Collegedunia NCERT Notes

The Ultimate NCERT Revision Guide for Class 12 Mathematics

Chapter 6: Application of Derivatives

What this chapter covers: the derivative is no longer just a symbol — here it becomes a tool. We use $\frac{dy}{dx}$ to track *rates of change*, to test whether a function is rising or falling, and to hunt down the highest and lowest values a quantity can take. The rationalised NCERT retains rate of change, monotonicity, and maxima/minima; *Tangents & Normals* and *Approximations using differentials* are included as **[JEE/NEET Extension]** sections because both remain heavily tested in entrance exams.

1 Rate of Change of Quantities

A derivative measures how fast one quantity changes when another changes. If y depends on x, then $\frac{dy}{dx}$ at a point gives the *instantaneous rate* at which y changes per unit change in x. When both x and y depend on a third variable — usually time t — the chain rule lets us convert between rates: this is the heart of every **related rates** problem in this chapter.

1.1 Derivative as a rate measurer

If $y = f(x)$, then $\frac{dy}{dx}$ at $x = x_0$ is the rate of change of y with respect to x at that point. The same idea generalises: if a quantity Q depends on time t, then $\frac{dQ}{dt}$ is the rate at which Q changes with time.

Rate of Change

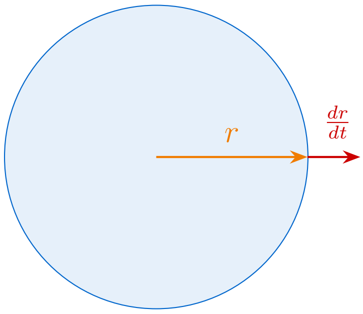
For $y = f(x)$, the rate of change of y w.r.t. x at $x = x_0$ is

$$\left.\frac{dy}{dx}\right|_{x=x_0} = f'(x_0).$$

If both x and y are functions of t, the chain rule gives

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \qquad \frac{dx}{dt} \neq 0.$$

Sign convention for rates. A positive rate means the quantity is increasing; a negative rate means it is decreasing. A radius growing at 2 cm/s means $\frac{dr}{dt} = +2$; a balloon *deflating* at 2 cm/s means $\frac{dr}{dt} = -2$.



Radius grows ⇒ Area grows: $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

The chain-rule bridge

Most rate problems give one rate and ask for another. The bridge is a **geometric or physical relation** between the two quantities (e.g. $A = \pi r^2$, $V = \frac{4}{3}\pi r^3$). Differentiate that relation w.r.t. time, plug in known values, solve.

Contents

1.2 Related rates — standard worked example

Problem. A balloon, in the shape of a sphere, is being inflated at the rate of 900 cm^3/s. Find the rate at which the radius is increasing when the radius is 15 cm.

Setup. Volume of a sphere: $V = \dfrac{4}{3}\pi r^3$.

Differentiate w.r.t. t.

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}.$$

Substitute $\frac{dV}{dt} = 900$ and $r = 15$:

$$900 = 4\pi(15)^2 \cdot \frac{dr}{dt} = 900\pi \cdot \frac{dr}{dt} \implies \frac{dr}{dt} = \frac{1}{\pi} \text{ cm/s}.$$

The 3-step recipe

(1) Identify all variables and the relation between them. (2) Differentiate the relation w.r.t. t. (3) Substitute the *instant* values *after* differentiating — never before.

1.3 Marginal cost and marginal revenue

In economics, if $C(x)$ is the total cost of producing x items, then

$$\text{Marginal Cost (MC)} = \frac{dC}{dx}.$$

This is the approximate cost of producing one additional item beyond the x-th. Similarly, if $R(x)$ is the total revenue,

$$\text{Marginal Revenue (MR)} = \frac{dR}{dx}.$$

Marginal Quantities

$$\text{MC} = \frac{dC}{dx}, \qquad \text{MR} = \frac{dR}{dx}, \qquad \text{Profit } P(x) = R(x) - C(x).$$

At maximum profit, $\frac{dP}{dx} = 0 \Rightarrow \text{MR} = \text{MC}$.

Why ride-share surge pricing exists

At peak hours the marginal cost of an additional ride (driver time, fuel, opportunity cost) rises sharply. Surge multipliers raise the marginal revenue per ride to match, keeping supply on the road. The condition MR $=$ MC is profit-optimal microeconomics, straight out of derivatives.

Plug *after*, not before

A common slip: students substitute $r = 15$ into $V = \frac{4}{3}\pi r^3$ *first*, getting a constant, then differentiate to get 0. You must differentiate the *symbolic* relation first, and only then substitute the instantaneous values.

1.4 A second worked example — shadow length

Problem. A man 2 m tall walks away from a lamp post 5 m tall at a speed of 1.2 m/s. How fast is the tip of his shadow moving along the ground?

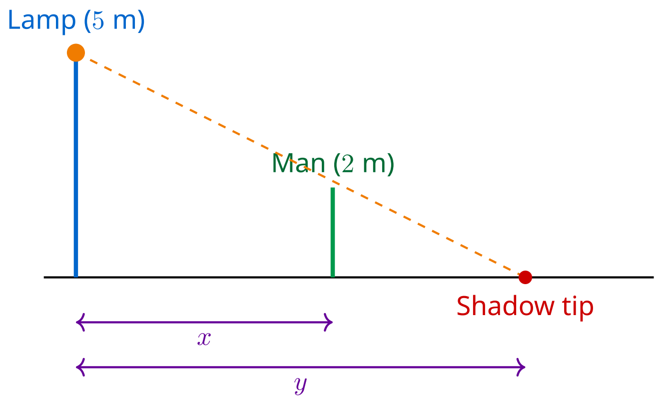
Setup. Let x be the man's distance from the post and y the distance of the shadow tip from the post. By similar triangles,

$$\frac{y}{5} = \frac{y-x}{2} \implies 2y = 5y - 5x \implies y = \frac{5x}{3}.$$

Differentiate w.r.t. t.

$$\frac{dy}{dt} = \frac{5}{3} \cdot \frac{dx}{dt} = \frac{5}{3}(1.2) = 2 \text{ m/s.}$$

The shadow tip moves at 2 m/s along the ground — faster than the man himself.



Why the shadow is faster

The shadow tip moves with speed $\frac{5}{3} \approx 1.67$ times the man's own speed, because the ratio comes from the lamp/man height ratio. Taller lamps slow the shadow down; shorter lamps speed it up.

2 Increasing and Decreasing Functions

The sign of the derivative tells us, at a glance, whether a function is climbing or descending. This single idea governs monotonicity, the location of extrema, and the shape of every curve we sketch.

2.1 Definitions

Let I be an interval in the domain of f.

- f is **increasing on** I if $x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)$ for all $x_1, x_2 \in I$.
- f is **strictly increasing on** I if $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$.

- f is **decreasing on** I if $x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2)$.
- f is **strictly decreasing on** I if $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$.

A function that is increasing on some intervals and decreasing on others is called *non-monotonic*; the points where it switches are exactly the critical points (Section 3).

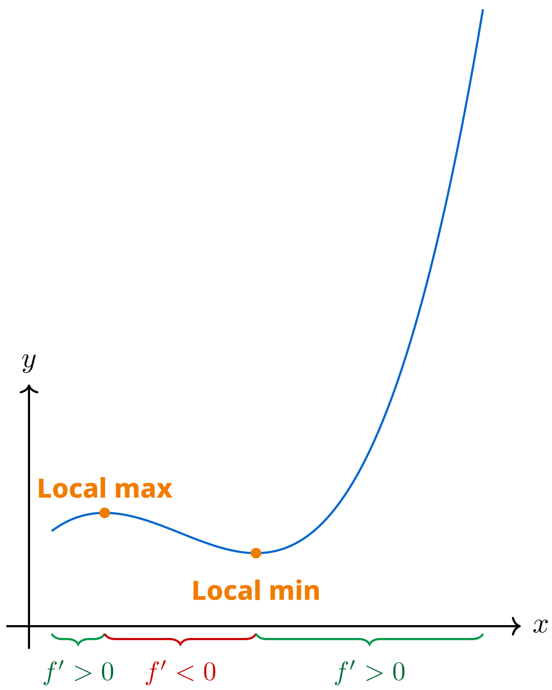
2.2 The first-derivative test for monotonicity

Monotonicity Theorem

Let f be continuous on $[a, b]$ and differentiable on (a, b). Then:

- $f'(x) > 0$ for all $x \in (a, b) \implies f$ is strictly increasing on $[a, b]$.
- $f'(x) < 0$ for all $x \in (a, b) \implies f$ is strictly decreasing on $[a, b]$.
- $f'(x) = 0$ for all $x \in (a, b) \implies f$ is constant on $[a, b]$.

The condition is sufficient, not necessary — $f(x) = x^3$ is strictly increasing on $\mathbb{R}$ even though $f'(0) = 0$. A single isolated zero of f' does not stop the function from being strictly increasing.



Sign of f' tells you the slope direction.

2.3 Standard procedure to find monotonic intervals

To find where f is increasing or decreasing:

1. Compute $f'(x)$.
2. Solve $f'(x) = 0$ to find **critical points**; also note where $f'(x)$ is undefined.

3. These points partition the real line (or the domain) into intervals.
4. Pick one test point in each interval, check the sign of f' there, and classify the interval as increasing $(+)$ or decreasing $(-)$.

Worked example. Find the intervals where $f(x) = x^3 - 3x^2 - 9x + 7$ is increasing or decreasing.

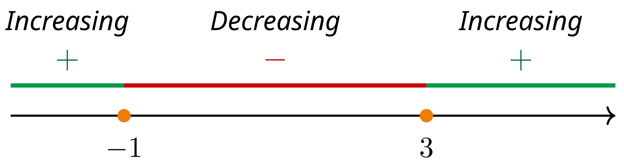
Step 1. $f'(x) = 3x^2 - 6x - 9 = 3(x-3)(x+1)$.

Step 2. $f'(x) = 0 \Rightarrow x = -1, 3$. These split $\mathbb{R}$ into $(-\infty, -1)$, $(-1, 3)$, $(3, \infty)$.

Step 3. Sign chart:

- On $(-\infty, -1)$: take $x = -2$, $f'(-2) = 3(-5)(-1) = +15 > 0 \quad \Rightarrow$ increasing.
- On $(-1, 3)$: take $x = 0$, $f'(0) = -9 < 0 \quad \Rightarrow$ decreasing.
- On $(3, \infty)$: take $x = 4$, $f'(4) = 3(1)(5) = +15 > 0 \quad \Rightarrow$ increasing.

So f is strictly increasing on $(-\infty, -1) \cup (3, \infty)$ and strictly decreasing on $(-1, 3)$.



Factor f' before sign-charting

If $f'(x)$ factors cleanly, the sign chart is almost free — just track the sign of each factor at each test point. Trying to evaluate a complicated polynomial at every test point is slower and error-prone.

2.4 Strict vs non-strict — a subtlety

The standard CBSE convention (followed in the NCERT 2026–27 syllabus) is:

- If $f'(x) \geq 0$ on I (with equality at only isolated points), f is increasing on I.
- If $f'(x) > 0$ throughout I, f is *strictly* increasing.

In board problems, "increasing" usually means "increasing in the inclusive sense" — include the endpoint where $f' = 0$ as long as it is isolated.

Open vs closed intervals

The intervals are usually reported *closed* at endpoints where $f' = 0$ (so $f(x) = x^3 - 3x^2 - 9x + 7$ is increasing on $(-\infty, -1]$ and $[3, \infty)$). But monotonicity at the endpoint of the *domain* requires you to check that the function is defined and continuous there.

3 Maxima and Minima

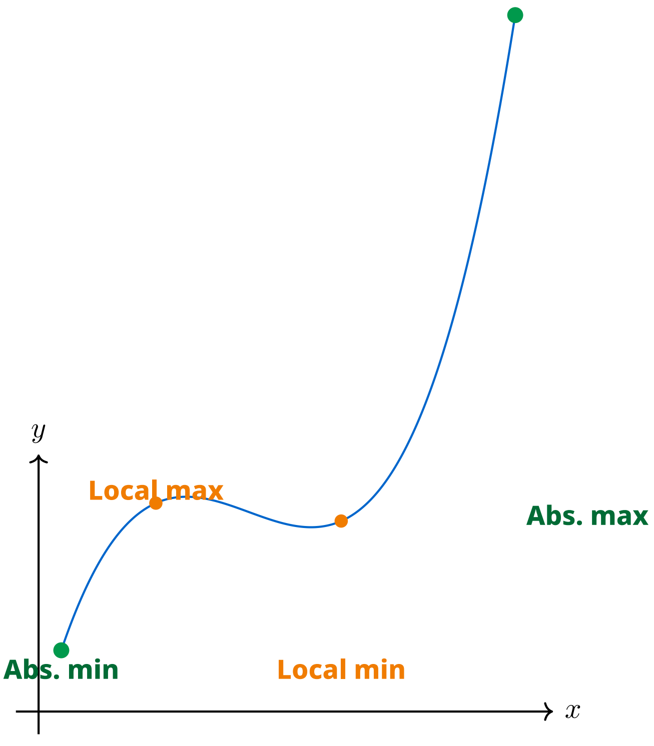
We now arrive at the chapter's central theme: finding the highest and lowest values a function attains. Two flavours appear — **local** (extremum compared to a small neighbourhood) and **absolute** (global extremum over the whole domain or a closed interval).

3.1 Definitions: local and absolute extrema

Let f be defined on an interval I and $c \in I$.

- c is a **point of local maximum** if there is an open interval around c on which $f(c) \geq f(x)$. The value $f(c)$ is the local maximum value.
- c is a **point of local minimum** if there is an open interval around c on which $f(c) \leq f(x)$. The value $f(c)$ is the local minimum value.
- c is a **point of absolute (global) maximum on** I if $f(c) \geq f(x)$ for every $x \in I$.
- Absolute minimum: similarly, with $\leq$ replaced.

Extremum (plural **extrema**) is the umbrella term for any maximum or minimum — local or absolute.



A local maximum in the interior need not be the absolute maximum. On a closed interval, the absolute extremum may sit at an endpoint and have nothing to do with the derivative.

Local vs absolute — in one sentence

A *local* extremum dominates only its neighbourhood; an *absolute* extremum dominates the entire interval under consideration.

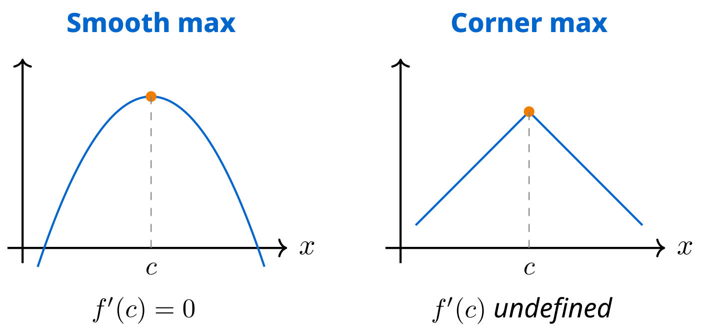
3.2 Critical points

A point c in the interior of the domain of f is called a **critical point** of f if either

$$f'(c) = 0 \qquad \text{or} \qquad f'(c) \text{ does not exist.}$$

Fermat's theorem. If f has a local extremum at an interior point c and $f'(c)$ exists, then $f'(c) = 0$.

Consequence. Every interior local extremum is a critical point. The converse is *false*: not every critical point is an extremum (e.g. $f(x) = x^3$ at $x = 0$ — $f'(0) = 0$ but it is an inflection, not an extremum).



"Critical point" has two flavours

Critical points are *not* only the roots of f'. They also include points where f' does not exist — corners, cusps, vertical tangents. Functions like $|x|$ at $x = 0$ have a critical point with no derivative.

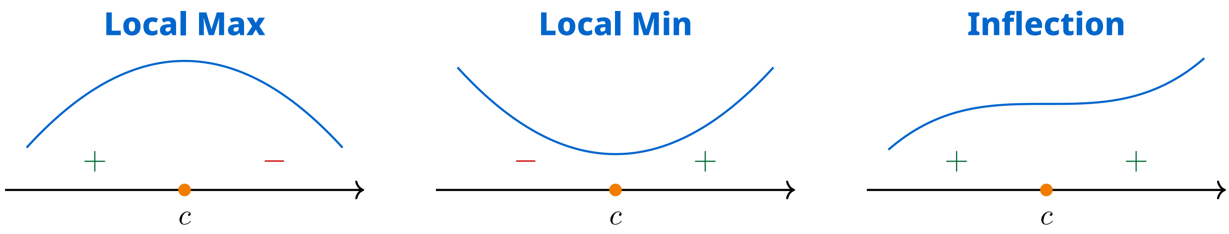
3.3 First Derivative Test

To classify a critical point c (with f continuous at c), inspect the sign of f' on either side.

First Derivative Test

At an interior critical point c:

- f' changes from $+$ to $-$ as x crosses c $\Rightarrow c$ is a **local maximum**.
- f' changes from $-$ to $+$ as x crosses c $\Rightarrow c$ is a **local minimum**.
- f' does *not* change sign at c $\Rightarrow c$ is a **point of inflection** (neither max nor min).



3.4 Second Derivative Test

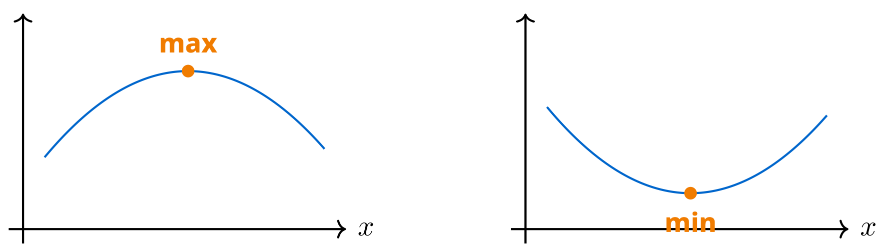
If computing the sign of f' on both sides feels heavy, the second derivative can do the work in one shot — provided it exists and is non-zero at the critical point.

Second Derivative Test

Let c be an interior critical point with $f'(c) = 0$ and $f''(c)$ defined.

- $f''(c) < 0$ $\Rightarrow$ f has a **local maximum** at c (curve concave down).
- $f''(c) > 0$ $\Rightarrow$ f has a **local minimum** at c (curve concave up).
- $f''(c) = 0$ $\Rightarrow$ test is **inconclusive**; fall back to the first derivative test.

Geometric intuition. f'' measures concavity. At a local max, the curve is shaped like ∩ — concave down — so $f'' < 0$. At a local min, it is shaped like ∪ — concave up — so $f'' > 0$.



$f''(c) < 0$ (concave down) *$f''(c) > 0$ (concave up)*

3.5 First vs Second Derivative Test — comparison

Aspect	First Derivative Test	Second Derivative Test
What it inspects	Sign of f' on *both sides* of c	Sign of f'' *at* c
Works when f'' doesn't exist	Yes	No
Works at corners/cusps	Yes	No (requires twice-differentiability)
Verdict on inflection	Detects it (no sign change)	Inconclusive if $f''(c) = 0$
Speed	Slower (need test points)	Faster (single evaluation)
Reliability	Always works for continuous f	Fails if $f''(c) = 0$
Best used when	Critical point is a corner or f'' is hard	f' and f'' are both clean

Pick the test by the situation

Polynomial with easy f'': use the second derivative test. Modulus / piecewise / fractional-power functions where f'' may not exist: use the first derivative test. If the second test is inconclusive ($f''(c) = 0$), *switch* to the first — don't keep computing higher derivatives unless the problem demands it.

Concavity sign rule

"Frown is max, smile is min." $f'' < 0$ means the curve frowns (∩, concave down) — a maximum. $f'' > 0$ means it smiles (∪, concave up) — a minimum.

4 Absolute Maximum and Minimum on a Closed Interval

When f is continuous on a closed and bounded interval $[a, b]$, the *Extreme Value Theorem* guarantees that f attains both an absolute maximum and an absolute minimum somewhere in $[a, b]$. Our job is to find them.

4.1 The closed-interval algorithm

Absolute Extrema on a Closed Interval

Let f be continuous on $[a, b]$ and differentiable on (a, b) (except possibly at finitely many points).

1. Find all critical points of f in (a, b) — points where $f' = 0$ or f' is undefined.
2. Evaluate f at each critical point.
3. Evaluate f at the endpoints a and b.
4. The largest of these values is the absolute maximum; the smallest is the absolute minimum.

This is sometimes called the **candidate test**: the absolute extrema can only happen at critical points or endpoints, so we evaluate f at every candidate and pick the winner.

4.2 Worked example

Problem. Find the absolute maximum and minimum of $f(x) = 2x^3 - 15x^2 + 36x + 1$ on $[1, 5]$.

Step 1. $f'(x) = 6x^2 - 30x + 36 = 6(x-2)(x-3)$. Critical points: $x = 2$ and $x = 3$, both in $(1, 5)$.

Step 2. Evaluate at critical points and endpoints:

- $f(1) = 2 - 15 + 36 + 1 = 24$
- $f(2) = 16 - 60 + 72 + 1 = 29$
- $f(3) = 54 - 135 + 108 + 1 = 28$
- $f(5) = 250 - 375 + 180 + 1 = 56$

Step 3. Absolute maximum $= 56$ at $x = 5$ (endpoint). Absolute minimum $= 24$ at $x = 1$ (endpoint).

Endpoint dominance

On a closed interval, the absolute extremum often sits at an *endpoint*, not at an interior critical point. Always evaluate both endpoints alongside the critical points — forgetting them is the most common mark-losing slip.

Don't apply the second derivative test for absolute extrema

The second derivative test classifies *local* extrema. On a closed interval, you must compare *values* at all critical points and both endpoints — a critical point may be a local max but still be smaller than $f(a)$ or $f(b)$.

5 Optimisation Problems

Optimisation is what makes derivatives an industrial tool. We translate a worded scenario into a function of one variable, then apply the maxima–minima machinery to extract the optimal value.

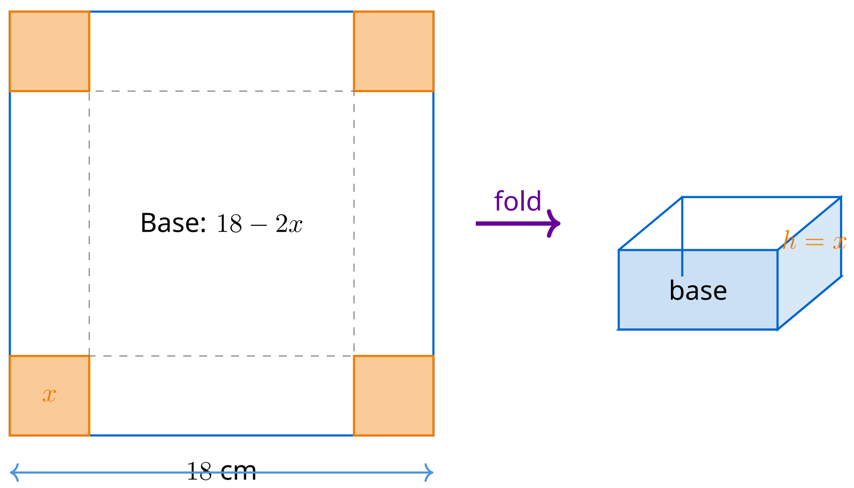
5.1 Standard recipe

1. Draw a clear diagram and label the variables.
2. Identify the quantity to be optimised (call it Q); express it as a function of the given variables.
3. Use the constraint(s) to reduce Q to a function of a single variable, say $Q = Q(x)$.
4. Note the natural domain of x from the physical constraints (lengths positive, $x \leq$ side length, etc.).
5. Find $\frac{dQ}{dx}$, solve $\frac{dQ}{dx} = 0$, get critical points.
6. Apply the second derivative test or compare values to identify the optimum.
7. Report the optimal value of Q, not just x.

5.2 Worked example — maximum volume of an open box

Problem. From a square sheet of side 18 cm, identical squares of side x cm are cut from each corner and the flaps folded up to make an open box. Find the value of x that maximises the volume.

Setup. After cutting and folding, the base of the box is a square of side $(18-2x)$ cm and the height is x cm.



Volume function.

$$V(x) = x(18 - 2x)^2, \qquad x \in (0, 9).$$

Differentiate.

$V'(x) = (18-2x)^2 + x \cdot 2(18-2x)\cdot(-2) = (18-2x)\big[(18-2x)-4x\big] = (18-2x)(18-6x).$

Setting $V'(x) = 0$: $x = 9$ (rejected, would mean no base) or $x = 3$.

Second derivative test.

$$V''(x) = -2(18-6x) + (18-2x)(-6) = -36 + 12x - 108 + 12x = 24x - 144.$$

At $x = 3$: $V''(3) = 72 - 144 = -72 < 0 \Rightarrow$ **maximum**.

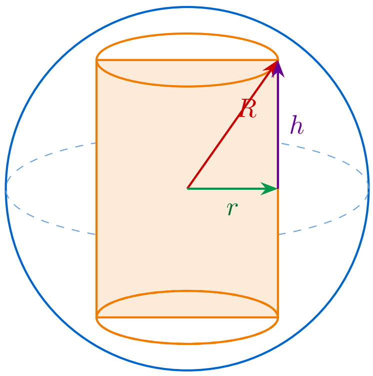
Maximum volume: $V(3) = 3 \cdot 12^2 = 432$ cm^3.

> **Packaging design**
>
> Every cardboard box on a shop shelf began as a flat sheet with a cut-and-fold pattern. Manufacturers run this exact derivative calculation to maximise capacity for a fixed sheet size — the difference between a 400 cm^3 and a 432 cm^3 box, scaled to millions of units, is a real cost saving.

5.3 Worked example — sphere inscribed in a cone

Problem. Show that the height of the cylinder of maximum volume inscribed in a sphere of radius R is $\dfrac{2R}{\sqrt{3}}$.

Setup. Let the cylinder have height $2h$ and radius r. By the right triangle inside the sphere, $r^2 + h^2 = R^2$.



$r^2 + h^2 = R^2$

Volume.

$$V = \pi r^2(2h) = 2\pi h(R^2 - h^2).$$

Differentiate.

$$\frac{dV}{dh} = 2\pi(R^2 - 3h^2).$$

$V' = 0 \Rightarrow h^2 = \dfrac{R^2}{3} \Rightarrow h = \dfrac{R}{\sqrt{3}}.$

Second derivative. $V''(h) = -12\pi h < 0$ at $h = R/\sqrt{3}$. Hence it is a maximum, and the height of the cylinder is

$$2h = \frac{2R}{\sqrt{3}}.$$

> **The trig substitution trick**
>
> For sphere/cylinder/cone optimisation, setting $r = R\sin\theta$, $h = R\cos\theta$ often kills the radical and converts the volume into a clean trigonometric polynomial. Try this when algebraic differentiation gets messy.

5.4 Worked example — wire bent into square and circle

Problem. A wire of length 28 cm is to be cut into two pieces. One piece is bent into a square and the other into a circle. What should be the lengths of the two pieces so that the combined area of the square and the circle is *minimum*?

Setup. Let the piece bent into the square have length x, so the circle gets length $(28 - x)$.

- Square: side $= x/4$, area $= x^2/16$.
- Circle: circumference $= 28 - x \Rightarrow$ radius $r = (28 - x)/(2\pi)$, area $= \pi r^2 = (28 - x)^2/(4\pi)$.

Total area.

$$A(x) = \frac{x^2}{16} + \frac{(28 - x)^2}{4\pi}, \qquad x \in [0, 28].$$

Differentiate.

$$A'(x) = \frac{x}{8} - \frac{28 - x}{2\pi}.$$

Setting $A'(x) = 0$:

$$\frac{x}{8} = \frac{28 - x}{2\pi} \implies \pi x = 4(28 - x) \implies x = \frac{112}{\pi + 4}.$$

Second derivative.

$$A''(x) = \frac{1}{8} + \frac{1}{2\pi} > 0.$$

So $x = \dfrac{112}{\pi + 4}$ cm is a minimum. The circle gets $28 - x = \dfrac{28\pi}{\pi + 4}$ cm.

> **"Minimum combined area" heuristic**
>
> When a fixed perimeter is split between shapes, the algebra always reduces to a sum-of-squares-like expression. $A'' > 0$ is automatic — you can skip the test and assert minimum once $A' = 0$ has a unique solution in the open domain.

5.5 Common optimisation templates

A few problem types recur every year. Memorise the setup, not the answer.

- **Rectangle in a circle/ellipse:** maximise area; use parametric form.
- **Wire bent into shapes:** fixed perimeter, vary the split between square and circle.
- **Largest cone in a sphere:** relate cone height to sphere radius via geometry.
- **Cylinder inscribed in a cone:** similar-triangles constraint.
- **Minimum surface area for fixed volume:** dial-up the symmetry — a cube minimises surface among rectangular boxes; a sphere minimises among all 3D shapes.

6 [JEE/NEET Extension] Tangents and Normals

The rationalised NCERT 2026–27 syllabus dropped tangents and normals from the chapter, but the topic remains a JEE Main/Advanced staple. The key idea: the derivative is the *slope of the tangent*.

6.1 Slope formulas

For a curve $y = f(x)$ at the point $P(x_0, y_0)$:

Tangent and Normal at (x_0, y_0)

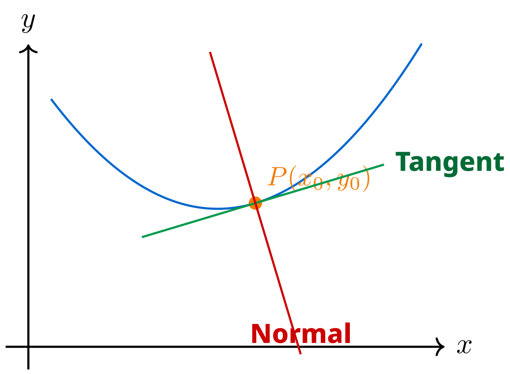
$$\text{Slope of tangent}: m_T = \left.\frac{dy}{dx}\right|_{(x_0,y_0)} = f'(x_0)$$

$$\text{Slope of normal}: m_N = -\frac{1}{f'(x_0)} \quad (\text{if } f'(x_0) \neq 0)$$

$$\text{Equation of tangent}: \quad y - y_0 = m_T(x - x_0)$$

$$\text{Equation of normal}: \quad y - y_0 = m_N(x - x_0)$$

The tangent is the line that "just touches" the curve at P; the normal is the line through P perpendicular to the tangent.



Special cases of slope.

- Tangent is *horizontal* if $f'(x_0) = 0$; equation $y = y_0$.
- Tangent is *vertical* if $f'(x_0) \to \infty$; equation $x = x_0$. Normal is then horizontal: $y = y_0$.
- Angle of the tangent with the x-axis: $\tan\theta = f'(x_0)$.

6.2 Angle between two curves

If two curves intersect at P and have tangent slopes m_1, m_2 there, the acute angle θ between them is

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|.$$

The curves are **orthogonal** (perpendicular) at P if $m_1 m_2 = -1$.

Two-curve intersection

First solve for the intersection point, then compute each curve's $\frac{dy}{dx}$ at that point, then apply the angle formula. Don't try to do all three at once.

7 [JEE/NEET Extension] Approximations using Differentials

A linear approximation built from the derivative lets us estimate $\sqrt{25.3}$, $(31)^{1/5}$, or $\sin 61^\circ$ without a calculator.

7.1 The differential dy

If $y = f(x)$ and Δx is a small change in x, define the **differential**

$$dy = f'(x)\,dx.$$

Here $dx = \Delta x$ is treated as an independent infinitesimal change. The differential dy is the *linear approximation* to the actual change $\Delta y = f(x + \Delta x) - f(x)$.

Linear Approximation

$$f(x + \Delta x) \approx f(x) + f'(x)\,\Delta x.$$

For small Δx, the right side is a very good estimate of the left.

Worked example. Approximate $\sqrt{25.3}$.

Take $f(x) = \sqrt{x}$, $x = 25$, $\Delta x = 0.3$.

$f(25) = 5$, $f'(x) = 1/(2\sqrt{x})$, so $f'(25) = 1/10$.

$$\sqrt{25.3} \approx 5 + \tfrac{1}{10}(0.3) = 5.03.$$

(True value: $5.02991\ldots$ — the approximation is accurate to 0.0001.)

7.2 Error estimates

If x is measured with an error Δx and $y = f(x)$:

- Absolute error in y: $\Delta y \approx f'(x)\,\Delta x$.
- Relative error in y: $\dfrac{\Delta y}{y} \approx \dfrac{f'(x)\,\Delta x}{f(x)}$.
- Percentage error: $\dfrac{\Delta y}{y} \times 100$%.

Example. The radius of a sphere is measured as 9 cm with an error of 0.03 cm. Estimate the percentage error in the calculated volume.

$V = \frac{4}{3}\pi r^3 \Rightarrow \dfrac{dV}{dr} = 4\pi r^2$. Hence

$$\frac{\Delta V}{V} = \frac{4\pi r^2\,\Delta r}{\frac{4}{3}\pi r^3} = \frac{3\,\Delta r}{r} = \frac{3 \times 0.03}{9} = 0.01 = 1\%.$$

Why $\times 3$?

For a sphere, $V \propto r^3$, so a small relative error in r *triples* when it propagates to V. In general, if $Q \propto x^n$, the relative error in Q is n times the relative error in x.

Don't confuse Δy and dy

Δy is the *exact* change in y; $dy = f'(x)\,dx$ is its *linear approximation*. They agree to first order but diverge for large Δx. Always state whether you're using the exact change or the differential approximation.

8 Quick Reference Summary

A condensed sheet for the night before the exam. If you remember nothing else, remember these.

8.1 Core formulas at a glance

Master Formula Sheet

Rate of change : $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

Monotonicity : $f' > 0 \Rightarrow$ increasing, $f' < 0 \Rightarrow$ decreasing

Critical point : $f'(c) = 0$ or $f'(c)$ undefined

First derivative test : f' sign change $+ \to - \Rightarrow$ max, $- \to + \Rightarrow$ min

Second derivative test : $f''(c) < 0 \Rightarrow$ max, $f''(c) > 0 \Rightarrow$ min

Absolute extrema on $[a, b]$: compare f at critical points and endpoints

Slope of tangent : $m_T = f'(x_0)$ [JEE/NEET]

Slope of normal : $m_N = -1/f'(x_0)$ [JEE/NEET]

Differential : $dy = f'(x)\,dx$ [JEE/NEET]

8.2 Standard derivatives you should know cold

Function	Derivative	Use case
x^n	nx^{n-1}	Power rule
$\sin x$, $\cos x$	$\cos x$, $-\sin x$	Sign flip on cosine
$\tan x$	$\sec^2 x$	≥ 1 always
e^x	e^x	Self-referential
$\ln x$	$1/x$	$x > 0$ only
$\sin^{-1} x$	$1/\sqrt{1-x^2}$	$\lvert x \rvert < 1$
$\tan^{-1} x$	$1/(1+x^2)$	All x
$\lvert x \rvert$	$\text{sgn}(x)$, undefined at 0	Critical pt at 0

8.3 Procedure cheat sheet

1. **Rate problem:** write the geometric relation $\to$ differentiate w.r.t. t $\to$ substitute instant values.
2. **Monotonicity:** find f' $\to$ solve $f' = 0$ $\to$ sign-chart the intervals.
3. **Local extrema:** find critical points $\to$ apply first or second derivative test.
4. **Absolute extrema on $[a, b]$:** find critical points in (a, b) $\to$ evaluate f there and at a, b $\to$ pick largest/smallest.
5. **Optimisation:** draw diagram $\to$ build $Q(x)$ using constraint $\to$ find domain $\to$ differentiate $\to$ classify critical points $\to$ report optimum value.
6. **Tangent/Normal [JEE]:** compute $f'(x_0)$ $\to$ tangent slope $= f'(x_0)$, normal slope $= -1/f'(x_0)$ $\to$ point-slope form.
7. **Approximation [JEE]:** pick a nearby "nice" x $\to$ compute $f(x)$, $f'(x)$ $\to$ apply

$$f(x + \Delta x) \approx f(x) + f'(x)\Delta x.$$

8.4 Common pitfalls — the one-page warning list

- Substituting instant values *before* differentiating in a rate problem $\rightarrow$ always differentiate first.
- Forgetting endpoints when finding absolute extrema on a closed interval.
- Treating "$f'(c) = 0$" as automatic proof of an extremum — it could be an inflection point.
- Applying the second derivative test at a corner or cusp where f'' doesn't exist.
- Using local-extremum logic when the question asks for absolute extremum.
- Forgetting that "critical point" includes points where f' is undefined, not just $f' = 0$.
- Mixing up tangent and normal slopes: they are *negative reciprocals*, not negatives of each other.
- Confusing Δy (exact change) with dy (linear approximation).

8.5 Final word

The derivative is one of the most powerful single ideas in school mathematics. In this chapter we cashed it in for three concrete payoffs: **rates** (how fast things change), **shape** (where a function rises, falls, peaks, dips), and **optima** (the best possible value of a designed quantity). Every JEE/NEET application of calculus you will meet — in physics, in chemistry, in economics — is some variation on these three themes.

Master the procedural recipes, draw the picture before reaching for algebra, and the chapter rewards you with marks that are very hard to lose.

Best of luck with your preparation!

Revise, practise, and trust your reasoning.