



Collegedunia NCERT Notes

The Ultimate NCERT Revision Guide for Class 12 Mathematics

Chapter 1: Relations and Functions

What this chapter covers: the formal language of relations on a set, the four properties (reflexive, symmetric, transitive, equivalence), and the three kinds of functions (injective, surjective, bijective). The rationalised NCERT keeps only these; we include *composition* and *inverse* as JEE/NEET extensions, clearly tagged, because they remain heavily tested in entrance exams.

1 Introduction and Preliminaries

Mathematics in Class 11 introduced sets, Cartesian products, and the idea of a relation as a subset of a Cartesian product. Class 12 lifts that machinery into a sharper, more structural view: we no longer just *describe* pairs — we *classify* the rules that pair things together, and use those classifications to build powerful objects like equivalence classes and bijective functions.

This chapter is the conceptual gateway to most of higher mathematics. Inverse trigonometry, calculus, linear algebra, group theory — all rest on the language of relations and functions.

1.1 Quick recap: Cartesian product

For two non-empty sets A and B , the **Cartesian product** is

$$A \times B = \{(a, b) : a \in A, b \in B\}.$$

If $|A| = m$ and $|B| = n$, then $|A \times B| = mn$. The product is in general *not* commutative: $A \times B \neq B \times A$ unless $A = B$ or one of them is empty.

Cartesian Product — Size

$$|A \times B| = |A| \cdot |B|, \quad |A \times A| = |A|^2.$$

A subset of $A \times B$ is a **relation** from A to B .

1.2 Relation: the formal definition

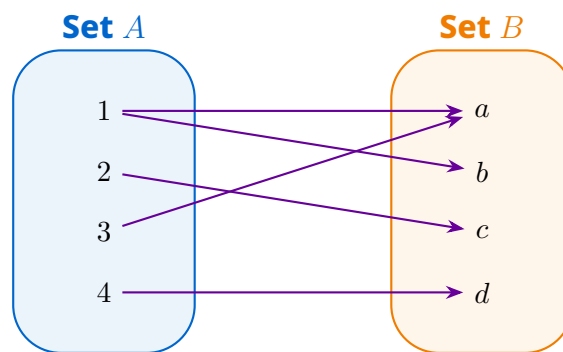
A **relation** R from set A to set B is any subset of $A \times B$. If $(a, b) \in R$ we write $a R b$ and say “ a is related to b .”

When $A = B$, we call R a **relation on** A .

Domain, Range, Codomain. For a relation $R \subseteq A \times B$:

- Domain of $R = \{a \in A : (a, b) \in R \text{ for some } b\}$
- Range of $R = \{b \in B : (a, b) \in R \text{ for some } a\}$
- Codomain of $R = B$ (the entire set we map *into*)

The range is always a subset of the codomain, and may be a proper subset.



$$\text{A relation: } R = \{(1, a), (1, b), (2, c), (3, a), (4, d)\}$$

Notice that element 1 relates to two elements of B . In a *relation* this is allowed; in a *function* (introduced next) it is not.

1.3 Function: a relation with a rule

A **function** $f : A \rightarrow B$ is a relation from A to B that satisfies two conditions:

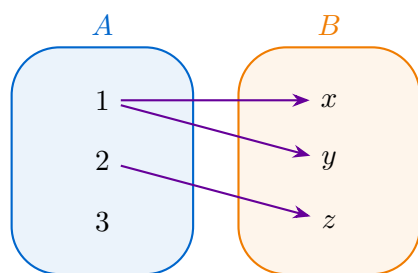
1. Every element of A has at least one image in B (**totality**).
2. Every element of A has *exactly one* image in B (**uniqueness**).

So a function is a special relation: every input gets exactly one output. We write $b = f(a)$ to denote that a maps to b .

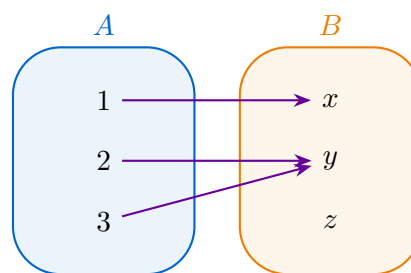
Function — Domain, Codomain, Range

For $f : A \rightarrow B$:

- Domain = A (the input set, in full)
- Codomain = B (the declared target set)
- Range = $f(A) = \{f(a) : a \in A\} \subseteq B$ (actual outputs)



Not a function
(1 has two images, 3 has none)



Is a function
(every input has exactly one image)

In the right panel, 2 and 3 share the same image y ; that is perfectly legal for a function. The illegal thing is one input branching to two outputs (left panel) or an input with no output at all.

Function vs Relation in one line

A **relation** pairs elements freely. A **function** insists each input has exactly one output. Every function is a relation; most relations are not functions.

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Codomain \neq Range

Students often write “the codomain of $f(x) = x^2, f : \mathbb{R} \rightarrow \mathbb{R}$ is $[0, \infty)$.” That’s the *range*. The codomain is whatever set we declared as the target — here, \mathbb{R} . Codomain is a *choice*; range is a *consequence*.

2 Types of Relations

The whole point of classifying relations is that certain combinations of properties give rise to extremely useful structures — especially **equivalence relations**, which let us partition a set into clean disjoint classes.

Throughout this section, R is a relation on a non-empty set A , i.e., $R \subseteq A \times A$.

2.1 Empty and Universal Relations

Two extreme cases bracket every other relation between them.

Empty relation: no element relates to anything. $R = \emptyset \subseteq A \times A$. Equivalently, aRb holds for no pair.

Universal relation: every element relates to every element. $R = A \times A$. Equivalently, aRb holds for every (a, b) .

Empty and Universal

$$R_{\text{empty}} = \emptyset \quad \text{and} \quad R_{\text{universal}} = A \times A.$$

These are sometimes called **trivial relations**.

Example. On $A = \{1, 2, 3\}$, define $R = \{(a, b) : a - b = 10\}$. No two elements of A differ by 10, so $R = \emptyset$ — the empty relation.

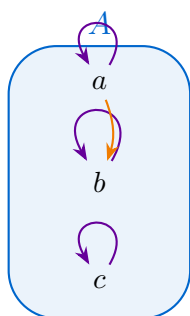
2.2 Reflexive Relation

A relation R on A is **reflexive** if every element relates to itself.

Reflexive

$$R \text{ is reflexive} \iff (a, a) \in R \quad \text{for every } a \in A.$$

Equivalently: aRa holds for all $a \in A$.



Each element has a self-loop (a, a) .
Extra pairs like (a, b) are allowed.

Examples.

- On \mathbb{R} , the relation \leq is reflexive: $x \leq x$ for every real x .
- On \mathbb{R} , the relation $<$ is *not* reflexive: $x < x$ is false.
- On the set of all lines in a plane, “is parallel to” is reflexive (every line is parallel to itself, by convention).

Reflexive needs every element

$R = \{(1, 1), (2, 2)\}$ on $A = \{1, 2, 3\}$ is **not reflexive** — the pair $(3, 3)$ is missing. Reflexive demands a self-loop on every element of A , no exceptions.

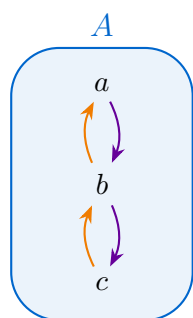
2.3 Symmetric Relation

A relation R on A is **symmetric** if whenever a relates to b , then b relates to a .

Symmetric

$$R \text{ is symmetric} \iff [(a, b) \in R \implies (b, a) \in R] \quad \forall a, b \in A.$$

Equivalently: $aRb \implies bRa$.



Every directed edge has a partner pointing the other way. (Self-loops are automatically symmetric.)

Examples.

- “ a is a sibling of b ” is symmetric (if a is b 's sibling, then b is a 's).
- “ a is the brother of b ” is *not* symmetric — if a is male and b is female, a is b 's brother but b is not a 's brother.
- On \mathbb{R} , the relation “ $|a - b| < 1$ ” is symmetric.
- “ a divides b ” on \mathbb{N} is *not* symmetric (2 divides 6 but 6 does not divide 2).

Symmetric in one check

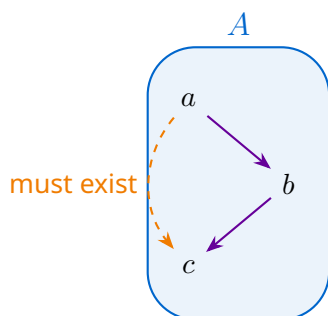
To prove R is symmetric, take an arbitrary $(a, b) \in R$ and *derive* $(b, a) \in R$ using the definition of R . Don't argue case by case unless A is small and listed.

2.4 Transitive Relation

A relation R on A is **transitive** if whenever a relates to b *and* b relates to c , then a relates to c .

Transitive

$$R \text{ is transitive} \iff [(a, b), (b, c) \in R \implies (a, c) \in R] \quad \forall a, b, c \in A.$$



If $a \rightarrow b$ and $b \rightarrow c$,
the short-cut $a \rightarrow c$
must also be in R .

Examples.

- “ \leq ” on \mathbb{R} is transitive: $a \leq b$ and $b \leq c \implies a \leq c$.
- “Is the mother of” is *not* transitive: A ’s mother is B , B ’s mother is C , but A ’s mother is not C (she’s the grandmother).
- “Divides” on \mathbb{N} is transitive: $a \mid b, b \mid c \implies a \mid c$.
- “Is perpendicular to” (on lines in a plane) is *not* transitive: $L_1 \perp L_2$ and $L_2 \perp L_3$ implies $L_1 \parallel L_3$, not $L_1 \perp L_3$.

“Vacuous truth” for transitivity

If you can’t find any (a, b) and (b, c) both in R for the *same* b , then transitivity holds vacuously. The empty relation is transitive for this reason — no premise, no obligation.

2.5 Equivalence Relation

A relation that is *simultaneously* reflexive, symmetric, and transitive is called an **equivalence relation**. These are the workhorses of mathematics — they let us treat related elements as “the same” for some specific purpose.

Equivalence Relation

R on A is an equivalence relation $\iff R$ is

1. Reflexive: $(a, a) \in R \ \forall a$
2. Symmetric: $(a, b) \in R \implies (b, a) \in R$
3. Transitive: $(a, b), (b, c) \in R \implies (a, c) \in R$

RST: the three checks

Equivalence = **R**eflexive + **S**ymmetric + **T**ransitive. Always check in this order; if any one fails, stop — the relation is not an equivalence.

Canonical example. On \mathbb{Z} , define $a \sim b$ iff $a - b$ is divisible by n (for a fixed $n \geq 2$). This is the relation of *congruence modulo n* , written $a \equiv b \pmod{n}$. It is an equivalence relation:

- Reflexive: $a - a = 0$ is divisible by n .

- Symmetric: if $n \mid (a - b)$ then $n \mid (b - a)$.
- Transitive: if $n \mid (a - b)$ and $n \mid (b - c)$, adding gives $n \mid (a - c)$.

2.6 Equivalence Classes and Partitions

Once we have an equivalence relation, we get a free gift: a **partition** of A into disjoint subsets.

Equivalence class of $a \in A$ is the set of all elements related to a :

$$[a] = \{x \in A : x \sim a\}.$$

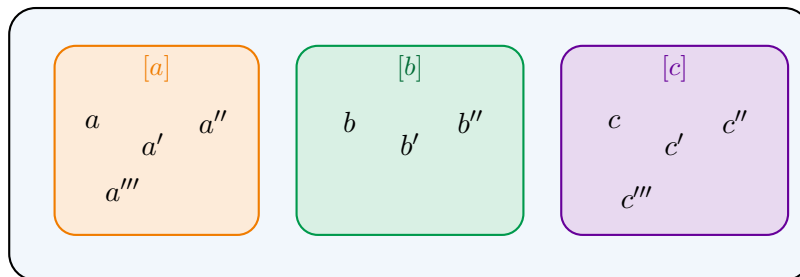
Fundamental Theorem on Equivalence Classes

For an equivalence relation \sim on A :

1. Every $a \in A$ lies in exactly one equivalence class (namely $[a]$).
2. Two classes $[a]$ and $[b]$ are either identical or disjoint — never overlapping in part.
3. The classes together cover all of A .

So the equivalence classes **partition** A .

Set A



Equivalence classes partition A into disjoint, exhaustive “blocks”.

Example (congruence mod 3). On \mathbb{Z} , congruence mod 3 has exactly three equivalence classes:

$$[0] = \{\dots, -6, -3, 0, 3, 6, \dots\},$$

$$[1] = \{\dots, -5, -2, 1, 4, 7, \dots\},$$

$$[2] = \{\dots, -4, -1, 2, 5, 8, \dots\}.$$

Every integer falls in exactly one of these three classes.

Why partitions matter

Grouping students by section, sorting files by month, bucketing IP addresses by subnet — these are all real-world uses of equivalence relations. Any time we “treat all members of a group as interchangeable” for some purpose, an

equivalence relation is silently at work.

Symmetric and Transitive does not imply Reflexive

A common faulty shortcut: "If aRb then bRa by symmetry, and then aRa by transitivity — so symmetric + transitive forces reflexive." The flaw: this only forces aRa for those a that already appear in *some* pair. If $a \in A$ but a is in no pair at all, the chain never starts, and (a, a) need not be in R . So you still must check reflexivity separately — it is independent of the other two.

3 Types of Functions

Functions split into three important classes based on how the inputs map to outputs: **one-one** (no two inputs share an output), **onto** (every codomain element is hit), and **bijective** (both at once). These are the building blocks of inverse functions, counting arguments, and the broader theory of cardinality.

Throughout this section, $f : A \rightarrow B$ is a function.

3.1 One-one (Injective) Function

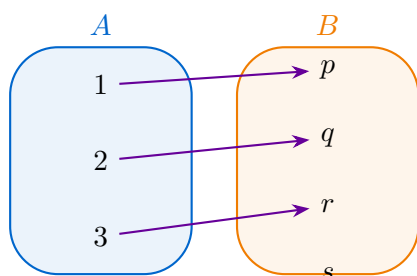
A function f is **one-one** (or **injective**) if distinct inputs always produce distinct outputs.

One-one (Injective)

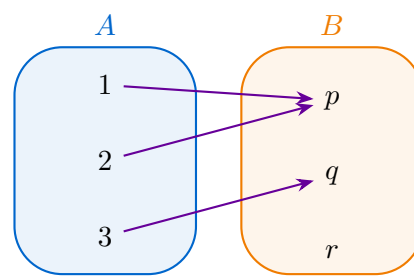
$$f \text{ is one-one} \iff [f(a_1) = f(a_2) \implies a_1 = a_2] \quad \forall a_1, a_2 \in A.$$

Equivalently (contrapositive): $a_1 \neq a_2 \implies f(a_1) \neq f(a_2)$.

The contrapositive form is usually easier to *visualise*; the direct form is usually easier to *prove* algebraically.



One-one (Injective)



Not one-one (many-to-one)

Algebraic test. To prove $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 2x + 5$ is one-one:

$$\begin{aligned} f(x_1) = f(x_2) &\implies 2x_1 + 5 = 2x_2 + 5 \\ &\implies 2x_1 = 2x_2 \\ &\implies x_1 = x_2. \end{aligned}$$

So f is one-one.

Functions that are NOT one-one (standard examples):

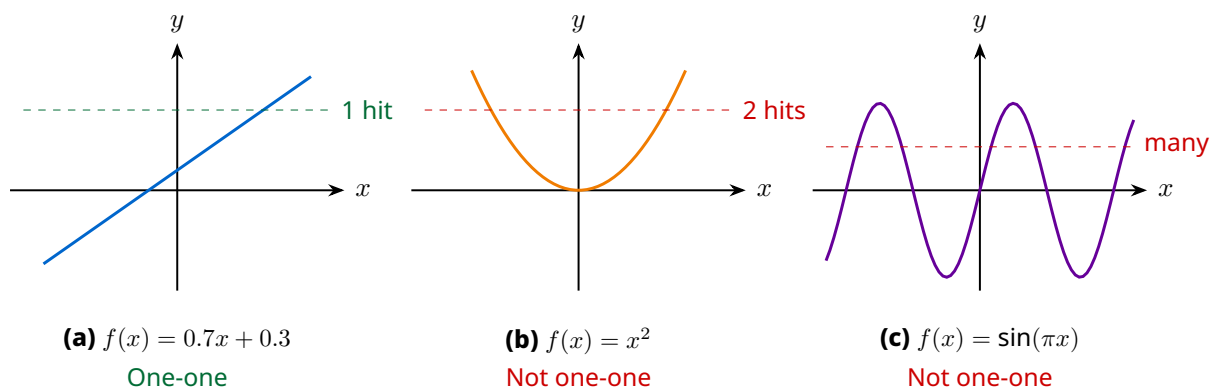
- $f(x) = x^2$ on \mathbb{R} : $f(-2) = f(2) = 4$ but $-2 \neq 2$.
- $f(x) = |x|$ on \mathbb{R} : $f(-3) = f(3) = 3$.
- $f(x) = \sin x$ on \mathbb{R} : $f(0) = f(\pi) = 0$.

3.2 Graphical (Horizontal Line) Test

For a function on \mathbb{R} given by its graph, the **horizontal line test** gives a quick visual check:

Horizontal Line Test for One-one

$f : \mathbb{R} \rightarrow \mathbb{R}$ is one-one \iff every horizontal line intersects the graph of f in **at most one point**.



Monotonic \implies one-one

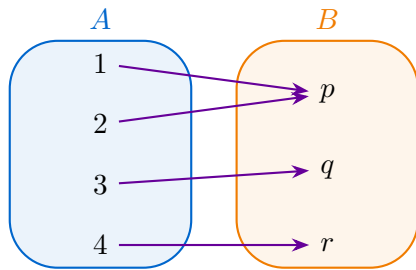
If f is *strictly increasing* or *strictly decreasing* on its domain, then f is automatically one-one. For a differentiable f , just check the sign of $f'(x)$: if $f'(x) > 0$ throughout (or $f'(x) < 0$ throughout), f is one-one. This is the cleanest test for calculus-style problems.

3.3 Onto (Surjective) Function

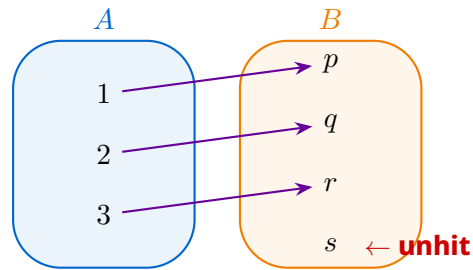
A function $f : A \rightarrow B$ is **onto** (or **surjective**) if every element of B is the image of *at least one* element of A . That is, the range equals the codomain.

Onto (Surjective)

$$f \text{ is onto} \iff \text{Range}(f) = B \iff \forall b \in B, \exists a \in A : f(a) = b.$$



Onto (surjective)
Every $b \in B$ is hit



Into (not onto)
 s has no preimage

A function that is *not* onto is sometimes called **into**. So every function is either onto or into — it's a partition of all functions.

Examples.

- $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2x + 5$: for any $y \in \mathbb{R}$, take $x = (y - 5)/2$ to get $f(x) = y$. So f is onto.
- $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^2$: there is no real x with $x^2 = -1$. So f is *not* onto. (But if we change the codomain to $[0, \infty)$, then it *is* onto.)
- $f : \mathbb{N} \rightarrow \mathbb{N}, f(n) = n + 1$: the output 1 is never produced (since $n \geq 1 \implies n + 1 \geq 2$). So f is *not* onto.

Onto depends on the declared codomain

$f(x) = x^2$ is onto if the codomain is $[0, \infty)$, but not onto if the codomain is \mathbb{R} . Whether a function is onto or not is meaningful only in relation to the codomain you stated. Always read the problem carefully for " $f : A \rightarrow B$ " to know what set you're mapping into.

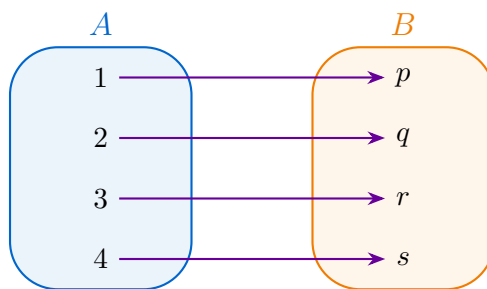
3.4 Bijective Function

A function that is *both* one-one and onto is called **bijective**.

Bijective

$$f \text{ is bijective} \iff f \text{ is one-one AND onto.}$$

For a bijection $f : A \rightarrow B$, every element of A pairs with exactly one element of B , and vice versa — a perfect pairing between the two sets.



Bijection: a perfect pairing

Bijection \iff Invertible

A function has an inverse if and only if it is bijective. This is the bridge to the next section, and the reason bijections matter so much.

Worked check. Show $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = 3x - 7$ is bijective.

One-one: $f(x_1) = f(x_2) \implies 3x_1 - 7 = 3x_2 - 7 \implies x_1 = x_2. \checkmark$

Onto: for $y \in \mathbb{R}$, take $x = (y + 7)/3 \in \mathbb{R}$; then $f(x) = 3 \cdot (y + 7)/3 - 7 = y. \checkmark$

So f is a bijection.

3.5 Standard Functions and their Types

Some functions appear so often you should memorise their type at sight. Domains assumed to be \mathbb{R} unless noted; codomain shown explicitly.

Function	Codomain	One-one?	Onto?
$f(x) = x$ (identity)	\mathbb{R}	Yes	Yes
$f(x) = ax + b, a \neq 0$	\mathbb{R}	Yes	Yes
$f(x) = x^2$	\mathbb{R}	No	No
$f(x) = x^2$	$[0, \infty)$	No (still)	Yes
$f(x) = x^3$	\mathbb{R}	Yes	Yes
$f(x) = x $	\mathbb{R}	No	No (image ≥ 0)
$f(x) = e^x$	\mathbb{R}	Yes	No (image > 0)
$f(x) = e^x$	$(0, \infty)$	Yes	Yes (bijection)
$f(x) = \log x (x > 0)$	\mathbb{R}	Yes	Yes
$f(x) = \sin x$	\mathbb{R}	No	No
$f(x) = \sin x, x \in [-\pi/2, \pi/2]$	$[-1, 1]$	Yes	Yes
$f(x) = \lfloor x \rfloor$ (GIF)	\mathbb{Z}	No	Yes

Restricting domain to make a bijection

Sine, cosine, and x^2 are not bijections on \mathbb{R} , but become bijections on suitable restricted domains. This is exactly how the inverse trig functions

$\sin^{-1}, \cos^{-1}, \tan^{-1}$ are defined in the next chapter. Remember the principal-value intervals — you'll need them.

Why injectivity matters

A national ID number must be **one-one**: two different people should never share the same ID, otherwise records get conflated. A passport mapping “person \rightarrow passport number” is engineered to be an injection. Insurance, banking, and government identity systems all rely on this property holding strictly.

4 Composition of Functions [JEE/NEET Extension]

This section is **not in the rationalised NCERT**, but appears extensively in JEE Main/Advanced and NEET. We cover it because every serious entrance exam candidate is expected to know it cold.

4.1 Definition of Composition

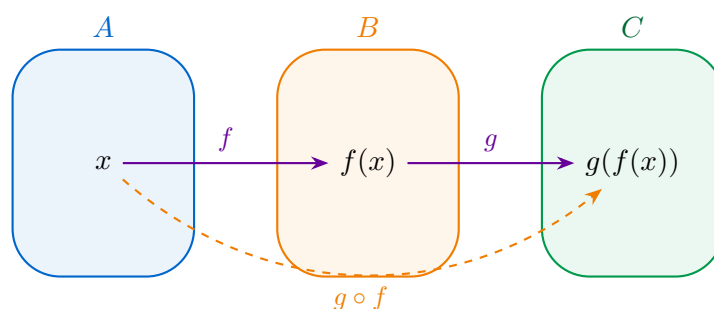
Given two functions $f : A \rightarrow B$ and $g : B \rightarrow C$, the **composition** $g \circ f : A \rightarrow C$ (read “ g composed with f ” or “ g of f ”) is the function defined by

$$(g \circ f)(x) = g(f(x)) \quad \text{for all } x \in A.$$

The output of f feeds in as the input of g . For $g \circ f$ to make sense, the codomain of f must lie inside the domain of g .

Composition

$$(g \circ f)(x) = g(f(x)), \quad \text{order matters: read right to left.}$$



Example. Let $f(x) = x + 2$ and $g(x) = x^2$. Then

$$(g \circ f)(x) = g(f(x)) = g(x + 2) = (x + 2)^2$$

while

$$(f \circ g)(x) = f(g(x)) = f(x^2) = x^2 + 2.$$

In general $g \circ f \neq f \circ g$. Composition is **not commutative**.

Order matters: $g \circ f \neq f \circ g$

$(g \circ f)(x)$ means “apply f first, then g .” Students often read left-to-right and apply g first — producing $f \circ g$ by accident. The trick: the function written *closer to x* acts *first*. Always.

4.2 Properties of Composition

1. **Associative:** $(h \circ g) \circ f = h \circ (g \circ f)$, whenever all compositions are defined.
2. **Identity acts trivially:** if I_A and I_B are identity functions on A and B , then $f \circ I_A = f$ and $I_B \circ f = f$.
3. **One-one preserved:** if f and g are both one-one, so is $g \circ f$.
4. **Onto preserved:** if f and g are both onto, so is $g \circ f$.
5. **Bijection preserved:** f, g bijective $\implies g \circ f$ bijective.

Inverse of a composition

If f and g are both bijections, then $g \circ f$ is invertible and

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}.$$

“Reverse the order when you invert” — like taking socks off before shoes.

Socks-and-shoes rule

Putting on: socks first, then shoes — $g \circ f$ (where f = socks, g = shoes). Taking off: shoes first, then socks — $f^{-1} \circ g^{-1}$. The order *reverses* when you invert.

4.3 Worked example

Let $f(x) = 2x + 3$ and $g(x) = x^2 + 1$. Find $f \circ g$, $g \circ f$, $f \circ f$, and verify $(g \circ f)(2)$ directly.

$$(f \circ g)(x) = f(g(x)) = f(x^2 + 1) = 2(x^2 + 1) + 3 = 2x^2 + 5$$

$$(g \circ f)(x) = g(f(x)) = g(2x + 3) = (2x + 3)^2 + 1 = 4x^2 + 12x + 10$$

$$(f \circ f)(x) = f(f(x)) = f(2x + 3) = 2(2x + 3) + 3 = 4x + 9$$

Check: $(g \circ f)(2) = 4(4) + 12(2) + 10 = 16 + 24 + 10 = 50$.

Direct: $f(2) = 7$, $g(7) = 49 + 1 = 50$. ✓

5 Inverse of a Function [JEE/NEET Extension]

Again, removed from the rationalised NCERT but heavily tested at JEE/NEET.

5.1 When does a function have an inverse?

A function $f : A \rightarrow B$ is called **invertible** if there exists a function $g : B \rightarrow A$ such that

$$g \circ f = I_A \quad \text{and} \quad f \circ g = I_B$$

where I_A, I_B are the identity functions on A and B . When such a g exists, it is unique; we call it the **inverse** of f and denote it f^{-1} .

Existence of Inverse

$f : A \rightarrow B$ has an inverse $\iff f$ is bijective.

This is why bijections are so important: they are precisely the invertible functions.

5.2 How to find an inverse

For a concrete $f : \mathbb{R} \rightarrow \mathbb{R}$ (or some subset), the recipe is:

1. Write $y = f(x)$.
2. Solve for x in terms of y .
3. The result $x = h(y)$ defines f^{-1} . Optionally swap $y \leftrightarrow x$ for the answer in standard form.

Example. Find f^{-1} for $f(x) = \frac{3x-2}{5}$.

$$y = \frac{3x-2}{5}$$

$$5y = 3x - 2$$

$$x = \frac{5y+2}{3}$$

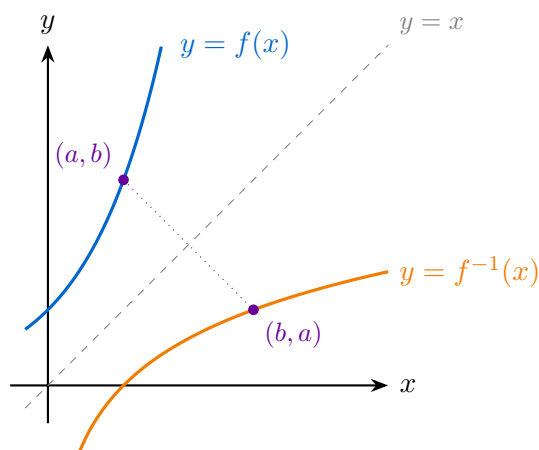
So $f^{-1}(y) = \frac{5y+2}{3}$, or in standard form $f^{-1}(x) = \frac{5x+2}{3}$.

The inverse undoes operations in reverse order

f first multiplies by 3, then subtracts 2, then divides by 5. f^{-1} first multiplies by 5, then adds 2, then divides by 3. The operations are inverted *and* their order is reversed — exactly like the socks-and-shoes rule.

5.3 Graphical view of the inverse

The graph of $y = f^{-1}(x)$ is the **reflection** of the graph of $y = f(x)$ across the line $y = x$.



If (a, b) lies on f , then (b, a) lies on f^{-1} — because $b = f(a)$ means $a = f^{-1}(b)$, swapping the roles of input and output.

5.4 Key properties of inverses

For bijective $f : A \rightarrow B$ and $g : B \rightarrow C$:

- $(f^{-1})^{-1} = f$.
- $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ (socks-and-shoes).
- Domain of $f^{-1} = \text{Codomain of } f$.
- Range of $f^{-1} = \text{Domain of } f$.

Inverse functions in cryptography

A public-key cipher is a function f that is *easy* to compute (encrypt) but whose inverse f^{-1} is *computationally infeasible* to find without a secret. The whole security of RSA, ECC, and modern internet banking rests on the existence of bijective functions whose inverse is hard to compute — a deep, beautiful corner of the same theory you're studying here.

6 Counting Relations and Functions

A category of problems that shows up in board MCQs, JEE, and NEET is: “How many relations / how many functions of type X exist between sets of given sizes?” The counting follows from clean combinatorial arguments.

Let $|A| = m$ and $|B| = n$ throughout.

6.1 Counting Relations on a set

A relation on A is a subset of $A \times A$. Since $|A \times A| = m^2$ and a set with m^2 elements has 2^{m^2} subsets,

$$\text{Number of relations on } A = 2^{m^2}.$$

For a relation from A to B (subset of $A \times B$, size mn):

$$\text{Number of relations from } A \text{ to } B = 2^{mn}.$$

Counting Relations

$$\text{All relations from } A \text{ to } B = 2^{mn}$$

$$\text{All relations on } A = 2^{m^2}$$

$$\text{Reflexive relations on } A = 2^{m^2-m}$$

$$\text{Symmetric relations on } A = 2^{m(m+1)/2}$$

Why the reflexive count is 2^{m^2-m} : reflexivity forces the m diagonal pairs (a, a) to be in R . The remaining $m^2 - m$ off-diagonal positions are each free choices, giving 2^{m^2-m} possibilities.

Why symmetric is $2^{m(m+1)/2}$: pair up (a, b) with (b, a) ; each pair is either both in or both out. The diagonal positions (a, a) are paired with themselves. Total “free choices” = m (diagonal) + $\binom{m}{2}$ (off-diagonal pairs) = $m(m+1)/2$.

6.2 Counting Functions

A function $f : A \rightarrow B$ assigns each of the m elements of A to one of the n elements of B — independent choices, so:

$$\text{Total functions from } A \text{ to } B = n^m.$$

Counting Functions

$$\text{Total functions } A \rightarrow B = n^m$$

$$\text{One-one functions } (m \leq n) = \frac{n!}{(n-m)!} = {}^n P_m$$

$$\text{Onto functions } (m \geq n) = \sum_{k=0}^n (-1)^k \binom{n}{k} (n-k)^m$$

$$\text{Bijections } (m = n) = n!$$

Why one-one is ${}^n P_m$: the first element of A has n choices in B , the second has $n-1$ (must avoid the first’s image), the third has $n-2$, etc. Product = $n(n-1) \cdots (n-m+1) = {}^n P_m$.

Why bijections need $m = n$: an injection requires $m \leq n$; a surjection requires $m \geq n$. Both at once force $m = n$. Then every one-one function is automatically onto (and vice versa), with count $n!$.

Small- m , small- n shortcuts

You won’t need the inclusion-exclusion onto formula for boards. JEE rarely

asks for it with $m > 4$ or $n > 4$. For small cases like $m = n = 3$, onto = bijection = $3! = 6$. For $m = 3, n = 2$: onto = $2^3 - 2 = 6$ (total functions minus the two constant ones).

Example. Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$. How many functions $A \rightarrow B$ are onto?
 Total functions = $2^3 = 8$. The non-onto functions are those that miss at least one element of B — that means everything maps to a , or everything maps to b : 2 such constant functions. Onto = $8 - 2 = 6$.

6.3 Counting equivalence classes

The number of equivalence classes of an equivalence relation \sim on A equals the number of *blocks* in the partition of A it induces. Standard fact:

$$|A| = \sum_{\text{classes } [a]} |[a]|.$$

Example. On \mathbb{Z} , congruence mod n gives exactly n equivalence classes: $[0], [1], \dots, [n - 1]$.

7 Quick Reference Summary

A one-stop lookup card. Everything in this chapter, condensed.

7.1 Definitions

Concept	Definition
Relation R on A	Any subset of $A \times A$
Reflexive	$(a, a) \in R$ for all $a \in A$
Symmetric	$(a, b) \in R \implies (b, a) \in R$
Transitive	$(a, b), (b, c) \in R \implies (a, c) \in R$
Equivalence relation	Reflexive AND Symmetric AND Transitive
Equivalence class $[a]$	$\{x \in A : x \sim a\}$
Function $f : A \rightarrow B$	Relation where every $a \in A$ has exactly one image
One-one (injective)	$f(a_1) = f(a_2) \implies a_1 = a_2$
Onto (surjective)	Range = B ; every $b \in B$ has a preimage
Bijjective	One-one AND Onto
Composition $(g \circ f)(x)$	$g(f(x))$
Invertible	Bijjective; admits f^{-1} such that $f \circ f^{-1} = I$

7.2 Key Formulas (Counting)

Counting Formulas — Memorise

For $|A| = m$, $|B| = n$:

$$\text{Relations from } A \text{ to } B = 2^{mn}$$

$$\text{Relations on } A = 2^{m^2}$$

$$\text{Reflexive relations on } A = 2^{m^2-m}$$

$$\text{Symmetric relations on } A = 2^{m(m+1)/2}$$

$$\text{Functions from } A \text{ to } B = n^m$$

$$\text{One-one functions } (m \leq n) = \frac{n!}{(n-m)!}$$

$$\text{Bijections } (m = n) = m!$$

$$\text{Onto functions } (m \geq n) = \sum_{k=0}^n (-1)^k \binom{n}{k} (n-k)^m$$

7.3 Property checklist for relations

When asked “Is R an equivalence relation?”, work through these in order:

1. **Reflexive:** pick an arbitrary $a \in A$ and verify $(a, a) \in R$.
2. **Symmetric:** assume $(a, b) \in R$ and derive $(b, a) \in R$.
3. **Transitive:** assume $(a, b), (b, c) \in R$ and derive $(a, c) \in R$.
4. If all three pass, declare R an equivalence relation. List the equivalence classes if asked.

7.4 Property checklist for functions

1. **Is it a function?** Check totality and uniqueness from the definition or arrow diagram.
2. **One-one?** Algebraically: $f(x_1) = f(x_2) \implies x_1 = x_2$. Graphically: horizontal line test. Calculus: strict monotonicity.
3. **Onto?** Take an arbitrary $y \in B$, solve $f(x) = y$ for $x \in A$, check that the solution actually lies in A .
4. **Bijective?** Both of the above. Then f^{-1} exists.

7.5 Final word

This chapter looks dense but is fundamentally three ideas:

- A **relation** is a set of pairs.
- A **function** is a relation where each input has exactly one output.
- Classifying relations (R, S, T) and functions (injective, surjective, bijective) lets us build the rest of mathematics on a rigorous foundation.

Master the definitions, do enough problems to feel comfortable producing the standard proofs (showing a relation is symmetric, showing a function is one-one), and the rest of Class 12 mathematics — inverse trigonometry, calculus, linear algebra — becomes notation-friendly rather than notation-heavy.

Best of luck with your preparation!

Revise, practise, and trust your reasoning.