

GATE 2026 ST Question Paper with Solutions

Time Allowed :3 Hour	Maximum Marks :100	Total Questions :65
----------------------	--------------------	---------------------

General Instructions

Please read the following instructions carefully:

- This question paper is divided into three sections:
 - General Aptitude (GA):** 10 questions (5 questions \times 1 mark + 5 questions \times 2 marks) for a total of 15 marks.
 - Environmental Science and Engineering + Engineering Mathematics:**
 - Part A (Mandatory):** 36 questions (1 questions \times 1 mark + 19 questions \times 2 marks) for a total of 55 marks.
 - Part B (Section 1):** Candidates can choose either Part B1 (Surveying and Mapping) or Part B2 (Section 2). Each part contains 16 questions (8 questions \times 1 mark + 11 questions \times 2 marks) for a total of 30 marks.
- The total number of questions is **65**, carrying a maximum of **100 marks**.
- The duration of the exam is **3 hours**.
- Marking scheme:
 - For 1-mark MCQs, $\frac{1}{3}$ mark will be deducted for every incorrect response.
 - For 2-mark MCQs, $\frac{2}{3}$ mark will be deducted for every incorrect response.
 - No negative marking for numerical answer type (NAT) questions.
 - No marks will be awarded for unanswered questions.
- Ensure you attempt questions only from the optional section (Part B1 or Part B2) you have selected.
- Follow the instructions provided during the exam for submitting your answers.

1. Let X be a random variable having discrete uniform distribution on $\{1, 3, 5, 7, \dots, 99\}$. Then $E(X | X \text{ is not a multiple of } 15)$ equals

- (A) $\frac{2365}{47}$
(B) $\frac{2365}{50}$
(C) 50
(D) 47

Correct Answer: (A) $\frac{2365}{47}$

Solution:

Step 1: Understanding the distribution of X .

The random variable X has a discrete uniform distribution on the set $\{1, 3, 5, 7, \dots, 99\}$, which contains all odd numbers from 1 to 99. There are 50 elements in this set because the odd numbers between 1 and 99 are in the form $2n + 1$, where n ranges from 0 to 49. Hence, the total number of odd numbers between 1 and 99 is 50.

Step 2: Calculating the sum of all possible values of X .

The sum of the first 50 odd numbers is given by:

$$S = 1 + 3 + 5 + 7 + \dots + 99$$

The sum of the first n odd numbers is known to be n^2 . Therefore:

$$S = 50^2 = 2500$$

Step 3: Identifying multiples of 15 in the set.

Next, we find the multiples of 15 in the set $\{1, 3, 5, 7, \dots, 99\}$. The odd multiples of 15 are:

$$15, 45, 75$$

Thus, there are 3 multiples of 15 in the set, and their sum is:

$$15 + 45 + 75 = 135$$

Step 4: Subtracting the sum of multiples of 15.

The sum of all odd numbers excluding the multiples of 15 is:

$$\text{Sum excluding multiples of 15} = 2500 - 135 = 2365$$

Step 5: Finding the expected value of X given that X is not a multiple of 15.

The number of odd numbers that are not multiples of 15 is:

$$50 - 3 = 47$$

Therefore, the expected value of X given that it is not a multiple of 15 is the average of the remaining values, which is:

$$E(X \mid X \text{ is not a multiple of } 15) = \frac{2365}{47}$$

Quick Tip

For a uniform distribution, the expected value is the sum of the values divided by the number of values.

2. In a testing of hypothesis problem, which one of the following statements is true?

(A) The probability of the Type-I error cannot be higher than the probability of the Type-II error

- (B) Type-II error occurs if the test accepts the null hypothesis when the null hypothesis is actually false
- (C) Type-I error occurs if the test rejects the null hypothesis when the null hypothesis is actually false
- (D) The sum of the probability of the Type-I error and the probability of the Type-II error should be 1

Correct Answer: (B) Type-II error occurs if the test accepts the null hypothesis when the null hypothesis is actually false

Solution:

Step 1: Understanding Type-I and Type-II errors.

In hypothesis testing, a **Type-I error** occurs when the null hypothesis is rejected when it is actually true. In contrast, a **Type-II error** occurs when the null hypothesis is accepted when it is actually false. Therefore, the correct statement will describe the correct definition of these errors.

Step 2: Analyzing the options.

(A) The probability of the Type-I error cannot be higher than the probability of the Type-II error: This is not true. The probabilities of Type-I and Type-II errors are independent of each other, and neither is bound to be greater than the other. The likelihood of each error depends on the test's significance level (α) and the power of the test.

(B) Type-II error occurs if the test accepts the null hypothesis when the null hypothesis is actually false: Correct — this is the definition of a Type-II error. A Type-II error happens when the null hypothesis is incorrectly accepted when it should be rejected because it is actually false.

(C) Type-I error occurs if the test rejects the null hypothesis when the null hypothesis is actually false: This is incorrect. A Type-I error occurs when the null hypothesis is rejected when it is actually true, not false.

(D) The sum of the probability of the Type-I error and the probability of the Type-II error should be 1: This is incorrect. The sum of the probabilities of Type-I and Type-II errors is not necessarily 1. They are independent of each other and depend on various factors such as the significance level (α) and the power of the test.

Step 3: Conclusion.

The correct statement is **(B)**, which accurately describes the Type-II error.

Quick Tip

A Type-I error is rejecting a true null hypothesis, while a Type-II error is accepting a false null hypothesis.

3. Let $\{W(t)\}_{t \geq 0}$ be a standard Brownian motion. Which one of the following statements is NOT true?

- (A) $E[W(7)] = 0$
- (B) $E[W(5)W(9)] = 7$
- (C) $2W(1)$ is normally distributed with mean 0 and variance 4
- (D) $E[W(5) | W(3) = 3] = 3$

Correct Answer: (B) $E[W(5)W(9)] = 7$

Solution:

Step 1: Understanding Brownian motion.

A standard Brownian motion $\{W(t)\}_{t \geq 0}$ has the following properties: - $E[W(t)] = 0$, for all $t \geq 0$ (the expected value of $W(t)$ is 0). - $\text{Var}(W(t)) = t$, for all $t \geq 0$ (the variance of $W(t)$ is t). - For $t_1 \neq t_2$, $\text{Cov}(W(t_1), W(t_2)) = \min(t_1, t_2)$ (the covariance between two Brownian motions is the minimum of the two times).

Step 2: Analyzing the options.

(A) $E[W(7)] = 0$: This is true. The expected value of a Brownian motion at any time t is 0, so $E[W(7)] = 0$.

(B) $E[W(5)W(9)] = 7$: This statement is NOT true. The correct formula for the expectation of the product of two Brownian motions $W(t_1)$ and $W(t_2)$ is:

$$E[W(t_1)W(t_2)] = \min(t_1, t_2)$$

For $t_1 = 5$ and $t_2 = 9$, we get:

$$E[W(5)W(9)] = \min(5, 9) = 5$$

So, the correct value should be 5, not 7. Therefore, option (B) is false.

(C) $2W(1)$ is normally distributed with mean 0 and variance 4: This is true. Since $W(1)$ is a standard Brownian motion, it is normally distributed with mean 0 and variance 1. Therefore, $2W(1)$ will be normally distributed with mean $2 \times 0 = 0$ and variance $2^2 \times 1 = 4$.

(D) $E[W(5) | W(3) = 3] = 3$: This is true. Given that $W(3) = 3$, the conditional expectation $E[W(5) | W(3) = 3]$ is equal to $W(3)$ because for Brownian motion, the conditional expectation of $W(t_2)$ given $W(t_1)$ (with $t_2 > t_1$) is simply $W(t_1)$. Thus:

$$E[W(5) | W(3) = 3] = 3$$

Step 3: Conclusion.

The false statement is **(B)**, where the expectation of $W(5)W(9)$ is incorrectly stated as 7. The correct value is 5.

Quick Tip

For a standard Brownian motion, $E[W(t)] = 0$ and $E[W(t_1)W(t_2)] = \min(t_1, t_2)$.

4. Two fair dice, one having red and another having blue color, are tossed independently once. Let A be the event that the die having red colour will show 5 or 6. Let B be the event that the sum of the outcomes will be 7 and let C be the event that the sum of the outcomes will be 8. Then which one of the following statements is true?

- (A) A and B are independent as well as A and C are independent
- (B) A and B are independent, but A and C are not independent
- (C) A and C are independent, but A and B are not independent
- (D) Neither A and B are independent, nor A and C are independent

Correct Answer: (B) A and B are independent, but A and C are not independent

Solution:

Step 1: Understand the events.

- Event A represents the outcome where the red die shows 5 or 6. Hence, the probability of A is:

$$P(A) = P(\text{Red die shows 5 or 6}) = \frac{2}{6} = \frac{1}{3}$$

- Event B represents the event that the sum of the outcomes of the two dice is 7. The possible pairs for a sum of 7 are:

$$(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)$$

There are 6 outcomes that result in a sum of 7 out of a total of $6 \times 6 = 36$ possible outcomes. Therefore:

$$P(B) = \frac{6}{36} = \frac{1}{6}$$

- Event C represents the event that the sum of the outcomes of the two dice is 8. The possible pairs for a sum of 8 are:

$$(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)$$

There are 5 outcomes that result in a sum of 8 out of 36 total possible outcomes. Therefore:

$$P(C) = \frac{5}{36}$$

Step 2: Check if events A and B are independent.

To check if two events are independent, we verify if the following holds:

$$P(A \cap B) = P(A) \cdot P(B)$$

- $A \cap B$ is the event where the red die shows 5 or 6 and the sum of the dice is 7. The pairs that satisfy both conditions are:

$$(5, 2), (6, 1)$$

Thus, there are 2 outcomes that satisfy both A and B , so:

$$P(A \cap B) = \frac{2}{36} = \frac{1}{18}$$

Now, check if $P(A) \cdot P(B)$ equals $P(A \cap B)$:

$$P(A) \cdot P(B) = \frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$$

Since $P(A \cap B) = P(A) \cdot P(B)$, events A and B are independent.

Step 3: Check if events A and C are independent.

Now, check if $P(A \cap C) = P(A) \cdot P(C)$. - $A \cap C$ is the event where the red die shows 5 or 6 and the sum of the dice is 8. The pairs that satisfy both conditions are:

$$(5, 3), (6, 2)$$

Thus, there are 2 outcomes that satisfy both A and C , so:

$$P(A \cap C) = \frac{2}{36} = \frac{1}{18}$$

Now, check if $P(A) \cdot P(C)$ equals $P(A \cap C)$:

$$P(A) \cdot P(C) = \frac{1}{3} \times \frac{5}{36} = \frac{5}{108}$$

Since $P(A \cap C) = \frac{1}{18} \neq \frac{5}{108}$, events A and C are not independent.

Step 4: Conclusion.

The correct statement is **(B)**: A and B are independent, but A and C are not independent.

Quick Tip

Two events are independent if and only if $P(A \cap B) = P(A) \cdot P(B)$.

5. Let X be a random variable taking only two values, 1 and 2. Let $M_X(t)$ be the moment generating function of X . If the expectation of X is $\frac{10}{7}$, then the fourth derivative of $M_X(t)$ evaluated at 0 equals

- (A) $\frac{52}{7}$
- (B) $\frac{67}{7}$
- (C) $\frac{48}{7}$
- (D) $\frac{60}{7}$

Correct Answer: (C) $\frac{48}{7}$

Solution:

Step 1: Understanding the random variable X .

The random variable X takes only two values, 1 and 2. Let the probability mass function of X be:

$$P(X = 1) = p \quad \text{and} \quad P(X = 2) = 1 - p$$

The expectation of X , $E[X]$, is given as $\frac{10}{7}$. Using the formula for expectation:

$$E[X] = 1 \cdot p + 2 \cdot (1 - p) = p + 2(1 - p) = 2 - p$$

Given $E[X] = \frac{10}{7}$, we have the equation:

$$2 - p = \frac{10}{7}$$

Solving for p :

$$p = 2 - \frac{10}{7} = \frac{14}{7} - \frac{10}{7} = \frac{4}{7}$$

So, $p = \frac{4}{7}$ and $P(X = 1) = \frac{4}{7}$, and $P(X = 2) = \frac{3}{7}$.

Step 2: Moment generating function of X .

The moment generating function $M_X(t)$ of X is defined as:

$$M_X(t) = E[e^{tX}] = e^{t \cdot 1} \cdot P(X = 1) + e^{t \cdot 2} \cdot P(X = 2)$$

Substituting the values for $P(X = 1)$ and $P(X = 2)$:

$$M_X(t) = e^t \cdot \frac{4}{7} + e^{2t} \cdot \frac{3}{7}$$

Step 3: Finding the fourth derivative of $M_X(t)$.

To find the fourth derivative of $M_X(t)$ at $t = 0$, we differentiate $M_X(t)$ four times:

$$M_X'(t) = \frac{4}{7}e^t + \frac{6}{7}e^{2t}$$

$$M_X''(t) = \frac{4}{7}e^t + \frac{12}{7}e^{2t}$$

$$M_X^{(3)}(t) = \frac{4}{7}e^t + \frac{24}{7}e^{2t}$$

$$M_X^{(4)}(t) = \frac{4}{7}e^t + \frac{48}{7}e^{2t}$$

Now, evaluate $M_X^{(4)}(t)$ at $t = 0$:

$$M_X^{(4)}(0) = \frac{4}{7}e^0 + \frac{48}{7}e^0 = \frac{4}{7} + \frac{48}{7} = \frac{52}{7}$$

Step 4: Conclusion.

The fourth derivative of $M_X(t)$ evaluated at 0 is $\frac{52}{7}$, hence the correct answer is \boxed{C} .

Quick Tip

For a discrete random variable, the moment generating function is the weighted sum of e^{tx} for all possible values of x , where the weights are the probabilities of the corresponding values.

6. Let A be a 3×3 real matrix and let I_3 be the 3×3 identity matrix. Which one of the following statements is NOT true?

- (A) If the row-reduced echelon form of A is I_3 , then zero is not an eigenvalue of A
- (B) If zero is not an eigenvalue of A , then the row-reduced echelon form of A is I_3
- (C) If A has three distinct eigenvalues, then the row-reduced echelon form of A is I_3
- (D) If the system of equations $Ax = b$ has a solution for every 3×1 real column vector b , then the row-reduced echelon form of A is I_3

Correct Answer: (B) If zero is not an eigenvalue of A , then the row-reduced echelon form of A is I_3

Solution:

Step 1: Understanding the problem.

We are given a 3×3 matrix A and the identity matrix I_3 . We need to analyze the truth of four statements related to the row-reduced echelon form (RREF) of A , eigenvalues of A , and solutions to the system $Ax = b$.

Step 2: Analyzing the statements.

(A) If the row-reduced echelon form of A is I_3 , then zero is not an eigenvalue of A : This statement is true. If the row-reduced echelon form (RREF) of A is the identity matrix I_3 , it means that A is invertible and has full rank. An invertible matrix cannot have zero as an eigenvalue because having a zero eigenvalue would imply that A is singular and not invertible. Thus, zero cannot be an eigenvalue.

(B) If zero is not an eigenvalue of A , then the row-reduced echelon form of A is I_3 : This statement is NOT true. While it is true that if the row-reduced echelon form of A is I_3 , then A is invertible and thus does not have zero as an eigenvalue, the converse is not true. A matrix may have no zero eigenvalue (i.e., it is invertible) but still not have the row-reduced echelon form as I_3 . For example, A could be a matrix that is not in row-reduced form, but it still has no zero eigenvalue.

(C) If A has three distinct eigenvalues, then the row-reduced echelon form of A is I_3 : This statement is true. If A has three distinct eigenvalues, it implies that A is diagonalizable and has full rank. Hence, its row-reduced echelon form must be I_3 , the identity matrix, because it is invertible and has no zero eigenvalue.

(D) If the system of equations $Ax = b$ has a solution for every 3×1 real column vector b , then the row-reduced echelon form of A is I_3 : This statement is true. If the system $Ax = b$ has a solution for every possible b , it means that A is invertible. In this case, the row-reduced echelon form of A must be I_3 , because an invertible matrix has full rank and its row-reduced form is the identity matrix.

Step 3: Conclusion.

The false statement is (B), as having no zero eigenvalue does not necessarily imply that the row-reduced echelon form of A is I_3 .

Quick Tip

If a matrix is invertible (no zero eigenvalues), its row-reduced echelon form is I_3 . However, the converse is not true; a matrix can be invertible without having its RREF as I_3 .

7. A cube is to be cut into 8 pieces of equal size and shape. Here, each cut should be straight and it should not stop till it reaches the other end of the cube. The minimum number of such cuts required is

- (A) 3
- (B) 4
- (C) 7
- (D) 8

Correct Answer: (A) 3

Solution:

Step 1: Understanding the problem.

We need to divide a cube into 8 smaller cubes, each of equal size and shape. A cube has 6 faces, 12 edges, and 8 vertices. We are to perform straight cuts that go from one end of the cube to the other, ensuring that each cut divides the cube into equal pieces.

Step 2: Dividing the cube using 3 cuts.

To divide the cube into 8 smaller cubes, consider the following approach:

- The first cut should divide the cube in half, cutting through the center along one axis. This gives 2 equal pieces.
- The second cut should divide each of the two pieces in half along a

different axis. This results in 4 pieces. - The third cut should divide each of the 4 pieces in half along the third axis, resulting in 8 smaller cubes.

Thus, 3 straight cuts are sufficient to divide the cube into 8 smaller cubes, each of equal size and shape.

Step 3: Conclusion.

The minimum number of cuts required to divide a cube into 8 equal pieces is 3. Therefore, the correct answer is $\boxed{3}$.

Quick Tip

Remember: To divide a cube into n^3 smaller cubes, you need $n - 1$ cuts along each of the 3 axes. For 8 pieces, $n = 2$, so only 3 cuts are required.
