Collegedunia NCERT Notes

The Ultimate NCERT Revision Guide for Class 12 Mathematics

Chapter 12: Linear Programming

What this chapter covers: the formulation of linear programming problems (LPPs) in two variables, the graphical (corner-point) method of finding the optimal value of a linear objective function subject to linear inequality constraints, and the distinction between bounded, unbounded and infeasible cases. We close with several fully worked manufacturing/diet/transport examples that mirror the patterns CBSE and JEE actually test.

1 Introduction and the LPP Idea

Class XI introduced systems of linear inequalities and their graphical solution sets. Class XII puts those half-planes to work: out of the infinitely many points that satisfy a set of constraints, we now want the *best* one — the point that maximises profit, or minimises cost, or stretches a limited resource the furthest. That "best" point is the answer to a **Linear Programming Problem (LPP)**.

Linear programming was born in the 1940s. The Russian mathematician **L. Kantorovich** (1939) and the American economist **F. L. Hitchcock** (1941) independently formulated the transportation problem; **G. B. Dantzig** (1947) gave the simplex algorithm; Kantorovich and **T. C. Koopmans** shared the 1975 Nobel Prize in Economics for this work. Today LPP underpins everything from airline crew scheduling to portfolio optimisation.

1.1 The motivating problem

A furniture dealer has a budget of Rs. $50{,}000$ and storage for at most 60 pieces. A table costs Rs. 2500 and yields a profit of Rs. 250; a chair costs Rs. 500 and yields a profit of Rs. 75. He can sell whatever he buys. How many tables and chairs should he buy to maximise his profit?

Let x = number of tables, y = number of chairs. The dealer cannot buy negative items, so $x \geq 0,\ y \geq 0$. His money is bounded by the budget and his storage is bounded by floor space:

$$2500x + 500y \leq 50000 \quad\iff\quad 5x + y \leq 100, \qquad x + y \leq 60.$$

His profit, the thing he wants to push as high as possible, is

$$Z = 250x + 75y.$$

The question “what (x, y) maximises Z while obeying every constraint?” is a textbook LPP.

The Three Ingredients of Every LPP

Every linear programming problem has exactly three parts:

- **Decision variables** x, y (here: number of tables, number of chairs). They are unknowns to be found, and almost always **non-negative**.
- **Objective function** $Z = ax + by$. A *linear* expression in the decision variables that is to be **maximised** (profit) or **minimised** (cost).
- **Constraints**: a set of *linear inequalities* (sometimes equations) in x and y that the variables must simultaneously satisfy.

Contents

1 Introduction and the LPP Idea **1**
1.1 The motivating problem 1
1.2 What “linear” means here 3
1.3 Optimisation problems in general 4

2 Mathematical Formulation of an LPP **4**
2.1 The five-step formulation recipe 4
2.2 Formulating the furniture problem completely 5
2.3 A diet-style formulation 5

3 Graphical Solution: The Corner-Point Method **6**
3.1 Feasible region, feasible solutions 6
3.2 Two foundational theorems 7
3.3 Why corners? An intuitive justification 7
3.4 The Corner-Point Method — algorithm 8
3.5 Solving the furniture problem 8

4 Bounded, Unbounded and Infeasible Cases **9**
4.1 Bounded feasible region 9
4.2 Unbounded feasible region 9

4.3 Infeasible problem 10
4.4 Multiple optimal solutions 11

5 Worked Examples 11
5.1 Example 1 — Bounded maximisation 11
5.2 Example 2 — Bounded minimisation (diet) 12
5.3 Example 3 — Both max and min, with multiple optima 13
5.4 Example 4 — Unbounded feasible region 14
5.5 Example 5 — Infeasible problem 15
5.6 Example 6 — A complete manufacturing problem 16
5.7 Example 7 — Diet problem, fully worked 17

6 Strategy, Pitfalls and Quick Reference 18
6.1 Exam strategy — a 6-minute LPP 18
6.2 Pitfalls compiled 19
6.3 Bounded vs unbounded vs infeasible — comparison 19
6.4 One-page formula and method recap 20
6.5 Where this chapter goes next 20

1.2 What “linear” means here

The word **linear** means every relation in the problem is a first-degree expression in the decision variables. No squares (x^2), no products of variables (xy), no fractions ($1/x$), no exponentials. Geometrically, every constraint cuts the plane along a straight line and every level set of Z is also a straight line. Without linearity the corner-point method breaks down and we would need calculus or numerical methods.

General Two-Variable LPP

$$\begin{aligned} \text{Optimise} \quad & Z = ax + by \\ \text{subject to} \quad & a_i x + b_i y \ \{\le, =, \ge\}\ c_i, \quad i = 1, 2, \dots, m \\ \text{and} \quad & x \ge 0, \ y \ge 0. \end{aligned}$$

Optimise means either maximise or minimise, depending on the problem. The conditions $x, y \ge 0$ are called **non-negative restrictions**; the remaining ones are the **problem constraints**.

Quick Tip

If the problem talks about quantities such as “number of items”, “hours of labour”, or “kilograms of food”, the non-negativity $x, y \ge 0$ is automatic — but

> you still must write it down. Examiners deduct marks for missing it.

1.3 Optimisation problems in general

A problem that seeks to maximise or minimise some quantity subject to constraints is called an **optimisation problem**. LPPs are the special case where both the objective and the constraints are linear. Other optimisation problems — minimising surface area for a given volume, fastest path through a graph — need different tools (calculus, dynamic programming, integer programming). The graphical method below works *only* when there are two decision variables and all relations are linear.

> **Real-World Application**
>
> The same furniture-dealer template scales up: airlines decide how many flights of each aircraft type to run, factories decide how many shifts of which product, hospitals decide nurse rosters. With dozens of variables and thousands of constraints the simplex method (1947) or its modern interior-point cousins solve LPPs in milliseconds.

2 Mathematical Formulation of an LPP

Formulation is half the marks. CBSE and JEE both reward students who can translate a word problem into the four-block form

$$\boxed{\text{Decision variables} \;\rightarrow\; \text{Objective} \;\rightarrow\; \text{Constraints} \;\rightarrow\; \text{Non-negativity}}$$

without skipping any step. The dealer problem above is one template; we now formalise the procedure and apply it to a fresh example.

2.1 The five-step formulation recipe

1. **Identify what is being chosen.** The decision variables x, y name the quantities the decision-maker controls.
2. **Identify the objective.** What is to be maximised (profit, output, sales) or minimised (cost, time, waste)? Write it as a linear function $Z = ax + by$.
3. **Translate every resource limit into an inequality.** Money, time, raw material, demand, supply — each gives one inequality.
4. **Add non-negativity.** $x \geq 0,\ y \geq 0$ unless the problem explicitly allows negative values.
5. **State the LPP cleanly.** “Maximise $Z = \ldots$ subject to $\ldots$” — with every constraint numbered.

Decision variables, Objective, Constraints

Decision variables x, y: the unknowns the problem wants you to choose.
Objective function $Z = ax + by$: the linear quantity to optimise; a, b are given constants.
Constraints: the inequalities (or equations) the decision variables must satisfy.
Non-negative restrictions: $x \geq 0,\ y \geq 0$. These are constraints too, but kept separate by tradition.

2.2 Formulating the furniture problem completely

For the dealer with Rs. $50{,}000$ and 60 pieces of storage:

$$\begin{aligned} \text{Maximise:}\quad & Z = 250x + 75y \\ \text{Subject to:}\quad & 5x + y \leq 100 \quad \text{(money, in units of Rs.}500\text{)} \\ & x + y \leq 60 \quad \text{(storage)} \\ & x \geq 0,\ y \geq 0. \end{aligned}$$

The four inequalities together define a region in the xy-plane — the *feasible region*. Every interior or boundary point of that region represents a *permissible* purchasing plan; the goal of Section 3 is to pick the plan that pushes Z to its highest value.

Quick Tip

Always scale constraints to nicer integers when possible. $2500x + 500y \leq 50000$ becomes $5x + y \leq 100$ after dividing by 500 — much easier to graph. Just don't divide by a variable or change the inequality direction.

2.3 A diet-style formulation

A dietician wishes to mix two foods F_1 and F_2. One kg of F_1 contains 3 units of vitamin A and 4 units of mineral; one kg of F_2 contains 6 units of vitamin A and 3 units of mineral. The mix must contain at least 80 units of vitamin A and 100 units of mineral. F_1 costs Rs. 5/kg and F_2 costs Rs. 7/kg. Minimise cost.

Let x, y = kg of F_1 and F_2. Then

$$\begin{aligned} \text{Minimise:}\quad & Z = 5x + 7y \\ \text{Subject to:}\quad & 3x + 6y \geq 80 \quad \text{(vitamin A)} \\ & 4x + 3y \geq 100 \quad \text{(mineral)} \\ & x \geq 0,\ y \geq 0. \end{aligned}$$

Note the inequalities are $\geq$ here, not $\leq$. Maximisation problems usually have $\leq$ constraints (resource ceilings); minimisation problems usually have $\geq$ constraints (nutritional floors). It is not a hard rule, but a useful pattern.

Maximisation vs Minimisation Patterns

- **Maximisation** (profit, output): constraints typically read *at most*, i.e. $\leq$. Feasible region is usually a **bounded** polygon hugging the origin.
- **Minimisation** (cost, time): constraints typically read *at least*, i.e. $\geq$. Feasible region is usually **unbounded**, extending away from the origin.

Common Mistake

Reading "at most" as $\geq$ or "at least" as $\leq$ is the single most common formulation error. Slow down: *at most 60* means ≤ 60, *at least 80* means ≥ 80. Sketch a number line if you must.

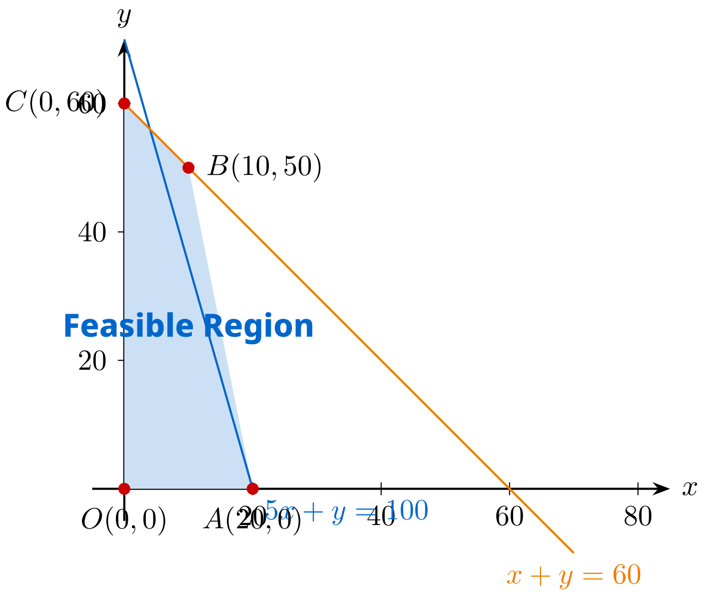
3 Graphical Solution: The Corner-Point Method

Once an LPP is formulated, we solve it by drawing the feasible region and inspecting its *corner points*. This works because of a beautiful structural fact about linear functions on convex polygons — their extreme values always sit at vertices.

3.1 Feasible region, feasible solutions

Feasible region: the set of all points (x, y) in the plane that satisfy every constraint of the LPP simultaneously — it is the intersection of all the half-planes defined by the constraints, together with the non-negative quadrant. Every point in the feasible region is called a **feasible solution**; every point outside it is an **infeasible solution**.

Optimal solution: a feasible solution that gives the optimal (max or min) value of the objective function Z is an **optimal solution**.



The four corners of the shaded region are $O(0,0)$, $A(20,0)$, $B(10,50)$ and $C(0,60)$. By inspection, no point of the plane outside this polygon satisfies all four inequalities simultaneously; every point inside or on the boundary does.

Feasible vs Infeasible

Feasible region = $\{(x,y) :$ all constraints hold$\}$.
Feasible solution = any single point in the feasible region.
Infeasible solution = any point that violates at least one constraint.
Optimal solution = the feasible solution at which Z attains its required optimum.

3.2 Two foundational theorems

Theorem 1 — Vertex Optimality

If the feasible region R of an LPP is non-empty and the objective function $Z = ax + by$ attains its optimum on R, then that optimum must occur at a **corner point (vertex)** of R.

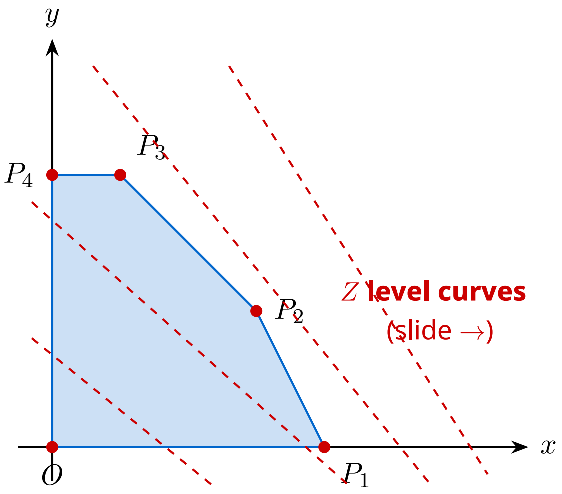
Theorem 2 — Bounded Feasible Region

If R is **bounded** (can be enclosed in some circle), then $Z = ax+by$ attains both a *maximum* and a *minimum* on R, and each occurs at some corner of R.

Remark. If R is *unbounded*, a maximum or minimum *may* fail to exist. But if it does exist, it still occurs at a corner. Section 4 explains the open-half-plane test that resolves the unbounded case.

3.3 Why corners? An intuitive justification

The level curves of $Z = ax + by$ are parallel straight lines $ax + by = k$ for various constants k. As k increases, the entire family slides in the same direction — perpendicular to the vector (a, b). Imagine pushing this family of parallel lines across the feasible region: the *last* point of the region they leave (or the *first* they enter) must be a vertex, because the boundary of a convex polygon is made of straight edges, and a straight line slid against another straight edge will leave it at a corner.



The dashed lines are members of the family $Z = ax + by =$ const. As Z grows, the line slides up-right. The *last* contact with the feasible region happens at one of the vertices — here P_2. That vertex is the maximiser.

3.4 The Corner-Point Method — algorithm

The Corner-Point Method (CPM)

1. Convert each constraint inequality into the equation of its boundary line. Draw all such lines on a coordinate plane.
2. Identify which side of each line the inequality holds (test with the origin if it is not on the line). Shade the intersection: this is the feasible region.
3. Find every **corner point** of the feasible region. A corner is the intersection of two boundary lines (or of a boundary line and an axis). Solve the two linear equations simultaneously.
4. Evaluate $Z = ax + by$ at each corner.
5. **Bounded region**: pick the largest value as max Z, smallest as min Z. **Unbounded region**: see Section 4.

Quick Tip

The fastest way to identify corners: list the constraint boundary lines, then pair them up. For m lines you get at most $\binom{m}{2}$ intersection points; only the ones that satisfy *all* the remaining constraints are real corners.

3.5 Solving the furniture problem

Recall: maximise $Z = 250x+75y$ over the polygon $O(0,0), A(20,0), B(10,50), C(0,60)$.

Corner point	$Z = 250x + 75y$	**Remark**
$O(0,0)$	0	
$A(20,0)$	5000	
$B(10,50)$	**6250**	**maximum**
$C(0,60)$	4500	

The maximum profit is Rs. 6250, attained at $B(10, 50)$: buy 10 tables and 50 chairs.

Real-World Application

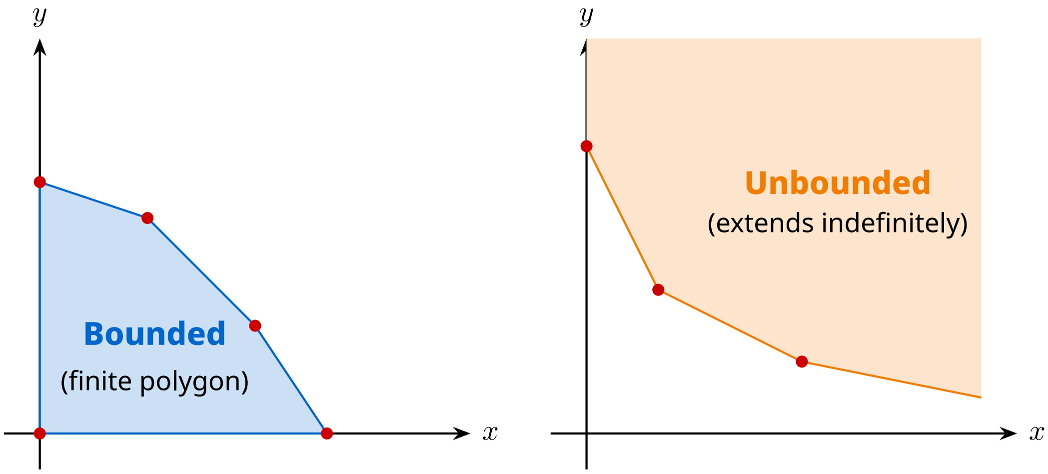
$B(10, 50)$ exhausts *both* resources — $5(10) + 50 = 100$ and $10 + 50 = 60$. This is typical: optimal LPP solutions usually push the binding constraints to equality, leaving zero slack. A solution with leftover money and leftover storage is almost always sub-optimal.

4 Bounded, Unbounded and Infeasible Cases

The corner-point method is uniform, but its conclusion depends on the geometry of the feasible region. Three qualitatively different situations arise.

4.1 Bounded feasible region

If the feasible region can be enclosed inside some circle of finite radius, it is **bounded**. By Theorem 2 both max Z and min Z exist and are attained at corner points. Just tabulate and pick.



4.2 Unbounded feasible region

An **unbounded** region extends without limit in some direction. The Remark to Theorem 2 then warns us: a maximum or minimum may not exist. Two cases to handle:

1. Compute Z at every corner. Suppose M is the largest such value.

2. Draw the open half-plane $ax + by > M$. If this half-plane has *no* point in common with the feasible region, then M is the true maximum. If it does share a point, then Z is unbounded above (no maximum exists).
3. Symmetrically for the minimum: if m is the smallest corner value, draw $ax + by < m$. If it overlaps the feasible region, no minimum exists; otherwise m is the minimum.

Open Half-Plane Test (Unbounded Region)

With $M = \max_{\text{corners}} Z$ and $m = \min_{\text{corners}} Z$:

- M is the maximum of Z iff $\{ax + by > M\} \cap R = \emptyset$.
- m is the minimum of Z iff $\{ax + by < m\} \cap R = \emptyset$.

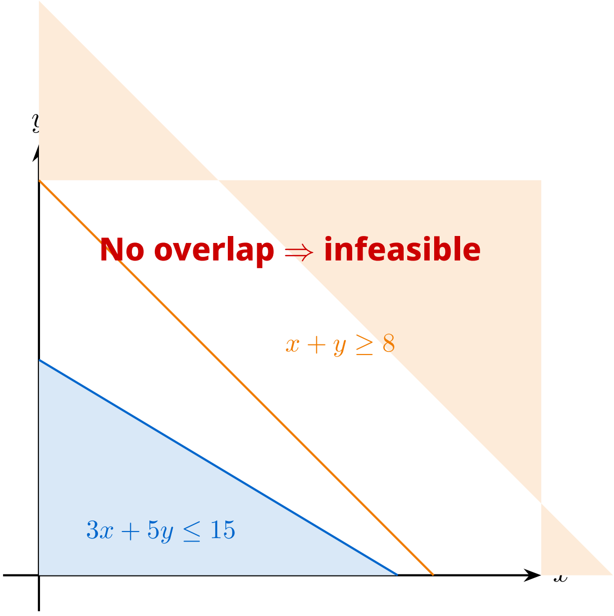
Otherwise the corresponding optimum does not exist.

Common Mistake

A common error is to assume the smallest corner value is always the minimum, even when the region is unbounded. Always check the open-half-plane condition for unbounded feasible regions, especially when the objective has a *negative* coefficient on a free-growing variable.

4.3 Infeasible problem

If the constraints contradict one another, no point satisfies them all simultaneously and the feasible region is **empty**. The problem has **no feasible solution**, hence no optimum.



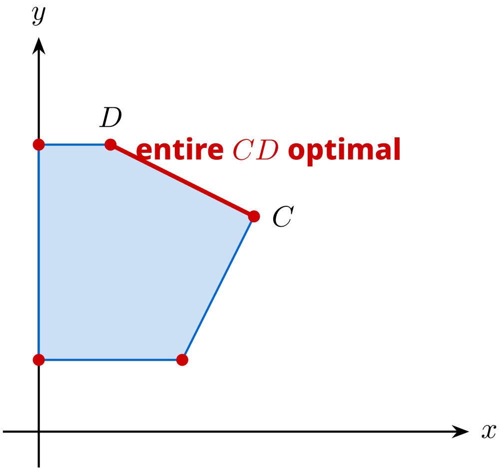
The two shaded half-planes do not meet: the constraint $x + y \geq 8$ rules out everything below the orange line, but $3x + 5y \leq 15$ rules out everything above the blue line. With $x, y \geq 0$, there is no common region.

Three Possible Outcomes

1. **Bounded feasible region** $\Rightarrow$ unique max and min, both at corners.
2. **Unbounded feasible region** $\Rightarrow$ max/min may exist (corner) or may not (open-half-plane test decides).
3. **Empty feasible region** $\Rightarrow$ no feasible solution; LPP has no answer.

4.4 Multiple optimal solutions

If two adjacent corners give the same optimal value of Z, then every point on the line segment joining them is also optimal. This happens when the objective function's level lines are parallel to one of the constraint boundary lines.



Quick Tip

If you ever get the same value of Z at two distinct corners, do not stop — state explicitly that "Z attains its optimum at every point of the line segment joining these corners." Examiners look for that wording.

5 Worked Examples

We now apply the corner-point method end-to-end on five problems — two from the NCERT example set, one minimisation, one unbounded case, and one infeasible case.

5.1 Example 1 — Bounded maximisation

Problem. Maximise $Z = 4x + y$ subject to $x + y \leq 50,\ 3x + y \leq 90,\ x \geq 0,\ y \geq 0$.

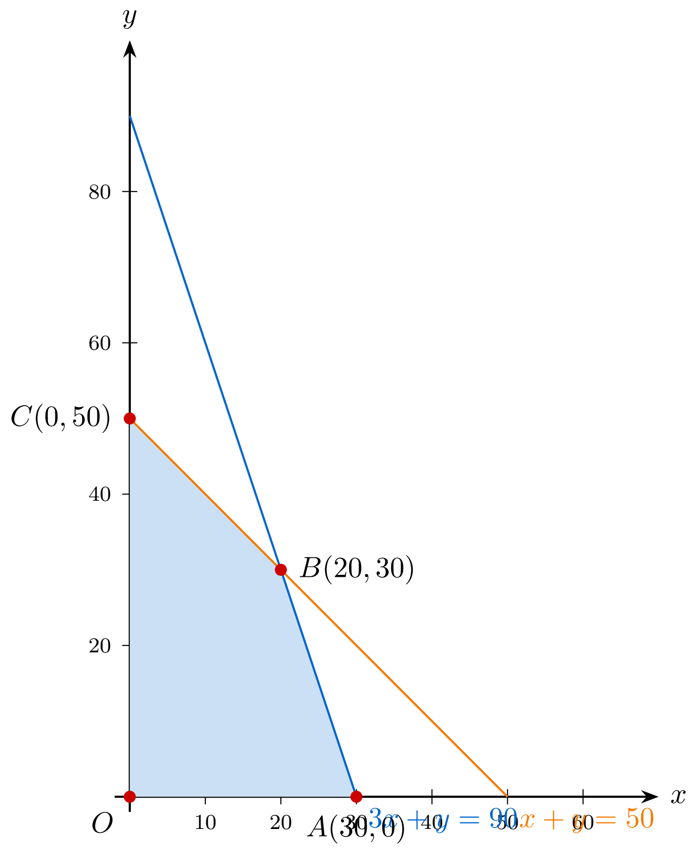
Step 1 — Boundary lines.

- $x + y = 50$: passes through $(50, 0)$ and $(0, 50)$.

- $3x + y = 90$: passes through $(30, 0)$ and $(0, 90)$.

Step 2 — Intersection of the two lines. Subtract: $2x = 40 \Rightarrow x = 20$, then $y = 30$. So they meet at $(20, 30)$.

Step 3 — Feasible region. The region is bounded by the two axes and the two lines, taking the side of each line that contains the origin (since $0 + 0 \le 50$ and $0 + 0 \le 90$ both hold).



Step 4 — Evaluate Z at corners.

Corner	$Z = 4x + y$
$O(0, 0)$	0
$A(30, 0)$	**120** ($\leftarrow$ maximum)
$B(20, 30)$	110
$C(0, 50)$	50

Conclusion. max $Z = 120$ at $(30, 0)$.

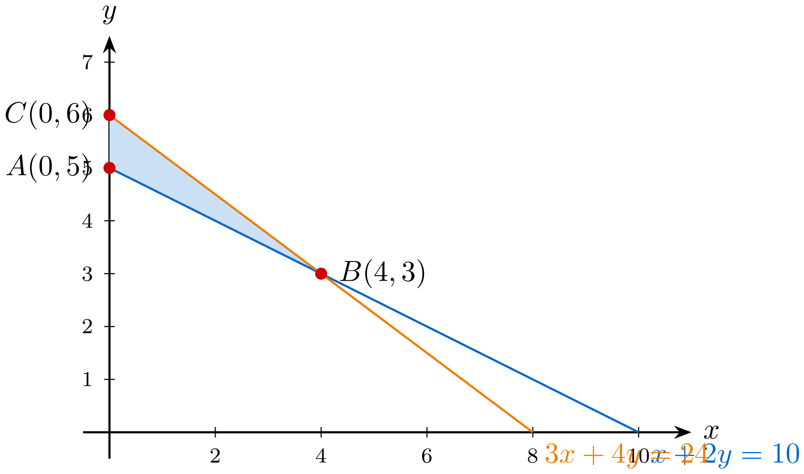
5.2 Example 2 — Bounded minimisation (diet)

Problem. Minimise $Z = 200x + 500y$ subject to $x + 2y \ge 10,\ 3x + 4y \le 24,\ x \ge 0,\ y \ge 0$.

Step 1 — Boundary lines. $x + 2y = 10$: $(10, 0)$ and $(0, 5)$. $3x + 4y = 24$: $(8, 0)$ and $(0, 6)$.

Step 2 — Intersection. From $x + 2y = 10$ we get $x = 10 - 2y$. Substitute into $3x + 4y = 24$: $30 - 6y + 4y = 24 \Rightarrow -2y = -6 \Rightarrow y = 3,\ x = 4$. Intersection: $(4, 3)$.

Step 3 — Feasible region. We need $x+2y \geq 10$ (above the lower line) *and* $3x+4y \leq 24$ (below the upper line), in the first quadrant. The feasible region is the small triangle $A(0, 5), B(4, 3), C(0, 6)$.



Step 4 — Evaluate Z.

Corner	$Z = 200x + 500y$
$A(0, 5)$	2500
$B(4, 3)$	**2300** (← minimum)
$C(0, 6)$	3000

Conclusion. min $Z = 2300$ at $(4, 3)$.

5.3 Example 3 — Both max and min, with multiple optima

Problem. Minimise and maximise $Z = 3x + 9y$ subject to $x + 3y \leq 60,\ x + y \geq 10,\ x \leq y,\ x \geq 0,\ y \geq 0$.

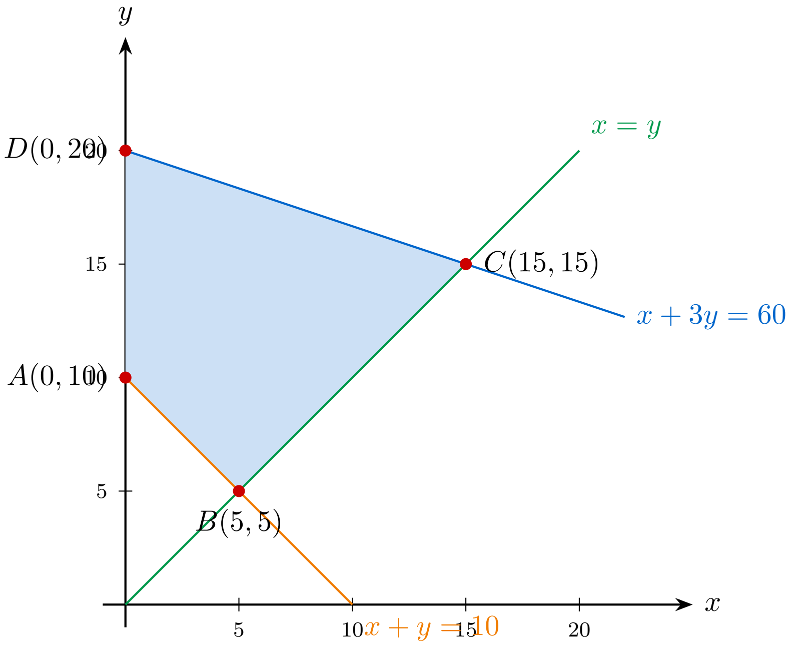
Step 1 — Boundary lines.

- $x + 3y = 60$: $(60, 0)$ and $(0, 20)$.
- $x + y = 10$: $(10, 0)$ and $(0, 10)$.
- $x = y$: the line through the origin at $45°$.

Step 2 — Corners (solve pairs simultaneously).

- $x + 3y = 60$ and y-axis $(x = 0)$: $(0, 20)$.
- $x + 3y = 60$ and $x = y$: $4y = 60 \Rightarrow y = 15, x = 15$: $(15, 15)$.
- $x + y = 10$ and $x = y$: $2x = 10 \Rightarrow x = 5, y = 5$: $(5, 5)$.
- $x + y = 10$ and y-axis $(x = 0)$: $(0, 10)$.

So the feasible region $ABCD$ has corners $A(0, 10),\ B(5, 5),\ C(15, 15),\ D(0, 20)$.



Step 3 — Evaluate $Z = 3x + 9y$.

Corner	$Z = 3x + 9y$	Remark
$A(0, 10)$	90	
$B(5, 5)$	**60**	minimum
$C(15, 15)$	**180**	maximum (tied)
$D(0, 20)$	**180**	maximum (tied)

Conclusion. min $Z = 60$ at $B(5, 5)$. Maximum $Z = 180$ is attained at *both* $C(15, 15)$ and $D(0, 20)$, hence at every point on the segment CD — the problem has **multiple optimal solutions**.

Memory Aid

"Tied corners ⇒ full edge." If two adjacent corners share the optimal Z, every point on the segment between them is also optimal — because Z is linear and constant along the level line that contains that edge.

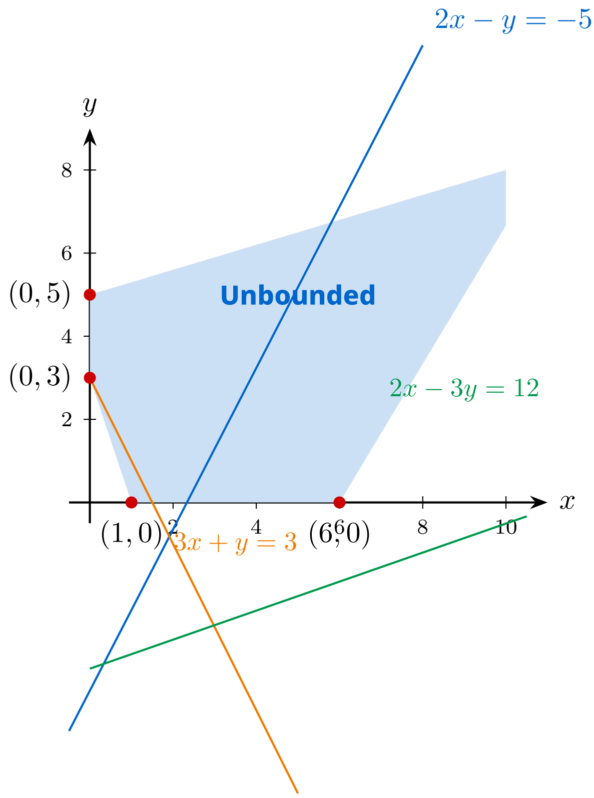
5.4 Example 4 — Unbounded feasible region

Problem. Determine graphically the minimum of $Z = -50x+20y$ subject to $2x-y \geq -5,\ 3x + y \geq 3,\ 2x - 3y \leq 12,\ x \geq 0,\ y \geq 0.$

Boundary lines and corners. Plotting the three lines together with the axes and computing the corner points one finds the corners

$A(1, 0),\ B(0, 3),\ C(0, 5),$
$D(1, 7)$ (intersection of $2x - y = -5$ and the $-5x + 2y = -30$ direction),
$E(6, 0).$

The relevant corners (boundary of the unbounded region adjacent to the origin's complement side) reduce to those listed by NCERT: $(0,5),\ (0,3),\ (1,0),\ (6,0)$.



Evaluate $Z=-50x+20y$.

Corner	$Z=-50x+20y$
$(0,5)$	100
$(0,3)$	60
$(1,0)$	-50
$(6,0)$	$-$**300** (← smallest)

Open-half-plane check. The smallest corner value is -300. Draw the half-plane $-50x+20y<-300$, i.e. $-5x+2y<-30$. Does it intersect the feasible region? Yes — e.g. the point $(10,0)$ lies in both. So Z can be pushed below -300 inside the region, hence **no minimum exists**.

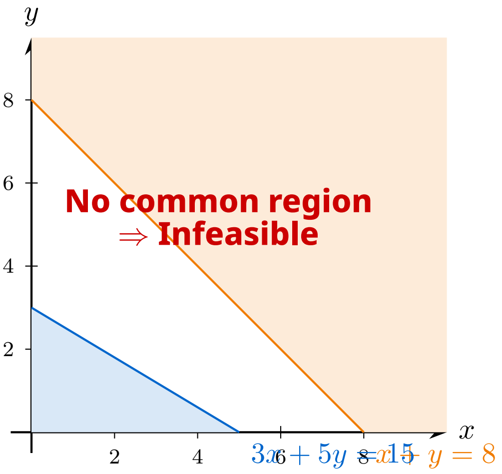
Take-Away

For unbounded regions, the smallest corner value is the minimum only if the open half-plane $ax+by<m$ has *no* common point with the feasible region. Always perform that check. The same logic governs maxima.

5.5 Example 5 — Infeasible problem

Problem. Minimise $Z=3x+2y$ subject to $x+y\geq 8,\ 3x+5y\leq 15,\ x\geq 0,\ y\geq 0$.

Sketch. $x + y = 8$ passes through $(8, 0)$ and $(0, 8)$; we want everything on or above this line. $3x + 5y = 15$ passes through $(5, 0)$ and $(0, 3)$; we want everything on or below it. In the first quadrant the two half-planes do not meet: the lower-right half-plane stops at $(5, 0)$–$(0, 3)$, while the upper-left one starts only at $(8, 0)$–$(0, 8)$.



Conclusion. The feasible region is empty. The LPP has **no solution**.

5.6 Example 6 — A complete manufacturing problem

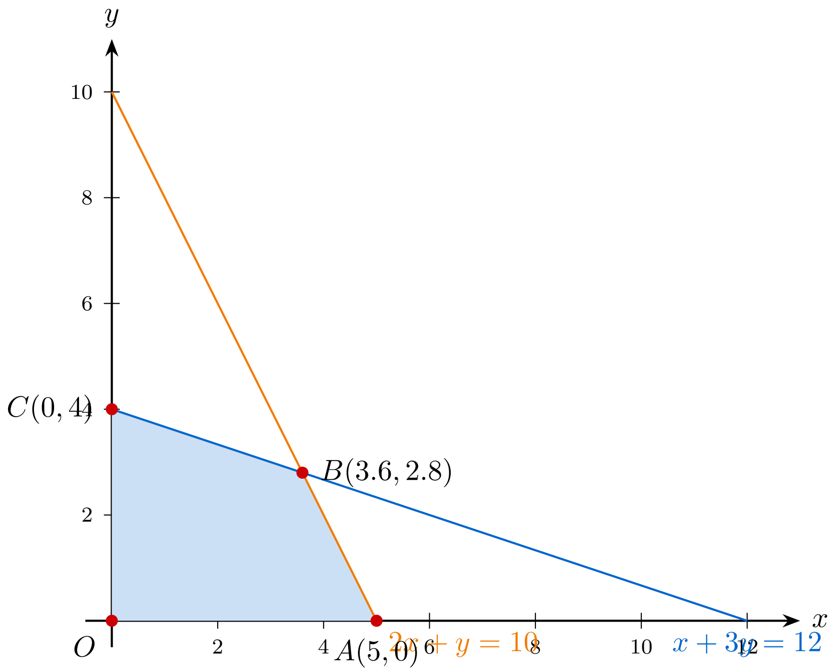
Problem. A factory makes two products P_1 and P_2. Each P_1 requires 1 hour on machine M_1 and 2 hours on machine M_2; each P_2 requires 3 hours on M_1 and 1 hour on M_2. M_1 runs at most 12 hours/day, M_2 at most 10 hours/day. Profit is Rs. 200 per P_1 and Rs. 500 per P_2. Find the daily production plan that maximises profit.

Formulation. Let x = units of P_1, y = units of P_2 per day.

$$\begin{aligned} \text{Maximise:}\quad & Z = 200x + 500y \\ \text{Subject to:}\quad & x + 3y \le 12 \quad (M_1 \text{ time}) \\ & 2x + y \le 10 \quad (M_2 \text{ time}) \\ & x \ge 0,\ y \ge 0. \end{aligned}$$

Boundary lines. $x + 3y = 12$ through $(12, 0)$, $(0, 4)$. $2x + y = 10$ through $(5, 0)$, $(0, 10)$. Intersection: solve $x + 3y = 12$ and $2x + y = 10$. From the second, $y = 10 - 2x$. Substitute: $x + 3(10 - 2x) = 12 \Rightarrow x + 30 - 6x = 12 \Rightarrow x = 18/5 = 3.6,\ y = 10 - 7.2 = 2.8$. So they meet at $(3.6, 2.8)$.

Feasible region. Corners $O(0, 0),\ A(5, 0),\ B(3.6, 2.8),\ C(0, 4)$.



Evaluate Z.

Corner	$Z = 200x + 500y$	Remark
$O(0,0)$	0	
$A(5,0)$	1000	
$B(3.6,2.8)$	**2120**	maximum
$C(0,4)$	2000	

Conclusion. Daily profit is maximised at Rs. 2120 by producing $x = 3.6$ units of P_1 and $y = 2.8$ units of P_2. If units must be integer-valued the answer rounds to the nearest feasible integer point — but that is an *integer* programming question, not a linear programming question.

Common Mistake

NCERT-level LPP does not require integer outputs. If the corner is $(3.6, 2.8)$, that is the answer for this chapter. “Number of items must be integer” arguments belong to Operations Research / Integer Programming, not to Class 12 LPP.

5.7 Example 7 — Diet problem, fully worked

Problem. A house owner mixes two foods, F and G. Food F costs Rs. 4 per kg and contains 3 units of vitamin and 4 units of mineral per kg. Food G costs Rs. 6 per kg and contains 6 units of vitamin and 3 units of mineral per kg. The mix must provide at least 80 units of vitamin and 100 units of mineral. Find the cost-minimising mix.

Formulation. $x, y \geq 0$ = kg of F, G.

$$\begin{aligned} \text{Minimise:} \quad & Z = 4x + 6y \\ \text{Subject to:} \quad & 3x + 6y \geq 80 \\ & 4x + 3y \geq 100 \\ & x \geq 0,\ y \geq 0. \end{aligned}$$

Corners. $3x + 6y = 80$ meets the x-axis at $(80/3, 0) \approx (26.67, 0)$ and y-axis at $(0, 40/3) \approx (0, 13.33)$. $4x + 3y = 100$ meets the x-axis at $(25, 0)$ and y-axis at $(0, 100/3) \approx (0, 33.33)$. To find the intersection, multiply the first equation by 2: $6x + 12y = 160$; subtract the second times 3: $12x + 9y = 300$. Eliminating: from $3x + 6y = 80$ get $x = (80 - 6y)/3$. Substitute into $4x + 3y = 100$: $4(80 - 6y)/3 + 3y = 100 \Rightarrow 320 - 24y + 9y = 300 \Rightarrow -15y = -20 \Rightarrow y = 4/3 \approx 1.33,\ x = (80 - 8)/3 = 24$.

Feasible region. It is unbounded above and to the right. The relevant corners are $(25, 0)$, $(24, 4/3)$ and $(0, 100/3)$.

Evaluate Z.

Corner	$Z = 4x + 6y$	
$(25, 0)$	100	
$(24, 4/3)$	$\mathbf{96} + \mathbf{8} = \mathbf{104}$	
$(0, 100/3)$	200	

The smallest corner value is $Z = 100$ at $(25, 0)$. Open-half-plane check: $4x + 6y < 100$. At, say, $(20, 0)$: $4(20) = 80 < 100$, but $4x + 3y = 80 \not\geq 100$ so $(20, 0)$ is *not* in the feasible region. Test the boundary of feasibility carefully — and in this case the half-plane has no point in common with the feasible region, so min $Z = 100$ at $(25, 0)$: use 25 kg of F and no G.

> **Real-World Application**
>
> Real diet-formulation software at hospitals and food companies runs huge LPPs with hundreds of nutrients and ingredient costs. The same five-step recipe scales: it just uses the simplex method instead of corner inspection.

6 Strategy, Pitfalls and Quick Reference

A short consolidation: what to do under time pressure in the exam, common errors, and a single-page summary.

6.1 Exam strategy — a 6-minute LPP

1. **Read once** and circle the numbers, units, and the word "maximise" or "minimise".
2. **Tabulate** the data (rows = products/foods/items, columns = resources). This catches sloppy formulations.

3. **Define** x, y in one sentence: "Let x = ...and y = ..." Marks here.
4. **Write** $Z = ax + by$ and every constraint on a fresh line.
5. **Plot**: each constraint line has x-intercept and y-intercept; mark them, draw the line, shade the correct side using the origin test.
6. **Corners:** list four to six points; solve simultaneously where lines meet.
7. **Table of Z values:** pick max or min.
8. **One-line conclusion** in the variables of the problem ("buy 10 tables and 50 chairs for Rs. 6250 profit").

Quick Tip

On the answer sheet, draw the graph *after* you have computed all the corner points algebraically. The graph then anchors and verifies your algebra rather than the other way around — and you will rarely re-draw it.

6.2 Pitfalls compiled

Common Mistake

Top six errors students make in LPP problems:

1. Translating "at most k" as $\geq k$ or vice versa.
2. Forgetting non-negativity $x, y \geq 0$.
3. Listing only some corners, missing the intersection of two interior lines.
4. For an unbounded region, declaring the smallest-corner Z-value as the minimum without the open-half-plane check.
5. Assuming integer answers — LPP allows fractional decision variables unless explicitly stated.
6. Reading the wrong side of the constraint line (use the origin to test whenever it is not on the line).

Memory Aid

"DOC-N" for LPP formulation order:
Decision variables $\to$ **O**bjective function $\to$ **C**onstraints $\to$ **N**on-negativity.
Never write a constraint before defining the variables it contains.

6.3 Bounded vs unbounded vs infeasible — comparison

Region type	Geometry	Behaviour of Z	Decision
Bounded	Closed convex polygon, can be enclosed in a circle	Both max and min exist; each at a corner	Tabulate; pick
Unbounded	Extends without limit in some direction	Max or min may not exist; corner value is candidate only	Apply open-half-plane test
Infeasible	Empty (constraints inconsistent)	Z has no domain	"No feasible solution"

6.4 One-page formula and method recap

LPP Quick Reference

General form (two variables)

$$\text{Optimise } Z = ax + by \text{ subject to } \sum a_i x + b_i y \ \{\leq, =, \geq\} \ c_i, \quad x, y \geq 0.$$

Corner-Point Method

1. Draw all constraint boundaries; shade the feasible region.
2. Find every corner (vertex) by simultaneous equations.
3. Tabulate Z at each corner.
4. Pick optimum (with open-half-plane test if the region is unbounded).

Key fact (Theorem 1): optimum of a linear Z on a feasible region, if attained, sits at a vertex.
Bounded $\Rightarrow$ **both max and min exist** (Theorem 2).

6.5 Where this chapter goes next

In Class 12 NCERT, LPP is restricted to two decision variables and the graphical method. Beyond that:

- **Simplex method** (Dantzig, 1947): an algebraic algorithm that handles any number of variables and constraints by pivoting between extreme points of the feasible polytope.
- **Duality theory**: every LPP (primal) has a partner LPP (dual) whose optimum equals the primal's — foundation of shadow pricing in economics.
- **Integer programming**: when decision variables must be whole numbers (people, planes), the polynomial-time LPP becomes NP-hard.
- **Interior-point methods** (Karmarkar, 1984): modern polynomial-time alternatives to the simplex method, widely used in commercial solvers.

Real-World Application

The 1975 Nobel Prize in Economic Sciences went jointly to Kantorovich (USSR)

> and Koopmans (USA) “for their contributions to the theory of optimum allocation of resources” — direct recognition of linear programming’s impact on economic planning.

Best of luck with your preparation!

Formulate carefully. Graph cleanly. Tabulate every corner.