Collegedunia NCERT Formula Sheet

The Ultimate Formula Reference for Class 12 Mathematics — Chapter 13

Chapter 13: Probability

Symbol / Term	Meaning
S	Sample space (set of all outcomes of a random experiment)
$E,\ F,\ A,\ B$	Events (subsets of S)
E' or E^c	Complement of event E (event "E does not occur")
$E \cap F$ (or EF)	Both E and F occur (simultaneous occurrence)
$E \cup F$	At least one of $E,\ F$ occurs
$P(E)$	Probability of event E; $0 \le P(E) \le 1$
$P(E \mid F)$	Conditional probability of E given F has occurred
$\{E_1, E_2, \ldots, E_n\}$	Partition of S: pairwise disjoint, exhaustive, each $P(E_i) > 0$
$n(A)$	Number of elementary events favourable to A

1 Conditional Probability

This section covers the definition of conditional probability, its three key properties (normalisation, complement, addition under a given event), and the multiplicative reformulation that links $P(E \cap F)$ to $P(F)$ via $P(E \mid F)$. Conditional probability **re-scales** the sample space to those outcomes consistent with the given information.

Conditional probability — definition

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}, \quad P(F) \neq 0$$

Equivalent counting form (equally likely outcomes):

$$P(E \mid F) = \frac{n(E \cap F)}{n(F)}$$

where $E, F \subseteq S$ are events; $n(\cdot)$ counts favourable elementary events.

Read as "probability of E **given** F". The given event F becomes the new sample space; only outcomes inside F are considered.

Properties of $P(\cdot \mid F)$

Let E, F, A, B be events with $P(F) \neq 0$.

(1) $P(S \mid F) = P(F \mid F) = 1$

(2) $P(E' \mid F) = 1 - P(E \mid F)$

(3) $P((A \cup B) \mid F) = P(A \mid F) + P(B \mid F) - P((A \cap B) \mid F)$

If A, B disjoint: $P((A \cup B) \mid F) = P(A \mid F) + P(B \mid F)$

Conditional probability **obeys all the usual probability axioms** once we restrict the sample space to F. Treat $P(\cdot \mid F)$ as a fresh probability measure.

Range and complement

$0 \leq P(E \mid F) \leq 1$

$P(E' \mid F) = 1 - P(E \mid F)$

Conditional probabilities lie in $[0, 1]$ just like ordinary probabilities. The complement rule under a given event mirrors the unconditional one.

$P(E \mid F)$ vs $P(F \mid E)$

These are **not equal** in general. $P(E \mid F) = \frac{P(E \cap F)}{P(F)}$, but $P(F \mid E) = \frac{P(E \cap F)}{P(E)}$. They share the same numerator but different denominators. Equal only when $P(E) = P(F)$.

2 Multiplication Theorem

The multiplication theorem rewrites $P(E \cap F)$ as a product of an unconditional and a conditional probability. It is the engine for **sequential experiments** (drawing without replacement, tree-diagram problems) and extends to any finite number of events.

Multiplication rule (two events)

$P(E \cap F) = P(E)\, P(F \mid E)$ provided $P(E) \neq 0$

$P(E \cap F) = P(F)\, P(E \mid F)$ provided $P(F) \neq 0$

Pick whichever conditioning is easier to compute. Use the first form when E happens **first** in time and influences F.

Multiplication rule (three events)

$P(E \cap F \cap G) = P(E)\, P(F \mid E)\, P(G \mid (E \cap F))$

Reads as a **chain**: probability of the first, times probability of the second given the first, times probability of the third given the first two. Extends naturally to n events.

Tree-diagram template

For a 2-stage experiment, label branches with conditional probabilities. The probability of any leaf = product of probabilities along the path from root. Sum over leaves with a given property to recover P of that event.

3 Independent Events

Two events are **independent** when the occurrence of one carries no information about the other — equivalently, knowing F does not change the probability of E. Independence is defined via products of probabilities; it is logically distinct from mutual exclusivity.

Independence — equivalent definitions

Events E, F are independent iff any one (hence all) of:

$P(E \cap F) = P(E)\, P(F)$

$P(E \mid F) = P(E)$ (when $P(F) > 0$)

$P(F \mid E) = P(F)$ (when $P(E) > 0$)

"Information about F does not change the probability of E." **Dependent** events satisfy $P(E \cap F) \neq P(E)\, P(F)$.

Independence under complementation

If E and F are independent, then so are:

E and F'; E' and F; E' and F'

Independence is **preserved under taking complements** on either side. Useful for "at least one" calculations.

At least one of two independent events

$P(\text{at least one of } A, B) = P(A \cup B) = 1 - P(A')\,P(B')$

$P(\text{none}) = P(A')\,P(B') = (1 - P(A))(1 - P(B))$

Switch to the **complement** ("none occurs") — it factorises cleanly for independent events. Generalises to n: $P(\text{none}) = \prod(1 - P(A_i))$.

Mutual independence of three events

A, B, C are mutually independent iff *all four* hold:

$P(A \cap B) = P(A)\,P(B)$
$P(A \cap C) = P(A)\,P(C)$
$P(B \cap C) = P(B)\,P(C)$
$P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$

Pairwise independence alone (only the first three) is **not** sufficient — the triple-product condition is independent and must be checked separately.

Independent $\neq$ Mutually Exclusive

Mutually exclusive: $E \cap F = \varnothing$, so $P(E \cap F) = 0$.
Independent: $P(E \cap F) = P(E)\,P(F)$.
If both $P(E), P(F) > 0$, they **cannot** be both independent and mutually exclusive. Mutual exclusivity refers to **outcomes**; independence refers to **probabilities**.

4 Total Probability & Partition

A **partition** $\{E_1, \ldots, E_n\}$ of S slices the sample space into disjoint exhaustive pieces. The theorem of total probability assembles $P(A)$ from the conditional probabilities of A on each piece, weighted by the partition probabilities. This is the workhorse that feeds Bayes' theorem.

Partition of a sample space

A set $\{E_1, E_2, \ldots, E_n\}$ of events is a **partition** of S if:
(a) $E_i \cap E_j = \varnothing$ for $i \neq j$ *(pairwise disjoint)*
(b) $E_1 \cup E_2 \cup \cdots \cup E_n = S$ *(exhaustive)*
(c) $P(E_i) > 0$ for every i
A simplest partition: $\{E, E'\}$ for any non-empty event E.

Theorem of Total Probability

If $\{E_1, E_2, \ldots, E_n\}$ partitions S and A is any event,

$$P(A) = \sum_{j=1}^{n} P(E_j)\,P(A \mid E_j)$$
$$= P(E_1)\,P(A \mid E_1) + \cdots + P(E_n)\,P(A \mid E_n)$$

$P(A)$ is a **weighted average** of $P(A \mid E_j)$, weights being the partition probabilities $P(E_j)$. Use when A can arise via several mutually exclusive "routes".

Two-event special case

For any event A and any event B with $0 < P(B) < 1$:

$P(A) = P(B)\,P(A \mid B) + P(B')\,P(A \mid B')$

The partition $\{B,\ B'\}$ is always available — handy for "defective/non-defective", "has-disease/healthy", "strike/no-strike" setups.

5 Bayes' Theorem

Bayes' theorem **inverts** conditional probability: given that effect A has been observed, find the probability that cause E_i produced it. The E_i are called *hypotheses*; $P(E_i)$ are *prior* probabilities and $P(E_i \mid A)$ are *posterior* probabilities.

Bayes' Theorem

Let $\{E_1, E_2, \ldots, E_n\}$ partition S and A be an event with $P(A) \neq 0$. Then for each $i = 1, 2, \ldots, n$:

$$P(E_i \mid A) = \frac{P(E_i)\,P(A \mid E_i)}{\sum_{j=1}^{n} P(E_j)\,P(A \mid E_j)}$$

Numerator: probability of the chosen cause-and-effect path. **Denominator**: total probability of A across all causes (theorem of total probability). Ratio = posterior probability of cause E_i.

Prior vs Posterior

Prior $P(E_i)$: probability of hypothesis E_i *before* observing A (a priori).
Likelihood $P(A \mid E_i)$: probability of evidence A given hypothesis E_i.
Posterior $P(E_i \mid A)$: probability of E_i *after* observing A (a posteriori).
Bayes' theorem is the **rule for updating priors into posteriors**.

Two-hypothesis form (most common in NCERT)

With partition $\{E, E'\}$:

$$P(E \mid A) = \frac{P(E)\,P(A \mid E)}{P(E)\,P(A \mid E) + P(E')\,P(A \mid E')}$$

Use this directly for **two-bag, two-machine, disease-test**, and "truth-teller / liar" problems. Always write down the four numbers $P(E), P(E'), P(A \mid E), P(A \mid E')$ first.

JEE/NEET Extension — Odds form of Bayes

$$\frac{P(E_i \mid A)}{P(E_k \mid A)} = \frac{P(E_i)}{P(E_k)} \cdot \frac{P(A \mid E_i)}{P(A \mid E_k)}$$

"Posterior odds = Prior odds $\times$ Likelihood ratio." Bypasses computing the full denominator when only ratios are needed.

PLP — Prior, Likelihood, Posterior

Prior $\times$ **L**ikelihood, normalised over all hypotheses, gives the **P**osterior.
Always list: priors $P(E_i)$, likelihoods $P(A \mid E_i)$, then plug into Bayes.

6 Random Variable

The chapter closes by introducing the idea of a **random variable** — a real-valued function on the sample space that turns qualitative outcomes into numbers. (The rationalised 2024–26 syllabus stops here; probability distribution tables, mean, variance and the binomial distribution have been removed.)

Random variable

A **random variable** X is a real-valued function whose domain is the sample space S of a random experiment, i.e., $X : S \to \mathbb{R}$.
Example: Toss a coin twice; $S = \{HH, HT, TH, TT\}$. Define X = number of heads. Then $X(HH) = 2$, $X(HT) = 1$, $X(TH) = 1$, $X(TT) = 0$.
Multiple random variables can be defined on the same sample space (e.g., heads $-$ tails).

Quick Reference — Chapter 13 Probability

Concept	**Formula / Statement**
Conditional probability	$P(E \mid F) = \dfrac{P(E \cap F)}{P(F)}, \quad P(F) \neq 0$
Range	$0 \leq P(E \mid F) \leq 1$
Complement (under given F)	$P(E' \mid F) = 1 - P(E \mid F)$
Addition (under given F)	$P((A \cup B) \mid F) = P(A \mid F) + P(B \mid F) - P((A \cap B) \mid F)$
Multiplication rule (two events)	$P(E \cap F) = P(E)\,P(F \mid E) = P(F)\,P(E \mid F)$
Multiplication rule (three events)	$P(E \cap F \cap G) = P(E)\,P(F \mid E)\,P(G \mid E \cap F)$
Independence (E, F)	$P(E \cap F) = P(E)\,P(F) \;\Leftrightarrow\; P(E \mid F) = P(E)$
Independence under complement	E, F indep. $\Rightarrow E, F'$ indep.; E', F indep.; E', F' indep.
At-least-one (independent)	$P(A \cup B) = 1 - P(A')\,P(B')$
Mutual indep. of A, B, C	All pairwise products *and* $P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$
Partition of S	$\{E_i\}$: pairwise disjoint, $\bigcup E_i = S$, each $P(E_i) > 0$
Total probability	$P(A) = \sum_{j=1}^{n} P(E_j)\,P(A \mid E_j)$
Bayes' theorem	$P(E_i \mid A) = \dfrac{P(E_i)\,P(A \mid E_i)}{\sum_j P(E_j)\,P(A \mid E_j)}$
Two-hypothesis Bayes	$P(E \mid A) = \dfrac{P(E)\,P(A \mid E)}{P(E)\,P(A \mid E) + P(E')\,P(A \mid E')}$
Random variable	$X : S \to \mathbb{R}$, a real-valued function on the sample space

Problem-Solving Decision Tree

When you see...	Reach for...
"Given that ...", "if it is known that ..."	Conditional probability $P(E \mid F) = P(E \cap F)/P(F)$
"Both ...", "and then ..." (sequential draws)	Multiplication rule (chain)
"Without replacement" draws	Multiplication rule with updated conditional probabilities
"With replacement" / "two independent tosses"	Independent events: $P(E \cap F) = P(E)\,P(F)$
"At least one" of independent events	Complement trick: $1 - \prod P(A_i')$
Several causes, one observed effect (find P(effect))	Total probability theorem
Effect observed, find P(cause) ("was it from machine B?")	Bayes' theorem
"Reliability of a test", "HIV test", sensitivity / specificity	Bayes' theorem with $\{E, E'\}$