Collegedunia NCERT Formula Sheet

Class 12 Mathematics — Chapter 12

Chapter 12: Linear Programming

Term	Meaning
Decision variables	The unknowns x, y whose values we seek (e.g. number of tables, chairs).
Objective function Z	Linear function $Z = ax + by$ to be maximised or minimised.
Constraints	Linear inequalities/equations restricting x, y (resources, demand, storage).
Non-negative restrictions	$x \geq 0$, $y \geq 0$ — standard for all NCERT LPPs.
Feasible region (FR)	Common region satisfying ALL constraints including $x, y \geq 0$.
Feasible solution	Any point inside or on the boundary of FR.
Infeasible solution	Any point outside FR (violates at least one constraint).
Optimal solution	Feasible point at which Z attains its maximum or minimum.

1 Linear Programming Problem (LPP)

This section sets up the language of an LPP — the objective function, the linear constraints, and what counts as a feasible vs. optimal solution. Every Class 12 LPP follows this standard form.

Standard form of an LPP (two variables)

Maximise (or minimise):

$Z = ax + by$

subject to: $a_i x + b_i y \{\leq, =, \geq\} c_i$

and $x \geq 0, \; y \geq 0$

where x, y = **decision variables**; a, b = constants in objective; a_i, b_i, c_i = constraint constants.

The objective and every constraint must be **linear** in x, y. Quadratic, product or modulus terms disqualify a problem as LPP.

What is an LPP?

A Linear Programming Problem seeks the **optimal value** (max or min) of a linear objective function of several non-negative variables, subject to a set of linear constraints. "Linear" means every relation involves only first powers of variables; "programming" means planning a course of action.

Objective function vs. constraints

Objective function $Z = ax + by$ is what we optimise — typically profit (to maximise) or cost (to minimise). **Constraints** are the restrictions on x, y from limited resources (capital, storage, raw material, labour, demand). Non-negativity $x, y \geq 0$ is always present because decision variables represent physical quantities.

Linear $\neq$ equal-sign

Linear constraints can be inequalities ($\leq, \geq$) **or** equations ($=$). Most NCERT LPPs use $\leq$ for resource limits (capital, storage) and $\geq$ for demand/diet requirements. Do not assume "constraint" means equation only.

2 Feasible & Infeasible Regions

This section defines the geometric arena of an LPP. Drawing the half-planes determined by every constraint and intersecting them yields the feasible region; everything else is infeasible.

Feasible region (FR)

FR $= \{(x, y)$: all constraints AND $x \geq 0, y \geq 0$ hold$\}$

where each linear inequality $ax + by \leq c$ (or $\geq c$) defines a closed half-plane.

FR is the **intersection of all such half-planes** including the first-quadrant constraint. It is always a **convex region** (possibly empty, a line segment, polygon, or unbounded polygon).

Feasible vs. infeasible solution

Feasible solution: any $(x, y) \in$ FR (inside or on boundary).

Infeasible solution: any $(x, y) \notin$ FR.

Optimal solution: $(x^*, y^*) \in$ FR that gives Z_{max} or Z_{min}.

Every point of FR is a candidate; only **corner points** can be optimal (Theorem 1). The numerical optimum is the value of Z at that corner.

Bounded vs. unbounded FR

Bounded: the FR can be enclosed inside some circle of finite radius — it is a closed polygon. **Unbounded:** the FR extends to infinity in at least one direction. Boundedness is decided by inspection of the graph; it controls whether max and min are both guaranteed (see Theorems 1 & 2).

Empty feasible region

If the constraint half-planes have **no common point**, the FR is empty and the LPP has *no feasible solution*. Always check the graph before evaluating Z. NCERT Example 5 ($x + y \geq 8$, $3x + 5y \leq 15$, $x, y \geq 0$) is empty.

3 Corner Point Method — Theorems

This section states the two fundamental theorems that justify the entire graphical method: the optimum, when it exists, must occur at a corner (vertex) of FR.

Theorem 1 — Optimum at a corner

Let R be the FR (convex polygon) of an LPP and $Z = ax + by$ the objective.

If Z has an **optimal value** (max or min) and x, y are subject to the linear constraints, then this optimum occurs at a **corner point (vertex)** of R.

Reduces an infinite search over R to a **finite check at vertices**. Foundation of the corner-point method.

Theorem 2 — Bounded FR

If R is **bounded**, then $Z = ax + by$ attains **both** a maximum and a minimum on R, and each occurs at a corner point of R.

For bounded FRs, the largest entry in the table of corner values is Z_{max} and the smallest is Z_{min}. No further checks needed.

Unbounded FR — extra check needed

If R is **unbounded**, max or min may not exist. If a candidate value M (or m) from corner points is to be the actual optimum:

- For **maximum** M: the open half-plane $ax + by > M$ must have **no point** in common with R.
- For **minimum** m: the open half-plane $ax + by < m$ must have **no point** in common with R.

If the half-plane intersects R, that extremum **does not exist**.

Multiple optimal solutions

If Z attains the same optimum at two corner points C_1 and C_2, then **every point on the line segment** C_1C_2 also gives that optimum. The LPP has *infinitely many* optimal solutions (NCERT Example 3: $Z = 3x + 9y$, max $= 180$ on segment CD).

4 Graphical Solution Procedure

This section lists the step-by-step recipe used for every NCERT LPP — plot, find corners, tabulate Z, pick the optimum (with the unbounded check if needed).

Corner Point Method — algorithm

Step 1. Draw the FR by plotting each constraint as a line and shading the correct half-plane; intersect to get FR.

Step 2. Find all **corner points** (vertices) — either by inspection or by solving the pair of boundary equations meeting at each vertex.

Step 3. Evaluate $Z = ax + by$ at every corner; tabulate.

Step 4a. If FR is **bounded:** largest value $= Z_{\max}$, smallest $= Z_{\min}$.

Step 4b. If FR is **unbounded:** let $M =$ largest, $m =$ smallest corner values. Check $ax + by > M$ (or $< m$) against FR — if no common point, M (m) is the optimum; otherwise it does not exist.

Convexity of FR guarantees Step 2's vertex list is finite. **Theorems 1–2** guarantee Steps 4a–b cover every case.

Worked example — tables & chairs

Constraints: $5x + y \le 100$, $x + y \le 60$, $x, y \ge 0$. FR = quadrilateral $OABC$ with vertices $O(0,0)$, $A(20,0)$, $B(10,50)$, $C(0,60)$.

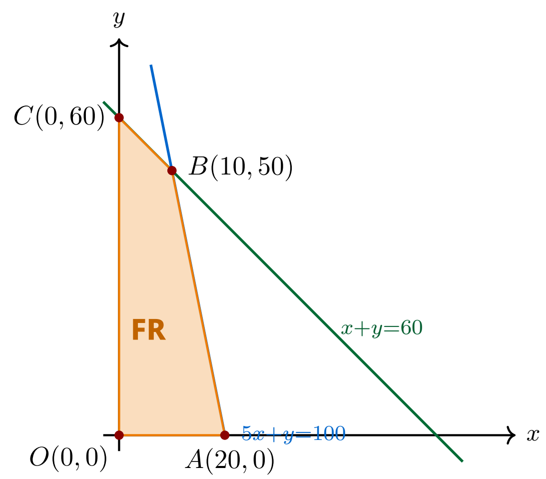
Vertex	$Z = 250x + 75y$
$O(0,0)$	0
$A(20,0)$	5000
$B(10,50)$	6250
$C(0,60)$	4500

$Z_{\max} = 6250$ at $B(10,50)$.

Buy **10 tables** and **50 chairs** for maximum profit Rs 6250.

5 Diagram — Bounded Feasible Region

A typical bounded FR for a 2-variable LPP with two resource constraints and non-negativity. Corners O, A, B, C are the only candidates for the optimum.



Shaded quadrilateral $OABC$ is the bounded FR. The optimum of any linear $Z = ax + by$ is attained at one of $\{O, A, B, C\}$.

6 Types of LPP (NCERT)

NCERT classifies LPPs by the real-life situation they model. The mathematics is identical — only the meaning of x, y and the form of constraints change.

Manufacturing problem

A firm produces **two products** using limited resources (labour, machine hours, raw material). Variables x, y = units of each product. Objective: **maximise profit** (or revenue). Constraints: resource availability $\leq$ given supply; non-negativity. (Furniture / tables-chairs problem is the canonical example.)

Diet problem

A diet must contain **at least** a prescribed amount of each nutrient using two food sources. Variables x, y = quantities of each food. Objective: **minimise cost** of the diet. Constraints: each nutrient $\geq$ minimum requirement; non-negativity. (Constraints use $\geq$ since intake must *exceed* thresholds.)

Transportation problem

Goods are shipped from sources (factories, godowns) to destinations (markets, depots) at known per-unit transport costs. Variables = quantities along each route. Objective: **minimise total transportation cost**. Constraints: supply at each source $=$ shipped out; demand at each destination $=$ shipped in; non-negativity.

JEE / Boards extension

Some LPPs also ask to **maximise revenue** (when profit per unit is unknown) or **minimise resource use** (when prices are fixed). The mathematical structure — linear Z + linear constraints + non-negativity — is identical. The corner-point method works unchanged.

7 Key Properties & Remarks

A short list of facts that follow from the graphical method and are frequently tested in MCQs and very-short-answer questions.

General features of every LPP

1. The feasible region is always a **convex** set (possibly unbounded).
2. Any local optimum on the FR is a **global** optimum (consequence of linearity + convexity).
3. The optimum is attained at **a vertex** of FR (Theorem 1).
4. If two corners give the same optimum, every point on the segment joining them also does.
5. Adding more constraints can only **shrink** the FR (cannot improve the optimum).

"No feasible region" vs. "No optimum"

No feasible region: constraints inconsistent — FR is empty; problem has no solution at all.

No optimum: FR exists but is unbounded in the direction of improvement — objective can grow without limit (or decrease without limit). Diagnose by graphing and applying the open-half-plane test for unbounded FRs.

FROCK — 5-step recall

Feasible region $\to$ **R**ead off corners $\to$ **O**bjective evaluated at each $\to$ **C**ompare values $\to$ **K**eep the best (with unbounded check if FR is open).

8 Quick Reference — Solved Snapshots

A consolidated look at the four standard outcomes that NCERT examples illustrate. Recognising which case you are in saves time in the exam.

Case map: bounded vs. unbounded

FR type	Conclusion
Bounded, non-empty	Both Z_{max} and Z_{min} exist; each at a corner
Unbounded, non-empty	Z_{max} / Z_{min} may or may not exist (apply open-half-plane test)
Empty (no FR)	No feasible solution; problem unsolvable
Z equal at 2 corners	Infinitely many optima on the joining segment

Exam-day checklist

1. State x, y and their meaning. 2. Write objective $Z = ax + by$. 3. List every constraint including $x, y \geq 0$. 4. Plot lines, shade FR, mark vertices. 5. Tabulate Z at vertices. 6. Quote the conclusion in words (e.g. "Buy 10 tables and 50 chairs for maximum profit Rs 6250").