Collegedunia NCERT Formula Sheet

Class 12 Mathematics (12th Maths) — NCERT 2026-27

Chapter 3: Matrices

Definitions | Types | Operations | Transpose | Symmetric/Skew | Elementary Operations | Inverse

Chapter Snapshot. A matrix is an ordered rectangular array of numbers/functions. This chapter formalises matrix **notation**, lists the standard **types**, defines the algebra of **addition**, **scalar multiplication** and **matrix multiplication** (with its non-commutative behaviour), introduces the **transpose** and the **symmetric/skew-symmetric** decomposition, and ends with **elementary row/column operations** as a route to the **inverse**. All identities below assume real entries and conformable orders.

1 Matrix, Order & Notation

This section fixes the language used in the rest of the chapter: what a matrix is, how its size is written, and how individual entries are addressed.

General $m \times n$ matrix

$$A = [a_{ij}]_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

where m = number of rows, n = number of columns, a_{ij} = entry in the i-th row and j-th column ($1 \le i \le m$, $1 \le j \le n$).

The pair (m, n) is the **order**. Two matrices have the same order only when both m and n match.

Number of entries & possible orders

Total entries in $A_{m\times n} = m \cdot n$

If a matrix has N entries, its possible orders are the ordered pairs (m, n) with $m \cdot n = N$.

Example: a matrix with 12 entries can be of order $1\times12,\ 2\times6,\ 3\times4,\ 4\times3,\ 6\times2,\ 12\times1$ — six orders in total.

Constructing a matrix from a rule

If $a_{ij} = f(i, j)$ is specified, build A by substituting each (i, j) pair. Example: $a_{ij} = \frac{1}{2}|i - 3j|$ gives the entries directly. The construction rule completely fixes the matrix once its order is known.

2 Types of Matrices

NCERT names eight standard families; each is a restriction on the order or on the entries.

Row, column, square, rectangular

Row matrix: order $1 \times n$, e.g. $[a_{11}\ a_{12}\ \cdots\ a_{1n}]$.

Column matrix: order $m \times 1$.

Square matrix: $m = n$; written as a square matrix of order n.

Rectangular: $m \neq n$.

In a square matrix of order n, the elements $a_{11}, a_{22}, \ldots, a_{nn}$ form the **principal** (or **leading**) **diagonal**.

Diagonal, scalar, identity, zero

Diagonal: square with $a_{ij} = 0$ for all $i \neq j$. Write **diag**$(d_1, d_2, \ldots, d_n)$.

Scalar: diagonal with $d_1 = d_2 = \cdots = d_n = k$.

Identity: scalar with $k = 1$, denoted I_n; $a_{ij} = \delta_{ij}$.

Zero (null): every entry is 0; denoted O (any order).

Every identity is scalar, every scalar is diagonal, every diagonal is square — but not conversely.

Equality of matrices

$A = B \iff$ (i) A and B have the **same order**, AND (ii) $a_{ij} = b_{ij}$ for every i, j.

Comparing two matrices of different orders is undefined — equality is checked entry-by-entry only after the order matches.

3 Addition & Scalar Multiplication

These two operations are defined entry-wise. Both require the operands to share a common order; the result keeps that same order.

Matrix addition

$A + B = [a_{ij} + b_{ij}]_{m\times n}$, where A, B are both $m \times n$.

Subtraction: $A - B = A + (-1)B = [a_{ij} - b_{ij}]$.

Addition is defined **only when the orders are identical**. If orders differ, $A + B$ does not exist.

Scalar multiplication

$kA = [k \cdot a_{ij}]_{m\times n}$, for any scalar $k \in \mathbb{R}$.

Negative of a matrix: $-A = (-1)A = [-a_{ij}]$.

Every entry is multiplied by the same scalar k; the order of the matrix is unchanged.

Properties: addition

Let A, B, C be $m \times n$ matrices, O the $m \times n$ zero matrix.

Commutative: $A + B = B + A$.

Associative: $(A + B) + C = A + (B + C)$.

Additive identity: $A + O = O + A = A$.

Additive inverse: $A + (-A) = O$.

Matrix addition behaves **exactly like ordinary number addition** on the set of $m \times n$ matrices.

Properties: scalar multiplication

For scalars $k, l \in \mathbb{R}$ and $m \times n$ matrices A, B:

$k(A + B) = kA + kB$

$(k + l)A = kA + lA$

$k(lA) = (kl)A = l(kA)$

$1 \cdot A = A, \quad 0 \cdot A = O$

Scalars distribute over matrix sums and over each other; they may be freely regrouped with the matrix factor.

4 Matrix Multiplication

*The most subtle operation here. It has a strict conformability rule and is generally **not commutative**.*

Conformability & product

Product AB is defined iff **(columns of A) = (rows of B)**.

If A is $m \times n$ and B is $n \times p$, then AB is $m \times p$ with

$$(AB)_{ij} = \sum_{k=1}^{n} a_{ik}\, b_{kj}$$

Read as: **row i of A dotted with column j of B**. The shared dimension n disappears in the product's order.

Non-commutativity

In general, $AB \neq BA$. Possible cases:

(i) AB defined, BA not defined (orders mismatch).

(ii) AB and BA both defined but of different orders.

(iii) AB and BA same order but $AB \neq BA$.

When $AB = BA$, the matrices are said to **commute**. Diagonal matrices of the same order always commute; the identity commutes with everything.

Properties of matrix multiplication

Assuming all products below are defined:

Associative: $(AB)C = A(BC)$

Distributive: $A(B + C) = AB + AC$, $(A + B)C = AC + BC$

Identity: $I_m\, A = A\, I_n = A$ for A of order $m \times n$

Scalar pull-out: $k(AB) = (kA)B = A(kB)$

The **cancellation law fails**: $AB = AC$ does not imply $B = C$, even when $A \neq O$.

Zero product without zero factor

For numbers, $ab = 0 \Rightarrow a = 0$ or $b = 0$.

For matrices this is **false**: $AB = O$ can occur with $A \neq O$ and $B \neq O$. Example: $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ gives $AB = O$ but $BA \neq O$.

Positive integer powers

For a square matrix A and $k \in \mathbb{N}$:

$$A^k = \underbrace{A \cdot A \cdots A}_{k \text{ times}}, \quad A^0 = I$$

$$A^m A^n = A^{m+n}, \quad (A^m)^n = A^{mn}$$

Powers are defined **only for square matrices** because the operand and product must have matching orders.

5 Transpose of a Matrix

The transpose swaps the row and column roles. It is the bridge to the symmetric/skew decomposition next.

Definition of transpose

If $A = [a_{ij}]_{m\times n}$, then

$$A' = A^T = [a_{ji}]_{n\times m}$$

Row i of A becomes column i of A'. The order flips from $m \times n$ to $n \times m$.

Properties of transpose

For matrices A, B of compatible orders and scalar k:

$(A')' = A$

$(kA)' = k\, A'$

$(A + B)' = A' + B'$

$(AB)' = B'\, A'$ **(reversal law)**

The product reverses order under transpose: $(ABC)' = C'B'A'$. This generalises to any finite number of factors.

6 Symmetric & Skew-Symmetric

A classification of square matrices via the transpose, with a clean decomposition theorem.

Symmetric & skew-symmetric

Let A be a square matrix.

Symmetric: $A' = A$, i.e. $a_{ij} = a_{ji}$ for all i, j.

Skew-symmetric: $A' = -A$, i.e. $a_{ij} = -a_{ji}$ for all i, j.

In a skew-symmetric matrix, the diagonal entries satisfy $a_{ii} = -a_{ii}$, hence $a_{ii} = 0$ — **every diagonal entry is zero**.

Decomposition theorem

Every square matrix A can be uniquely written as the sum of a symmetric matrix P and a skew-symmetric matrix Q:

$$A = P + Q, \quad P = \frac{1}{2}(A + A'), \quad Q = \frac{1}{2}(A - A')$$

Checks: $P' = P$ and $Q' = -Q$.

$\frac{1}{2}(A + A')$ is always symmetric and $\frac{1}{2}(A - A')$ is always skew-symmetric, regardless of A.

Useful corollaries

$A + A'$ is always symmetric; $A - A'$ is always skew-symmetric; AA' and $A'A$ are always symmetric. For a skew-symmetric A of odd order, $\det(A) = 0$ (JEE extension via Chapter 4).

7 Elementary Operations & Inverse

NCERT introduces six elementary operations (three on rows, three on columns) and uses them to test invertibility and to compute the inverse.

Six elementary operations

On rows (R) or columns (C) of a matrix:

1. Interchange: $R_i \leftrightarrow R_j$ (or $C_i \leftrightarrow C_j$).

2. Scaling: $R_i \to kR_i$, $k \neq 0$ (or $C_i \to kC_i$).

3. Addition of a multiple: $R_i \to R_i + kR_j$, $j \neq i$ (or $C_i \to C_i + kC_j$).

Every elementary operation is **reversible**: each has an inverse operation of the same type that undoes it.

Inverse of a square matrix

A square matrix A of order n is **invertible** if there exists a square matrix B of order n with

$$AB = BA = I_n$$

B is then unique and is denoted A^{-1}. So $AA^{-1} = A^{-1}A = I$.

Inverse exists **only for square matrices** and only when the defining equation has a solution; such a matrix is also called **non-singular**.

Uniqueness & reversal law for inverse

Uniqueness: the inverse of an invertible matrix is unique.

Reversal: if A and B are invertible of the same order,

$$(AB)^{-1} = B^{-1}A^{-1}$$

Same order-reversal pattern as transpose: $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$, mirroring $(ABC)' = C'B'A'$.

Inverse via elementary row operations

To find A^{-1} by elementary row operations, start with

$$A = I \cdot A$$

Apply a sequence of elementary **row** operations to both sides until the LHS becomes I. The matrix that has accumulated on the RHS where I stood is A^{-1}:

$$I = A^{-1} \cdot A$$

If at any stage one entire row of the LHS becomes zero, A^{-1} **does not exist**. The same procedure works with column operations using $A = A \cdot I$.

Don't mix rows & columns

While computing A^{-1} from $A = IA$, use **only row operations** throughout. If you start with $A = AI$ instead, use **only column operations**. Mixing the two in a single computation destroys the bookkeeping and gives a wrong inverse.

Adjoint formula (Ch. 4 link)

The standard non-NCERT-Ch. 3 route is via determinants: if $\det(A) \neq 0$,

$$A^{-1} = \frac{1}{\det(A)}\,\text{adj}(A).$$

Use this when A is small (2×2 or 3×3); use elementary operations for larger or hand-friendly cases.

Reversal twins

$(AB)' = B'A'$ and $(AB)^{-1} = B^{-1}A^{-1}$.

Same shape, same flip: transpose and inverse both **reverse the order** of a product. Pair them in memory to avoid sign/order slips.

Quick Reference — Chapter 3 Matrices

Compact summary of every named identity used above

Concept	Statement / Formula
Matrix	$A = [a_{ij}]_{m\times n}$; order $= (m, n)$; entries $= m \cdot n$
Row / column matrix	order $1 \times n$ / $m \times 1$
Square matrix	$m = n$; diagonal $a_{11}, \ldots, a_{nn}$
Diagonal matrix	square with $a_{ij} = 0$ for $i \neq j$
Scalar matrix	diagonal with all diagonal entries equal
Identity matrix I_n	scalar with 1's on the diagonal; $a_{ij} = \delta_{ij}$
Zero matrix O	every entry 0
Equality	same order and $a_{ij} = b_{ij}$ for all i, j
Addition	$A + B = [a_{ij} + b_{ij}]$ (same order required)
Scalar multiplication	$kA = [k\, a_{ij}]$
Multiplication	$(AB)_{ij} = \sum_k a_{ik}\, b_{kj}$; $A_{m\times n}\, B_{n\times p} = C_{m\times p}$
Non-commutativity	$AB \neq BA$ in general
Powers	$A^m A^n = A^{m+n}$, $(A^m)^n = A^{mn}$, $A^0 = I$
Identity in $\times$	$I_m A = A I_n = A$
Transpose	$A' = [a_{ji}]_{n\times m}$
Transpose laws	$(A')' = A$, $(kA)' = kA'$, $(A + B)' = A' + B'$, $(AB)' = B'A'$
Symmetric	$A' = A$
Skew-symmetric	$A' = -A$; all diagonal entries $= 0$
Decomposition	$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$ (symm. + skew)
Elementary ops	$R_i \leftrightarrow R_j$; $R_i \to kR_i$ ($k \neq 0$); $R_i \to R_i + kR_j$ (and column analogues)
Invertible matrix	square A with $AB = BA = I$; $B = A^{-1}$ unique
Inverse reversal	$(AB)^{-1} = B^{-1}A^{-1}$
Inverse via row ops	start $A = IA$; row-reduce LHS to I; RHS becomes A^{-1}
Adjoint route (Ch. 4)	$A^{-1} = \frac{1}{\det(A)}$ adj(A) when $\det(A) \neq 0$