MH Board Class 12 Mathematics and Statistics Question Paper with Solutions(Memory Based)

Time Allowed :3 Hours	Maximum Marks :70	Total questions :37

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Please check that this question paper contains 23 printed pages.
2. Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
3. Please check that this question paper contains 37 questions.
4. 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

1. Write the dual of the statement $p \wedge [\neg q \vee (p \wedge q) \vee \neg r]$.

Solution:

Concept: In propositional logic, the **dual** of a statement is obtained by:

- Replacing every $\wedge$ (AND) with $\vee$ (OR)
- Replacing every $\vee$ (OR) with $\wedge$ (AND)
- Negations remain unchanged
- Constants (if present): $0 \leftrightarrow 1$

No structural rearrangement is required—only operator swapping.

Step 1: Given expression

$$p \wedge [\neg q \vee (p \wedge q) \vee \neg r]$$

Step 2: Replace outer AND with OR

$$p \vee [\neg q \vee (p \wedge q) \vee \neg r]$$

Step 3: Convert inner ORs to ANDs and AND to OR

$$p \vee [\neg q \wedge (p \vee q) \wedge \neg r]$$

Final Answer: The dual is

$$\boxed{p \vee [\neg q \wedge (p \vee q) \wedge \neg r]}$$

Quick Tip

To find the dual quickly, just swap $\wedge \leftrightarrow \vee$ everywhere while keeping negations the same.

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2. Find the area of a triangle with vertices $A(1, 2, 3)$, $B(2, 3, 4)$, and $C(3, 4, 5)$.

Solution:

Concept: The area of a triangle formed by three points in 3D space is given by:

$$\text{Area} = \frac{1}{2} \left| \vec{AB} \times \vec{AC} \right|$$

where:

- $\vec{AB} = B - A$
- $\vec{AC} = C - A$
- The magnitude of the cross product gives the area of the parallelogram.

Step 1: Find vectors

$$\vec{AB} = (2 - 1, 3 - 2, 4 - 3) = (1, 1, 1)$$

$$\vec{AC} = (3 - 1, 4 - 2, 5 - 3) = (2, 2, 2)$$

Step 2: Compute cross product

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{vmatrix}$$

Since the rows are proportional, the determinant is:

$$\vec{AB} \times \vec{AC} = (0, 0, 0)$$

Step 3: Find area

$$\text{Area} = \frac{1}{2} \times 0 = 0$$

Final Answer:

$$\boxed{0}$$

Explanation: The vectors $\vec{AB}$ and $\vec{AC}$ are parallel, meaning the three points are collinear. Hence, they do not form a triangle, and the area is zero.

Quick Tip

If two side vectors are scalar multiples, the triangle is degenerate and its area is zero.

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3. Differentiate $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x.

Solution:

Concept: Use the derivative formula:

$$\frac{d}{dx}[\cos^{-1}(u)] = \frac{-1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

Also use the identity:

$$\frac{1-x^2}{1+x^2} = \cos(2\tan^{-1} x)$$

This simplifies the expression significantly.

Step 1: Let

$$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

Using identity:

$$\frac{1-x^2}{1+x^2} = \cos(2\tan^{-1} x)$$

So,

$$y = \cos^{-1}[\cos(2\tan^{-1} x)]$$

Step 2: Simplify inverse cosine

$$y = 2\tan^{-1} x$$

Step 3: Differentiate

$$\frac{dy}{dx} = 2 \cdot \frac{1}{1+x^2}$$

Final Answer:

$$\boxed{\frac{2}{1+x^2}}$$

Explanation: Recognizing the trigonometric identity avoids complicated differentiation and makes the problem straightforward.

Quick Tip

Expressions like $\frac{1-x^2}{1+x^2}$ often relate to $\cos(2\tan^{-1}x)$. Use identities to simplify inverse trig problems.

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4. Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$.

Solution:

Concept: The principal value range of the inverse sine function is:

$$-\frac{\pi}{2} \leq \sin^{-1}(x) \leq \frac{\pi}{2}$$

We need the angle within this interval whose sine equals $-\frac{1}{2}$.

Step 1: Recall standard values

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

Step 2: Use odd property of sine Since sine is an odd function:

$$\sin^{-1}(-x) = -\sin^{-1}(x)$$

So,

$$\sin^{-1}\left(-\frac{1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right)$$

Step 3: Substitute known value

$$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

Final Answer:

$$\boxed{-\frac{\pi}{6}}$$

Explanation: The angle must lie in the principal range $[-\frac{\pi}{2}, \frac{\pi}{2}]$, so the negative acute angle is chosen.

Quick Tip

Always check the principal value range before giving answers for inverse trigonometric functions.

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5. Write the converse, inverse, and contrapositive of: "If voltage increases, then current decreases".

Solution:

Concept: Let the statement be of the form:

$$\text{If } p \text{ then } q$$

where:

- Converse: If $q \to p$
- Inverse: If $\neg p \to \neg q$
- Contrapositive: If $\neg q \to \neg p$

Step 1: Identify statements

- p: Voltage increases
- q: Current decreases

Step 2: Write required forms

Converse: If current decreases, then voltage increases.

Inverse: If voltage does not increase, then current does not decrease.

Contrapositive: If current does not decrease, then voltage does not increase.

Explanation: The converse swaps hypothesis and conclusion, the inverse negates both parts, and the contrapositive swaps and negates both. The contrapositive is logically equivalent to the original statement.

Quick Tip

Remember: Contrapositive = Swap + Negate both parts. It is always logically equivalent to the original statement.

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6. Evaluate $\int \frac{x+2}{2x^2+6x+5}\,dx$ using the substitution or partial fraction method.

Solution:

Concept: When the numerator resembles the derivative of the denominator, use substitution:

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + C$$

If not exact, split the numerator appropriately.

Step 1: Let

$$I = \int \frac{x+2}{2x^2+6x+5}dx$$

Differentiate denominator:

$$\frac{d}{dx}(2x^2+6x+5) = 4x+6$$

We express numerator in terms of $4x+6$:

$$x+2 = \frac{1}{4}(4x+6) + \frac{1}{2}$$

Step 2: Split the integral

$$I = \int \frac{\frac{1}{4}(4x+6)}{2x^2+6x+5}dx + \int \frac{\frac{1}{2}}{2x^2+6x+5}dx$$

$$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{2}\int \frac{dx}{2x^2+6x+5}$$

Step 3: First integral (log form)

$$= \frac{1}{4}\ln|2x^2+6x+5|$$

Step 4: Second integral (complete square)

$$2x^2 + 6x + 5 = 2(x^2 + 3x) + 5$$

$$= 2\left[(x + \tfrac{3}{2})^2 + \tfrac{1}{4}\right]$$

So,

$$\int \frac{dx}{2x^2 + 6x + 5} = \frac{1}{2}\int \frac{dx}{(x + \frac{3}{2})^2 + (\frac{1}{2})^2}$$

Using:

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

$$= \frac{1}{2} \cdot 2\tan^{-1}(2x + 3) = \tan^{-1}(2x + 3)$$

Step 5: Multiply constant

$$\frac{1}{2} \times \tan^{-1}(2x + 3) = \frac{1}{2}\tan^{-1}(2x + 3)$$

Final Answer:

$$\boxed{\frac{1}{4}\ln|2x^2 + 6x + 5| + \frac{1}{2}\tan^{-1}(2x + 3) + C}$$

Explanation: The numerator was decomposed into a derivative form plus a constant. One part gave a logarithm, while the remaining quadratic produced an inverse tangent after completing the square.

Quick Tip

If numerator is close to derivative of denominator, split it into derivative form + remainder to simplify integration.

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7. A die is tossed twice. If a "success" is getting a number greater than 4, find the probability distribution of the number of successes.

Solution:

Concept: A success is getting a number greater than 4 (i.e., 5 or 6). So probability of success:

$$P(S) = \frac{2}{6} = \frac{1}{3}$$

Probability of failure:

$$P(F) = \frac{4}{6} = \frac{2}{3}$$

Since the die is tossed twice independently, this is a binomial distribution with:

$$n = 2, \quad p = \frac{1}{3}$$

Step 1: Let X = number of successes in two tosses. Possible values: $X = 0, 1, 2$

Step 2: Use binomial probability formula

$$P(X = k) = \binom{2}{k} p^k (1-p)^{2-k}$$

For X = 0:

$$P(0) = (1-p)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

For X = 1:

$$P(1) = 2 \cdot \frac{1}{3} \cdot \frac{2}{3} = \frac{4}{9}$$

For X = 2:

$$P(2) = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

Step 3: Probability distribution

X	$P(X)$
0	$\frac{4}{9}$
1	$\frac{4}{9}$
2	$\frac{1}{9}$

Final Answer: The probability distribution is:

$$P(0) = \frac{4}{9}, \quad P(1) = \frac{4}{9}, \quad P(2) = \frac{1}{9}$$

Explanation: Each toss is independent with success probability $\frac{1}{3}$. Hence, the number of successes follows a binomial distribution.

Quick Tip

Repeated independent trials with two outcomes follow a binomial distribution. Identify n and p first.

8. The population of a town grows at a rate proportional to its size. If it grows from 40,000 to 60,000 in 40 years, what will it be in another 20 years?

Solution:

Concept: Growth proportional to population follows exponential growth:

$$\frac{dP}{dt} = kP$$

Solution:

$$P(t) = P_0 e^{kt}$$

We use given values to find k, then predict future population.

Step 1: Given data

$$P_0 = 40000, \quad P(40) = 60000$$

Using:

$$P(40) = 40000e^{40k}$$

$$60000 = 40000e^{40k}$$

$$\frac{3}{2} = e^{40k}$$

$$k = \frac{1}{40} \ln\left(\frac{3}{2}\right)$$

Step 2: Population after another 20 years Total time = 60 years

$$P(60) = 40000e^{60k}$$

Substitute k:

$$P(60) = 40000\left(e^{40k}\right)^{3/2}$$

Since $e^{40k} = \frac{3}{2}$,

$$P(60) = 40000\left(\frac{3}{2}\right)^{3/2}$$

Step 3: Simplify

$$\left(\frac{3}{2}\right)^{3/2} = \sqrt{\frac{27}{8}}$$

$$P(60) = 40000 \cdot \sqrt{\frac{27}{8}}$$

Approximate:

$$\sqrt{\frac{27}{8}} \approx 1.837$$

$$P(60) \approx 40000 \times 1.837 \approx 73,480$$

Final Answer:

$$\boxed{P \approx 73,500 \text{ (approximately)}}$$

Explanation: Exponential growth means the growth factor remains constant. Using the growth ratio from the first 40 years allows prediction of future population.

Quick Tip

For exponential growth problems, find the growth factor first, then raise it to the required time ratio.

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9. Prove that $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$**.**

Solution:

Concept: Use trigonometric substitution:

$$x = a\sin\theta$$

This simplifies expressions of the form $\sqrt{a^2 - x^2}$.

Step 1: Substitute Let:

$$x = a\sin\theta \quad \Rightarrow \quad dx = a\cos\theta\, d\theta$$

Then,

$$\sqrt{a^2 - x^2} = \sqrt{a^2 - a^2\sin^2\theta} = a\cos\theta$$

Step 2: Transform the integral

$$\int \sqrt{a^2 - x^2}\, dx = \int (a\cos\theta)(a\cos\theta\, d\theta)$$

$$= a^2 \int \cos^2\theta\, d\theta$$

Step 3: Use identity

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$= \frac{a^2}{2} \int (1 + \cos 2\theta) d\theta$$

$$= \frac{a^2}{2} \left(\theta + \frac{\sin 2\theta}{2} \right) + C$$

Step 4: Simplify

$$= \frac{a^2}{2}\theta + \frac{a^2}{4} \sin 2\theta + C$$

Using:

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

$$= \frac{a^2}{2}\theta + \frac{a^2}{2} \sin\theta\cos\theta + C$$

Step 5: Back-substitute Since:

$$\sin\theta = \frac{x}{a}, \quad \cos\theta = \frac{\sqrt{a^2 - x^2}}{a}, \quad \theta = \sin^{-1}\left(\frac{x}{a}\right)$$

So,

$$\sin\theta\cos\theta = \frac{x}{a} \cdot \frac{\sqrt{a^2 - x^2}}{a} = \frac{x\sqrt{a^2 - x^2}}{a^2}$$

Substitute back:

$$\frac{a^2}{2}\theta + \frac{a^2}{2} \cdot \frac{x\sqrt{a^2 - x^2}}{a^2}$$

$$= \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{a^2 - x^2} + C$$

Final Result:

$$\boxed{\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + C}$$

Explanation: Trigonometric substitution converts the radical into a simple trigonometric integral, which is then transformed back into algebraic form.

Quick Tip

For integrals of the form $\sqrt{a^2 - x^2}$, always try substitution $x = a\sin\theta$.

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10. Prove the Section Formula for internal division in vectors.

Solution:

Concept: The section formula gives the position vector of a point dividing a line segment internally in a given ratio. If a point P divides the line joining points A and B internally in the ratio $m : n$, then its position vector is a weighted average of the vectors of A and B.

Let:

- Position vector of $A = \vec{a}$
- Position vector of $B = \vec{b}$
- Position vector of $P = \vec{p}$

Suppose P divides AB internally in the ratio $m : n$, i.e.,

$$AP : PB = m : n$$

Step 1: Express vectors along the line Since P lies on line AB,

$$\vec{AP} = \vec{p} - \vec{a}, \quad \vec{PB} = \vec{b} - \vec{p}$$

Given ratio:

$$\frac{|\vec{AP}|}{|\vec{PB}|} = \frac{m}{n}$$

So in vector form:

$$\vec{p} - \vec{a} = \frac{m}{n}(\vec{b} - \vec{p})$$

Step 2: Solve algebraically

$$n(\vec{p} - \vec{a}) = m(\vec{b} - \vec{p})$$

$$n\vec{p} - n\vec{a} = m\vec{b} - m\vec{p}$$

Bring like terms together:

$$n\vec{p} + m\vec{p} = m\vec{b} + n\vec{a}$$

$$(m+n)\vec{p} = m\vec{b} + n\vec{a}$$

Step 3: Find position vector of P

$$\vec{p} = \frac{m\vec{b} + n\vec{a}}{m+n}$$

Final Result (Section Formula): If a point divides a line segment internally in the ratio $m : n$, then:

$$\boxed{\vec{p} = \frac{m\vec{b} + n\vec{a}}{m+n}}$$

Explanation: The formula shows that the point dividing the segment lies between the two endpoints and is a weighted average of their position vectors. The weights depend on the opposite segments of the ratio.

Quick Tip

In internal division, multiply each endpoint vector by the opposite ratio and divide by the sum of ratios.
