

Solutions

A solution is a homogeneous mixture of two or more chemically non-reacting substances whose composition can be varied within certain limits.

Solvent : ~~solute~~ component in larger amount.

Solute : the component in smaller amount.

Binary solution : one solute + one solvent.

• Types of Solutions

Based on the physical state of solute & solvent, 9 types of solutions are possible :

- ① Gas in gas : air ($O_2 + N_2 + \dots$)
- ② Gas in liq : O_2 in water, soda
- ③ Gas in sol : H_2 in Pd, Pt
- ④ Liq in gas : moisture in air
- ⑤ Liq in liq : alcohol in water
- ⑥ Liq in sol : amalgam (Hg in Na)
- ⑦ Sol in gas * : camphor in air
- ⑧ Sol in liq : salt in water
- ⑨ Sol in sol : alloys (brass, bronze)

Most common type : solid / liquid in liquid.

Concentration of Solutions

Concentration = amount of solute in a given quantity of solution / solvent. Several methods:

(1) Mass Percentage (w/w)

$$w/w \% = (\text{mass of solute} / \text{mass of soln}) \times 100$$

*

<-e.g. 10% glucose soln : 10 g glucose in
<-90 g water (total 100 g soln).

(2) Volume Percentage (v/v)

$$v/v \% = (\text{vol. of solute} / \text{vol. of soln}) \times 100$$

35% v/v aq. ethylene glycol : antifreeze in
car radiators (freezes at $-17.6\text{ }^{\circ}\text{C}$).

(3) Mass by Volume (w/V)

$$w/V \% = (\text{mass of solute} / \text{vol. soln}) \times 100$$

<-used in medicine & pharmacy.

Note : w/V depends on temperature (V changes).

Concentration Units (contd.)

(4) Parts per Million (ppm)

For very dilute solutions (trace amounts).

$$\text{ppm} = (\text{mass solute} / \text{mass soln}) \times 10^6$$

e.g. 5.8 g of dissolved O₂ in 1000 kg sea water = 5.8 ppm. 1 ppm F⁻ in drinking water prevents tooth decay ; 1.5 ppm causes mottling. Pollutants in air also expressed in ppm.

(5) Mole Fraction (x)

$$x_A = n_A / (n_A + n_B)$$

<- mole fraction
<- of A

For binary mixture : $x_A + x_B = 1$

useful in relating P, V to composition

(~~Henry~~ Dalton's law, Raoult's law).

Independent of temperature. No units.

Molarity & Molality

(6) Molarity (M)

Moles of solute dissolved per litre of solution.

$$M = n_{\text{solute}} / V \text{ (in L)}$$

<- unit : mol/L
<- = M

$$\text{Also } M = (w_B \times 1000) / (M_B \times V \text{ mL})$$

Drawback : depends on T (volume* expands / contracts with T). Hence NOT temp-independent.

(7) Molality (m)

Moles of solute per kg of solvent.

$$m = n_{\text{solute}} / W_{\text{solvent}} \text{ (kg)}$$

<- unit : mol / kg
<- = m

$$\text{Also } m = (w_B \times 1000) / (M_B \times W_A \text{ g})$$

Mass independent of T \Rightarrow molality is temperature ~~dependent~~ independent.

Preferred in colligative property calculations.

Solved Example 1

Q. Calculate molarity of solution containing 5 g NaOH in 450 mL of solution.

Solution :

$$M_{\text{NaOH}} = 23 + 16 + 1 = 40 \text{ g/mol}$$

$$\text{Moles of NaOH} = 5 / 40 = 0.125 \text{ mol}$$

$$\text{Volume} = 450 \text{ mL} = 0.450 \text{ L}$$

Using definition of molarity :

$$M = (\text{moles of solute}) / (\text{vol. in L})$$

$$= 0.125 / 0.450 = 0.278 \text{ mol/L}$$

$$\text{Final answer : } M = 0.278 \text{ M}$$

Quick check

$$1 \text{ L of soln contains } 0.278 \text{ mol} = 0.278 \times 40$$

$$= 11.1 \text{ g NaOH. } 450 \text{ mL would contain}$$

$$11.1 \times 0.45 = 5.0 \text{ g. Verified.}$$

Solved Example 2:

Q. A solution of glucose in water is 10% w/w.
Calculate mole fraction of glucose and water.

Solution :

Take 100 g of solution.

Mass of glucose = 10 g ; mass of water = 90 g.

M of glucose ($C_6H_{12}O_6$) = 180 g/mol

M of water (H_2O) = 18 g/mol

$$n_{glu} = 10 / 180 = 0.0556 \text{ mol}$$

$$n_{H_2O} = 90 / 18 = 5.00 \text{ mol}$$

Mole fraction of glucose :

$$x_{glu} = 0.0556 / (0.0556 + 5.00)$$

$$= 0.0556 / 5.0556 = 0.011$$

$$x_{H_2O} = 1 - 0.011 = 0.989$$

Answer : $x_{glu} = 0.011$; $x_{H_2O} = 0.989$

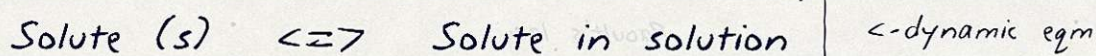
Solubility

Solubility = max amount of a substance that dissolves in a given solvent at a specified T.

Depends on : nature of solute / solvent, temperature and pressure.

Solid in Liquid

Saturated solution : solute - dissolved solute in dynamic equilibrium with ~~undissolved~~ dissolved solute.



Like dissolves like :

polar solute -- polar solvent (NaCl - H₂O)

non-polar -- non-polar (naphthalene - C₆H₆)

Effect of T

IF dissolution endothermic ($\Delta H > 0$) :

solubility increases with T (KNO₃).

+ IF exothermic ($\Delta H < 0$) :

solubility decreases with T (Ce₂(SO₄)₃).

Pressure : negligible effect on solid solubility.

Solubility of Gas in Liquid

Strongly affected by Temperature & Pressure.

Henry's Law

At constant T , solubility of a gas in a liquid is directly proportional to its partial pressure.

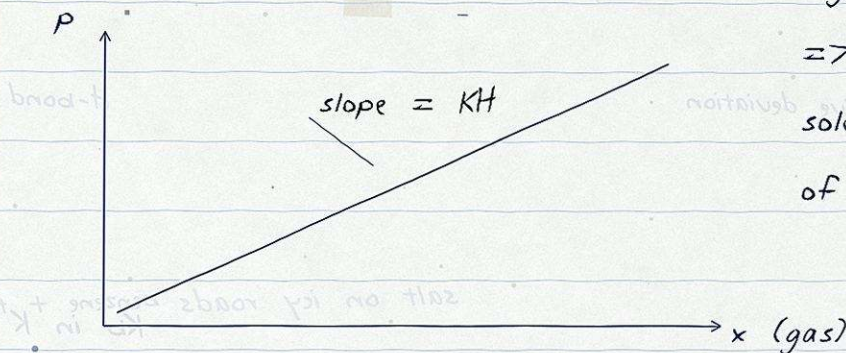
$$p = K_H \cdot x$$

$\leftarrow K_H = \text{Henry's law}$
 $\leftarrow \text{constant.}$

$p =$ partial pressure of gas above soln

$x =$ mole fraction of gas in soln

Plot : p vs x



Higher K_H

\Rightarrow lower

solubility

of gas.

Henry's Law - Applications

KH varies with T & gas nature

KH (gas) increases with T \Rightarrow solubility of gas decreases with rise in T.

(Aquatic life prefers cold water - more dissolved O₂.)

Some KH values (293 K)

| Gas | KH / kbar |
|-----------------|-------------------------------|
| He | 144.97 |
| H ₂ | 69.16 |
| N ₂ | 76.48 |
| O ₂ | 34.86 |
| CO ₂ | 1.67 (least KH, most soluble) |

Real-life uses

- ① Soda water : sealed at high p CO₂.
- ② Scuba divers : He + O₂ mix, not air, to avoid the 'bends' (N₂ bubbles in blood on rapid ascent).
- ③ High altitudes : low pO₂ \Rightarrow anoxia. Climbers suffer weakness (low O₂ in blood).

Solved Example 3

Q. N_2 gas is bubbled through water at 293 K. If $p(N_2) = 0.987$ bar, calculate moles of N_2 dissolved in 1 L water. $K_H(N_2) = 76.48$ kbar.

Solution :

By Henry's law : $p = K_H \cdot x$

$$x_{N_2} = p / K_H \\ = 0.987 / (76480) = 1.29 \times 10^{-5}$$

$$1 \text{ L water} = 1000 \text{ g} \Rightarrow 1000/18 = 55.55 \text{ mol}$$

$$x_{N_2} = n_{N_2} / n_{H_2O} \quad (\text{since } n_{N_2} \ll n_{H_2O})$$

$$n_{N_2} = x_{N_2} \times n_{H_2O} \\ = 1.29 \times 10^{-5} \times 55.55$$

$$n(N_2) = 7.16 \times 10^{-4} \text{ mol}$$

Very little N_2 dissolves at 1 atm in pure water - because $K_H(N_2)$ is high.

Vapour Pressure - Raoult's Law

Vapour Pressure (VP)

Pressure exerted by vapour in equilibrium with its liquid at a given temperature.

Raoult's Law (liq-liq solution)

For a solution of volatile liquids, the partial VP of each component is directly proportional to its mole fraction in the solution.

$$p_A = p_A^* x_A$$

$$p_B = p_B^* x_B$$

$\leftarrow p^* = \text{VP of}$
 $\leftarrow \text{pure component}$

By Dalton's law of partial pressures :

$$p_{\text{total}} = p_A + p_B = p_A^* x_A + p_B^* x_B$$

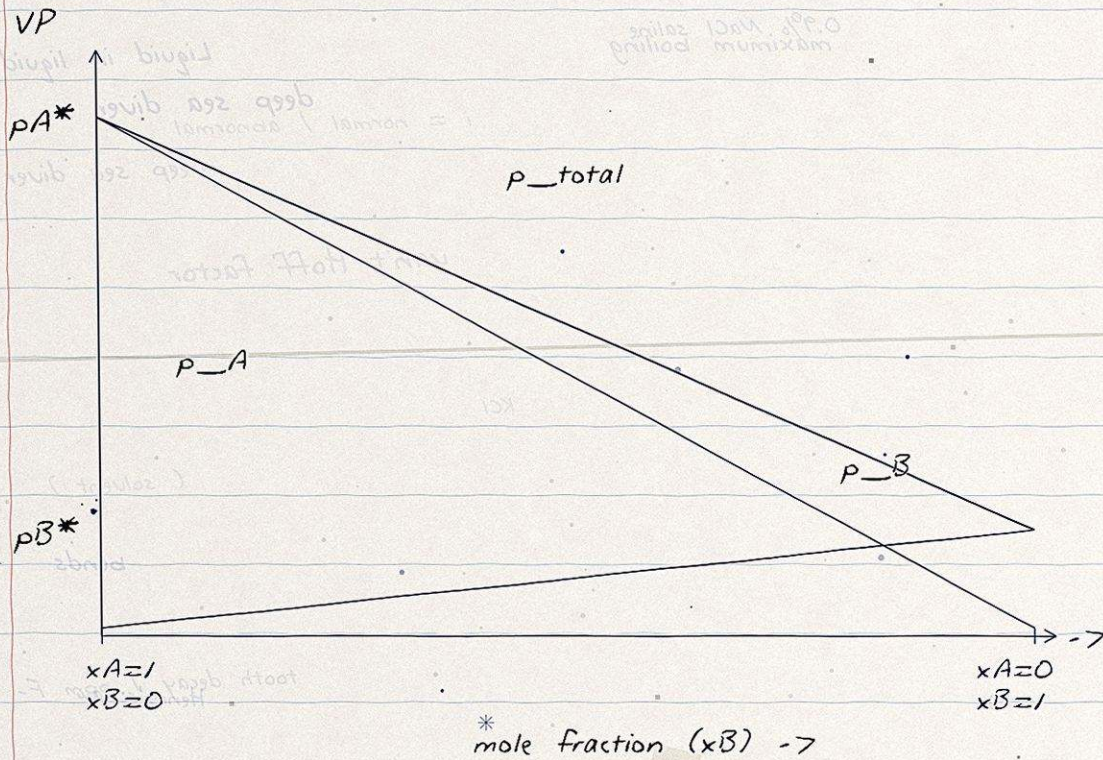
Since $x_A = 1 - x_B$:

$$p_{\text{total}} = p_A^* + (p_B^* - p_A^*) x_B$$

$\Rightarrow p_{\text{total}}$ varies linearly with x_B .

VP vs Mole Fraction Plot

For an ideal binary solution :



All three plots are STRAIGHT lines for an ideal solution (obeys Raoult's law over all compositions and at all temperatures).

Ideal & Non-ideal Solutions

Ideal Solution

Obeys Raoult's law for ALL compositions and all temperatures. Conditions :

- ① $p_A = p_A^* x_A$ and $p_B = p_B^* x_B$
- ② $\Delta_{\text{mix}} H = 0$ (no heat change)
- ③ $\Delta_{\text{mix}} V = 0$ (no vol. change)
- ④ $F(A-A) = F(B-B) = F(A-B)$

Examples : n-hexane + n-heptane

benzene + toluene

$\text{CCl}_4 + \text{SiCl}_4$ (chlorides)

Non-ideal Solution *

Does NOT obey Raoult's law.

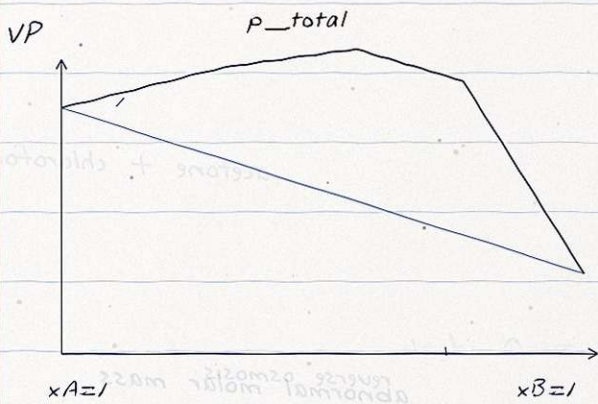
$F(A-B) \neq F(A-A), F(B-B) \Rightarrow$ shows deviation.

Two types :

- ① + ve dev : $F(A-B) < F(A-A), F(B-B)$
VP higher than predicted by Raoult's law.
- ② - ve dev : $F(A-B) > F(A-A), F(B-B)$
VP lower than predicted by Raoult's law.

Deviation Plots

(a) Positive Deviation



Examples :

ethanol + acetone

CCl₄ + C₆H₆

CS₂ + acetone

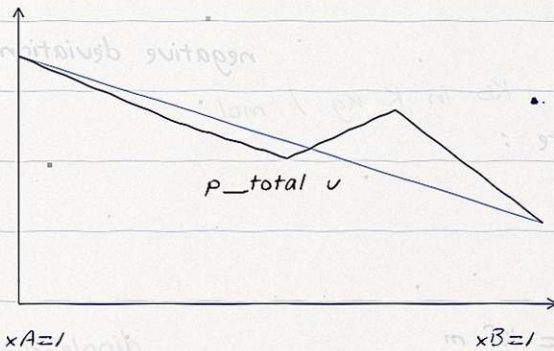
EtOH + H₂O

*

H-bonding in pure ethanol breaks =>

more vapours.

(b) Negative Deviation



Examples :

CHCl₃ + acetone

HCl + H₂O

HNO₃ + H₂O

New H-bond btwn

CHCl₃ & CH₃-CO-CH₃

=> VP lowered.

Azeotropes

Liquid mixtures that distill at a constant boiling temperature without change in composition (vapour & liquid have same x).

Cannot be separated by simple distillation.

Two kinds

① Minimum BP : * + ve deviation systems

e.g. EtOH + H₂O at 95.4% EtOH boils

at 351.15 K (< both pure components).

② Maximum BP : - ve deviation systems

e.g. HNO₃ (68%) + H₂O boils at 393.5 K

(> both pure components).

Summary Table

| Type | VP | delta H | delta - V | Azeotrope |
|-------|--------|---------|-----------|-----------|
| Ideal | Raoult | 0 | 0 | none |
| + dev | higher | > 0 | > 0 | Min-BP |
| - dev | lower | < 0 | < 0 | Max-BP |

Solid Solute in Liquid Solvent

Non-volatile solute does NOT contribute to VP.

$$\text{So } p_{\text{solution}} = p_1 = p_1^* x_1 \quad (\text{Raoult})$$

where 1 = solvent ; non-vol. solute (2) has $p_2 = 0$.

Relative Lowering of VP

VP of solvent in solution < VP of pure solvent.

$$(p_1^* - p) / p_1^* = x_2$$

<- depends only on
<- x_2 (colligative)

Derivation :

$$p_1 = p_1^* x_1 = p_1^* (1 - x_2)$$

$$p_1^* - p_1 = p_1^* x_2$$

$$(p_1^* - p) / p_1^* = x_2$$

For dilute solution ($n_2 \ll n_1$) :

$$x_2 = n_2 / n_1 = (w_2 / M_2) / (w_1 / M_1)$$

$$\Rightarrow (p_1^* - p) / p_1^* = (w_2 M_1) / (M_2 w_1)$$

Allows molar mass determination of solute.

Colligative Properties

Properties of dilute solutions that depend **ONLY** on the number of solute particles, and **NOT** on their nature.

Four colligative properties :

- ① Relative lowering of VP
- ② Elevation of boiling point
- ③ Depression of freezing point
- ④ Osmotic pressure

(2) Elevation of BP

BP of solvent rises when a non-volatile solute is added. $\Delta T_b = T_b - T_b^*$.

$$\Delta T_b = K_b m$$

$\leftarrow K_b = \text{molal BP}$
 $\leftarrow \text{elev. const.}$

K_b units : $\text{K kg} / \text{mol}$ (or $^{\circ}\text{C kg} / \text{mol}$).

$$K_b = (R M_b T_b^{*2}) / (1000 \Delta H_{\text{vap}}^*)$$

$K_b (\text{water}) = 0.52 \text{ K kg/mol}$; $K_b (\text{C}_6\text{H}_6) = 2.53$

Depression of Freezing Point

FP of solvent lowers when a non-volatile solute is added. $\Delta T_f = T_f^* - T_f$.

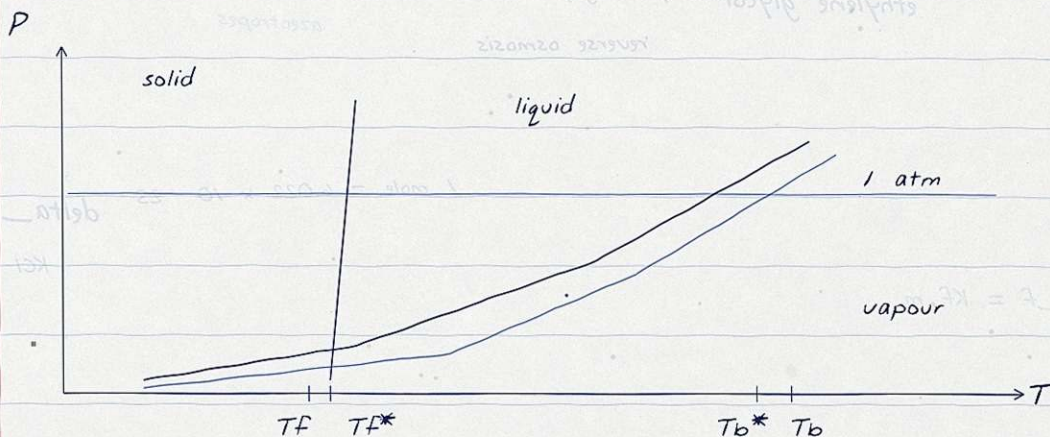
$$\Delta T_f = K_f m$$

$\leftarrow K_f = \text{molal FP}$
 $\leftarrow \text{depr. const.}$

$$K_f = (R M_f T_f^{*2}) / (1000 \Delta H_{fus})$$

$K_f (\text{water}) = 1.86 \text{ K kg/mol}$; $K_f (\text{C}_6\text{H}_6) = 5.12$

Phase diagram (qualitative)



Solution curve sits below pure solvent curve.

Osmosis & Osmotic Pressure

Osmosis : spontaneous flow of solvent molecules from a pure solvent (or dilute soln) to a concentrated soln through a semipermeable membrane (SPM).

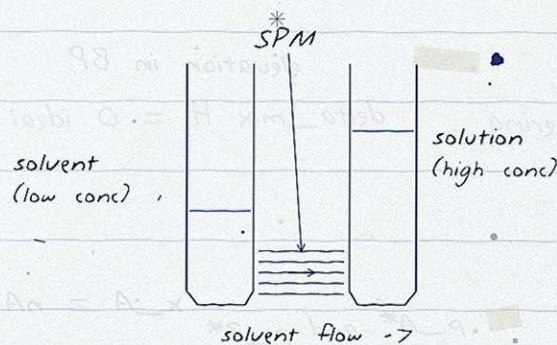
Osmotic pressure (π) = extra pressure that must be applied on the solution side to JUST stop osmosis.

$$\pi = C R T = (n_2/V) R T$$

<- Van't Hoff
<- equation

C = molar conc. (mol/L) ; $R = 0.0821$ L atm/mol K

T in Kelvin ; π in atm.



h gives a measure of osmotic pressure.

Osmosis - Terminology & Uses

Isotonic, Hyper-, Hypotonic

(1) **Isotonic** : two soln of equal π .

e.g. 0.9% NaCl & blood plasma at 310 K.

(2) **Hypertonic** : soln with higher π .

Water exits cell \rightarrow shrinks (crenation).

(3) **Hypotonic** : soln with lower π .

Water enters cell \rightarrow bursts (haemolysis).

Reverse Osmosis

If $P_{\text{applied}} > \pi$ on the solution side,

solvent flows from solution \rightarrow pure side.

Used in ~~desolintization~~ desalination of sea water.

Why π is preferred (not ΔT_b / T_f)

(i) Magnitude of π is large \Rightarrow measurable

even for very dilute solutions.

(ii) Measured at room T (unlike BP / FP).

(iii) Suitable for proteins, polymers which

decompose at high T.

Solved Example 4

Q. 18 g of glucose ($C_6H_{12}O_6$) is dissolved in 1 kg of water in a saucepan. At what T will water boil at 1 atm? $K_b(H_2O) = 0.52 \text{ K kg/mol}$.

Solution :

$$M(C_6H_{12}O_6) = 6(12) + 12(1) + 6(16) = 180 \text{ g/mol}$$

$$\text{moles of glucose} = 18 / 180 = 0.10 \text{ mol}$$

$$\text{mass of solvent} = 1 \text{ kg}$$

$$m = 0.10 / 1 = 0.10 \text{ mol / kg}$$

$$\text{Using } \Delta T_b = K_b \times m :$$

$$\Delta T_b = 0.52 \times 0.10 = 0.052 \text{ K}$$

$$\text{Boiling point of pure water} = 373.15 \text{ K}$$

$$T_b (\text{soln}) = 373.15 + 0.052 = 373.20 \text{ K}$$

So water boils at 373.20 K (100.05 C).

Solved Example 5

Q. 1.00 g of a non-electrolyte solute dissolved in 50 g of benzene lowers the FP by 0.40 K. $K_f(\text{benzene}) = 5.12 \text{ K kg/mol}$. Find M_2 .

Solution :

Use formula relating ΔT_f with M_2 :

$$\Delta T_f = K_f \times (1000 w_2) / (M_2 w)$$

$$M_2 = (K_f \times 1000 \times w_2) / (\Delta T_f \times w)$$

Substitute : $K_f = 5.12$, $w_2 = 1.00 \text{ g}$,

$\Delta T_f = 0.40$, $w = 50 \text{ g}$.

$$M_2 = (5.12 \times 1000 \times 1.00) / (0.40 \times 50)$$

$$= 5120 / 20$$

$$M_2 = 256 \text{ g / mol}$$

Likely S_8 ($8 \times 32 = 256$). Sulphur dissolves in C_6H_6 .

Solved Example 6

Q. 200 cm³ of an aqueous soln of a protein contains 1.26 g. of the protein. The osmotic pressure of this soln at 300 K is 2.57×10^{-3} bar. Calculate the molar mass of the protein.

Solution :

$$\pi = (w_2 R T) / (M_2 V)$$

$$\Rightarrow M_2 = (w_2 R T) / (\pi V)$$

Convert units :

$$w_2 = 1.26 \text{ g}$$

$$V = 200 \text{ mL} = 0.200 \text{ L}$$

$$R = 0.0831 \text{ L bar / mol K}$$

$$T = 300 \text{ K} \quad *$$

$$\pi = 2.57 \times 10^{-3} \text{ bar}$$

$$M_2 = (1.26 \times 0.0831 \times 300) / (2.57 \times 10^{-3} \times 0.200)$$

$$= 31.41 / (5.14 \times 10^{-4})$$

$$M_2 = 6.11 \times 10^4 \text{ g / mol}$$

Typical for biomolecules. Osmotic method works.

Abnormal Molar Mass

Molar mass calculated from a colligative property is the 'colligative' or 'apparent' molar mass M_2 (apparent).

IF solute DISSOCIATES (e.g. NaCl, KCl) :

of particles $>$ # of formula units
 observed $\Delta T >$ calculated ΔT
 apparent $M_2 <$ true M_2 .

IF solute ASSOCIATES (e.g. C_6H_5COOH in C_6H_6) :

of particles $<$ # of formula units
 observed $\Delta T <$ calculated ΔT
 apparent $M_2 >$ true M_2 .

Van't Hoff factor (i)

$$i = (\text{observed coll. prop}) / (\text{calc. coll. prop})$$

$$i = (\text{calc. } M_2) / (\text{observed } M_2)$$

$$i = (\text{No. of particles after dissoc / assoc}) / (\text{No. of particles before})$$

$i > 1$: dissociation ; $i < 1$: association.

Modified Colligative Formulas

Including Van't Hoff factor i :

$$(p_i^* - p) / p_i^* = i x_2$$

$$\Delta T_b = i K_b m$$

$$\Delta T_f = i K_f m$$

$$p_i = i C R T$$

Degree of dissociation / association (alpha)

For dissociation : $A_n \rightarrow n A$ ($n =$ particles after)

$$\alpha = (i - 1) / (n - 1)$$

For association : $n A \rightarrow A_n$ (1 particle from n)

$$\alpha = (1 - i) (n) / (n - 1)$$

Examples : i (NaCl) 2 ; i (BaCl₂) 3
 i (C₆H₅COOH in C₆H₆) 0.5 (dimer)

Solved Example 7

Q. 2 g of benzoic acid (C_6H_5COOH) is dissolved in 25 g of benzene which shows a depression in FP of 1.62 K. $K_f(C_6H_6) = 4.9 \text{ K kg/mol}$.

Calculate i and degree of association.

Solution :

$$M(C_6H_5COOH) = 122 \text{ g/mol}$$

$$m = (2 / 122) \times (1000 / 25) = 0.656 \text{ mol/kg}$$

$$\Delta T_f(\text{calc}) = K_f m = 4.9 \times 0.656 = 3.22 \text{ K}$$

$$\Delta T_f(\text{obs}) = 1.62 \text{ K}$$

$$i = 1.62 / 3.22 = 0.503$$

Since $i < 1 \Rightarrow$ association (forms dimer, $n=2$)

$$\alpha = (1 - i) \times n / (n - 1) = (1 - 0.503) \times 2 / (2 - 1)$$

$$= 2 \times 0.497 = 0.994 \quad 99.4 \%$$

Nearly all benzoic acid dimerises in benzene.

Solved Example 8

Q. Calculate the osmotic pressure at 300 K of a 0.0100 M aqueous KCl solution. Assume complete dissociation. $R = 0.0821 \text{ L atm/mol K}$.

Solution :



$$\pi = i C R T$$

$$\pi = 2 \times 0.0100 \times 0.0821 \times 300$$

$$= 2 \times 0.0100 \times 24.63$$

$$= 0.493 \text{ atm}$$

$$\pi = 0.493 \text{ atm}$$

Compare with non-electrolyte (glucose) at same

C : $\pi = 0.246 \text{ atm}$. So KCl gives 2x more π .

Because 1 mol KCl \rightarrow 2 mol particles.

Chapter Summary

Key formulae

- * $M = n / V \text{ (L)}$; $m = n / W \text{ (kg)}$
- * $x_A = n_A / (n_A + n_B)$
- * Henry : $p = K_H \times$
- * Raoult : $p_i = p_i^* x_i$
- * $(p^* - p) / p^* = x_2$ (rel. lower)
- * $\Delta T_b = i K_b m$
- * $\Delta T_f = i K_f m$
- * $p_i = i C^* R T$
- * $\alpha_{\text{dissoc}} = (i - 1) / (n - 1)$

Important constants

$$K_b \text{ (H}_2\text{O)} = 0.52 \text{ K kg/mol}$$

$$K_f \text{ (H}_2\text{O)} = 1.86 \text{ K kg/mol}$$

$$K_b \text{ (C}_6\text{H}_6) = 2.53 \text{ K kg/mol}$$

$$K_f \text{ (C}_6\text{H}_6) = 5.12 \text{ K kg/mol}$$

$$R = 0.0821 \text{ L atm/(mol K)} = 0.0831 \text{ L bar/(mol K)}$$

Common pitfalls

1. molarity changes with T ; molality doesn't.
2. forget Van't Hoff i for electrolytes ?
3. mass of SOLVENT in molality, not solution.
4. R values for atm vs bar differ - check units.