Collegedunia NCERT Formula Sheet

Class 12 Mathematics (12th Maths) — NCERT 2026-27

Chapter 8: Application of Integrals

Area Under a Curve | Area between Two Curves | Standard Regions

Chapter Snapshot. The definite integral developed in Chapter 7 here becomes a tool for measuring **plane areas**. The chapter has two sections: (i) area enclosed by a curve, the x-axis (or y-axis), and two vertical (or horizontal) bounding lines; and (ii) area between two curves. Throughout, the rule is the same — **integrate the larger function minus the smaller, between the intersection points**.

1 Area Under a Curve

A curve sits above (or below) one of the coordinate axes; the region between curve, axis, and two vertical/horizontal lines has a signed area given by a single definite integral.

Strip above the x-axis

If $f(x) \geq 0$ on $[a, b]$, the area A enclosed by $y = f(x)$, the x-axis, and the lines $x = a$, $x = b$ is

$$A = \int_a^b f(x)\,dx$$

Area equals the definite integral when the curve is on or above the axis; units of area follow from the units of x and y.

Strip below the x-axis

If $f(x) \leq 0$ on $[a, b]$, the definite integral is non-positive. The geometric area is

$$A = \left|\int_a^b f(x)\,dx\right| = -\int_a^b f(x)\,dx$$

Area is always reported as a positive number — take the absolute value of any negative definite integral.

Mixed-sign strip

If f changes sign at $c \in (a, b)$, split:

$$A = \int_a^c |f(x)|\,dx + \int_c^b |f(x)|\,dx$$

Each piece is integrated with the correct sign; failing to split is the classic "zero area" mistake when the integrand is antisymmetric.

Strip with x as function of y

If a region sits between $x = g(y)$, the y-axis, and the horizontal lines $y = c$, $y = d$ with $g(y) \geq 0$,

$$A = \int_c^d g(y)\,dy$$

Use this orientation when the region is more naturally bounded by horizontal lines than vertical ones — e.g. areas described as functions of y such as $x = y^2$.

Element of area

NCERT motivates the area integral by approximating the region with thin vertical strips of width dx and height $|f(x)|$, each contributing $dA = |f(x)|\,dx$. Summing as $dx \to 0$ recovers $\int_a^b |f(x)|\,dx$.

Sign-induced cancellation

The bare integral $\int_{-a}^{a} x^3\,dx = 0$ does **not** mean the area is zero — the integrand is odd and its integral cancels across the symmetric interval. The geometric area between $y = x^3$ and the x-axis from $-a$ to a is $\int_{-a}^{a} |x^3|\,dx = a^4/2$.

2 Standard Curve-Areas

A handful of NCERT-worked regions whose answers are worth memorising.

Area of a circle

For $x^2 + y^2 = a^2$, the area enclosed equals

$$A = 4\int_0^a \sqrt{a^2 - x^2}\,dx = \pi a^2$$

Integrate the first quadrant ($y = \sqrt{a^2 - x^2}$, $0 \le x \le a$) and multiply by 4 using the curve's x- and y-axis symmetry.

Area of an ellipse

For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, integrate the first-quadrant branch $y = b\sqrt{1 - x^2/a^2}$ and use four-fold symmetry:

$$A = 4\int_0^a b\sqrt{1 - x^2/a^2}\,dx = \pi ab$$

πab generalises the circle ($a = b$). The major and minor semi-axes appear symmetrically; areas scale linearly in each.

Area under a parabola

For $y^2 = 4ax$ bounded by $x = h$,

$$A = 2\int_0^h 2\sqrt{ax}\,dx = \frac{8}{3}\sqrt{a}\,h^{3/2}$$

Use the symmetric upper and lower branches; the area scales as $h^{3/2}$, a steeper growth than a linear strip.

Quarter of an ellipse

For the first-quadrant portion of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ enclosed with the two axes,

$$A_Q = \int_0^a b\,\sqrt{1 - x^2/a^2}\,dx = \frac{\pi ab}{4}$$

One-fourth of the total ellipse area πab. The other three quadrants follow by reflection across the axes.

Area of a triangle (linear curves)

If a triangle has vertices on lines $y = m_1x + c_1$ and $y = m_2x + c_2$ between $x = a$ and $x = b$,

$$A = \int_a^b \big|(m_1x + c_1) - (m_2x + c_2)\big|\,dx$$

Integration confirms the elementary $\frac{1}{2} \cdot \text{base} \cdot \text{height}$ — useful as a sanity check on the area-between-curves machinery.

Symmetry shortcut

If the region has axis symmetry, integrate only one half (or quarter) and multiply. The circle, the ellipse, and every even function above the x-axis all benefit from this.

3 Area Between Two Curves

When two curves enclose a region, the area is the integral of the upper minus the lower curve.

Two curves of x

If $f(x) \geq g(x)$ on $[a, b]$, the area between $y = f(x)$ and $y = g(x)$ from $x = a$ to $x = b$ is

$$A = \int_a^b [f(x) - g(x)]\, dx$$

"Top minus bottom", integrated across the interval. The limits a and b are the x-coordinates of the curves' intersections (or any prescribed bounding lines).

Curves crossing inside the interval

If $f - g$ changes sign at $x = c \in (a, b)$, split:

$$A = \int_a^c [f - g]\, dx \;+\; \int_c^b [g - f]\, dx$$

Whichever curve is on top changes across c — flip the sign convention in each sub-interval so the integrand stays non-negative.

Two curves of y

If $u(y) \geq v(y)$ on $[c, d]$, the area between $x = u(y)$ and $x = v(y)$ from $y = c$ to $y = d$ is

$$A = \int_c^d [u(y) - v(y)]\, dy$$

Use the y-form when the region is bounded by horizontal lines or when the curves are more cleanly written as $x = u(y)$.

Picking the orientation

Sketch first. Vertical strips (dx) work when curves are easily written as $y = f(x)$ with one always above the other in x. Horizontal strips (dy) work when curves are easily written as $x = g(y)$ or when the upper curve flips inside the interval.

Recipe — area between two curves

Step 1. Sketch both curves and identify the enclosed region.

Step 2. Find the intersections (set $f(x) = g(x)$).

Step 3. Identify which curve is on top on each sub-interval.

Step 4. Integrate top minus bottom on each piece; sum.

Step 5. Take absolute value at the end if a piece gave a negative result by an oversight in the top-vs-bottom call.

Step 1 is non-negotiable. Skipping the sketch is the single highest-frequency source of wrong-sign and wrong-limits errors.

Top minus bottom

Area between curves = **integral of (top − bottom)** between the intersection x-values. If you draw the picture, you cannot get the sign wrong.

Limits before integrand

Solve $f = g$ **first** to fix the limits a and b. Setting up the integrand without first finding the intersections often leads to using the wrong limits — usually the algebraic bounds rather than the geometric ones.

Worked template — line & parabola

For $y = x^2$ (parabola) and $y = x + 2$ (line). Solve $x^2 = x + 2 \Rightarrow x = -1, 2$. Line is on top:

$$A = \int_{-1}^{2} [(x + 2) - x^2]\, dx = \tfrac{9}{2}$$

Substituting the formula directly with intersections as limits gives the area in one step — no sketching of strips required after Steps 1–4 of the recipe.

Region between two parabolas

For $y^2 = x$ and $x^2 = y$. Intersections at $(0,0)$ and $(1,1)$; for $0 \le x \le 1$ the upper curve is $y = \sqrt{x}$, the lower is $y = x^2$:

$$A = \int_0^1 \left[\sqrt{x} - x^2\right] dx = \frac{1}{3}$$

Standard NCERT region; tests both the top-vs-bottom decision and the basic power-rule integral.

Why "top minus bottom"?

For a thin vertical strip at horizontal coordinate x with width dx, the strip's height is the vertical distance between the two curves, $f(x) - g(x)$, when $f \ge g$. Element of area $dA = (f-g)\,dx$. Summing gives the integral; reversing the sign of $f - g$ in a sub-interval simply re-assigns "top" to the other curve.

Quick Reference — Chapter 8 Application of Integrals

Compact summary of every named identity used above

Concept	Statement / Formula
Area, $f \geq 0$ on $[a, b]$	$A = \int_a^b f(x)\,dx$
Area, $f \leq 0$	$A = \left|\int_a^b f(x)\,dx\right|$
Sign-change at c	split at c, take $|f|$ on each piece
Strip as $x = g(y)$	$A = \int_c^d g(y)\,dy$
Element of area	$dA = |f(x)|\,dx$
Circle $x^2 + y^2 = a^2$	$A = \pi a^2$
Ellipse	$A = \pi ab$
Parabola $y^2 = 4ax$, up to $x = h$	$A = \frac{8}{3}\sqrt{a}\,h^{3/2}$
Two y-curves, $f \geq g$	$A = \int_a^b [f(x) - g(x)]\,dx$
Curves crossing at c	split, switch top/bottom at c
Two x-curves, $u \geq v$	$A = \int_c^d [u(y) - v(y)]\,dy$
Recipe	sketch $\to$ intersections $\to$ pick top $\to$ integrate $\to$ sum