Collegedunia NCERT Formula Sheet

Class 12 Mathematics (12th Maths) — NCERT 2026-27

Chapter 6: Application of Derivatives

Rate of Change | Increasing/Decreasing | Tangent & Normal | Approximation | Maxima/Minima | Derivative Tests

Chapter Snapshot. The derivative, introduced in Chapter 5, here becomes a tool to extract information about a function. This chapter uses $f'(x)$ to read off **rate of change**, **monotonic** (increasing/decreasing) behaviour, equations of **tangents and normals**, small-change **approximations**, and the **absolute and local extrema** of a function on an interval — closed via the **first** and **second derivative tests**.

1 Rate of Change

A derivative is, before anything else, an instantaneous rate. If a variable changes with time (or with any other parameter), $\frac{d}{dt}$ *gives the speed of that change.*

Derivative as a rate

If $y = f(x)$, $\left.\frac{dy}{dx}\right|_{x=x_0}$ = instantaneous rate of change of y w.r.t. x at $x = x_0$.

Units of dy/dx are (units of y) per (unit of x). If y and x both depend on t, use the chain rule: $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$.

Marginal cost & marginal revenue

For a cost function $C(x)$ and revenue $R(x)$ where x = units produced/sold:

$$\text{MC} = \frac{dC}{dx}, \quad \text{MR} = \frac{dR}{dx}$$

Marginal cost is the approximate cost of producing one extra unit; **marginal revenue** the approximate revenue from selling one extra unit.

Related rates — standard examples

Sphere: $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$.

Sphere surface: $S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r \frac{dr}{dt}$.

Cube: $V = a^3 \Rightarrow \frac{dV}{dt} = 3a^2 \frac{da}{dt}$.

General recipe: write the geometric/physical relation, differentiate both sides w.r.t. t, then plug in the known rate.

Sign of dy/dx

A positive rate $dy/dx > 0$ at x_0 means y is increasing there; negative means decreasing; zero means a momentary stationary point. The sign is the central diagnostic for everything that follows in the chapter.

2 Increasing & Decreasing

A derivative-based test that classifies an interval, not just a single point.

Monotonicity test

Let f be continuous on $[a, b]$ and differentiable on (a, b).

$f'(x) > 0$ on $(a, b) \Rightarrow f$ **strictly increasing** on $[a, b]$.

$f'(x) < 0$ on $(a, b) \Rightarrow f$ **strictly decreasing** on $[a, b]$.

$f'(x) = 0$ for all $x \in (a, b) \Rightarrow f$ constant on $[a, b]$.

Weak versions ($\geq$ and $\leq$) give **non-decreasing** and **non-increasing** respectively. The strict/weak distinction matters in JEE.

Method — find monotonic intervals

Step 1. Compute $f'(x)$.

Step 2. Solve $f'(x) = 0$; mark these **critical points** on the number line.

Step 3. Test the sign of f' in each sub-interval between consecutive critical points (and at $\pm\infty$ for unbounded domains).

Step 4. Read off the increasing/decreasing intervals from the signs.

Include endpoints only when they belong to the domain. Open vs closed intervals follow NCERT convention: include if the function is defined and the result extends continuously.

Useful corollary

If $f'(x) \geq 0$ on (a, b) and $f'(x) = 0$ at **only finitely many** points, then f is still strictly increasing on $[a, b]$. This lets isolated stationary points coexist with strict monotonicity.

3 Tangents & Normals

The derivative provides the slope; coordinate geometry provides the line.

Slope of tangent / normal

At a point (x_0, y_0) on $y = f(x)$:

Slope of tangent $m_T = f'(x_0) = \left.\dfrac{dy}{dx}\right|_{x_0}$.

Slope of normal $m_N = -\dfrac{1}{f'(x_0)}$ (when $f'(x_0) \neq 0$).

Tangent and normal are **perpendicular** at the point of contact: $m_T\, m_N = -1$.

Equation of tangent at (x_0, y_0)

$$y - y_0 = f'(x_0)\,(x - x_0)$$

Vertical tangent: if $f'(x_0)$ is infinite, tangent is the vertical line $x = x_0$.

Point-slope form of the line through (x_0, y_0) with slope m_T.

Equation of normal at (x_0, y_0)

$$y - y_0 = -\frac{1}{f'(x_0)}\,(x - x_0)$$

Horizontal normal: if $f'(x_0) = 0$, normal is the vertical line $x = x_0$.

Same point-slope template, slope replaced by $-1/f'(x_0)$. Special cases handle the slopes 0 and ∞.

Angle of intersection of two curves

At a common point, if the slopes of the tangents to the two curves are m_1, m_2, the angle θ between them satisfies

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Curves are **orthogonal** iff $m_1 m_2 = -1$.

Direct lift from coordinate geometry — the curves' tangents at the intersection point inherit the line-angle formula.

4 Approximation & Differentials

For small changes in the input, the derivative gives a quick first-order estimate of the change in the output.

Differentials

dx = small change in x (independent).

$dy = f'(x)\,dx$ — the differential of y.

dy is the change in the tangent-line approximation, not in f itself. The two agree to first order.

Approximation formula

For a small change Δx near $x = x_0$,

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\,\Delta x$$

The error is of order $(\Delta x)^2$. Use small Δx — typically $|\Delta x| \lesssim 0.05\,|x_0|$ for reliable accuracy.

Worked-style — $\sqrt{a+h}$

$\sqrt{a+h} \approx \sqrt{a} + \dfrac{h}{2\sqrt{a}}$. For $\sqrt{25.3}$: take $a = 25,\ h = 0.3$ to get $5 + 0.03 = 5.03$.

5 Maxima & Minima

*NCERT distinguishes **local** extrema (best in a neighbourhood) from **absolute** extrema (best on the entire interval) and uses two derivative tests.*

Definitions

On a domain D:

Local maximum at c: $f(c) \geq f(x)$ for all x near c.

Local minimum at c: $f(c) \leq f(x)$ for all x near c.

Absolute max/min on D: $f(c)$ is the largest/smallest value of f on the entire D.

"Local" = in some neighbourhood; "absolute" = over the whole domain. An absolute extremum is automatically a local extremum if c is an interior point.

Critical / stationary points

c is a **critical point** of f if $f'(c) = 0$ or $f'(c)$ does not exist (and c is in the domain).

By **Fermat's theorem**, every interior local extremum is a critical point. The converse is false — not every critical point is an extremum.

First-derivative test

Let c be a critical point of f, with f continuous near c. Examine f' on either side:

Local max: f' changes from $+$ to $-$ at c.

Local min: f' changes from $-$ to $+$ at c.

Neither (inflection): f' keeps the same sign.

Always applicable when f is just continuous around c. Required whenever the second-derivative test is inconclusive.

Second-derivative test

Let $f''(c)$ exist with $f'(c) = 0$.

$f''(c) < 0$: local maximum at c.

$f''(c) > 0$: local minimum at c.

$f''(c) = 0$: test is **inconclusive**; fall back to the first-derivative test or higher derivatives.

Fast when applicable. The sign of f'' reflects concavity: down ($-$) at a peak, up ($+$) at a valley.

Absolute extrema on $[a, b]$

For a continuous f on a closed bounded interval $[a, b]$:

Step 1. Find all critical points of f in (a, b).

Step 2. Evaluate f at every critical point AND at both endpoints a, b.

Step 3. The largest of these values is the **absolute maximum**; the smallest is the **absolute minimum**.

Extreme Value Theorem guarantees both exist when f is continuous on a closed bounded interval. Endpoints **must** be checked — they are not critical points but can host the absolute extreme.

Endpoint trap

For absolute extrema on $[a, b]$, never skip $f(a)$ and $f(b)$. Even when f' never vanishes inside (a, b), the absolute max/min is then attained at one of the endpoints — missing it gives a wrong answer outright.

Recipe — applied optimisation

Read the problem → choose the variable x → express the quantity $Q(x)$ to be optimised → note the domain (often a closed interval) → solve $Q'(x) = 0$ inside the domain → apply 1st or 2nd derivative test → compare with endpoint values → state the answer in problem units.

FDT vs SDT

First-Derivative Test = sign **change** across c.

Second-Derivative Test = sign of $f''(c)$ **at** c.

“Change” is wider — works whenever f is continuous; “at” is faster but needs f'' to exist and be non-zero.

Quick Reference — Chapter 6 Application of Derivatives

Compact summary of every named identity used above

Concept	**Statement / Formula**
Rate of change	$\frac{dy}{dx}$ at x_0
Time-linked rates	$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$
Marginal cost / revenue	MC $= C'(x)$, MR $= R'(x)$
Sphere volume rate	$dV/dt = 4\pi r^2\, dr/dt$
Monotonicity	$f' > 0$ inc.; $f' < 0$ dec.; $f' = 0$ const.
Slope of tangent	$m_T = f'(x_0)$
Slope of normal	$m_N = -1/f'(x_0)$
Tangent equation	$y - y_0 = f'(x_0)(x - x_0)$
Normal equation	$y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)$
Angle between curves	$\tan\theta = \left\lvert\frac{m_1 - m_2}{1 + m_1 m_2}\right\rvert$
Orthogonal curves	$m_1 m_2 = -1$
Differential	$dy = f'(x)\,dx$
Linear approximation	$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\,\Delta x$
Critical point	$f'(c) = 0$ or $f'(c)$ undefined
Fermat's theorem	interior local extremum $\Rightarrow$ critical point
First-derivative test	sign change of f' at c: $+\to-$ max, $-\to+$ min
Second-derivative test	$f'(c) = 0$; $f''(c) < 0$ max, $f''(c) > 0$ min, $= 0$ inconclusive
Absolute extrema on $[a, b]$	compare f at critical points and endpoints
EVT	continuous f on closed bounded $[a, b]$ attains max & min