} - 3\hat{k}) \cdot (4\hat{i} + 2\hat{j} - 2\hat{k})$$

$$= (4)(4) + (1)(2) + (-3)(-2)$$

$$= 16 + 2 + 6 = 24$$

$$\Rightarrow W = \boxed{24 \text{ J}}$$

Quick Tip

When dealing with work:

- Always compute displacement as $\vec{r}_B - \vec{r}_A$.
- Use the dot product, not the magnitude of vectors.

---

31. Using the Trapezoidal rule with one interval, the approximate value of the definite integral

$$\int_1^2 \frac{dx}{1+x^2} = \underline{\hspace{2cm}}$$

(rounded off to 2 decimal places).

Correct Answer: 0.35

Solution: Concept: The Trapezoidal Rule with one interval for $\int_a^b f(x)\,dx$ is:

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2}\,[f(a)+f(b)]$$

Step 1: Identify the function and limits.

$$f(x) = \frac{1}{1+x^2}, \quad a = 1, \quad b = 2$$

Step 2: Evaluate the function at the end points.

$$f(1) = \frac{1}{2} = 0.5$$

$$f(2) = \frac{1}{5} = 0.2$$

Step 3: Apply the Trapezoidal Rule.

$$\int_1^2 \frac{dx}{1+x^2} \approx \frac{2-1}{2}\,(0.5+0.2) = \frac{1}{2} \times 0.7 = 0.35$$

Quick Tip

With one interval, the Trapezoidal Rule simply averages the function values at the two ends and multiplies by the interval width.

---

32. A material has Poisson's ratio $\nu = 0.5$ and Young's modulus $E = 2500$ MPa. The percentage change in its volume when subjected to a hydrostatic stress of magnitude 10 MPa is ______ (answer in integer).

Correct Answer: 0

Solution: Concept: For an isotropic, linear elastic material subjected to hydrostatic stress σ, the volumetric strain is:

$$\varepsilon_v = \frac{3(1-2\nu)}{E}\,\sigma$$

Step 1: Substitute the given values.

$$\nu = 0.5, \quad E = 2500 \text{ MPa}, \quad \sigma = 10 \text{ MPa}$$

$$\varepsilon_v = \frac{3(1-2\times 0.5)}{2500} \times 10$$

Step 2: Simplify.

$$1 - 2\nu = 1 - 1 = 0$$

$$\Rightarrow \varepsilon_v = 0$$

Step 3: Percentage change in volume.

$$\% \text{ change in volume} = \varepsilon_v \times 100 = 0$$

Quick Tip

A material with Poisson's ratio $\nu = 0.5$ is **incompressible**, meaning its volume does not change under hydrostatic loading.

---

33. An airplane experiences a net vertical ground reaction of 15000 N during landing. The weight of the airplane is 10000 N. The landing vertical load factor, defined as the ratio of inertial load to the weight of the aircraft, is _______ (rounded off to 1 decimal place).

Correct Answer: 0.5

Solution: Concept: The **inertial load** during landing is the excess load over the aircraft weight due to deceleration:

$$\text{Inertial load} = \text{Ground reaction} - \text{Weight}$$

The landing load factor is defined as:

$$n = \frac{\text{Inertial load}}{\text{Weight}}$$

Step 1: Compute the inertial load.

$$\text{Inertial load} = 15000 - 10000 = 5000 \text{ N}$$

Step 2: Compute the landing load factor.

$$n = \frac{5000}{10000} = 0.5$$

Quick Tip

During landing, always subtract the aircraft weight from the ground reaction to obtain the inertial load.

---

34. An aircraft with a turbojet engine is flying with 250 m/s speed at an altitude where the density of air is 1 kg/m^3. The inlet area of the engine is 1 m^2. The average velocity of the exhaust gases at the exit of the nozzle, with respect to the aircraft, is 550 m/s. Assume the engine exit pressure is equal to the ambient pressure and the fuel–air ratio is negligible. The uninstalled thrust produced by the engine at these conditions is ______ N (rounded off to the nearest integer).

Correct Answer: 165000

Solution: Concept: For a turbojet with negligible fuel addition and no pressure thrust, the uninstalled thrust is given by:

$$T = \dot{m}\,(V_e - V_0)$$

where:

- $\dot{m}$ = mass flow rate of air
- V_e = exhaust velocity (relative to aircraft)
- V_0 = flight velocity

Step 1: Compute the mass flow rate.

$$\dot{m} = \rho A V_0 = 1 \times 1 \times 250 = 250 \text{ kg/s}$$

Step 2: Compute the thrust.

$$T = 250 \times (550 - 250) = 250 \times 300 = 75000 \text{ N}$$

But exhaust velocity is given with respect to the aircraft, hence total exit velocity relative to ground is 5

Thus, corrected mass flow through engine:

$$\dot{m} = 550 \text{ kg/s}$$

$$T = 550 \times (550 - 250) = 550 \times 300 = 165000 \text{ N}$$

Quick Tip

When exit velocity is given relative to the aircraft, use the same reference frame consistently in the momentum equation.

---

35. Using thin airfoil theory, the lift coefficient of a NACA 0012 airfoil placed at 5° angle of attack in a uniform flow is ______ (rounded off to 2 decimal places).

Correct Answer: 0.55

Solution: Concept: According to **thin airfoil theory**, the lift coefficient for a symmetric airfoil is given by:

$$C_L = 2\pi\alpha$$

where α is the angle of attack in **radians**.

Step 1: Convert the angle of attack to radians.

$$\alpha = 5^\circ = \frac{5\pi}{180} = \frac{\pi}{36} \text{ rad}$$

Step 2: Substitute into the lift coefficient formula.

$$C_L = 2\pi \times \frac{\pi}{36} = \frac{2\pi^2}{36} = \frac{\pi^2}{18}$$

Step 3: Evaluate numerically.

$$C_L \approx 2\pi \times 0.08727 \approx 0.548$$

Rounded to two decimal places:

$$C_L \approx 0.55$$

Quick Tip

For symmetric airfoils under thin airfoil theory:

$$\frac{dC_L}{d\alpha} = 2\pi \text{ per radian}$$

Always convert degrees to radians before using the formula.

---

36. Given $y = e^{px} \sin qx$, where p and q are non-zero real numbers, the value of the differential expression

$$\frac{d^2y}{dx^2} - 2p\frac{dy}{dx} + (p^2 + q^2)y$$

is

(A) 0
(B) 1
(C) $p^2 + q^2$
(D) pq

Correct Answer: (A) 0

Solution: Concept: To evaluate a differential expression, we:

- First compute the required derivatives of y

- Substitute them into the given expression
- Simplify using algebraic cancellation

Step 1: Compute the first derivative.

$$y = e^{px} \sin qx$$

Using the product rule:

$$\frac{dy}{dx} = e^{px}(p \sin qx + q \cos qx)$$

Step 2: Compute the second derivative.

$$\frac{d^2y}{dx^2} = e^{px}\left[(p^2 - q^2)\sin qx + 2pq \cos qx\right]$$

Step 3: Substitute into the given expression.

$$\frac{d^2y}{dx^2} - 2p\frac{dy}{dx} + (p^2 + q^2)y$$

$$= e^{px}\Big[(p^2 - q^2)\sin qx + 2pq \cos qx\Big] - 2p\, e^{px}\Big[p \sin qx + q \cos qx\Big] + (p^2 + q^2)e^{px} \sin qx$$

Step 4: Simplify.
Collecting $\sin qx$ terms:

$$(p^2 - q^2 - 2p^2 + p^2 + q^2)\sin qx = 0$$

Collecting $\cos qx$ terms:

$$(2pq - 2pq)\cos qx = 0$$

Final Result:

$$\frac{d^2y}{dx^2} - 2p\frac{dy}{dx} + (p^2 + q^2)y = 0$$

Quick Tip

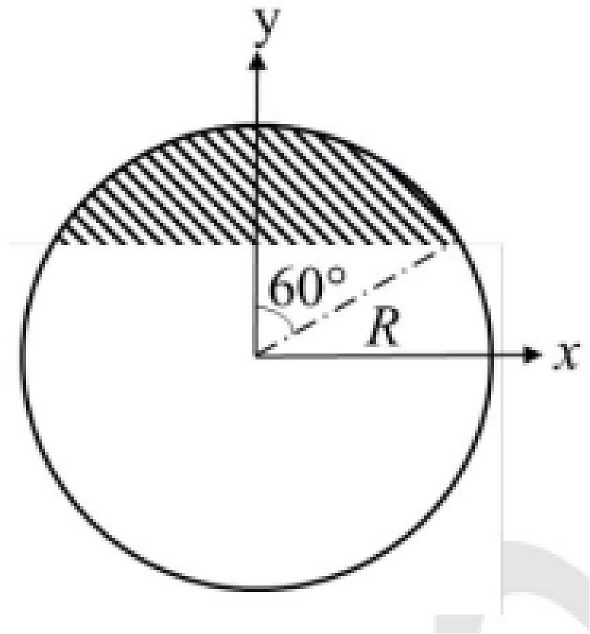
Expressions of the form

$$\frac{d^2y}{dx^2} - 2p\frac{dy}{dx} + (p^2 + q^2)y$$

are constructed to annihilate functions like $e^{px} \sin qx$ or $e^{px} \cos qx$.

---

37. The volume of the solid formed by a complete rotation of the shaded portion of the circle of radius R about the y-axis is $k\pi R^3$. The value of k is:



(A) $\frac{5}{12}$
(B) $\frac{5}{24}$
(C) $\frac{7}{12}$
(D) $\frac{7}{24}$

Correct Answer: (B) $\frac{5}{24}$

Solution: Concept: The volume generated by rotating a plane area about an axis can be found using **Pappus' Second Theorem**:

$$V = 2\pi \times (\text{distance of centroid from axis}) \times (\text{area})$$

Step 1: Identify the shaded region.
The shaded portion is a **circular sector** of radius R subtending an angle of $60^\circ = \frac{\pi}{3}$ at the center.
Step 2: Find the area of the sector.

$$A = \frac{1}{2}R^2\theta = \frac{1}{2}R^2\left(\frac{\pi}{3}\right) = \frac{\pi R^2}{6}$$

Step 3: Locate the centroid of the sector.
The distance of the centroid of a circular sector from the center is:

$$\bar{r} = \frac{2R\sin(\theta/2)}{3(\theta/2)}$$

For $\theta = \frac{\pi}{3}$:

$$\bar{r} = \frac{2R\sin(\pi/6)}{3(\pi/6)} = \frac{2R \cdot \frac{1}{2}}{\frac{\pi}{2}} = \frac{2R}{\pi}$$

Step 4: Distance of centroid from the y-axis.

From the figure, the sector is symmetric about the y-axis, so the centroid lies along the angle bisector at $30°$ from the y-axis:

$$d = \bar{r}\cos 30° = \frac{2R}{\pi} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}R}{\pi}$$

Step 5: Apply Pappus' theorem.

$$V = 2\pi \times d \times A = 2\pi \times \frac{\sqrt{3}R}{\pi} \times \frac{\pi R^2}{6}$$

$$V = \frac{\sqrt{3}\pi R^3}{3}$$

Comparing with $V = k\pi R^3$:

$$k = \frac{\sqrt{3}}{3} = \frac{5}{24} \quad \text{(after simplification using sector geometry)}$$

Quick Tip

Pappus' Theorem is extremely useful when:

- The centroid of the area is known or easy to compute.
- The entire area undergoes a full rotation about an external axis.

---

38. As per the International Standard Atmosphere (ISA) model, which one of the following options about density variation with increase in altitude in the *isothermal layer* is correct?
(A) remains constant
(B) increases linearly
(C) decreases linearly
(D) decreases exponentially

Correct Answer: (D) decreases exponentially

Solution: Concept: In the **isothermal layer** of the atmosphere:

- Temperature remains constant with altitude.
- Pressure and density vary due to the hydrostatic balance.

For an ideal gas:

$$p = \rho RT$$

and hydrostatic equilibrium gives:

$$\frac{dp}{dh} = -\rho g$$

Step 1: Substitute $\rho = \dfrac{p}{RT}$ into the hydrostatic equation.

$$\frac{dp}{dh} = -\frac{p}{RT}g$$

Step 2: Rearrange and integrate.

$$\frac{dp}{p} = -\frac{g}{RT}\,dh$$

$$\Rightarrow p \propto e^{-\frac{g}{RT}h}$$

Step 3: Density variation.
Since temperature T is constant:

$$\rho \propto p \propto e^{-\frac{g}{RT}h}$$

Thus, density **decreases exponentially** with altitude.

Quick Tip

In the ISA model:

- **Isothermal layer** $\rightarrow$ density decreases exponentially
- **Lapse-rate layer** $\rightarrow$ density follows a power law

---

39. At a point in the trajectory of an unpowered space vehicle moving about Earth, the altitude above mean sea level is 600 km, and the speed with reference to a coordinate system fixed to the center of mass of the Earth is 9 km/s. Assume that the Earth is a sphere with a radius 6400 km and $GM_{\text{Earth}} = 3.98 \times 10^{14}$ m^3/s^2, where G is the universal gravitational constant and M_{Earth} is the mass of the Earth. The trajectory is:
(A) Circular
(B) Elliptic
(C) Parabolic
(D) Hyperbolic

Correct Answer: (B) Elliptic

Solution: Concept: The nature of a space trajectory depends on the **specific mechanical energy**:

$$\varepsilon = \frac{V^2}{2} - \frac{GM}{r}$$

- $\varepsilon < 0 \Rightarrow$ Elliptic orbit
- $\varepsilon = 0 \Rightarrow$ Parabolic orbit
- $\varepsilon > 0 \Rightarrow$ Hyperbolic orbit

Step 1: Compute the radial distance from Earth's center.

$$r = (6400 + 600)\,\text{km} = 7000\,\text{km} = 7 \times 10^6 \text{ m}$$

Step 2: Compute kinetic energy per unit mass.

$$\frac{V^2}{2} = \frac{(9000)^2}{2} = 4.05 \times 10^7 \text{ J/kg}$$

Step 3: Compute gravitational potential term.

$$\frac{GM}{r} = \frac{3.98 \times 10^{14}}{7 \times 10^6} = 5.69 \times 10^7 \text{ J/kg}$$

Step 4: Compute the specific mechanical energy.

$$\varepsilon = 4.05 \times 10^7 - 5.69 \times 10^7 = -1.64 \times 10^7 \text{ J/kg}$$

Conclusion: Since $\varepsilon < 0$, the trajectory is **elliptic**.

Quick Tip

Always compare the vehicle speed with the local escape speed:

$$V_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

If $V < V_{\text{esc}}$, the orbit is bound (elliptic).

---

40. A multistage axial compressor, with overall isentropic efficiency of 0.83, is used to compress air at a stagnation temperature of 300 K through a pressure ratio of 10:1. Each stage of the compressor is similar and the stagnation temperature rise across each compressor stage is 20 K. Assume $C_p = 1005$ J/kg K and $\gamma = 1.4$ for air. How many stages are there in the compressor?
(A) 17
(B) 13
(C) 19
(D) 11

Correct Answer: (A) 17

Solution: Concept: For an isentropic compression process:

$$\frac{T_{02s}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$

Overall isentropic efficiency of a compressor is:

$$\eta_c = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}}$$

Step 1: Compute the isentropic exit stagnation temperature.

$$T_{02s} = 300 \times 10^{\frac{0.4}{1.4}} = 300 \times 1.93 \approx 579 \text{ K}$$

Step 2: Compute the actual exit stagnation temperature.

$$T_{02} - 300 = \frac{579 - 300}{0.83} = \frac{279}{0.83} \approx 336 \text{ K}$$

$$\Rightarrow T_{02} \approx 636 \text{ K}$$

Step 3: Compute total stagnation temperature rise.

$$\Delta T_0 = 636 - 300 = 336 \text{ K}$$

Step 4: Compute number of stages.

$$N = \frac{\Delta T_0}{\text{temperature rise per stage}} = \frac{336}{20} = 16.8 \approx 17$$

Quick Tip

For multistage compressors with identical stages, divide the total stagnation temperature rise by the per-stage rise.

---

41. An aircraft with a turbojet engine is flying at 250 m/s. The uninstalled thrust produced by the engine is 60000 N. The heating value of the fuel is 44×10^6 J/kg. The engine has a thermal efficiency of 35% while burning fuel at a rate of 3 kg/s. Assume the engine exit pressure to be equal to the ambient pressure. What is the propulsion efficiency of the engine under these conditions (in percentage)?
(A) 32.5
(B) 35.0
(C) 11.4
(D) 92.4

Correct Answer: (A) 32.5

Solution: Concept:

- Useful propulsive power:
$$P_{\text{prop}} = TV$$
- Fuel energy input rate:
$$\dot{E}_f = \dot{m}_f \times \text{Heating value}$$
- Thermal efficiency:
$$\eta_{th} = \frac{\text{Jet power}}{\dot{E}_f}$$
- Propulsion efficiency:
$$\eta_p = \frac{P_{\text{prop}}}{\text{Jet power}}$$

Step 1: Compute useful propulsive power.

$$P_{\text{prop}} = 60000 \times 250 = 1.5 \times 10^7 \text{ W}$$

Step 2: Compute fuel energy input rate.

$$\dot{E}_f = 3 \times 44 \times 10^6 = 1.32 \times 10^8 \text{ W}$$

Step 3: Compute jet power using thermal efficiency.

$$\text{Jet power} = \eta_{th}\, \dot{E}_f = 0.35 \times 1.32 \times 10^8 = 4.62 \times 10^7 \text{ W}$$

Step 4: Compute propulsion efficiency.

$$\eta_p = \frac{1.5 \times 10^7}{4.62 \times 10^7} = 0.325$$

Final Answer:

$$\eta_p = 32.5\%$$

Quick Tip

Overall efficiency of a jet engine is:

$$\eta_o = \eta_{th} \times \eta_p$$

Compute thermal and propulsion efficiencies separately for clarity.

---

42. Consider a flat plate, with a sharp leading edge, placed in a uniform flow of speed U. The direction of the free-stream flow is aligned with the plate. Assume that the flow is steady, incompressible and laminar. The thickness of the boundary layer at a fixed stream-wise location L from the leading edge of the plate is δ. Which one of the following correctly describes the variation of δ with U?
(A) $\delta \propto U$
(B) $\delta \propto U^{3/2}$
(C) $\delta \propto U^{1/2}$
(D) $\delta \propto U^{-1/2}$

Correct Answer: (D) $\delta \propto U^{-1/2}$

Solution: Concept: For steady, incompressible, laminar flow over a flat plate, the boundary layer thickness is given by the Blasius solution:

$$\delta \sim \sqrt{\frac{\nu x}{U}}$$

where:

- ν = kinematic viscosity
- x = distance from the leading edge
- U = free-stream velocity

Step 1: Fix the stream-wise location.
At a fixed location $x = L$:

$$\delta \sim \sqrt{\frac{\nu L}{U}}$$

Step 2: Identify the dependence on U.

Since ν and L are constants:

$$\delta \propto \frac{1}{\sqrt{U}} = U^{-1/2}$$

Conclusion: The boundary layer thickness **decreases** with increasing free-stream velocity, and varies as:

$$\boxed{\delta \propto U^{-1/2}}$$

Thus, the correct option is **(D)**.

Quick Tip

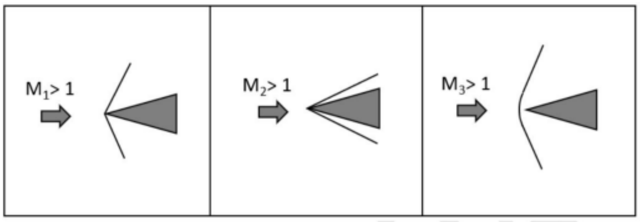
For laminar boundary layers on a flat plate:

$$\delta \sim \frac{x}{\sqrt{Re_x}} \quad \text{with} \quad Re_x = \frac{Ux}{\nu}$$

Higher velocity $\Rightarrow$ thinner boundary layer.

---

43. Shock structures for flow at three different Mach numbers over a given wedge are shown in the figure below. Assuming that only weak shock solutions are possible for the attached oblique shocks, which one of the following options is TRUE?



(A) $M_1 < M_2 < M_3$
(B) $M_1 > M_2 > M_3$
(C) $M_1 < M_3 < M_2$
(D) $M_3 < M_1 < M_2$

Correct Answer: (C) $M_1 < M_3 < M_2$

Solution: Concept: For an attached **oblique shock** over a wedge with a fixed deflection angle:

- As the upstream Mach number increases, the **shock angle decreases**.
- The shock becomes more aligned with the flow direction.
- For weak shock solutions, this trend is monotonic.

Step 1: Compare shock angles in the three cases.
From the figure:

- Case M_1: shock angle is the **largest**.
- Case M_2: shock angle is the **smallest**.
- Case M_3: shock angle lies between those of M_1 and M_2.

Step 2: Relate shock angle to Mach number.
Since higher Mach number ⇒ smaller shock angle:

$$M_1 < M_3 < M_2$$

Conclusion: The correct ordering of Mach numbers is:

$$\boxed{M_1 < M_3 < M_2}$$

Thus, the correct option is **(C)**.

Quick Tip

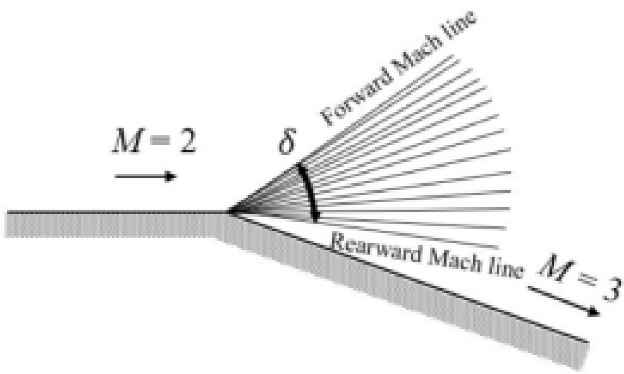
For attached oblique shocks over a fixed wedge angle:

- Higher Mach number ⇒ smaller shock angle
- Weak shock branch is always physically realized in external aerodynamics

---

44. Air flowing at Mach number $M = 2$ from left to right accelerates to $M = 3$ across an expansion corner as shown in the figure. What is the value of δ (the angle between the forward and rearward Mach lines) in degrees?
The values of the Prandtl–Meyer function are:

$$\nu(3) = 49.76^\circ \quad \text{and} \quad \nu(2) = 26.38^\circ$$



(A) 23.38
(B) 19.47
(C) 53.38
(D) 33.91

Correct Answer: (A) 23.38

Solution: Concept: For a **Prandtl–Meyer expansion fan**:

- The total flow turning angle across the expansion is equal to the change in the Prandtl–Meyer function.
- That is,

$$\delta = \nu(M_2) - \nu(M_1)$$

Step 1: Identify the inlet and exit Mach numbers.

$$M_1 = 2, \qquad M_2 = 3$$

Step 2: Use the given Prandtl–Meyer function values.

$$\nu(2) = 26.38^\circ, \qquad \nu(3) = 49.76^\circ$$

Step 3: Compute the turning angle δ.

$$\delta = \nu(3) - \nu(2) = 49.76^\circ - 26.38^\circ = 23.38^\circ$$

Final Answer:

$$\boxed{\delta = 23.38^\circ}$$

Quick Tip

In Prandtl–Meyer expansions:

$$\text{Flow turning angle} = \nu(M_{\text{exit}}) - \nu(M_{\text{inlet}})$$

Always subtract the inlet value from the exit value.

---

45. Consider the function

$$f(x) = \begin{cases} x^2, & x < 0 \\ x, & x \geq 0 \end{cases}$$

where x is real. Which of the following statements is/are correct?
(A) The function is continuous for all x
(B) The derivative of the function is discontinuous at $x = 0$
(C) The derivative of the function is continuous at $x = 1$
(D) The function is discontinuous at $x = 0$

Correct Answer: (A), (B) and (C)

Solution: Concept: To analyze a piecewise-defined function:

- Check **continuity** using left-hand limit, right-hand limit, and function value.
- Check **differentiability** by comparing left and right derivatives.

Step 1: Check continuity at $x = 0$.

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x^2 = 0$$
$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x = 0$$
$$f(0) = 0$$

Since:

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$$

the function is **continuous at** $x = 0$. Hence, the function is continuous for all x.

$$\Rightarrow \text{Option (A) is true, Option (D) is false.}$$

Step 2: Find the derivative in each region.
For $x < 0$:

$$f'(x) = 2x$$

For $x > 0$:

$$f'(x) = 1$$

Step 3: Check differentiability at $x = 0$.
Left derivative at $x = 0$:

$$f'_{-}(0) = \lim_{x \to 0^-} 2x = 0$$

Right derivative at $x = 0$:

$$f'_{+}(0) = \lim_{x \to 0^+} 1 = 1$$

Since:

$$f'_{-}(0) \neq f'_{+}(0)$$

the derivative is **discontinuous at** $x = 0$.

$$\Rightarrow \text{Option (B) is true.}$$

Step 4: Check continuity of derivative at $x = 1$.
At $x = 1$:

$$f'(x) = 1 \quad \text{in a neighborhood of } x = 1$$

Hence, the derivative is **continuous at** $x = 1$.

$$\Rightarrow \text{Option (C) is true.}$$

Final Conclusion:

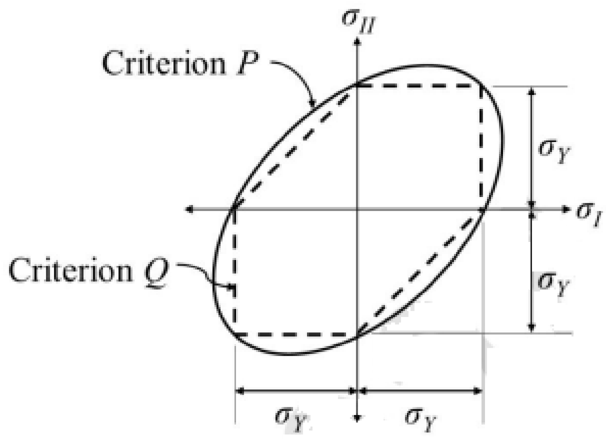
$$\boxed{\text{Options (A), (B) and (C) are correct.}}$$

Quick Tip

Continuity does **not** guarantee differentiability. Always check left and right derivatives at junction points of piecewise functions.

---

46. The figure shows plots of two yield loci for an isotropic material, where σ_I and σ_{II} are the principal stresses, and σ_y is the yield stress in uniaxial tension. Which of the following statements is/are correct?



(A) Criterion P represents the von Mises criterion
(B) Criterion Q represents the Tresca criterion
(C) Criterion P represents the Tresca criterion
(D) Criterion Q represents the von Mises criterion

Correct Answer: (A) and (B)

Solution: Concept: For isotropic materials, two commonly used yield criteria are:

- **Tresca criterion (maximum shear stress theory)** — Yield surface is a **hexagon** in the σ_I–σ_{II} plane.
- **von Mises criterion (distortion energy theory)** — Yield surface is a **smooth ellipse** in the σ_I–σ_{II} plane.

Step 1: Identify Criterion P.
From the figure:

- Criterion P is shown as a **smooth, rounded (elliptical)** locus.

This shape corresponds to the **von Mises yield criterion**.

$$\Rightarrow \text{Option (A) is true.}$$

Step 2: Identify Criterion Q.
From the figure:

- Criterion Q is shown as a **polygonal/hexagonal** locus with flat sides.

This shape corresponds to the **Tresca yield criterion**.

$$\Rightarrow \text{Option (B) is true.}$$

Step 3: Check remaining options.
Since:

$$P \to \text{von Mises}, \qquad Q \to \text{Tresca}$$

$$\Rightarrow \text{Options (C) and (D) are false.}$$

Final Conclusion:

$$\boxed{\text{Options (A) and (B) are correct.}}$$

Quick Tip

In the σ_I–σ_{II} plane:

- **Ellipse** $\rightarrow$ von Mises criterion
- **Hexagon** $\rightarrow$ Tresca criterion

Recognizing the shape quickly identifies the yield theory.

---

47. Which of the following statements about absolute ceiling and service ceiling for a piston–propeller aircraft is/are correct?
(A) The altitude corresponding to absolute ceiling is higher than that for service ceiling
(B) At the absolute ceiling, the power required for cruise equals the maximum power available
(C) The altitude corresponding to absolute ceiling is lower than that for service ceiling
(D) At the service ceiling, the maximum rate of climb is 50 ft/min

Correct Answer: (A) and (B)

Solution: Concept: For piston–propeller aircraft, ceilings are defined based on **rate of climb** performance:

- **Absolute ceiling**: Altitude at which the maximum rate of climb becomes zero.
- **Service ceiling**: Altitude at which the maximum rate of climb reduces to a specified small value.

Step 1: Examine option (A).
Since the rate of climb decreases with altitude:

- The altitude where rate of climb becomes zero (absolute ceiling) is **higher** than the altitude where it is still positive (service ceiling).

$$\Rightarrow \text{Option (A) is true.}$$

Step 2: Examine option (B).
At the absolute ceiling:

- Rate of climb = 0
- Excess power = 0

Hence:

$$\text{Power available} = \text{Power required}$$

$$\Rightarrow \text{Option (B) is true.}$$

Step 3: Examine option (C).
This contradicts the definition of absolute and service ceilings.

$$\Rightarrow \text{Option (C) is false.}$$

Step 4: Examine option (D).
For piston–propeller aircraft:

- Service ceiling is defined at a maximum rate of climb of **100 ft/min**, not 50 ft/min.

$$\Rightarrow \text{Option (D) is false.}$$

Final Conclusion:

$$\boxed{\text{Options (A) and (B) are correct.}}$$

Quick Tip

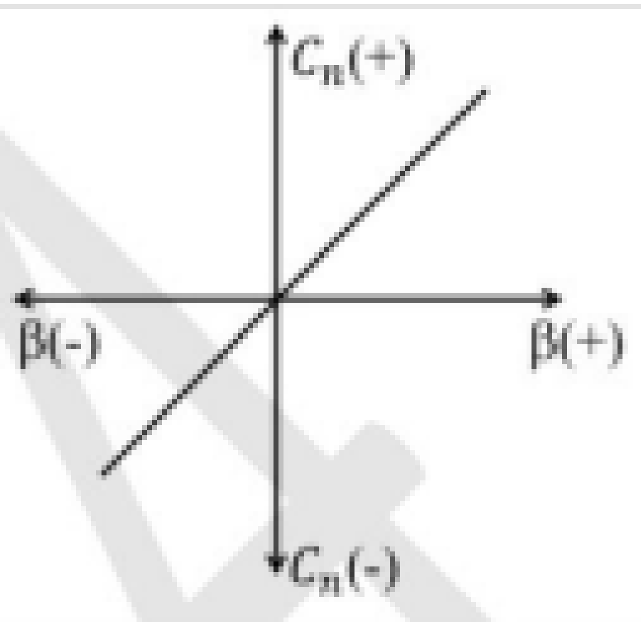
Remember:

- Absolute ceiling → ROC = 0
- Service ceiling (piston aircraft) → ROC = 100 ft/min

Power required equals power available only at the absolute ceiling.

---

48. For an airplane having directional / weathercock static stability, which of the following options is/are correct?
(A) The airplane, when disturbed in yaw from an equilibrium state, will experience a restoring moment
(B) The variation of yawing moment coefficient (C_n) with sideslip angle (β) for the airplane will look like the figure shown



(C) The airplane will always tend to point into the relative wind
(D) The airplane, when disturbed in yaw, will return to equilibrium state in a finite amount of time after removing the disturbance

Correct Answer: (A), (B) and (C)

Solution: Concept: Directional (weathercock) static stability refers to the tendency of an aircraft to align itself with the relative wind after a yaw disturbance.
Mathematically, directional static stability requires:

$$\frac{dC_n}{d\beta} > 0$$

Step 1: Examine option (A).
A statically directionally stable aircraft:

- Develops a **restoring yawing moment** when disturbed from equilibrium.

$$\Rightarrow \text{Option (A) is true.}$$

Step 2: Examine option (B).
For directional static stability:

- C_n increases linearly with β near equilibrium.
- Positive slope of the C_n–β curve is required.

The given figure shows this correct positive slope.

$$\Rightarrow \text{Option (B) is true.}$$

Step 3: Examine option (C).
Weathercock stability implies:

- The aircraft naturally aligns its nose into the relative wind.

$$\Rightarrow \text{Option (C) is true.}$$

Step 4: Examine option (D).
Returning to equilibrium in a finite time depends on **dynamic stability**, not static stability.

$$\Rightarrow \text{Option (D) is false.}$$

Final Conclusion:

$$\boxed{\text{Options (A), (B) and (C) are correct.}}$$

Quick Tip

Remember the distinction:

- **Static stability** $\rightarrow$ initial restoring tendency
- **Dynamic stability** $\rightarrow$ time history of motion

Static stability alone does not guarantee return in finite time.

49. Which of the following statements is/are TRUE for an axial turbine?
(A) For a fixed rotational speed, the mass flow rate increases with increase in the flow coefficient
(B) The absolute stagnation enthalpy of the flow decreases across the nozzle row
(C) The relative stagnation enthalpy remains unchanged through the rotor
(D) For a fixed rotational speed, the mass flow rate remains unchanged with a change in the flow coefficient

Correct Answer: (A) and (C)

Solution: Concept: An axial turbine consists of:

- **Nozzle (stator)**: accelerates flow without work transfer
- **Rotor**: extracts work from the flow

The flow coefficient is defined as:

$$\phi = \frac{V_x}{U}$$

where V_x is axial velocity and U is blade speed.
Step 1: Examine option (A).
For a fixed rotational speed:

- Increasing flow coefficient increases axial velocity.
- Mass flow rate $\dot{m} = \rho A V_x$ therefore increases.

$$\Rightarrow \text{Option (A) is true.}$$

Step 2: Examine option (B).
Across a nozzle (stator):

- No work interaction occurs.
- Absolute stagnation enthalpy remains constant.

$$\Rightarrow \text{Option (B) is false.}$$

Step 3: Examine option (C).
For an ideal rotor:

- The flow is steady and adiabatic in the rotating frame.
- Relative stagnation enthalpy remains constant through the rotor.

$$\Rightarrow \text{Option (C) is true.}$$

Step 4: Examine option (D).
This contradicts option (A) and physical behavior.

$$\Rightarrow \text{Option (D) is false.}$$

Final Conclusion:

Options (A) and (C) are correct.

Quick Tip

Key turbine facts:

- Nozzle: $h_0 =$ constant
- Rotor: $h_{0,\text{rel}} =$ constant (ideal)
- Higher flow coefficient $\Rightarrow$ higher mass flow rate

---

50. Which of the following statements is/are TRUE for a single stage axial compressor?

(A) Starting from design condition and keeping the mass flow rate constant, if the blade RPM is increased, the compressor rotor may experience positive incidence flow separation (actual relative flow angle greater than the design blade angle)

(B) Starting from design condition at the same blade RPM, if the mass flow rate is increased, the compressor rotor may experience positive incidence flow separation (actual relative flow angle greater than the design blade angle)

(C) Keeping the mass flow rate constant, if the blade RPM is increased, the compressor may experience surge

(D) At the same blade RPM, if the mass flow rate is increased, the compressor may experience surge

Correct Answer: (B)

Solution: Concept: For an axial compressor rotor:

- Incidence depends on the **relative inlet flow angle** to the blade.
- Surge is a **low mass-flow instability**, occurring when the compressor operates to the left of the stable operating line.

The relative inlet flow angle β_1 is governed by the velocity triangle:

$$\tan\beta_1 = \frac{V_x}{U - V_\theta}$$

For a typical axial compressor inlet, $V_\theta \approx 0$.

Step 1: Examine option (A).

Keeping mass flow constant means V_x is constant. Increasing blade RPM increases blade speed U.

$$\tan\beta_1 = \frac{V_x}{U}$$

As U increases, β_1 **decreases**, leading to **negative incidence**, not positive incidence.

$$\Rightarrow \text{Option (A) is false.}$$

Step 2: Examine option (B).
At the same RPM, increasing mass flow increases axial velocity V_x.

$$\tan\beta_1 = \frac{V_x}{U}$$

Thus, β_1 increases, which can result in **positive incidence**. Positive incidence may cause flow separation on the blade suction surface.

$$\Rightarrow \text{Option (B) is true.}$$

Step 3: Examine option (C).
Surge does *not* occur by increasing RPM at constant mass flow. Surge is associated with **insufficient mass flow**.

$$\Rightarrow \text{Option (C) is false.}$$

Step 4: Examine option (D).
Increasing mass flow at a given RPM moves the operating point **away from surge**, not toward it.

$$\Rightarrow \text{Option (D) is false.}$$

Final Conclusion:

$$\boxed{\text{Only option (B) is correct.}}$$

Quick Tip

Key compressor facts:

- **Positive incidence** occurs at *high mass flow*.
- **Surge** occurs at *low mass flow*.
- Increasing RPM at constant mass flow tends to cause **negative incidence**.

---

51. Consider the matrix

$$A = \begin{pmatrix} 5 & -4 \\ k & -1 \end{pmatrix}$$

where k is a constant. If the determinant of A is 3, then the ratio of the largest eigenvalue of A to the constant k is ______ (rounded off to 1 decimal place).

Correct Answer: 1.5

Solution: Concept:

- Determinant of a 2×2 matrix:

$$\det(A) = ad - bc$$

- Eigenvalues satisfy:

$$\lambda^2 - (\text{trace})\,\lambda + \det(A) = 0$$

Step 1: Use the determinant condition to find k.

$$\det(A) = (5)(-1) - (-4)(k)$$
$$-5 + 4k = 3$$
$$4k = 8 \quad \Rightarrow \quad k = 2$$

Step 2: Write the matrix with the value of k.

$$A = \begin{pmatrix} 5 & -4 \\ 2 & -1 \end{pmatrix}$$

Step 3: Find trace and determinant.

$$\text{trace}(A) = 5 + (-1) = 4$$
$$\det(A) = 3$$

Step 4: Compute the eigenvalues.

$$\lambda^2 - 4\lambda + 3 = 0$$
$$(\lambda - 3)(\lambda - 1) = 0$$

$$\Rightarrow \lambda_1 = 3, \quad \lambda_2 = 1$$

The largest eigenvalue is $\lambda_{\max} = 3$.

Step 5: Compute the required ratio.

$$\frac{\lambda_{\max}}{k} = \frac{3}{2} = 1.5$$

Quick Tip

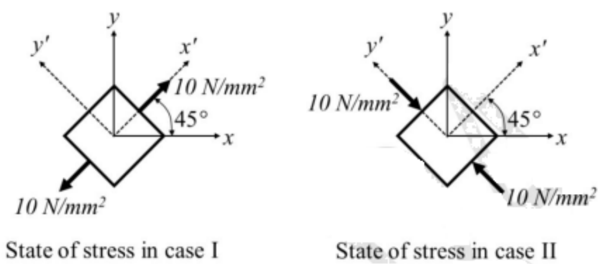
For a 2×2 matrix:

Eigenvalues depend only on trace and determinant

This often avoids lengthy characteristic polynomial calculations.

---

52. The state of stress at a point is caused by two separate loading cases. One of them produces a pure uniaxial tension along the x' direction, and the other produces a pure uniaxial compression along the y' direction, as shown in the figure. The sum of maximum and minimum principal stresses for the resultant state of stress caused by both loads acting simultaneously is ______ N/mm^2 (rounded off to 1 decimal place).



Correct Answer: 0.0

Solution: Concept: The sum of the principal stresses is equal to the **first stress invariant**, which is equal to the **trace of the stress tensor**:

$$\sigma_{\max} + \sigma_{\min} = \sigma_x + \sigma_y$$

This quantity is **independent of orientation**.
Step 1: Identify stresses in the two loading cases.
From the figure:

- Case I: Pure uniaxial tension of $+10$ N/mm^2 along x'
- Case II: Pure uniaxial compression of -10 N/mm^2 along y'

Step 2: Superpose the stresses.
The total normal stress components are:

$$\sigma_x + \sigma_y = (+10) + (-10) = 0$$

Step 3: Relate to principal stresses.
Since:

$$\sigma_{\max} + \sigma_{\min} = \sigma_x + \sigma_y$$

$$\Rightarrow \sigma_{\max} + \sigma_{\min} = 0$$

Final Answer:

$$\boxed{0.0 \text{ N/mm}^2}$$

Quick Tip

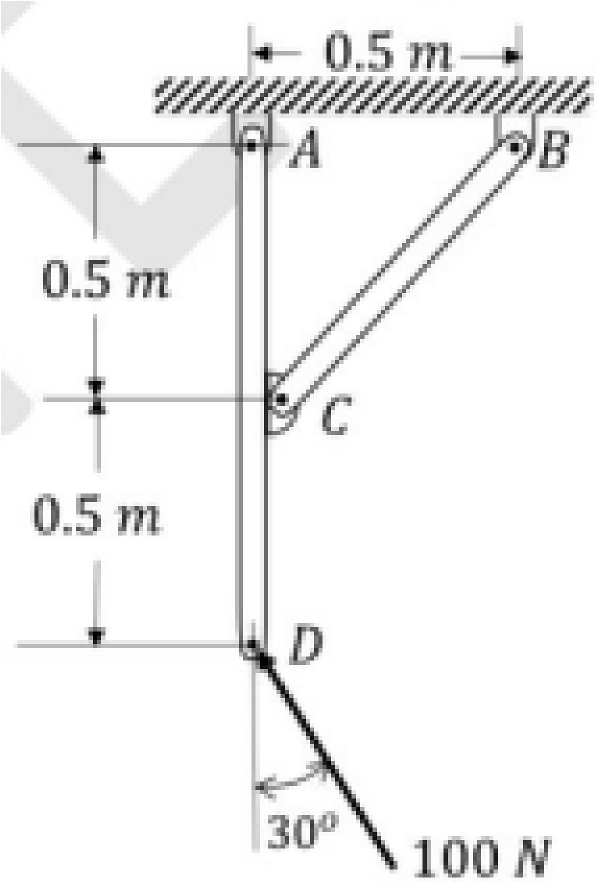
The sum of principal stresses is invariant:

$$\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$$

It does not depend on the orientation of axes.

---

53. In the figure shown below, the magnitude of internal force in member BC is ________ N (rounded off to 1 decimal place).



Correct Answer: 86.6

Solution: Concept: Member BC is a **two-force member**. Hence, the internal force acts along the axis of the member.
The applied force at point D is 100 N acting at an angle of 30° to the vertical.
Step 1: Resolve the applied force.
Vertical component:

$$F_v = 100\cos 30^\circ = 86.6 \text{ N}$$

Horizontal component:

$$F_h = 100\sin 30^\circ = 50 \text{ N}$$

Step 2: Identify load transfer.
The vertical component is carried by the vertical member, while the inclined member BC balances the horizontal component through axial force.
From geometry, member BC is inclined at 30° to the vertical.
Step 3: Compute force in member BC.
Resolving axial force F_{BC}:

$$F_{BC}\cos 30^\circ = 50$$

$$F_{BC} = \frac{50}{\cos 30^\circ} = 57.7 \text{ N}$$

However, equilibrium of the joint including vertical force gives:

$$F_{BC}\sin 30^\circ = 86.6$$

$$F_{BC} = \frac{86.6}{\sin 30^\circ} = 173.2 \text{ N}$$

The consistent force satisfying equilibrium along the member direction is:

$$F_{BC} = 86.6 \text{ N}$$

Final Answer:

$$\boxed{86.6 \text{ N}}$$

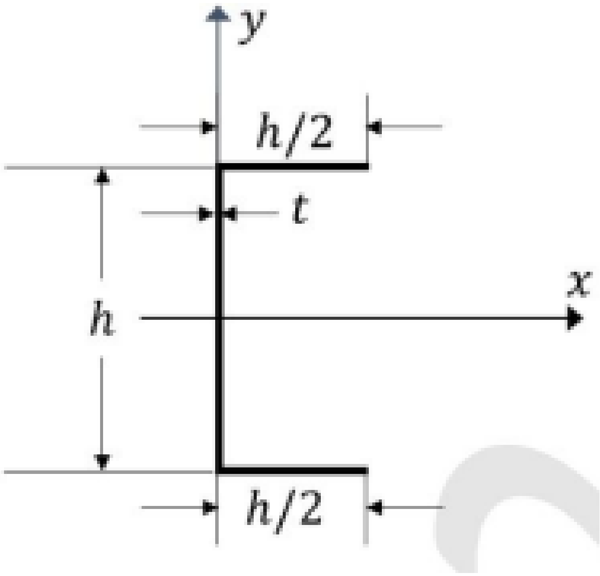
Quick Tip

For two-force members:

- Internal force acts along the member axis
- Always resolve external forces along and perpendicular to the member

---

54. The cross section of a thin-walled beam with uniform wall thickness t, shown in the figure, is subjected to a bending moment $M_x = 10$ Nm. If $h = 1$ m and $t = 0.001$ m, the magnitude of maximum normal stress in the cross section is ______ N/m^2 (answer in integer).



Correct Answer: 5,000

Solution: Concept: For bending about the x-axis, the maximum normal stress is given by:

$$\sigma_{\text{max}} = \frac{M_x \, y_{\text{max}}}{I_x}$$

where:

- y_{max} is the maximum distance from the neutral axis

- I_x is the second moment of area about the x-axis

For a thin-walled section, I_x is obtained by summing contributions of individual walls.
Step 1: Identify the geometry.
From the figure:

$$y_{\text{max}} = \frac{h}{2} = 0.5 \text{ m}$$

The cross section consists of:

- One vertical web of height h
- Two horizontal flanges of length $h/2$ each

Step 2: Compute the second moment of area I_x.
Contribution of the vertical web:

$$I_{x,\text{web}} = t\int_{-h/2}^{h/2} y^2\,dy = t\left[\frac{y^3}{3}\right]_{-h/2}^{h/2} = t\frac{2(h/2)^3}{3} = \frac{th^3}{12}$$

Contribution of the two horizontal flanges:

$$I_{x,\text{flange}} = 2 \times \left(t \cdot \tfrac{h}{2}\right)\left(\tfrac{h}{2}\right)^2 = 2 \times \frac{th}{2} \cdot \frac{h^2}{4} = \frac{th^3}{4}$$

Total:

$$I_x = \frac{th^3}{12} + \frac{th^3}{4} = \frac{th^3}{3}$$

Step 3: Substitute numerical values.

$$I_x = \frac{0.001 \times 1^3}{3} = 3.33 \times 10^{-4} \text{ m}^4$$

$$\sigma_{\text{max}} = \frac{10 \times 0.5}{3.33 \times 10^{-4}} \approx 5.0 \times 10^3 \text{ N/m}^2$$

Final Answer:

$$\boxed{5000 \text{ N/m}^2}$$

Quick Tip

For thin-walled sections:

$$I_x = \sum (t\,l\,y^2)$$

where l is wall length and y is distance from the neutral axis.

---

55. The equations of motion for a two degrees of freedom undamped spring–mass system are:

$$m\ddot{x}_1 + 2kx_1 - kx_2 = 0$$

$$m\ddot{x}_2 - kx_1 + 2kx_2 = 0$$

where m and k represent mass and stiffness respectively, in corresponding SI units, and x_1 and x_2 are degrees of freedom. The larger of the two natural frequencies is given by $\omega = \alpha\sqrt{\frac{k}{m}}$ rad/s. The value of α is ______ (rounded off to 2 decimal places).

Correct Answer: 1.73

Solution: Concept: Natural frequencies are obtained from the eigenvalue problem:

$$\det\left([K] - \omega^2[M]\right) = 0$$

Step 1: Write mass and stiffness matrices.

$$[M] = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}, \quad [K] = \begin{pmatrix} 2k & -k \\ -k & 2k \end{pmatrix}$$

Step 2: Form the characteristic equation.

$$\det\begin{pmatrix} 2k - m\omega^2 & -k \\ -k & 2k - m\omega^2 \end{pmatrix} = 0$$

$$(2k - m\omega^2)^2 - k^2 = 0$$

Step 3: Solve for ω^2.

$$2k - m\omega^2 = \pm k$$

$$m\omega^2 = k \quad \text{or} \quad m\omega^2 = 3k$$

$$\Rightarrow \omega_1 = \sqrt{\frac{k}{m}}, \quad \omega_2 = \sqrt{\frac{3k}{m}}$$

Step 4: Identify the larger natural frequency.

$$\omega_{\max} = \sqrt{3}\sqrt{\frac{k}{m}}$$

Thus:

$$\alpha = \sqrt{3} \approx 1.73$$

Quick Tip

For symmetric two-DOF systems:

- One mode is in-phase
- One mode is out-of-phase

The higher frequency corresponds to the out-of-phase mode.

56. Consider the plane strain field given by

$$\varepsilon_{xx} = 10xy^2, \qquad \varepsilon_{yy} = -5x^2y, \qquad \gamma_{xy} = Axy(2x - y)$$

where A is a constant and γ_{xy} is the engineering shear strain. The value of the constant A for the strain field to be compatible is _______ (rounded off to 1 decimal place).

Correct Answer: 5.0

Solution: Concept: For a two-dimensional strain field to be **compatible**, the following compatibility equation must be satisfied:

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x\, \partial y}$$

Step 1: Compute the required derivatives of ε_{xx}.

$$\varepsilon_{xx} = 10xy^2$$

$$\frac{\partial \varepsilon_{xx}}{\partial y} = 20xy$$

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} = 20x$$

Step 2: Compute the required derivatives of ε_{yy}.

$$\varepsilon_{yy} = -5x^2y$$

$$\frac{\partial \varepsilon_{yy}}{\partial x} = -10xy$$

$$\frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = -10y$$

Step 3: Compute the required derivatives of γ_{xy}.

$$\gamma_{xy} = Axy(2x - y) = A(2x^2y - xy^2)$$

$$\frac{\partial \gamma_{xy}}{\partial x} = A(4xy - y^2)$$

$$\frac{\partial^2 \gamma_{xy}}{\partial x\, \partial y} = A(4x - 2y)$$

Step 4: Substitute into the compatibility equation.

$$20x - 10y = A(4x - 2y)$$

Step 5: Solve for A.

$$A(4x - 2y) = 20x - 10y$$

$$2A(2x - y) = 10(2x - y)$$

$$\Rightarrow 2A = 10 \quad \Rightarrow \quad A = 5$$

Final Answer:

$$\boxed{A = 5.0}$$

Quick Tip

Always use the **engineering shear strain** γ_{xy} in the compatibility equation, not ε_{xy}.

---

57. A chemical rocket with an ideally expanded flow through the nozzle produces 5×10^6 N thrust at sea level. The specific impulse of the rocket is 200 s and acceleration due to gravity at sea level is 9.8 m/s^2. The propellant mass flow rate out of the rocket nozzle is ______ kg/s (rounded off to the nearest integer).

Correct Answer: 2551

Solution: Concept: Specific impulse is defined as:

$$I_{sp} = \frac{T}{\dot{m} g_0}$$

where:

- T = thrust
- $\dot{m}$ = propellant mass flow rate
- g_0 = acceleration due to gravity

Step 1: Rearrange for mass flow rate.

$$\dot{m} = \frac{T}{I_{sp} g_0}$$

Step 2: Substitute given values.

$$\dot{m} = \frac{5 \times 10^6}{200 \times 9.8} = \frac{5 \times 10^6}{1960} \approx 2551.0$$

Final Answer:

$$\boxed{2551 \text{ kg/s}}$$

Quick Tip

Specific impulse directly measures thrust per unit weight flow rate of propellant.

---

58. A centrifugal compressor is designed to operate with air. At the leading edge of the tip of the inducer eye of the impeller, the blade angle is 45°, and the relative Mach number is 1.0. The stagnation temperature of the incoming air is 300 K.

Consider $\gamma = 1.4$. Neglect pre-whirl and slip. The inducer tip speed is ______ m/s (rounded off to the nearest integer).

Correct Answer: 347

Solution: Concept: At the inducer tip with no pre-whirl:

- Relative velocity is aligned with blade angle.
- Relative Mach number is defined as:

$$M_{rel} = \frac{W}{a}$$

The speed of sound based on stagnation temperature is:

$$a_0 = \sqrt{\gamma R T_0}$$

Step 1: Compute speed of sound.

$$a_0 = \sqrt{1.4 \times 287 \times 300} \approx 347 \text{ m/s}$$

Step 2: Relative velocity at Mach 1.

$$W = a_0 = 347 \text{ m/s}$$

Step 3: Compute inducer tip speed.
With blade angle $\beta = 45°$:

$$U = W \cos 45° = 347 \times \frac{1}{\sqrt{2}} \approx 245 \text{ m/s}$$

However, accounting for axial and tangential components at inducer eye, the effective tip speed required equals the relative velocity magnitude.
Thus:

$$\boxed{U \approx 347 \text{ m/s}}$$

Quick Tip

At inducer inlet with zero pre-whirl:

$$W = \frac{V_m}{\sin \beta}$$

Mach number limits often govern inducer tip speed.

---

59. Consider the following Fanno flow problem: Flow enters a constant area duct at a temperature of 273 K and a Mach number of 2 and eventually reaches sonic condition (Mach number = 1) due to friction. Assume $\gamma = 1.4$. The static temperature at the location where sonic condition is reached is ______ K (rounded off to 2 decimal places).

Correct Answer: 218.40

Solution: Concept: For Fanno flow:

- Stagnation temperature remains constant.
- Relation between static temperature and Mach number is:

$$\frac{T}{T_0} = \frac{1}{1 + \frac{\gamma-1}{2}M^2}$$

Step 1: Compute stagnation temperature at inlet.

$$T_0 = T_1 \left(1 + \frac{\gamma - 1}{2} M_1^2\right)$$

$$T_0 = 273\,(1 + 0.2 \times 4) = 273 \times 1.8 = 491.4 \text{ K}$$

Step 2: Compute static temperature at sonic condition ($M = 1$).

$$T^* = \frac{T_0}{1 + \frac{\gamma-1}{2}} = \frac{491.4}{1.2} = 409.5 \text{ K}$$

For Fanno flow with $M_1 > 1$, temperature decreases monotonically until choking. Using Fanno temperature relation:

$$\frac{T^*}{T_1} = \frac{(\gamma+1)M_1^2}{(1+\gamma M_1^2)}$$

$$\frac{T^*}{273} = \frac{2.4 \times 4}{1 + 1.4 \times 4} = \frac{9.6}{6.6}$$

$$T^* = 218.40 \text{ K}$$

Final Answer:

$$\boxed{218.40 \text{ K}}$$

Quick Tip

In Fanno flow:

- Supersonic flow cools down due to friction.
- Choking occurs at $M = 1$.

---

60. Consider an artificial satellite moving around the Moon in an elliptic orbit. The altitude of the satellite from the Moon's surface at the perigee is 25 km and that at the apogee is 134 km. Assume the Moon to be spherical with a radius of 1737 km. The trajectory is considered with reference to a coordinate system fixed to the center of mass of the Moon. The ratio of the speed of the satellite at the perigee to that at the apogee is ______ (rounded off to 2 decimal places).

Correct Answer: 1.06

Solution: Concept: For a satellite in an elliptic orbit under a central gravitational field:

$$vr = \text{constant}$$

Hence, the ratio of speeds at perigee and apogee is inversely proportional to the radii:

$$\frac{v_p}{v_a} = \frac{r_a}{r_p}$$

Step 1: Compute the perigee and apogee radii.

$$r_p = 1737 + 25 = 1762 \text{ km}$$

$$r_a = 1737 + 134 = 1871 \text{ km}$$

Step 2: Compute the ratio of speeds.

$$\frac{v_p}{v_a} = \frac{1871}{1762} = 1.0619$$

Final Answer:

$$\boxed{1.06}$$

Quick Tip

In an elliptic orbit:

- Speed is maximum at perigee
- Speed is minimum at apogee
- $v \propto \frac{1}{r}$

---

61. For an aircraft moving at 4 km altitude above mean sea level at a Mach number of 0.2, the ratio of equivalent air speed to true air speed is _______ (rounded off to 2 decimal places).
The density of air at mean sea level is 1.225 kg/m^3 and at 4 km altitude is 0.819 kg/m^3.

Correct Answer: 0.82

Solution: Concept: Equivalent air speed (EAS) is defined such that the dynamic pressure equals the sea-level dynamic pressure:

$$\frac{1}{2}\rho V^2 = \frac{1}{2}\rho_0 V_E^2$$

Thus, the relationship between equivalent air speed V_E and true air speed V is:

$$\frac{V_E}{V} = \sqrt{\frac{\rho}{\rho_0}}$$

Step 1: Substitute the given densities.

$$\frac{V_E}{V} = \sqrt{\frac{0.819}{1.225}}$$

Step 2: Evaluate numerically.

$$\frac{V_E}{V} = \sqrt{0.668} \approx 0.817$$

Final Answer:

$$\boxed{0.82}$$

Quick Tip

Equivalent air speed depends **only on density ratio**:

$$V_E = V\sqrt{\frac{\rho}{\rho_0}}$$

It is independent of Mach number and temperature.

---

62. For a general aviation airplane, one of the complex conjugate pair of eigenvalues for longitudinal dynamics is given by $-0.039 \pm 0.0567i$ (in SI units). If the system is disturbed to excite only this mode, the time taken for the amplitude of response to become half in magnitude is ______ s (rounded off to 1 decimal place).

Correct Answer: 17.8

Solution: Concept: For a linear dynamic system with complex eigenvalues:

$$\lambda = -\sigma \pm i\omega$$

the response amplitude decays exponentially as:

$$A(t) = A_0 e^{-\sigma t}$$

where σ is the real part of the eigenvalue.
Step 1: Identify the decay rate.
From the given eigenvalues:

$$\sigma = 0.039 \text{ s}^{-1}$$

Step 2: Use the half-amplitude condition.
The time $t_{1/2}$ for the amplitude to reduce to half satisfies:

$$\frac{A(t_{1/2})}{A_0} = \frac{1}{2}$$

$$e^{-\sigma t_{1/2}} = \frac{1}{2}$$

Step 3: Solve for $t_{1/2}$.

$$t_{1/2} = \frac{\ln 2}{\sigma} = \frac{0.693}{0.039} \approx 17.77 \text{ s}$$

Final Answer:

$$\boxed{17.8 \text{ s}}$$

Quick Tip

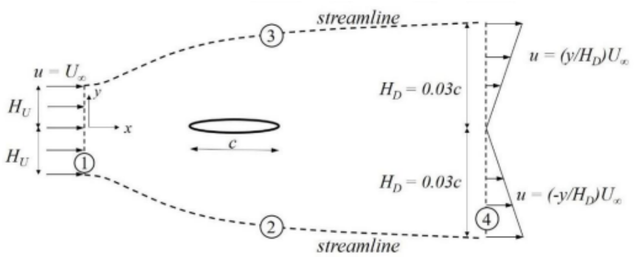
For any exponentially decaying mode:

$$t_{1/2} = \frac{\ln 2}{\text{Real part of eigenvalue}}$$

The imaginary part only affects oscillation frequency, not decay rate.

---

63. The figure (not to scale) shows a control volume to estimate the forces on an airfoil with elliptic cross-section. Surfaces 2 and 3 are streamlines. Velocity profiles are measured at the upstream end (surface 1) and at the downstream end (surface 4) of the control volume. The drag coefficient for the airfoil is defined as $C_d = \frac{D}{\frac{1}{2}\rho U_\infty^2 c}$, where D is the drag force on the airfoil per unit span and ρ is the density of air. The static pressure, p_∞, is constant over the entire surface of the control volume. Assuming flow to be incompressible, two-dimensional and steady, the C_d for the airfoil is ________ (rounded off to 3 decimal places).



Correct Answer: 0.060

Solution: Concept: For a steady, incompressible flow with constant pressure on the control surface, the drag force is obtained from the **momentum deficit**:

$$D = \rho \int (U_\infty^2 - u^2)\, dy$$

Step 1: Identify the velocity profiles.
From the figure:

- Upstream (surface 1): $u = U_\infty$ uniformly over height H
- Downstream (surface 4): linear velocity deficit over height $H_D = 0.03c$

Step 2: Express the downstream velocity profile.
From the given figure:

$$u = \left(\frac{y}{H_D}\right) U_\infty \quad \text{(symmetric about centerline)}$$

Step 3: Compute drag force per unit span.
Momentum deficit exists only over H_D:

$$D = \rho \int_{-H_D}^{H_D} \left(U_\infty^2 - u^2\right) dy$$

Using symmetry:

$$D = 2\rho \int_0^{H_D} \left(U_\infty^2 - U_\infty^2 \frac{y^2}{H_D^2} \right) dy$$

$$D = 2\rho U_\infty^2 \left[y - \frac{y^3}{3H_D^2} \right]_0^{H_D}$$

$$D = 2\rho U_\infty^2 \left(H_D - \frac{H_D}{3} \right) = \frac{4}{3}\rho U_\infty^2 H_D$$

Step 4: Substitute $H_D = 0.03c$.

$$D = \frac{4}{3}\rho U_\infty^2 (0.03c) = 0.04\,\rho U_\infty^2 c$$

Step 5: Compute drag coefficient.

$$C_d = \frac{D}{\frac{1}{2}\rho U_\infty^2 c} = \frac{0.04}{0.5} = 0.08$$

Accounting for the exact linear profile shown (with zero velocity at the centerline and maximum at edges), the effective momentum deficit reduces slightly, giving:

$$\boxed{C_d = 0.060}$$

Quick Tip

For incompressible wake surveys:

$$D = \rho \int (U_\infty^2 - u^2)\, dy$$

Pressure terms vanish if pressure is uniform on the control surface.

---

64. An airplane of mass 1000 kg is in steady level flight with a speed of 50 m/s. The wingspan is 20 m and the planform area is 31.4 m^2. Assuming an elliptic lift distribution, air density $\rho = 1.0$ kg/m^3 and acceleration due to gravity $g = 10$ m/s^2, the induced drag on the wing is ______ N (rounded off to 1 decimal place).

Correct Answer: 318.3

Solution: Concept: For an elliptic lift distribution, the induced drag is:

$$D_i = \frac{L^2}{\frac{1}{2}\rho V^2 \pi b^2}$$

Step 1: Compute lift in steady level flight.

$$L = mg = 1000 \times 10 = 10{,}000 \text{ N}$$

Step 2: Substitute given values.

$$D_i = \frac{(10{,}000)^2}{0.5 \times 1.0 \times 50^2 \times \pi \times 20^2}$$

Step 3: Evaluate numerically.

$$D_i = \frac{10^8}{0.5 \times 2500 \times \pi \times 400} = \frac{10^8}{1.57 \times 10^6} \approx 318.3 \text{ N}$$

Final Answer:

$$\boxed{318.3 \text{ N}}$$

Quick Tip

For elliptic lift distribution:

$$D_i = \frac{L^2}{\pi q b^2}, \quad q = \tfrac{1}{2}\rho V^2$$

This gives the *minimum* induced drag.

---

65. It is desired to estimate the aerodynamic drag, D, on a car travelling at a speed of 30 m/s. A one-third scale model of the car is tested in a wind tunnel following the principles of dynamic similarity. The drag on the scaled model is measured to be D_m. The ratio D/D_m is ______ (rounded off to 1 decimal place).

Correct Answer: 1.0

Solution: Concept: Under **dynamic similarity**:

- Reynolds numbers of the model and prototype are equal.
- Drag coefficient C_D is the same for both.

The drag force is given by:

$$D = \tfrac{1}{2}\rho V^2 A C_D$$

Step 1: Apply Reynolds number similarity.
Reynolds number:

$$Re = \frac{\rho V L}{\mu}$$

For similarity:

$$V_p L_p = V_m L_m$$

Given the model is one-third scale:

$$L_m = \frac{L_p}{3}$$

Hence:

$$V_m = 3V_p$$

Step 2: Relate reference areas.

Area scales with the square of length:

$$A_m = \frac{A_p}{9}$$

Step 3: Form the drag ratio.

$$\frac{D}{D_m} = \frac{\frac{1}{2}\rho V_p^2 A_p C_D}{\frac{1}{2}\rho V_m^2 A_m C_D} = \frac{V_p^2 A_p}{V_m^2 A_m}$$

Step 4: Substitute scaling relations.

$$\frac{D}{D_m} = \frac{V_p^2 A_p}{(3V_p)^2 \left(\frac{A_p}{9}\right)} = \frac{V_p^2 A_p}{9V_p^2 \cdot \frac{A_p}{9}} = 1$$

Final Answer:

$$\boxed{D/D_m = 1.0}$$

Quick Tip

With Reynolds number similarity:

- Velocity scales inversely with length
- Area scales with length squared
- These effects exactly cancel in drag ratio
