Collegedunia NCERT Formula Sheet

Class 12 Mathematics (12th Maths) — NCERT 2026-27

Chapter 7: Integrals

Indefinite Integration | Standard Integrals | Substitution | Partial Fractions | Integration by Parts | Special Integrals | Definite Integrals | Properties | Evaluation

Chapter Snapshot. Integration is the inverse of differentiation. The chapter opens with the **indefinite integral** (an antiderivative plus an arbitrary constant), tabulates **standard integrals**, lays out the three workhorse techniques — **substitution**, **partial fractions**, and **integration by parts** (with the **ILATE** priority) — then catalogues **special integrals**. The second half builds the **definite integral**, the **Fundamental Theorem of Calculus**, and the **properties** that simplify symmetric and piecewise integrals.

1 Indefinite Integral

*If $F'(x) = f(x)$, then F is an antiderivative of f. The collection of **all** antiderivatives is the indefinite integral.*

Definition & notation

$$\int f(x)\,dx = F(x) + C$$

where $F'(x) = f(x)$ and C is an arbitrary constant of integration. Two antiderivatives of f differ by a constant. The symbol $\int \cdot\, dx$ reads "the family of antiderivatives w.r.t. x".

Linearity

$$\int [k_1 f(x) + k_2 g(x)]\,dx = k_1 \int f\,dx + k_2 \int g\,dx$$

Constants pull out of integrals and sums split — exactly mirroring derivatives. Linearity is the foundation of every reduction technique below.

Why $+C$?

Differentiation kills constants ($\frac{d}{dx}c = 0$), so undoing it recovers a constant up to a free additive choice. Always include $+C$ in every indefinite-integral answer.

2 Standard Integrals

A core table — every later technique reduces ultimately to one of these.

Power, exponential, log

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

The case $n = -1$ is the lone power-rule exception — it gives $\ln|x|$, not a power of x. The absolute value handles negative x.

Trigonometric

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

Sign pattern: the three co-functions (**cos**, **cot**, **csc**) introduce the minus. Each row is the inverse of a Chapter 5 derivative.

Inverse trigonometric

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$$

$$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} |x| + C$$

These three are companion to the Chapter 5 inverse-trig derivatives. The other inverse-trig integrals differ only by sign.

$\sec x$, $\csc x$, $\tan x$, $\cot x$

$$\int \tan x\,dx = \ln|\sec x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\,dx = \ln|\csc x - \cot x| + C$$

Standard NCERT-listed integrals; the $\sec x$ and $\csc x$ results follow from the rationalising trick $\sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x}$.

3 Integration by Substitution

A direct rewrite of the chain rule: replace a chunk of the integrand with a new variable so the integral collapses to a standard form.

Substitution rule

If $u = g(x)$ is differentiable,

$$\int f(g(x))\,g'(x)\,dx = \int f(u)\,du$$

Pick u so that $du = g'(x)\,dx$ absorbs the "extra" factor in the integrand. Substitution is the reverse of the chain rule.

Two essential substitution corollaries

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$$

$$\int f'(x)\,[f(x)]^n\,dx = \frac{[f(x)]^{n+1}}{n+1} + C \quad (n \neq -1)$$

These two patterns close out most "inner-function" integrals: spot f' in the numerator, then read off the log or the power directly.

Trig power reduction

$$\sin^2 x = \frac{1-\cos 2x}{2}, \qquad \cos^2 x = \frac{1+\cos 2x}{2}.$$

Power-of-trig integrals collapse to standard ones via these identities.

4 Partial Fractions

A rational integrand $P(x)/Q(x)$ with $\deg P < \deg Q$ splits into a sum of simpler fractions, each on the standard-integral list.

Standard decompositions

Linear distinct:
$\frac{1}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$

Repeated linear:
$\frac{1}{(x-a)^2} = \frac{A}{x-a} + \frac{B}{(x-a)^2}$

Irreducible quadratic:
$\frac{1}{(x-a)(x^2+bx+c)} = \frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$

Compare coefficients (or substitute convenient x values) to find $A, B, C, \ldots$, then integrate each piece using the standard table.

Degree check first

Before decomposing, ensure $\deg P < \deg Q$. If not, do **polynomial long division** first; only the remainder needs partial-fractions treatment.

5 Integration by Parts

Inverse of the product rule. Choose one factor to differentiate and the other to integrate.

Integration-by-parts formula

$$\int u\,dv = uv - \int v\,du$$

or equivalently, with $u = u(x)$, $v = v(x)$,

$$\int u(x)\,v'(x)\,dx = u(x)\,v(x) - \int u'(x)\,v(x)\,dx$$

Choose u to be the factor whose derivative is *simpler* (or eventually vanishes), and v' the factor easy to integrate.

ILATE rule

Priority order for picking u:

Inverse trig → **L**ogarithmic → **A**lgebraic → **T**rigonometric → **E**xponential.

The function class earlier in ILATE becomes u; the later one becomes dv.

Two key IBP identities

$$\int e^x \left[f(x) + f'(x)\right] dx = e^x f(x) + C$$

$$\int \ln x\,dx = x \ln x - x + C$$

The first identity short-circuits IBP when the integrand has the $e^x(f + f')$ shape; the second is the canonical NCERT IBP example.

6 Special Integrals

A short catalogue of templates whose answers should be memorised — each is needed in standard NCERT problems.

Quadratic denominators

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

Three sister forms — sign of a^2 relative to x^2 picks the answer family. Memorise the position of the $\frac{1}{2a}$ factor.

Surd denominators

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|x + \sqrt{x^2+a^2}\right| + C$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x + \sqrt{x^2-a^2}\right| + C$$

Inside the radical: $-x^2 \to \sin^{-1}$; $+x^2 \to \ln$. The Chapter 7 surd-integral suite.

Integrals of surds

With $\sqrt{a^2-x^2}$ in the integrand: $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$.

With $\sqrt{x^2+a^2}$: $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right| + C$.

With $\sqrt{x^2-a^2}$: $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$.

Each has two pieces — a polynomial $\frac{x}{2}\sqrt{\cdot}$ plus the corresponding inverse-trig or $\ln$ contribution from above.

$\sqrt{ax^2+bx+c}$ and $1/\sqrt{ax^2+bx+c}$

Complete the square: $ax^2+bx+c = a\left[(x+b/2a)^2 + (c/a - b^2/4a^2)\right]$, substitute $u = x + b/2a$, then read off the answer from the standard radical table above.

7 Definite Integral

*Attaches limits a and b to an antiderivative and gives a single **number**. The connector to areas, averages, and signed displacement.*

Definite integral — limit-of-sums

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} h\sum_{r=1}^{n} f(a+rh)$$

where $h = (b-a)/n$. Riemann-sum definition. NCERT presents this as the rigorous starting point; in practice we evaluate definite integrals via the next box.

Fundamental Theorem of Calculus

FTC I (existence): If f is continuous on $[a,b]$, then $A(x) = \int_a^x f(t)\,dt$ is differentiable and $A'(x) = f(x)$.

FTC II (evaluation): If F is any antiderivative of f on $[a,b]$,

$$\int_a^b f(x)\,dx = F(b) - F(a) = \left[F(x)\right]_a^b$$

FTC II converts definite-integral evaluation into a two-point look-up of any antiderivative — no Riemann-sum work needed.

Substitution in definite integrals

If $u = g(x)$ with g continuous and differentiable,

$$\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

Substitute the **limits** too — failing to update them is the single most common slip in definite-integral substitution.

8 Properties — Definite

Seven NCERT-numbered properties that simplify symmetric, periodic, and piecewise integrals — often without computing an antiderivative.

P1 & P2 — variable name & reversal

$$\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

The dummy variable can be renamed; swapping the limits flips the sign.

P3 — splitting at an interior point

For $a < c < b$,

$$\int_a^b f\,dx = \int_a^c f\,dx + \int_c^b f\,dx$$

Essential for piecewise-defined integrands — split at every break-point of f.

P4 — reflection $a + b - x$

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

Swaps the integrand under reflection about the midpoint. The single most powerful definite-integral trick — turns hostile integrands into manageable ones.

P5 — symmetric interval

For an even f: $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$.

For an odd f: $\int_{-a}^{a} f(x)\,dx = 0$.

Even: $f(-x) = f(x)$; odd: $f(-x) = -f(x)$. Cuts the work in half on symmetric intervals — and saves it entirely for odd integrands.

P6 — doubling rule on $[0, 2a]$

$$\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$$

Consequence: if $f(2a-x) = f(x)$, the integral equals $2\int_0^a f(x)\,dx$; if $f(2a-x) = -f(x)$, it is 0. Extends the symmetric-interval idea to $[0, 2a]$. Watch for **sin** x on $[0, 2\pi]$ — classic test case.

Periodic functions

If f has period T, $\int_0^{nT} f(x)\,dx = n\int_0^T f(x)\,dx$ for $n \in \mathbb{N}$. Shifts of the interval by any multiple of T leave the integral unchanged.

Limit update under substitution

When substituting $u = g(x)$ in a definite integral, the limits change to $g(a)$ and $g(b)$ — do NOT leave them as a and b. Forgetting this is the single highest-frequency mistake in board exams.

Quick Reference — Chapter 7 Integrals

Compact summary of every named identity used above

Concept	Statement / Formula
Indefinite integral	$\int f\,dx = F + C$ where $F' = f$
Linearity	$\int (k_1 f + k_2 g)\,dx = k_1 \int f + k_2 \int g$
Power	$\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C\ (n \neq -1)$
$1/x$	$\int \dfrac{dx}{x} = \ln\|x\| + C$
Exponential	$\int e^x\,dx = e^x,\ \int a^x\,dx = \dfrac{a^x}{\ln a}$
$\sin, \cos$	$-\cos x,\ \sin x$
$\sec^2, \csc^2$	$\tan x,\ -\cot x$
$\sec\tan, \csc\cot$	$\sec x,\ -\csc x$
$\tan, \cot$	$\ln\|\sec x\|,\ \ln\|\sin x\|$
$\sec x$	$\ln\|\sec x + \tan x\|$
$\csc x$	$\ln\|\csc x - \cot x\|$
Substitution	$\int f(g(x))g'(x)\,dx = \int f(u)\,du$
Log shortcut	$\int \dfrac{f'(x)}{f(x)}\,dx = \ln\|f(x)\|$
Power shortcut	$\int f'\,f^n\,dx = \dfrac{f^{n+1}}{n+1}$
Partial fractions	decompose P/Q when $\deg P < \deg Q$
Integration by parts	$\int u\,dv = uv - \int v\,du$
ILATE	u order: I $\to$ L $\to$ A $\to$ T $\to$ E
$e^x(f + f')$ identity	$\int e^x(f + f')\,dx = e^x f + C$
$\int \ln x\,dx$	$x \ln x - x + C$
$1/(x^2 - a^2)$	$\frac{1}{2a} \ln\left\|\frac{x-a}{x+a}\right\|$
$1/(a^2 - x^2)$	$\frac{1}{2a} \ln\left\|\frac{a+x}{a-x}\right\|$
$1/(x^2 + a^2)$	$\frac{1}{a} \tan^{-1}(x/a)$
$1/\sqrt{a^2 - x^2}$	$\sin^{-1}(x/a)$
$1/\sqrt{x^2 \pm a^2}$	$\ln\|x + \sqrt{x^2 \pm a^2}\|$
FTC II	$\int_a^b f = F(b) - F(a)$
Substitution (def.)	limits become $g(a),\ g(b)$
P1, P2	$\int_a^b f(t)dt$; swap-limits flips sign
P3	split at $c \in (a, b)$
P4	$\int_a^b f(x) = \int_a^b f(a + b - x)$

P5 — even/odd	for even f: equals $2\int_0^a f$; for odd f: equals 0
P6 — $[0, 2a]$	split via $f(2a - x)$
Periodic	$\int_0^{nT} f = n\int_0^T f$