Collegedunia NCERT Formula Sheet

The Ultimate Formula Reference for Class 12 Mathematics — Chapter 9

Chapter 9: Differential Equations

Chapter Roadmap — What You Need to Know

Topic	Core idea
Order & Degree	Highest derivative present; highest power of that derivative (poly form only).
Solutions	General (with arbitrary constants $=$ order) and particular (constants fixed).
Formation of DE	Differentiate n times to eliminate n arbitrary constants $\Rightarrow$ DE of order n.
Variable Separable	Rearrange to $h(y)\,dy = g(x)\,dx$, then integrate both sides.
Homogeneous DE	$\frac{dy}{dx} = F\left(\frac{y}{x}\right)$; substitute $y = vx$ to separate variables.
Linear DE	$\frac{dy}{dx} + Py = Q$; multiply by IF $= e^{\int P\,dx}$; solution $y \cdot \text{IF} = \int Q \cdot \text{IF}\,dx + C$.

1 Definition, Order & Degree

This section covers what a differential equation (DE) is, the difference between ordinary and partial DEs, and the two integer descriptors — **order** and **degree** — that classify every DE.

What is a differential equation?

An equation that involves the derivative(s) of a dependent variable with respect to one (or more) independent variable(s). If only one independent variable is involved, it is an **ordinary** differential equation (ODE). NCERT Ch 9 deals exclusively with ODEs.

Standard derivative notation

$$\frac{dy}{dx} = y', \quad \frac{d^2y}{dx^2} = y'', \quad \frac{d^3y}{dx^3} = y'''$$

For higher orders: $y_n = \frac{d^ny}{dx^n}$.

Use the **prime notation** for compactness, especially when stating order and degree.

Order of a DE

Order $=$ order of the **highest** derivative appearing in the DE.

Example: $\left(\frac{d^3y}{dx^3}\right) + x^2\left(\frac{d^2y}{dx^2}\right)^3 = 0$ has order 3.

Order is always a **positive integer**. It tells you how many arbitrary constants the general solution will carry.

Degree of a DE

Defined **only** when the DE is a polynomial in its derivatives. Then

Degree $=$ highest power of the highest-order derivative.

Example: $2\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$: order 2, degree 1.

If derivatives appear inside **sin**, **cos**, **log**, $e^{(\cdot)}$ or under radicals that cannot be cleared, the degree is **not defined**.

Order vs Degree — read carefully

Degree is the power of the **highest-order** derivative, *not* the highest power among all derivatives. In $\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^5 = 0$, order $= 2$, degree $= 1$ (not 5).

Worked check: degree undefined

$$\frac{dy}{dx} + \sin\left(\frac{dy}{dx}\right) = 0$$

Not a polynomial in y' $\Rightarrow$ degree **not defined**; order $= 1$.

Same goes for $y'' + e^{y'} = 0$ — order is 2, but degree cannot be assigned.

2 General & Particular Solutions

This section defines what counts as a *solution* of a DE, and the difference between a family of solutions (general) and a single member of the family (particular).

Solution of a DE

$y = \phi(x)$ is a solution of a DE if substituting $\phi(x)$ and its derivatives makes LHS $=$ RHS.

The curve $y = \phi(x)$ is the **integral curve** (or **solution curve**) of the DE.

General & particular solution

General solution: contains exactly n arbitrary constants, where n $=$ order of the DE.

Particular solution: obtained from the general solution by assigning specific values to those constants (using initial/boundary conditions).

The general solution represents a **whole family** of curves; the particular solution picks the **one curve** satisfying a given point/condition.

Quick verification recipe

To verify $y = \phi(x)$ solves a DE: compute $y', y'', \dots$ up to the order; substitute into LHS; simplify; check LHS $=$ RHS.

3 Formation of a Differential Equation

This section explains how to construct a DE from a family of curves by eliminating the arbitrary constants. (This subsection is abridged in the 2026–27 NCERT reprint, but the technique is still routinely tested.)

Formation by elimination of constants

If a family of curves has n **arbitrary constants**, then:

- Differentiate n times with respect to x.
- Eliminate the n constants using the original equation and the n derived equations.

$\Rightarrow$ resulting equation is a DE of **order** n.

Number of arbitrary constants $=$ order of the resulting DE. This is the **key counting principle**.

Worked pattern: 2-constant family

Family $y = a\cos x + b\sin x$ (two constants a, b):

$y' = -a\sin x + b\cos x$

$$y'' = -a\cos x - b\sin x = -y$$
$$\Rightarrow \frac{d^2y}{dx^2} + y = 0 \quad \text{(order 2)}$$
Two constants $\Rightarrow$ differentiate **twice** $\Rightarrow$ order 2 DE.

4 Method 1 — Variable Separable

This section gives the simplest solution technique: when the DE can be rearranged so that x-stuff sits with dx and y-stuff sits with dy, you just integrate both sides.

Variable separable form

$$\frac{dy}{dx} = h(y)\,g(x)$$

Separate: $\frac{dy}{h(y)} = g(x)\,dx \quad (h(y) \neq 0)$

Integrate: $\int \frac{dy}{h(y)} = \int g(x)\,dx + C$

Works whenever the RHS **factors** into a function of x times a function of y. The arbitrary constant C is added once, on **either side**.

Solution template

$$H(y) = G(x) + C$$

where $H(y)$ is an antiderivative of $\frac{1}{h(y)}$ and $G(x)$ is an antiderivative of $g(x)$.

Apply the initial condition $y(x_0) = y_0$ at this stage to find C and get the **particular solution**.

Worked: exponential growth/decay

$$\frac{dP}{dt} = kP \Rightarrow \frac{dP}{P} = k\,dt \Rightarrow \log|P| = kt + C_1$$
$$\Rightarrow P = P_0\,e^{kt}$$

Used in NCERT for **continuous compounding** (bank interest, $k = r/100$) and **bacterial growth**.

Don't forget the constant

After integrating both sides, add $+C$ **once**. Forgetting C makes the answer a particular solution by accident; double-adding (one on each side) only renames the constant but confuses examiners.

5 Method 2 — Homogeneous DE

This section handles DEs of the form $\frac{dy}{dx} = F(y/x)$. The trick is to substitute $y = vx$, which converts the DE into a separable one in v and x.

Homogeneous function (degree n)

$F(\lambda x, \lambda y) = \lambda^n F(x, y)$ for every nonzero λ.

A DE $\frac{dy}{dx} = F(x, y)$ is **homogeneous** iff F is a homogeneous function of **degree zero** — equivalently, F can be written purely in terms of y/x (or x/y).

Standard substitution

For $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$: put $y = vx$, so $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

DE becomes: $v + x\frac{dv}{dx} = g(v)$

$$\Rightarrow \frac{dv}{g(v) - v} = \frac{dx}{x} \quad \text{(now separable)}$$

After integrating in v and x, **back-substitute** $v = y/x$ to express the answer in the original variables.

Alternate substitution

If the DE is given as $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$, put $x = vy$, so $\frac{dx}{dy} = v + y\frac{dv}{dy}$.

Use this orientation when x is the easier dependent variable (e.g., when the DE is naturally polynomial in x rather than y).

General solution template

$$\int \frac{dv}{g(v)-v} = \int \frac{dx}{x}+C = \log|x|+C$$

Then replace v by $\frac{y}{x}$.

Final answer is typically an **implicit relation** between x and y, often containing **log** terms.

Test before solving

Always verify the DE is homogeneous *before* substituting $y = vx$. Quick test: replace $x \to \lambda x, y \to \lambda y$; if RHS is unchanged ($\lambda^0$ factor), it is homogeneous.

6 Method 3 — Linear DE

This section gives the most powerful first-order method: any DE that can be put in the linear form $\frac{dy}{dx} + P(x)\,y = Q(x)$ is solved by multiplying through by an **integrating factor**.

Linear DE: standard form

$$\frac{dy}{dx} + P(x)\,y = Q(x)$$

where P, Q are functions of x only (or constants).

Linear means y and y' appear to the **first power** only, and never multiplied together. Coefficients depend on x only.

Integrating Factor (IF)

$\text{IF} = e^{\int P(x)\,dx}$

Multiplying the DE by IF makes the LHS exactly $\frac{d}{dx}(y \cdot \text{IF})$ — so the equation becomes directly integrable.

General solution

$$y \cdot \text{IF} = \int Q(x) \cdot \text{IF}\,dx + C$$

i.e. $$y\,e^{\int P\,dx} = \int Q(x)\,e^{\int P\,dx}\,dx + C$$

Three-step recipe: (i) write in standard form, (ii) compute IF, (iii) write the solution and integrate.

Linear DE in x (alternate form)

$$\frac{dx}{dy} + P_1(y)\,x = Q_1(y)$$

$\text{IF} = e^{\int P_1(y)\,dy}$, solution:

$$x \cdot \text{IF} = \int Q_1(y) \cdot \text{IF}\,dy + C$$

Use when the DE is linear in x as a function of y — often spotted when y^2, $\sin y$, etc. appear as coefficients with dy alone.

Common IF shortcuts

- $\int \frac{1}{x}\,dx = \log|x| \Rightarrow \text{IF} = x$
- $\int \cot x\,dx = \log|\sin x| \Rightarrow \text{IF} = \sin x$
- $\int \tan x\,dx = \log|\sec x| \Rightarrow \text{IF} = \sec x$
- $\int k\,dx = kx \Rightarrow \text{IF} = e^{kx}$

Memorise these four — they cover the bulk of board-exam linear DEs.

IF-recipe: "P-IF-Q-Integrate"

Put DE in standard form $\to$ find **IF** $= e^{\int P\,dx}$ $\to$ multiply by **Q**·IF $\to$ **Integrate**. Solution: $y \cdot \text{IF} = \int Q \cdot \text{IF}\,dx + C$.

7 Solving Strategy — Which Method to Pick

This section is a one-glance decision tree to identify the right method when a DE first appears.

Quick decision rules

- RHS factors as $g(x)h(y) \Rightarrow$ **Variable Separable**
- RHS is a function of y/x alone $\Rightarrow$ **Homogeneous**; sub $y = vx$
- DE fits $y' + P(x)y = Q(x) \Rightarrow$ **Linear**; IF $= e^{\int P\,dx}$
- DE fits $x' + P_1(y)x = Q_1(y) \Rightarrow$ **Linear in x**; IF $= e^{\int P_1\,dy}$

If two tests pass, prefer **Separable** (it is cleanest). Linear and Homogeneous are mutually exclusive for first-order DEs in NCERT scope.

JEE Extension — Bernoulli's Equation

$\frac{dy}{dx} + P(x)y = Q(x)\,y^n$, $n \neq 0, 1$.
Substitute $v = y^{1-n}$ to reduce to linear: $\frac{dv}{dx} + (1-n)Pv = (1-n)Q$.
Outside NCERT 12, but common in JEE.

JEE Extension — Exact Equations

$M(x,y)\,dx + N(x,y)\,dy = 0$ is **exact** when $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. Solution: find F with $F_x = M, F_y = N$; then $F(x,y) = C$. (Removed from NCERT 2024–26, still in JEE syllabus.)

8 Applications — Growth, Decay & Geometry

This section lists the classic application templates — bank interest, bacterial growth, radioactive decay, and curve-tangent problems — all of which reduce to one of the three methods above.

Continuous growth / decay

$\frac{dN}{dt} = kN \Rightarrow N(t) = N_0\,e^{kt}$
where $N_0 = N(0)$; $k > 0$ growth, $k < 0$ decay.

Compound interest: $P(t) = P_0\,e^{rt/100}$, r = rate per year. **Doubling time:** $t = \frac{\log 2}{k}$.

Slope-of-tangent problems

"Slope of tangent at (x,y) equals $f(x,y)$" translates to
$\frac{dy}{dx} = f(x,y)$
Then identify which of the three methods applies. **Pass-through point** (x_0, y_0) fixes the arbitrary constant.

Rate-proportional models

If "rate of change is proportional to quantity present": $\frac{dQ}{dt} = kQ$.

If "proportional to difference from equilibrium Q_e": $\frac{dQ}{dt} = k(Q_e - Q)$ (Newton's law of cooling type).

First is separable $\rightarrow$ exponential. Second is linear: IF $= e^{kt}$.

Quick Reference — Three Methods Compared

Aspect	Variable Separable	Homogeneous	Linear (1st order)
Standard form	$\frac{dy}{dx} = g(x)\,h(y)$	$\frac{dy}{dx} = F\left(\frac{y}{x}\right)$	$\frac{dy}{dx} + P(x)y = Q(x)$
Identifying test	RHS factors into pure-x $\times$ pure-y	RHS depends only on y/x ratio	y, y' first-power, no $y^2, y\,y'$
Trick / substitution	none — just separate	$y = vx$ (or $x = vy$)	multiply by IF $= e^{\int P\,dx}$
After step	$\int \frac{dy}{h(y)} = \int g(x)\,dx + C$	$\int \frac{dv}{g(v) - v} = \log\lvert x\rvert + C$	$y \cdot \text{IF} = \int Q \cdot \text{IF}\,dx + C$
Back-substitute?	no	yes: $v \to y/x$ at the end	no
Typical NCERT use	growth, decay, simple PDEs	curves with $x^2 + y^2,\ xy,\ y/x$	$y' + (P)y = Q$, IF tricks

Common Integrating Factors — Memorise These

$P(x)$	$\int P\,dx$	**IF** $= e^{\int P\,dx}$
$1/x$	$\log\lvert x\rvert$	x
$-1/x$	$-\log\lvert x\rvert$	$1/x$
k (constant)	kx	e^{kx}
$\cot x$	$\log\lvert\sin x\rvert$	$\sin x$
$\tan x$	$\log\lvert\sec x\rvert$	$\sec x$
$\sec x$	$\log\lvert\sec x + \tan x\rvert$	$\sec x + \tan x$
$2x$	x^2	e^{x^2}

Order & Degree — Worked Examples

Differential equation	Order	Degree
$\frac{dy}{dx} - \cos x = 0$	1	1
$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$	2	1
$y''' + y^2 + e^{y'} = 0$	3	not defined
$\frac{d^4y}{dx^4} + \sin(y'') = 0$	4	not defined
$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$	2	not defined