Collegedunia NCERT Formula Sheet

Class 12 Mathematics (12th Maths) — NCERT 2026-27

Chapter 5: Continuity & Differentiability

Continuity | Algebra of Derivatives | Chain Rule | Implicit | Inverse Trig | Exponential & Logarithmic | Logarithmic Diff. | Parametric | Second Derivative | MVT & Rolle

Chapter Snapshot. This chapter extends Class 11 differentiation to richer function classes. It pins down **continuity** on a domain via one-sided limits, lifts the Class 11 algebra of derivatives to **composites** (chain rule), then handles **implicit**, **inverse trigonometric**, **exponential**, **logarithmic**, **logarithmic-differentiation**, and **parametric** forms. It closes with the **second derivative** and two foundational existence theorems — **Rolle's** and the **Mean Value Theorem**.

1 Continuity

A function is continuous at c when its graph has no break there. NCERT pins this down via a three-condition limit test.

Continuity at a point

f is continuous at $x = c$ iff all three hold:

(i) $f(c)$ is defined,

(ii) $\lim_{x\to c} f(x)$ exists,

(iii) $\lim_{x\to c} f(x) = f(c)$.

Equivalent one-line form: $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$ — **LHL** = **RHL** = **value**.

Continuity on an interval

f is continuous on an open interval (a, b) if it is continuous at every point of (a, b).

On $[a, b]$, additionally require right-continuity at a ($\lim_{x\to a^+} f(x) = f(a)$) and left-continuity at b ($\lim_{x\to b^-} f(x) = f(b)$).

"Continuous on its domain" means the three-condition test holds at **every** point of the domain.

Algebra of continuous functions

If f, g are continuous at c, so are:

$f \pm g,\quad f\,g,\quad k\,f,\quad \frac{f}{g}$ (provided $g(c) \neq 0$).

Composite: if f is continuous at c and g is continuous at $f(c)$, then $g \circ f$ is continuous at c.

Polynomials, rational functions (away from zeros of denominator), $\sin x, \cos x, e^x$ are continuous on $\mathbb{R}$; $\log x$ is continuous on $(0, \infty)$.

Discontinuity types

A function fails the three-condition test in three ways: (a) $f(c)$ undefined, (b) limit fails to exist (jump or oscillation), (c) limit exists but $\neq f(c)$ (removable). NCERT treats these collectively as "discontinuity at c".

2 Differentiability & Derivatives

Differentiability is a stronger condition than continuity — it demands the limit of the slope of the chord to exist.

Derivative — first principle

$$f'(c) = \lim_{h\to 0} \frac{f(c+h) - f(c)}{h}$$

provided the limit exists (one number, finite).

Geometrically the limit is the **slope of the tangent** to $y = f(x)$ at $x = c$.

Continuity ⇐ Differentiability

If f is differentiable at c, then f is continuous at c.

Converse is **false**: $f(x) = |x|$ is continuous at 0 but not differentiable there. Continuity is necessary, not sufficient.

Algebra of derivatives

For differentiable u, v and constant k:

$(u \pm v)' = u' \pm v'$

$(k\,u)' = k\,u'$

Product rule: $(u\,v)' = u'\,v + u\,v'$

Quotient rule: $\left(\frac{u}{v}\right)' = \frac{u'\,v - u\,v'}{v^2}$, $v \neq 0$

Same as Class 11 — extended here to every elementary function listed below.

Algebraic & trigonometric derivatives

$\frac{d}{dx}(x^n) = n\,x^{n-1}$

$\frac{d}{dx}(\sin x) = \cos x$, $\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$, $\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$, $\frac{d}{dx}(\csc x) = -\csc x \cot x$

The negative sign sits on the three "co-functions": **cos**, **cot**, **csc**.

Exponential & log derivatives

$\frac{d}{dx}(e^x) = e^x$, $\frac{d}{dx}(a^x) = a^x \ln a$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$ $(x > 0)$, $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

Natural exponential is the unique function equal to its own derivative; that fact pins down all other exponential/log derivatives.

Inverse-trig derivatives

$(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$

$(\cos^{-1} x)' = -\frac{1}{\sqrt{1-x^2}}$, $|x| < 1$

$(\tan^{-1} x)' = \frac{1}{1+x^2}$

$(\cot^{-1} x)' = -\frac{1}{1+x^2}$

$(\sec^{-1} x)' = \frac{1}{|x|\sqrt{x^2-1}}$

$(\csc^{-1} x)' = -\frac{1}{|x|\sqrt{x^2-1}}$, $|x| > 1$

Each "co-inverse" is the negative of its partner. The $|x|$ factor in $\sec^{-1}$ / $\csc^{-1}$ comes from the principal-branch convention.

$|x|$ inside the radical

For $\sec^{-1}$ and $\csc^{-1}$, the denominator is $|x|\sqrt{x^2-1}$, **not** $x\sqrt{x^2-1}$. Dropping the absolute value gives the wrong sign for $x < -1$.

3 Chain Rule & Implicit Form

The two tools that take you from standard derivatives to derivatives of arbitrary composites and equations.

Chain rule

If $y = f(u)$ and $u = g(x)$ are both differentiable,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = f'(u) \cdot g'(x)$$

Multi-step: $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$

Differentiate the outer function leaving the inner alone, then multiply by the derivative of the inner. Works for any finite chain.

Implicit differentiation

For an equation $F(x, y) = 0$, differentiate both sides with respect to x, treating y as a function of x, then solve algebraically for $\frac{dy}{dx}$.

Key step: $\frac{d}{dx}(g(y)) = g'(y)\,\frac{dy}{dx}$

Use when y cannot be solved explicitly in terms of x — e.g. $x^2 + y^2 = a^2$, $\sin(xy) = x + y$.

Inverse trig via chain rule

$\frac{d}{dx}(\sin^{-1}(f(x))) = \frac{f'(x)}{\sqrt{1 - f(x)^2}}$. Same wrapper for the other five inverse-trig derivatives — chain rule does the work.

4 Logarithmic Differentiation

Two situations crash through algebraic differentiation: variable-base variable-exponent ($y = u^v$) and products of many factors. Logarithmic differentiation handles both.

Logarithmic differentiation — recipe

Given $y = f(x)$ involving products, quotients, or $u(x)^{v(x)}$, take ln both sides:

$$\ln y = \ln f(x)$$

Differentiate implicitly: $\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(\ln f(x))$

Solve: $\frac{dy}{dx} = y \cdot \frac{d}{dx}(\ln f(x))$

Converts products to sums and powers to products — turning a long differentiation into short ones.

Standard case: $y = u(x)^{v(x)}$

$\ln y = v(x)\, \ln u(x)$ gives

$$\frac{dy}{dx} = u^v \left[v' \ln u + \frac{v\,u'}{u} \right]$$

Required whenever both the base and the exponent depend on x (e.g. $y = x^{\sin x}$). Power rule and exponential rule each fail by themselves.

5 Parametric & 2nd Derivative

When the curve is given by $x = f(t)$, $y = g(t)$, differentiate without eliminating t. Higher derivatives apply the operator d/dx repeatedly.

Parametric derivative

If $x = f(t)$ and $y = g(t)$ with $f'(t) \neq 0$,

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{g'(t)}{f'(t)}$$

Cleanly handles curves like $x = a\cos t$, $y = a\sin t$ without having to solve for y as a function of x.

Parametric second derivative

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{1}{dx/dt} \cdot \frac{d}{dt}\left(\frac{dy}{dx}\right)$$

Warning: d^2y/dx^2 is **not** the simple ratio of the two t-second-derivatives. Always differentiate dy/dx first with respect to t and then divide by dx/dt once more.

Second derivative

$$f''(x) = \frac{d}{dx}(f'(x)) = \frac{d^2y}{dx^2}$$

$f'(x)$ tracks the slope; $f''(x)$ tracks how the slope itself changes — the basis for concavity and Chapter 6's second-derivative test.

Parametric pitfall

$\frac{d^2y}{dx^2} \neq \frac{d^2y/dt^2}{d^2x/dt^2}$. Always differentiate dy/dx as a function of t first, then divide by dx/dt a second time.

6 Mean Value Theorems

Two existence theorems — Rolle's is a special case of the MVT. They sit at the foundation of nearly every monotonicity, root-counting and inequality argument in Chapter 6.

Rolle's theorem

If f is
(i) continuous on $[a, b]$,
(ii) differentiable on (a, b), and
(iii) $f(a) = f(b)$,
then there exists $c \in (a, b)$ with

$$f'(c) = 0$$

At least one stationary point lies strictly between the equal-value endpoints. The theorem is an **existence** statement — c need not be unique.

Mean Value Theorem (Lagrange)

If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists $c \in (a, b)$ with

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

At some interior point the **instantaneous slope equals the average slope** across the interval. Rolle's theorem is the special case $f(a) = f(b)$.

Reading the MVT

Geometrically: somewhere between a and b the tangent to the graph is parallel to the chord joining $(a, f(a))$ and $(b, f(b))$. All three hypotheses are essential — drop any one and the conclusion can fail.

Standard MVT inequality

For $0 < a < b$, applying MVT to $f(x) = \ln x$ gives $\frac{b-a}{b} < \ln \frac{b}{a} < \frac{b-a}{a}$. A representative JEE/NEET application of MVT to derive inequalities.

CCC-D triple check

For every limit problem in this chapter ask: **Continuous**? **Continuous on closed interval**? **Differentiable on open interval**? If yes, Rolle/MVT applies. Three Cs, one D — that's all the bookkeeping the theorems need.

Quick Reference — Chapter 5 Continuity & Differentiability

Compact summary of every named identity used above

Concept	Statement / Formula
Continuity at c	$\lim_{x\to c^-} f = \lim_{x\to c^+} f = f(c)$
Algebra of continuity	sum / product / quotient ($g \neq 0$) / composite preserve continuity
Derivative — first principle	$f'(c) = \lim_{h\to 0} \dfrac{f(c+h)-f(c)}{h}$
Differentiability $\Rightarrow$ continuity	true; converse false ($\lvert x\rvert$ at 0)
$(u \pm v)'$, $(uv)'$, $(u/v)'$	sum/product/quotient rules
$(x^n)'$	nx^{n-1}
$(\sin x)', (\cos x)'$	$\cos x, \ -\sin x$
$(\tan x)', (\cot x)'$	$\sec^2 x, \ -\csc^2 x$
$(\sec x)', (\csc x)'$	$\sec x \tan x, \ -\csc x \cot x$
$(e^x)', (a^x)'$	$e^x, \ a^x \ln a$
$(\ln x)', (\log_a x)'$	$1/x, \ 1/(x \ln a)$
$(\sin^{-1} x)', (\cos^{-1} x)'$	$\pm 1/\sqrt{1-x^2}$
$(\tan^{-1} x)', (\cot^{-1} x)'$	$\pm 1/(1+x^2)$
$(\sec^{-1} x)', (\csc^{-1} x)'$	$\pm 1/(\lvert x\rvert\sqrt{x^2-1})$
Chain rule	$\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$
Implicit	differentiate $F(x,y)=0$, solve for dy/dx
Log differentiation	$y = u^v \Rightarrow \ln y = v \ln u$
$y = u^v$ derivative	$u^v\left[v' \ln u + \frac{vu'}{u}\right]$
Parametric dy/dx	$g'(t)/f'(t)$
Parametric d^2y/dx^2	$\dfrac{1}{f'(t)}\dfrac{d}{dt}(g'(t)/f'(t))$
Second derivative	$f''(x) = \dfrac{d^2y}{dx^2}$
Rolle's theorem	cont. $[a,b]$, diff. (a,b), $f(a)=f(b) \Rightarrow \exists\, c: \ f'(c)=0$
MVT	cont. $[a,b]$, diff. $(a,b) \Rightarrow \exists\, c: \ f'(c) = \dfrac{f(b)-f(a)}{b-a}$