

Collegedunia NCERT Formula Sheet

Class 12 (12th) Mathematics — NCERT 2026-27 / Latest Edition

Chapter 2: Inverse Trigonometric Functions

Principal Value Branches (Domain \rightarrow Range)

Function	Domain	Principal Value Branch (Range)
$y = \sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \tan^{-1} x$	\mathbb{R}	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1} x$	\mathbb{R}	$(0, \pi)$
$y = \sec^{-1} x$	$\mathbb{R} \setminus (-1, 1)$	$[0, \pi] \setminus \left\{\frac{\pi}{2}\right\}$
$y = \csc^{-1} x$	$\mathbb{R} \setminus (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \setminus \{0\}$

1 Basic Definitions & Existence

This section establishes what an inverse trigonometric function means, why trigonometric functions must be restricted to principal branches to admit inverses, and the standard notation used throughout the chapter.

What is an inverse trig function?

A function $f : A \rightarrow B$ is invertible only if it is one-one and onto. Sine, cosine and the other trigonometric functions are periodic, hence many-one on \mathbb{R} . By restricting each to a chosen **principal branch** (see table on page 1) we obtain a bijection, and its inverse is the corresponding inverse trigonometric function. So $\sin^{-1} x$ returns the unique angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is x .

Defining identities

$$\sin^{-1}(\sin y) = y, \quad y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\sin(\sin^{-1} x) = x, \quad x \in [-1, 1]$$

$$\cos^{-1}(\cos y) = y, \quad y \in [0, \pi]$$

$$\cos(\cos^{-1} x) = x, \quad x \in [-1, 1]$$

Inverses act as **true left/right inverses only on the principal branch**. Outside that branch you must first reduce the inner angle into the branch before cancelling.

Notation caution

$$\sin^{-1} x \neq (\sin x)^{-1} = \frac{1}{\sin x}$$

Alternative notation: $\sin^{-1} x = \mathbf{arcsin} x$

The superscript -1 here denotes **func-**

tion inversion, not a reciprocal. Use **arcsin, arccos, arctan** in ambiguous contexts.

Branch slip

Writing $\sin^{-1}(\sin \frac{3\pi}{4}) = \frac{3\pi}{4}$ is wrong because $\frac{3\pi}{4} \notin [-\frac{\pi}{2}, \frac{\pi}{2}]$. Reduce first: $\sin \frac{3\pi}{4} = \sin(\pi - \frac{3\pi}{4}) = \sin \frac{\pi}{4}$, so the answer is $\frac{\pi}{4}$.

2 Property 1 — Self-cancellation Rules

These four identities tell you when an inverse trig function and the corresponding trig function cancel cleanly. They are nothing more than the defining identities restated, but knowing the precise validity range is the most-tested point.

Sine & cosine self-cancellation

$$\sin^{-1}(\sin x) = x, \quad x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$$

$$\sin(\sin^{-1} x) = x, \quad x \in [-1, 1]$$

$$\cos^{-1}(\cos x) = x, \quad x \in [0, \pi]$$

$$\cos(\cos^{-1} x) = x, \quad x \in [-1, 1]$$

Reading direction matters. **Outer-then-inner** ($\sin^{-1} \sin$) requires the inner angle to lie in the principal branch; **inner-then-outer** requires only that the original x be in the function's domain $[-1, 1]$.

Tangent & cotangent self-cancellation

$$\tan^{-1}(\tan x) = x, \quad x \in (-\frac{\pi}{2}, \frac{\pi}{2})$$

$$\tan(\tan^{-1} x) = x, \quad x \in \mathbb{R}$$

$$\cot^{-1}(\cot x) = x, \quad x \in (0, \pi)$$

$$\cot(\cot^{-1} x) = x, \quad x \in \mathbb{R}$$

Tangent and cotangent are surjective onto \mathbb{R} , so $\tan(\tan^{-1} x)$ holds for **every real** x without restriction.

Secant & cosecant self-cancellation

$$\sec^{-1}(\sec x) = x, \quad x \in [0, \pi] \setminus \{\frac{\pi}{2}\}$$

$$\sec(\sec^{-1} x) = x, \quad |x| \geq 1$$

$$\csc^{-1}(\csc x) = x, \quad x \in [-\frac{\pi}{2}, \frac{\pi}{2}] \setminus \{0\}$$

$$\csc(\csc^{-1} x) = x, \quad |x| \geq 1$$

Excluded points ($\frac{\pi}{2}$ for sec, 0 for csc) are exactly where the underlying trig value is undefined.

Reducing out-of-branch angles

Use $\sin \theta = \sin(\pi - \theta)$, $\cos \theta = \cos(-\theta) = \cos(2\pi - \theta)$, $\tan \theta = \tan(\theta - n\pi)$ to push the inner angle into the principal branch **before** applying the inverse.

3 Property 2 — Negative Argument (Odd / Even Behaviour)

How does each inverse trig function react to the sign of its argument? The sine-family is odd; the cosine-family obeys a reflection rule about $\pi/2$ (or π).

Sine, tan, cosec — odd functions

$$\sin^{-1}(-x) = -\sin^{-1} x, \quad x \in [-1, 1]$$

$$\tan^{-1}(-x) = -\tan^{-1} x, \quad x \in \mathbb{R}$$

$$\csc^{-1}(-x) = -\csc^{-1} x, \quad |x| \geq 1$$

Their principal branches are **symmetric about 0**, so flipping the input sign flips the output sign.

Cosine, cot, sec — reflection rule

$$\cos^{-1}(-x) = \pi - \cos^{-1} x, \quad x \in [-1, 1]$$

$$\cot^{-1}(-x) = \pi - \cot^{-1} x, \quad x \in \mathbb{R}$$

$$\sec^{-1}(-x) = \pi - \sec^{-1} x, \quad |x| \geq 1$$

Their principal branches sit in $[0, \pi]$, which is **symmetric about $\frac{\pi}{2}$** , **not 0**, hence the $\pi - (\cdot)$ reflection.

4 Property 3 — Reciprocal Argument

Replacing x by $\frac{1}{x}$ swaps each inverse trig function with its cofunction-inverse partner. These identities are how secant, cosecant and cotangent inverses are usually computed in practice.

Reciprocal identities

$$\csc^{-1} x = \sin^{-1}(1/x), |x| \geq 1$$

$$\sec^{-1} x = \cos^{-1}(1/x), |x| \geq 1$$

$$\cot^{-1} x = \tan^{-1}(1/x), x > 0$$

$$\cot^{-1} x = \pi + \tan^{-1}(1/x), x < 0$$

The split for \cot^{-1} is the catch: because \cot^{-1} 's range is $(0, \pi)$ (always positive) but $\tan^{-1} \frac{1}{x}$ becomes negative when $x < 0$, you must add π to land in the correct branch.

5 Property 4 — Complementary Angle Identities

Each pair of cofunctions adds up to a right angle (or to $\pi/2$ in radian terms). Useful for converting between sin/cos inverses or tan/cot inverses in a single step.

Cofunction sums

$$\sin^{-1} x + \cos^{-1} x = \pi/2$$

$$\text{for } x \in [-1, 1]$$

$$\tan^{-1} x + \cot^{-1} x = \pi/2$$

$$\text{for } x \in \mathbb{R}$$

$$\sec^{-1} x + \csc^{-1} x = \pi/2$$

$$\text{for } |x| \geq 1$$

Each identity reflects the right-triangle fact: the two acute angles in a right triangle are complementary, and one is the sine-inverse while the other is the cosine-inverse of the same side ratio.

Pair them up

Three complementary pairs, all summing to $\pi/2$: **(sin, cos)**, **(tan, cot)**, **(sec, csc)** — same partners as the cofunction identities you know from Class 11 trigonometry.

6 Addition & Subtraction Formulas

These are the workhorse identities for combining or splitting inverse trig expressions in exam questions. Each formula carries a side-condition on the input range — getting that wrong is the most common error.

 \tan^{-1} addition

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}, \quad xy < 1$$

$$\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1} \frac{x+y}{1-xy}, \quad x > 0, y > 0, xy > 1$$

$$\tan^{-1} x + \tan^{-1} y = -\pi + \tan^{-1} \frac{x+y}{1-xy}, \quad x < 0, y < 0, xy > 1$$

When $xy > 1$ the basic formula gives an angle outside $(-\frac{\pi}{2}, \frac{\pi}{2})$, so a $\pm\pi$ correction restores the principal branch.

 \tan^{-1} subtraction

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy},$$

$$xy > -1$$

Mirror image of the addition rule; remember the sign in the denominator flips from $-$ to $+$.

 \sin^{-1} addition / subtraction

$$\sin^{-1} x + \sin^{-1} y = \sin^{-1} \left(x\sqrt{1-y^2} + y\sqrt{1-x^2} \right),$$

$$\text{if } x, y \geq 0 \text{ and } x^2 + y^2 \leq 1$$

$$\sin^{-1} x - \sin^{-1} y = \sin^{-1} \left(x\sqrt{1-y^2} - y\sqrt{1-x^2} \right),$$

$$\text{if } x, y \geq 0 \text{ and } x^2 + y^2 \leq 1$$

Derive by setting $\sin^{-1} x = A$, $\sin^{-1} y = B$, then $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ with $\cos A = \sqrt{1-x^2}$.

 \cos^{-1} addition / subtraction

$$\cos^{-1} x + \cos^{-1} y = \cos^{-1} \left(xy - \sqrt{1-x^2}\sqrt{1-y^2} \right),$$

$$\text{if } x, y \geq 0$$

$$\cos^{-1} x - \cos^{-1} y = \cos^{-1} \left(xy + \sqrt{1-x^2}\sqrt{1-y^2} \right),$$

$$\text{if } x \leq y, \text{ both in } [-1, 1]$$

Companion to the sine identities, derived from $\cos(A \pm B)$. Mind the **sign flip** in the radical term between $+$ and $-$ versions.

The $xy < 1$ trap

Students reflexively write $\tan^{-1} 2 + \tan^{-1} 3 = \tan^{-1} \frac{5}{-5} = \tan^{-1}(-1) = -\frac{\pi}{4}$. But $xy = 6 > 1$ and both positive, so the correct value is $\pi + \tan^{-1}(-1) = \frac{3\pi}{4}$.

7 Double-Angle (2θ) Formulas

Setting $y = x$ in the addition formulas (or applying $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$ identities to $\theta = \tan^{-1} x$) yields the following one-variable identities, heavily used in integration and simplification.

$2 \tan^{-1}$ family

$$2 \tan^{-1} x = \tan^{-1} \left(\frac{2x}{1-x^2} \right)$$

for $|x| < 1$

$$2 \tan^{-1} x = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

for $|x| \leq 1$

$$2 \tan^{-1} x = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$$

for $x \geq 0$

The three forms come from $\tan 2\theta$, $\sin 2\theta$, $\cos 2\theta$ rewritten with $t = \tan \theta$. Outside the stated ranges, $\pm\pi$ correction terms appear.

$2 \sin^{-1}$ and $2 \cos^{-1}$

$$2 \sin^{-1} x = \sin^{-1} (2x\sqrt{1-x^2}),$$

$$-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$$

$$2 \cos^{-1} x = \cos^{-1} (2x^2 - 1), \quad 0 \leq x \leq 1$$

Derived from $\sin 2\theta = 2 \sin \theta \cos \theta$ and $\cos 2\theta = 2 \cos^2 \theta - 1$.

Triple-angle (JEE extension)

$$3 \sin^{-1} x = \sin^{-1} (3x - 4x^3), \quad x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

$$3 \cos^{-1} x = \cos^{-1} (4x^3 - 3x), \quad x \in \left[\frac{1}{2}, 1\right]$$

$$3 \tan^{-1} x = \tan^{-1} \frac{3x - x^3}{1 - 3x^2},$$

$$-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

From triple-angle trig identities applied to $\theta = \sin^{-1} x$ etc. Range restrictions are tight; outside them you pick up $\pm\pi$ or $\pm 2\pi$.

8 Substitution Forms (Conversion Between Inverses)

Inverse trig expressions can be rewritten as another inverse trig function using a right-triangle substitution. Memorise these — they shortcut entire JEE problems.

Standard substitution table

For $x \in [-1, 1]$, let $\theta = \sin^{-1} x$. Then:

$$\sin \theta = x, \quad \cos \theta = \sqrt{1-x^2}$$

$$\tan \theta = \frac{x}{\sqrt{1-x^2}}$$

Therefore:

$$\sin^{-1} x = \cos^{-1} \sqrt{1-x^2}$$

$$= \tan^{-1} \frac{x}{\sqrt{1-x^2}}$$

$$= \cot^{-1} \frac{\sqrt{1-x^2}}{x}$$

Visualise a right triangle with opposite = x and hypotenuse = 1. Every other ratio falls out by Pythagoras.

\tan^{-1} conversions

For $x \geq 0$, with $\theta = \tan^{-1} x$:

$$\tan^{-1} x = \sin^{-1} \frac{x}{\sqrt{1+x^2}}$$

$$= \cos^{-1} \frac{1}{\sqrt{1+x^2}}$$

$$= \sec^{-1} \sqrt{1+x^2}$$

$$= \csc^{-1} \frac{\sqrt{1+x^2}}{x}$$

Right triangle with opposite = x , adjacent = 1 , hypotenuse = $\sqrt{1+x^2}$. For $x < 0$ adjust signs / add π as needed.

9 Sum to π — Special Identities

A few high-yield identities that appear repeatedly in past papers. They follow from the addition formulas with a specific algebraic relation among arguments.

Three-term tangent sum

If $x + y + z = xyz$, then

$$\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$$

If $xy + yz + zx = 1$ (with $x, y, z > 0$), then

$$\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$$

Both identities come from the tangent triple-sum formula. They convert symmetric algebraic conditions into clean angle sums.

Useful angle values

$$\arcsin(1/2) = \pi/6$$

$$\arcsin(\sqrt{3}/2) = \pi/3$$

$$\arccos(1/2) = \pi/3$$

$$\arccos(\sqrt{3}/2) = \pi/6$$

$$\arctan(1) = \pi/4$$

$$\arctan(\sqrt{3}) = \pi/3$$

$$\arctan(1/\sqrt{3}) = \pi/6$$

These are the standard-angle inverses to recall **on sight**. Negative arguments flip sign for sin/tan; reflect about $\pi/2$ for cos.

JEE/NEET extension — derivative formulas

Although derivatives are studied in Chapter 5, frequent JEE shortcuts use:

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}, |x| < 1$$

$$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}, x \in \mathbb{R}$$

$$\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}, |x| > 1.$$

10 Graph & Range Reminders

A compact graphical summary of all six principal-branch graphs you should be able to sketch in seconds.

Monotonicity at a glance

Increasing on their domains:

$\sin^{-1} x$, $\tan^{-1} x$, $\sec^{-1} x$ (on $x > 1$)

Decreasing on their domains:

$\cos^{-1} x$, $\cot^{-1} x$, $\csc^{-1} x$ (on $x > 1$)

Sine-family graphs pass through the origin with positive slope; cosine-family graphs descend from π (at $x = -1$) through $\pi/2$ (at $x = 0$) to 0 (at $x = 1$).

Asymptotes for \tan^{-1} and \cot^{-1}

$$\lim_{x \rightarrow +\infty} \tan^{-1} x = \frac{\pi}{2}, \quad \lim_{x \rightarrow -\infty} \tan^{-1} x = -\frac{\pi}{2}$$

$$\lim_{x \rightarrow +\infty} \cot^{-1} x = 0, \quad \lim_{x \rightarrow -\infty} \cot^{-1} x = \pi$$

\tan^{-1} and **\cot^{-1}** are the only inverse trig functions with horizontal asymptotes — their domains stretch over all of \mathbb{R} .

Quick Reference — All Properties on One Page

Property	Identity	Valid for
Self-cancel (sin)	$\sin^{-1}(\sin x) = x$	$x \in [-\pi/2, \pi/2]$
Self-cancel (cos)	$\cos^{-1}(\cos x) = x$	$x \in [0, \pi]$
Self-cancel (tan)	$\tan^{-1}(\tan x) = x$	$x \in (-\pi/2, \pi/2)$
Negative (sin)	$\sin^{-1}(-x) = -\sin^{-1} x$	$x \in [-1, 1]$
Negative (cos)	$\cos^{-1}(-x) = \pi - \cos^{-1} x$	$x \in [-1, 1]$
Negative (tan)	$\tan^{-1}(-x) = -\tan^{-1} x$	$x \in \mathbb{R}$
Reciprocal (csc)	$\csc^{-1} x = \sin^{-1}(1/x)$	$ x \geq 1$
Reciprocal (sec)	$\sec^{-1} x = \cos^{-1}(1/x)$	$ x \geq 1$
Reciprocal (cot, $x > 0$)	$\cot^{-1} x = \tan^{-1}(1/x)$	$x > 0$
Cofunction (sin/cos)	$\sin^{-1} x + \cos^{-1} x = \pi/2$	$x \in [-1, 1]$
Cofunction (tan/cot)	$\tan^{-1} x + \cot^{-1} x = \pi/2$	$x \in \mathbb{R}$
Cofunction (sec/csc)	$\sec^{-1} x + \csc^{-1} x = \pi/2$	$ x \geq 1$
Sum (tan)	$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$	$xy < 1$
Difference (tan)	$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$	$xy > -1$
Double (tan \rightarrow tan)	$2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}$	$ x < 1$
Double (tan \rightarrow sin)	$2 \tan^{-1} x = \sin^{-1} \frac{2x}{1+x^2}$	$ x \leq 1$
Double (tan \rightarrow cos)	$2 \tan^{-1} x = \cos^{-1} \frac{1-x^2}{1+x^2}$	$x \geq 0$
Double (sin)	$2 \sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})$	$ x \leq 1/\sqrt{2}$
Double (cos)	$2 \cos^{-1} x = \cos^{-1}(2x^2 - 1)$	$0 \leq x \leq 1$

Master these eighteen identities and the principal-value table on page 1 — together they cover every Class 12 board question and the vast majority of JEE-Main problems on Inverse Trigonometric Functions.