



Collegedunia NCERT Formula Sheet

Class 12 Mathematics — Chapter 1

Chapter 1: Relations and Functions

Symbol / Term	Meaning
$A \times B$	Cartesian product: $\{(a, b) : a \in A, b \in B\}$
$R \subseteq A \times A$	Relation R defined on set A
$a R b$ or $(a, b) \in R$	" a is related to b " under R
ϕ	Empty relation (no pair related)
$A \times A$	Universal relation (every pair related)
I_A or id_A	Identity function on A : $I_A(x) = x$
$f : X \rightarrow Y$	Function f from domain X to codomain Y
$g \circ f$ (or gof)	Composition: $(g \circ f)(x) = g(f(x))$
f^{-1}	Inverse function (exists iff f is bijective)
$[a]$	Equivalence class of a
$ A $ or $n(A)$	Cardinality (number of elements) of A

1 Relations: Basics & Types

This section gathers the definition of a relation as a subset of a Cartesian product and the named types of relations — empty, universal, reflexive, symmetric, transitive — that combine into an equivalence relation.

Relation on a set

A **relation** R from set A to set B is any subset of $A \times B$:

$$R \subseteq A \times B, \quad (a, b) \in R \Leftrightarrow a R b$$

If $A = B$, R is called a **relation on A** .

A relation simply lists which ordered pairs

(a, b) are "related". Equality, \leq , "is parallel to", "divides" are all relations.

Empty & universal relations

Empty relation: $R = \phi \subset A \times A$ — no element related to any.

Universal relation: $R = A \times A$ — every element related to every other.

Both are called **trivial relations**. Example: on a boys' school set, " a is sister of b " is empty; "heights of a and b differ by < 3 m" is universal.

Reflexive, symmetric, transitive

A relation R on A is:

Reflexive: $(a, a) \in R \forall a \in A$

Symmetric: $(a_1, a_2) \in R \Rightarrow (a_2, a_1) \in R$

Transitive: $(a_1, a_2), (a_2, a_3) \in R \Rightarrow (a_1, a_3) \in R$

Check all three independently — a relation can satisfy any subset. Counter-examples often kill one property at a time.

Equivalence relation

R is an **equivalence relation** on A iff R is **reflexive, symmetric AND transitive**.

Classic examples:

- " $a \equiv b \pmod{n}$ " on \mathbb{Z} (n divides $a - b$)
- " T_1 is congruent to T_2 " on triangles
- " $L_1 \parallel L_2$ " on lines in a plane

Equivalence relations **partition** the set into disjoint equivalence classes. Drop any one of the three properties and the partition breaks.

Equivalence class of an element

For an equivalence relation R on X and $a \in X$,

$$[a] = \{x \in X : (a, x) \in R\}.$$

Properties: (i) $a \in [a]$; (ii) $[a] = [b] \Leftrightarrow (a, b) \in R$; (iii) any two classes are either **equal or disjoint**; (iv) $\bigcup_a [a] = X$.

On \mathbb{Z} with $R : \{(a, b) : 3 \mid a - b\}$, the classes are $[0], [1], [2]$ — the three residue sets mod 3.

Symmetric \neq Reflexive

$R = \{(1, 2), (2, 1)\}$ on $\{1, 2, 3\}$ is symmetric but **not reflexive** (no $(1, 1), (2, 2), (3, 3)$) and **not transitive** (need $(1, 1)$ from $(1, 2), (2, 1)$). Each property must be verified separately on *every* required pair — one missing pair is enough to disqualify.

2 Functions: Types

This section covers the function vocabulary — one-one (injective), onto (surjective), bijective — plus the standard counting facts used in MCQs.

Function $f : X \rightarrow Y$

A relation $f \subseteq X \times Y$ is a **function** iff every $x \in X$ has **exactly one** image $y = f(x) \in Y$.

- Domain = X • Codomain = Y
- Range = $\{f(x) : x \in X\} \subseteq Y$

Every function is a relation; not every relation is a function. The vertical-line test (one x , one y) is the visual check.

One-one (injective)

$f : X \rightarrow Y$ is **one-one** iff distinct inputs give distinct outputs:

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \quad \forall x_1, x_2 \in X$$

Otherwise f is called **many-one**.

Standard test: assume $f(x_1) = f(x_2)$ and try to derive $x_1 = x_2$. Graphically: every horizontal line meets $y = f(x)$ in **at most one** point.

Onto (surjective)

$f : X \rightarrow Y$ is **onto** iff every $y \in Y$ has a pre-image in X :

$$\forall y \in Y, \exists x \in X \text{ s.t. } f(x) = y$$

Equivalent statement: **Range**(f) = Y (codomain).

To prove onto, pick an arbitrary $y \in Y$ and **construct** an $x \in X$ with $f(x) = y$. If the codomain is shrunk to **Range**(f), every function becomes onto.

Bijjective = one-one + onto

$f : X \rightarrow Y$ is **bijjective** (a one-one correspondence) iff it is **both** one-one and onto.

- f bijjective $\Leftrightarrow f^{-1}$ exists (Theorem).
- On a **finite** set X , $f : X \rightarrow X$ is one-one \Leftrightarrow onto.

“Bijjective” pairs each input with a unique output and uses up every element of the

codomain — a perfect matching.

Standard examples to remember

- $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 2x$: **one-one and onto**.
- $f : \mathbb{N} \rightarrow \mathbb{N}$, $f(x) = 2x$: one-one but **not onto** (odd \mathbb{N} missed).
- $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^2$: **neither** ($f(-1) = f(1)$); negatives unreached).
- $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = |x|$, $f(x) = [x]$, $\text{sgn}(x)$: **neither** one-one nor onto.
- Identity $I_A(x) = x$: always bijjective on A .

JEE/NEET Extension: counting

For finite sets with $|A| = m$, $|B| = n$:

- $|A \times B| = mn$
- Number of relations from A to B : 2^{mn}
- Number of functions $A \rightarrow B$: n^m
- Number of one-one functions ($m \leq n$):

$${}^n P_m = \frac{n!}{(n-m)!}$$
- Number of onto functions $\{1, \dots, n\} \rightarrow \{1, \dots, n\}$: $n!$
- Number of bijections $A \rightarrow A$ with $|A| = n$: $n!$

3 Composition of Functions

This section covers function composition, its non-commutativity, associativity, and its interaction with one-one/onto properties.

Composition $g \circ f$

For $f : A \rightarrow B$ and $g : B \rightarrow C$, the composition $g \circ f : A \rightarrow C$ is defined by

$$(g \circ f)(x) = g(f(x)) \quad \forall x \in A$$

Read **right to left**: apply f first, then g .

The codomain of f must match (or sit inside) the domain of g for $g \circ f$ to exist. Otherwise the composition is undefined.

Properties of composition

- **Non-commutative** (in general): $f \circ g \neq g \circ f$
- **Associative**: $h \circ (g \circ f) = (h \circ g) \circ f$
- Identity: $f \circ I_A = f = I_B \circ f$ for $f : A \rightarrow B$
- f, g one-one $\Rightarrow g \circ f$ one-one
- f, g onto $\Rightarrow g \circ f$ onto
- f, g bijjective $\Rightarrow g \circ f$ bijjective

Example: $f(x) = \cos x$, $g(x) = 3x^2$ on \mathbb{R} give $g \circ f = 3 \cos^2 x$ but $f \circ g = \cos(3x^2)$ — clearly different.

Order matters in $g \circ f$

$(g \circ f)(x) = g(f(x))$ — apply the **inner** function (f) first, then the **outer** (g). Reading the symbol left-to-right and computing $f(g(x))$ is the most common slip in MCQs. The notation gof used by NCERT means the same as $g \circ f$.

4 Invertible Functions

This section covers invertibility — existence of f^{-1} , the bijection criterion, and the standard $f^{-1} \circ f = I$ identity.

Invertible function

$f : X \rightarrow Y$ is **invertible** iff there exists $g : Y \rightarrow X$ with

$$g \circ f = I_X \quad \text{and} \quad f \circ g = I_Y$$

g is unique; called the **inverse** f^{-1} .

Both identity conditions are required. An f with only a left-inverse may not be onto; with only a right-inverse may not be one-one.

Bijection criterion

$f : X \rightarrow Y$ is **invertible iff f is bijective** (one-one and onto).

Then $f^{-1} : Y \rightarrow X$ exists and:

$$f^{-1}(y) = x \Leftrightarrow f(x) = y$$

Useful shortcut: to prove f invertible without finding f^{-1} explicitly, just show f is one-one and onto. Then existence is guaranteed.

Finding f^{-1} algebraically

Recipe for $f : X \rightarrow Y$ invertible:

1. Write $y = f(x)$.
2. Solve for x in terms of y : $x = g(y)$.
3. Then $f^{-1}(y) = g(y)$ (rename to $f^{-1}(x)$ if desired).

Example: $f : \mathbb{N} \rightarrow Y$, $f(x) = 4x + 3$, $Y = \{4x + 3 : x \in \mathbb{N}\}$

$$\Rightarrow f^{-1}(y) = \frac{y - 3}{4}$$

Always verify $f^{-1} \circ f(x) = x$ and $f \circ f^{-1}(y) = y$ to catch algebra errors.

Inverse properties

For invertible f, g with composable domains:

$$(f^{-1})^{-1} = f$$

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1} \quad (\text{order flips!})$$

$$I_A^{-1} = I_A$$

The order-flip rule for $(g \circ f)^{-1}$ is the analogue of $(AB)^{-1} = B^{-1}A^{-1}$ for matrices — “unwrap” the outermost operation first.

JEE/NEET Extension: equivalence relations on small sets

Number of equivalence relations on a set with n elements equals the **Bell number** B_n :

$$B_1 = 1, B_2 = 2, B_3 = 5, B_4 = 15, B_5 = 52.$$

On $\{1, 2, 3\}$ there are **5** equivalence relations; on $\{1, 2, 3\}$ containing both $(1, 2)$ and $(2, 1)$ there are exactly **2** (the smallest one with classes $\{1, 2\}, \{3\}$, and the universal one). On $\{1, 2, 3\}$, the number of relations

containing $(1, 2)$ and $(2, 3)$ that are reflexive and transitive but **not** symmetric is **3**.

RST \Rightarrow Equivalence

Reflexive + Symmetric + Transitive = Equivalence.

Lose any letter, lose the partition. “**RST** stays — the set partitions.”

Quick Reference — Chapter 1 at a Glance

Concept	Definition / Key Statement
Relation R on A	Any subset of $A \times A$; $(a, b) \in R$ means " $a R b$ ".
Empty relation	$R = \phi$; no element related to any.
Universal relation	$R = A \times A$; every pair related.
Reflexive	$(a, a) \in R$ for all $a \in A$.
Symmetric	$(a, b) \in R \Rightarrow (b, a) \in R$.
Transitive	$(a, b), (b, c) \in R \Rightarrow (a, c) \in R$.
Equivalence relation	Reflexive + Symmetric + Transitive; partitions A into classes.
Equivalence class $[a]$	$\{x \in A : (a, x) \in R\}$; classes are equal or disjoint.
Function $f : X \rightarrow Y$	Each $x \in X$ has exactly one image $f(x) \in Y$.
One-one (injective)	$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.
Onto (surjective)	Range = Y ; every $y \in Y$ has a pre-image.
Bijjective	One-one and onto; equivalent to invertibility.
Composition $g \circ f$	$(g \circ f)(x) = g(f(x))$; associative, generally not commutative.
Identity I_A	$I_A(x) = x$; $f \circ I_A = f = I_B \circ f$.
Invertibility	$\exists g$ with $g \circ f = I_X$, $f \circ g = I_Y$; iff f is bijective.
$(g \circ f)^{-1}$	$= f^{-1} \circ g^{-1}$ (order flips).
$ A \times B $	$ A \cdot B $.
# relations $A \rightarrow B$	$2^{ A \cdot B }$.
# functions $A \rightarrow B$	$ B ^{ A }$.
# bijections $A \rightarrow A$	$n!$ where $n = A $.
# equivalence relations on $\{1, \dots, n\}$	Bell number B_n : 1, 2, 5, 15, 52, ...

Coverage: NCERT Class 12 Maths Chapter 1 (post-rationalisation): 1.1 Introduction, 1.2 Types of Relations, 1.3 Types of Functions, 1.4 Composition of Functions and Invertible Function. Binary operations (pre-rationalisation 1.4 onwards) are no longer in syllabus and have been omitted.