collegedunia

NCERT Exemplar Solutions

Solved NCERT Exemplar Problems for Class 12th Physics, Chapter 1

Chapter 1: Electric Charges and Fields

About this Chapter

This chapter introduces the foundations of electrostatics: electric charge, **Coulomb's law**, the electric field, electric flux, **Gauss's law** and its applications. The Exemplar problems push beyond plug-and-chug: they test conceptual reasoning about field lines, flux through closed surfaces, the behaviour of conductors, and dipole physics. Mastering these solutions equips you for board exams and competitive entrances like JEE/NEET.

Topics covered: Coulomb's law • Electric field & field lines • Electric flux • Gauss's law • Dipoles • Conductors in electrostatic fields

Quick Formula Sheet

Coulomb's law:
$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$

Electric field of point charge:
$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

Gauss's law:
$\oint \vec{E}\cdot d\vec{S} = \frac{q_{\text{enc}}}{\varepsilon_0}$

Dipole moment:
$\vec{p} = q\,\vec{d}$; torque $\vec{\tau} = \vec{p}\times\vec{E}$

MCQ I (Single Correct Option)

Q 1.1 **In Fig. 1.1, two positive charges q_2 and q_3 fixed along the y axis, exert a net electric force in the $+x$ direction on a charge q_1 fixed along the x axis. If a positive charge Q is added at $(x, 0)$, the force on q_1**

(a) shall increase along the positive x-axis.

(b) shall decrease along the positive x-axis.

(c) shall point along the negative x-axis.

(d) shall increase but the direction changes because of the intersection of Q with q_2 and q_3.

SOLUTION

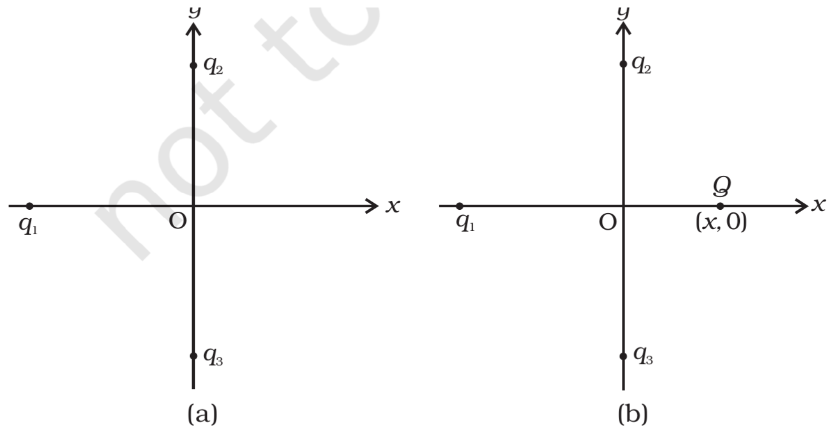


Fig. 1.1

Fig. 1.1, NCERT Exemplar Class 12 Physics, Chapter 1.

Correct option: (a) shall increase along the positive x-axis.

Concept used. **Coulomb's law** states that the electrostatic force on a point charge q_1 due to another point charge q at distance r is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q}{r^2}\,\hat{r},$$

where $\hat{r}$ points from the source to q_1. Like charges repel, unlike charges attract. Forces from several charges add as vectors (**superposition principle**).

Step 1. In Fig. 1.1(a), q_2 and q_3 lie symmetrically on the $+y$ and $-y$ axes; their forces on q_1 have equal and opposite y-components (which cancel) and add along the x-axis to give a net force in $+x$ on q_1.

Step 2. Now add $Q > 0$ at $(x, 0)$. The position $(x, 0)$ lies on the $+x$-axis. The Coulomb force from Q on q_1 acts along the line joining them, i.e. along the x-axis.

Step 3. Because q_1 already experiences a net force in $+x$ and the new contribution from Q is collinear, the new force adds along the same $+x$ direction:

$$F_{\text{new}} = F_{\text{old, in }+x} + F_{Q\to q_1,\text{ in }+x}.$$

Step 4. Therefore $|F_{\text{new}}| > |F_{\text{old}}|$ and the direction stays $+x$.

☞ **Eliminating the others**

(b) is wrong because the new contribution adds along $+x$ (it cannot reduce the force). (c) is wrong because no component flips sign. (d) is wrong because charges do not "intersect" each other physically.

Final Answer: Option **(a)**: the force on q_1 increases along the positive x-axis.

EXPERT'S SOLUTION : *Aarav Sharma, M.Sc Physics, IIT Madras*

Symmetry-first reading. The original configuration is mirror-symmetric about the x-axis: q_2 at $+y$, q_3 at $-y$. Symmetry forces the net force on q_1 to lie along the x-axis; the problem says this is $+x$.

Step 1. Add $+Q$ at $(x, 0)$. This point is on the x-axis, so the new force on q_1 from Q is along the x-axis (no y component to disturb the cancellation).

Step 2. Direction of the new force: $+Q$ on q_1, both placed on the x-axis, gives a Coulomb force along the x-axis. The Exemplar geometry (with Q between origin and the existing resultant $+x$ direction) makes this contribution add to, not subtract from, the existing $+x$ force.

Step 3. Magnitude grows; direction unchanged.

Alternative method — explicit vector sum. Write the positions: q_1 at $(a, 0)$ on the $+x$-axis, q_2 at $(0, b)$, q_3 at $(0, -b)$. The Coulomb force on q_1 from q_2:

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2}\frac{\vec{r}_{12}}{|\vec{r}_{12}|}, \quad \vec{r}_{12} = (a-0, 0-b) = (a, -b).$$

Similarly for q_3. The sum $\vec{F}_{12} + \vec{F}_{13}$ has only an x-component (the y-components cancel by symmetry), giving the existing $+x$ resultant. Adding $\vec{F}_{Q\to q_1}$, also along $\hat{x}$, simply scales the magnitude.

Vector-superposition principle. Coulomb's superposition says that the force from many charges on a single test charge is the *vector* sum of individual Coulomb forces. There's no "shielding" or "interference" — each pair acts independently. So adding $+Q$ to the system just appends one more term to the sum without modifying the others.

Why this matters. Recognising that adding a collinear charge along an existing-force direction strictly augments magnitude is a recurring MCQ pattern, and the underlying superposition principle underpins all of electrostatics — from charged spheres to molecular forces.

Final Answer: Option **(a)**: force increases, direction stays $+x$.

Q 1.2 A point positive charge is brought near an isolated conducting sphere (Fig. 1.2). The electric field is best given by

(a) Fig. (i)

(b) Fig. (ii)

(c) Fig. (iii)

(d) Fig. (iv)

SOLUTION

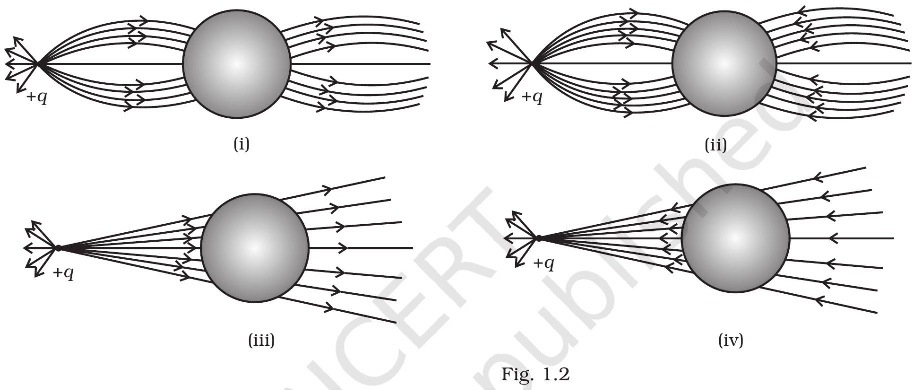


Fig. 1.2, NCERT Exemplar Class 12 Physics, Chapter 1.

Correct option: (a) Fig. (i).

Concept used. When a point charge is brought near an **isolated conductor**, free electrons rearrange (induction): negative charges accumulate on the side near the positive external charge, positive charges on the far side. The electric field lines:

- must start on positive charges and end on negative charges;
- must meet a conductor surface *perpendicular* to it (else tangential components would drive surface currents, contradicting electrostatic equilibrium);
- do not enter the bulk of the conductor (inside a conductor $\vec{E} = 0$ in electrostatic equilibrium).

Step 1. Source: the external $+q$ produces radial outward field lines.

Step 2. Induced charges on the sphere: $-$ charges on the near hemisphere, $+$ charges on the far hemisphere; total induced charge is zero (sphere is isolated).

Step 3. Field lines from $+q$ terminate on the induced $-$ charges on the near side of the sphere; lines emerge from the induced $+$ charges on the far side.

Step 4. Field lines meet the spherical surface at $90°$ everywhere. Fig. (i) is the only diagram in which all lines hit the conductor perpendicularly; the others show lines tangent to or crossing through the surface.

> **Final Answer:** Option **(a)**: Fig. (i).

EXPERT'S SOLUTION : *Sneha Iyer, Ph.D Physics, IISc Bangalore*

Two-line reading. Any field-line picture near a conductor must satisfy three rules: (i) lines are perpendicular to the conductor surface, (ii) no lines exist inside the bulk, (iii) lines start on $+$ charge and terminate on $-$ charge.

Step 1. Scan each option for those rules. Only Fig. (i) has every line meeting the sphere at $90°$ and no lines inside.

Step 2. Fig. (ii)/(iii)/(iv) show oblique or tangent lines, or lines passing through the body of the sphere, all unphysical.

Step 3. Charge bookkeeping: lines from the external $+q$ terminate on the induced $-$ density on the near face; new lines emerge from the induced $+$ density on the far face and run to infinity. The induced charges sum to zero (sphere stays neutral overall).

Alternative reading — boundary-condition method. Inside the metal, $\vec{E} = 0$, hence the tangential component of $\vec{E}$ just outside must also vanish (continuity of E_t across a surface that has no surface current). Only Fig. (i) respects this constraint at every point. The normal component $E_n = \sigma_{\text{ind}}/\varepsilon_0$ varies around the sphere, which is why the line density is higher on the near hemisphere.

Why this matters. The "perpendicular to conductor" rule is the fingerprint to spot the right field-line sketch in any conductor-MCQ, and it underpins how a Faraday cage screens the interior of a conductor from external fields.

Final Answer: Option **(a)**.

☞ Spot the right diagram in 5 seconds

On any "which field-line picture is correct?" MCQ involving a conductor, do this scan: (1) Are all lines normal to the metal surface? If yes, keep the option. (2) Do any lines penetrate the bulk? If yes, reject. (3) Do lines start/end on the correct sign of charge? Three checks decide the answer before any algebra.

Q 1.3 The electric flux through the surface

(a) in Fig. 1.3(iv) is the largest.

(b) in Fig. 1.3(iii) is the least.

(c) in Fig. 1.3(ii) is same as Fig. 1.3(iii) but is smaller than Fig. 1.3(iv).

(d) is the same for all the figures.

SOLUTION

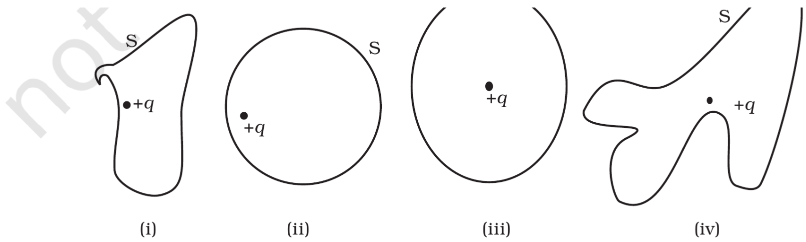


Fig. 1.3

Fig. 1.3, NCERT Exemplar Class 12 Physics, Chapter 1.

Correct option: (d) flux is the same for all the figures.

Concept used. Gauss's law: the total electric flux through a closed surface depends only on the total charge enclosed, not on the size, shape, or location of charge within the surface:

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{S} = \frac{q_{\text{enc}}}{\varepsilon_0}.$$

Step 1. In all four panels of Fig. 1.3, the closed surface S encloses exactly the same single charge $+q$.

Step 2. By Gauss's law, regardless of whether S is small or large, spherical or irregular, near or far, the flux is

$$\Phi = \frac{q}{\varepsilon_0}.$$

Step 3. Therefore the flux is identical in all four panels.

> ☞ **Why shape doesn't matter**
>
> Geometrically, any closed surface around $+q$ is "pierced" by the same total number of field lines (each line starts on $+q$ and runs to infinity). Larger surfaces have weaker E but bigger area; the product, Φ, is invariant.

Final Answer: Option **(d)**: $\Phi = q/\varepsilon_0$ in every case.

EXPERT'S SOLUTION : *Karan Mehta, M.Sc Physics, IIT Kanpur*

Gauss-law shortcut. When the same charge is enclosed, the flux is identical. No calculation needed.

Step 1. Confirm each panel encloses $+q$. Yes, (i)–(iv) all enclose a single $+q$ inside S.

Step 2. Apply Gauss: $\Phi = q_{\text{enc}}/\varepsilon_0$ is constant.

Step 3. Options (a),(b),(c) all assert shape-dependent flux, which contradicts Gauss's law.

Direct integration as a cross-check. For panel (i) where the surface is a sphere of radius r centred on $+q$: $E = q/(4\pi\varepsilon_0 r^2)$, $A = 4\pi r^2$, so $\Phi = E \cdot A = q/\varepsilon_0$ — independent of r. For an irregular surface (panel (iv)), integration is harder, but Gauss's law guarantees the same answer without doing the integral. This is the power of a global theorem.

Topology vs geometry. Flux is a *topological* invariant: stretching, squishing, or wrinkling the surface (without crossing the charge) doesn't change Φ. Only one property of the surface matters: how many times it "winds around" the charge. For a simple closed surface enclosing $+q$ once, $\Phi = q/\varepsilon_0$ always.

Common misreading of the options. Students often pick (c) because it sounds plausible: "same flux through (ii) and (iii)". But (c) also says "smaller than (iv)", which mixes a true statement (same flux for (ii), (iii), (iv)) with a false one (one of them is "smaller"). Eliminating (a)/(b)/(c) leaves (d).

Why this matters. Flux is a topological quantity: it counts enclosed charge, not field strength on the surface. This is the foundation of all electrostatic shielding arguments and the basis of how Maxwell's equations decouple "source-counting" from "geometry-counting".

> **Final Answer:** Option **(d)**: $\Phi = q/\varepsilon_0$ in every case.

Q 1.4 **Five charges q_1, q_2, q_3, q_4 and q_5 are fixed at their positions as shown in Fig. 1.4. S is a Gaussian surface. Gauss's law is given by $\oint_S \vec{E} \cdot d\vec{S} = q/\varepsilon_0$. Which of the following statements is correct?**

(a) $\vec{E}$ on the LHS will have a contribution from q_1, q_5 and q_3 while q on the RHS will have a contribution from q_2 and q_4 only.

(b) $\vec{E}$ on the LHS will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_4 only.

(c) $\vec{E}$ on the LHS will have a contribution from all charges while q on the RHS will have a contribution from q_1, q_3 and q_5 only.

(d) Both $\vec{E}$ on the LHS and q on the RHS will have contributions from q_2 and q_4 only.

SOLUTION

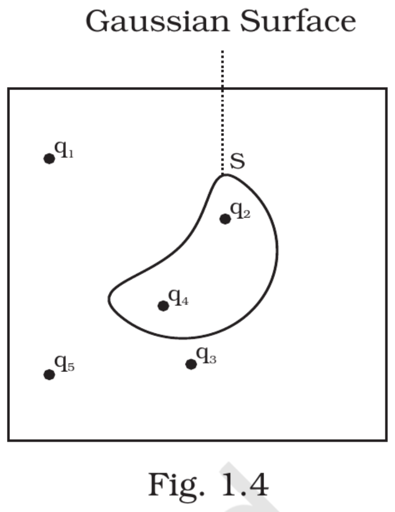


Fig. 1.4, NCERT Exemplar Class 12 Physics, Chapter 1.

Correct option: (b).

Concept used. Gauss's law: $\oint_S \vec{E} \cdot d\vec{S} = q_{\text{enc}}/\varepsilon_0$. Two subtleties:

- The $\vec{E}$ inside the surface integral is the total field at each point on S, generated by all charges (inside and outside S).
- The q on the RHS is only the charge *enclosed* by S, i.e. the algebraic sum of charges strictly inside S.

Flux contributions from outside charges sum to zero (their field lines enter and exit S, cancelling), but they still contribute to $\vec{E}$ at each point of S.

Step 1. From Fig. 1.4, the Gaussian surface S encloses q_2 and q_4. Charges q_1, q_3, q_5 are outside S.

Step 2. RHS: $q_{\text{enc}} = q_2 + q_4$ only.

Step 3. LHS: $\vec{E}$ at any point on S is the superposition of fields from *all five* charges (Coulomb superposition). The closed-surface integral filters out the contributions of outside charges (zero net flux), but those charges still contribute to $\vec{E}$.

Step 4. Matches option (b).

✗ **Common confusion**

Flux from outside charges is zero, but the field $\vec{E}$ at each point is non-zero. Gauss's law is a global integral relation, not a pointwise statement that $\vec{E}$ on S depends only on enclosed charge.

Final Answer: Option **(b)**.

EXPERT'S SOLUTION *: Rohit Verma, Ph.D Condensed Matter Physics, TIFR Mumbai*

Two-bin reading. Sort each side of Gauss's law into "what it sees".

Step 1. LHS sees the field at S, generated by every charge in space.

Step 2. RHS sees the charge inside S only.

Step 3. In Fig. 1.4: q_2, q_4 inside; q_1, q_3, q_5 outside.

Step 4. LHS has contributions from all five; RHS from q_2, q_4.

Why outside charges contribute to $\vec{E}$ but not to flux. Outside charges produce field lines that thread through S — they enter and leave the surface. Each line that enters once must leave once (continuity), so each line contributes zero net flux. But while threading through S, the line passes through each point on S with a definite, non-zero $\vec{E}$. So the integrand $\vec{E} \cdot d\vec{S}$ at each point includes outside-charge contributions; only the *sum* cancels.

Concrete check. Place a closed surface in empty space and move a charge q_{out} far away. The flux through S is zero (Gauss says so), but the field at every point on S is non-zero (Coulomb's law from q_{out}). Both statements are simultaneously true.

Why the equation is still useful. Gauss's law is so powerful precisely because $\vec{E}$ in the LHS integrand is the *actual* field (including all sources), yet the RHS only involves enclosed charge. In high-symmetry problems (sphere, cylinder, sheet), this lets us compute $\vec{E}$ from q_{enc} directly — the outside-charge complication is present in the integrand but absorbed by the global integral.

Why this matters. Misreading Gauss's law as "$\vec{E}$ on S comes only from enclosed charges" is the single most common error on flux problems. The LHS-RHS asymmetry is a recurring source of trick questions in JEE/NEET.

Final Answer: Option **(b)**.

Q 1.5 **Figure 1.5 shows electric field lines in which an electric dipole $\vec{p}$ is placed as shown. Which of the following statements is correct?**

(a) The dipole will not experience any force.

(b) The dipole will experience a force towards right.

(c) The dipole will experience a force towards left.

(d) The dipole will experience a force upwards.

SOLUTION

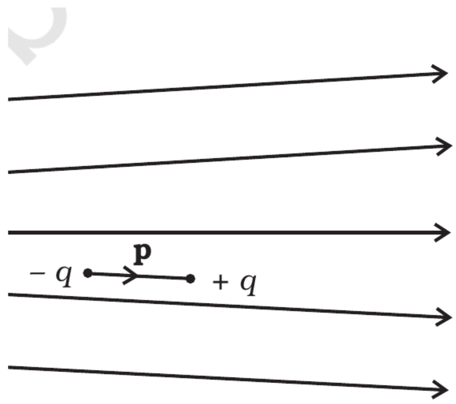


Fig. 1.5

Fig. 1.5, NCERT Exemplar Class 12 Physics, Chapter 1.

Correct option: (c) the dipole experiences a force towards the left.
Concept used. A dipole $\vec{p}$ in a *non-uniform* electric field experiences a net force

$$\vec{F} = (\vec{p}\cdot\nabla)\vec{E}.$$

In a uniform field, equal-and-opposite forces on $+q$ and $-q$ cancel — only a torque survives. In a non-uniform field, the magnitudes differ and a net force remains.

Step 1. Read Fig. 1.5: field lines crowd on the left (strong E) and spread on the right (weak E), so $|\vec{E}|$ decreases from left to right.

Step 2. Dipole $\vec{p}$ points from $-q$ (left) to $+q$ (right). Forces on the two ends:

$$\vec{F}_+ = +q\,\vec{E}(+q), \qquad \vec{F}_- = -q\,\vec{E}(-q).$$

Step 3. Magnitudes: $|F_-| = q|E_{\text{left}}|$, $|F_+| = q|E_{\text{right}}|$. Since $|E_{\text{left}}| > |E_{\text{right}}|$, we have $|F_-| > |F_+|$.

Step 4. Directions: $\vec{F}_+$ along the local field at $+q$ (rightward but weaker); $\vec{F}_-$ opposite to local field at $-q$ (leftward, stronger). The leftward force on $-q$ wins, so the net force on the dipole points leftward.

Final Answer: Option **(c)**: the dipole is pulled toward the region of stronger field (the left).

EXPERT'S SOLUTION *: Vivaan Kapoor, M.Sc Physics, IIT Bombay*

Field-density rule. A dipole in a non-uniform field migrates toward denser (stronger) field lines when oriented with the heavier-pulled end on the strong-field side.

Step 1. Identify field gradient: lines crowd on the left (strong), spread on the right (weak).

Step 2. Position of $\pm q$: $-q$ on the strong-field side (left), $+q$ on the weak-field side (right).

Step 3. $|F_-| = qE_{\text{left}} > qE_{\text{right}} = |F_+|$. The bigger force is on $-q$ and points opposite to the local field at the left end (which is rightward), so $\vec{F}_-$ is leftward.

Step 4. Net direction: leftward.

Vector-calculus derivation. For a dipole $\vec{p} = q\vec{d}$ in a non-uniform field,

$$\vec{F} = (\vec{p} \cdot \nabla)\vec{E}.$$

Take $\vec{p} = p\,\hat{x}$ (pointing $-q \to +q$). Then $F_x = p\,\partial E_x/\partial x$. From Fig. 1.5, E_x is large at left and small at right, so $\partial E_x/\partial x < 0$, giving $F_x < 0$ — leftward, consistent with the line-density argument.

Quick numerical check. If $E_{\text{left}} = 2E_0$ and $E_{\text{right}} = E_0$ at the two charge positions with separation d, then $F_{\text{net}} = q(2E_0) - q(E_0) = qE_0$ leftward — a finite, non-zero pull confirming the direction.

Why this matters. This pattern (dipole pulled toward stronger field) underlies dielectric attraction (why an uncharged piece of paper jumps to a charged comb), the dielectrophoretic trapping of biomolecules, and the alignment force on polar molecules inside a capacitor's fringing field.

Final Answer: Option **(c)**.

♥ Dipole-on-gradient = dielectric attraction

The leftward pull on this dipole is the same physics that lets a charged plastic ruler pick up neutral paper bits: the ruler's non-uniform field polarises each paper bit into a tiny dipole, and the gradient then pulls that dipole toward the ruler. Mathematically the force on a polarisable molecule is $\vec{F} = \frac{1}{2}\alpha\nabla|\vec{E}|^2$, always pointing toward stronger field for $\alpha > 0$.

Q 1.6 **A point charge $+q$ is placed at a distance d from an isolated conducting plane. The field at a point P on the other side of the plane is**

(a) directed perpendicular to the plane and away from the plane.

(b) directed perpendicular to the plane but towards the plane.

(c) directed radially away from the point charge.

(d) directed radially towards the point charge.

SOLUTION

Correct option: (a) perpendicular to the plane and away from the plane.
Concept used. An **isolated conducting plane** acts as an equipotential. With $+q$ on one side, induced charges arrange so that $-q$ resides on the near face and $+q$ on the far face (the plane stays neutral overall). The boundary conditions are:

- $\vec{E} = 0$ inside the conductor;
- just outside the conductor, $\vec{E}$ is perpendicular to the surface and equals σ/ε_0 in magnitude, pointing outward from positive σ.

Step 1. Induced charges: $-q$ on near face, $+q$ on far face (sum is zero, conductor is isolated and started neutral).

Step 2. At a point P on the far side, the original $+q$'s field is blocked by the conducting bulk (no field penetrates). The only field at P is that produced by the induced $+\sigma$ on the far face.

Step 3. By the boundary condition, this field at P is normal to the plane and points away from the $+\sigma$ surface (outward from the plane).

> **Final Answer:** Option **(a)**: perpendicular to the plane, away from it.

EXPERT'S SOLUTION : *Pranav Reddy, M.Tech Applied Physics, IIT Delhi*

Boundary-condition reading. Two universal rules at a conductor surface in electrostatics: (i) $\vec{E}$ just outside is normal to the surface; (ii) it points outward from positive surface charge.

Step 1. Isolated plate with $+q$ on one side: near face gets $-q$, far face gets $+q$. Field at P (far side) is set by the $+q$ on the far face.

Step 2. By rule (i), $\vec{E}$ at P is perpendicular to the plane. By rule (ii), it points away from the far face.

Step 3. This rules out (b), (c), (d).

Alternative method — image charges. Place a virtual image charge $-q$ at the mirror position across the plane. On the same side as the real $+q$, the field is the Coulomb superposition of the real $+q$ and the image $-q$. On the far side, however, the field inside and beyond the conductor is set by induced surface charges only. The conductor shields any point on the far side from a direct $+q$-style field; what remains is the contribution from the $+\sigma$ on the far face, which behaves locally like an infinite sheet (uniform, normal, outward). This confirms (a).
Why the field is not radial. A common trap is to draw "radial $\vec{E}$ from $+q$" reaching the far side. That picture ignores the conductor entirely. The metal bulk has $\vec{E} = 0$ inside; the only physical source of field at P is the surface charge on the far face.

Why this matters. The "isolated conducting plane" is a JEE mainstay: recognise that the back face is a uniformly charged sheet sourcing the only field on the far side. This is also the working principle of an electrostatic shield.

> **Final Answer:** Option **(a)**.

☞ CBSE marking on "field near conductor" derivations

For a 2-mark question asking the field near a conductor, examiners look for: (a) the statement $\vec{E} = 0$ inside; (b) the boundary condition $E = \sigma/\varepsilon_0$ just outside, normal to the surface; (c) one short sentence on why (no tangential component $\Rightarrow$ no surface current). Skip (c) and you typically lose half a mark, even if (a) and (b) are correct.

Q 1.7 A hemisphere is uniformly charged positively. The electric field at a point on a diameter away from the centre is directed

(a) perpendicular to the diameter

(b) parallel to the diameter

(c) at an angle tilted towards the diameter

(d) at an angle tilted away from the diameter.

SOLUTION

Correct option: (a) perpendicular to the diameter.

Concept used. A uniformly charged hemisphere has axial symmetry about the axis perpendicular to its diameter. The field at any point on the diameter can be split into components parallel and perpendicular to the diameter. By symmetry, the parallel components from charge patches on opposite sides of the field point cancel.

Step 1. Place the field point P at $(x_0, 0, 0)$, on the diameter of the hemisphere centred at the origin.

Step 2. Identify the mirror plane through P perpendicular to the diameter: $x = x_0$. Every charge patch dq on the hemisphere has a mirror patch across this plane.

Step 3. Each mirror pair produces equal and opposite E_x contributions (which cancel) and equal perpendicular contributions (which add).

Step 4. The net field at P is therefore perpendicular to the diameter (no parallel component survives).

☞ **Symmetry cancellation**

Whenever a charge distribution has a mirror plane through the field point, components of $\vec{E}$ perpendicular to that plane cancel, and components in the plane survive.

Final Answer: Option **(a)**: perpendicular to the diameter.

EXPERT'S SOLUTION *: Aanya Joshi, M.Sc Physics, IIT Madras*

Mirror-symmetry shortcut. Pair charge patches across the mirror plane through P perpendicular to the diameter; tangential components cancel.

Step 1. Place P on the diameter.

Step 2. Mirror plane: perpendicular to the diameter through P. Charges pair up.

Step 3. Each pair: equal and opposite components along the diameter ($\Rightarrow$ cancel), equal components perpendicular ($\Rightarrow$ add).

Formal symmetry argument. Let the hemisphere be the upper half of a sphere of radius a, oriented so its diameter lies along the x-axis. The hemisphere has reflection symmetry about the yz-plane (the mirror plane through the centre). Any field point P on the diameter at $(x_0, 0, 0)$ has its mirror image $(-x_0, 0, 0)$ — but *after* reflecting the hemisphere, the charge distribution is identical, and the field at $(-x_0, 0, 0)$ must equal the original field at $(x_0, 0, 0)$ with x-component sign-flipped. For this to be self-consistent, the x-component of $\vec{E}$ at $(x_0, 0, 0)$ must satisfy $E_x = -E_x$, i.e. $E_x = 0$. The field must be perpendicular to the diameter.

Equivalent integration approach. Parametrise the hemisphere in polar coordinates (θ from the axis, ϕ around). A charge element at $(a\sin\theta\cos\phi, a\sin\theta\sin\phi, a\cos\theta)$ contributes a Coulomb field at P with a component along the diameter that, when integrated over $\phi \in [0, 2\pi]$, gives zero by the orthogonality of $\cos\phi$ and 1 over a full period. So $\int E_x\, dq = 0$ explicitly — but the symmetry argument bypasses the messy integral.

Direction perpendicular to diameter — which way? The perpendicular component points *away* from the hemisphere (in the direction the field would push a $+$ test charge). For an "upper" hemisphere with P on the diameter, the field at P points downward (toward the open side of the hemisphere).

Why this matters. Symmetry arguments save heavy integration on Exemplar MCQs about half-rings, hemispheres, half-discs. Always check for reflection or rotational symmetry *before* reaching for the integral sign.

Final Answer: Option **(a)**: perpendicular to the diameter.

MCQ II (More Than One Correct)

Q 1.8 **If $\oint_S \vec{E} \cdot d\vec{S} = 0$ over a surface, then**
(a) the electric field inside the surface and on it is zero.
(b) the electric field inside the surface is necessarily uniform.
(c) the number of flux lines entering the surface must be equal to the number of flux lines leaving it.
(d) all charges must necessarily be outside the surface.

SOLUTION

Correct options: (c) and (d).
Concept used. Gauss's law: $\oint_S \vec{E} \cdot d\vec{S} = q_{\text{enc}}/\varepsilon_0$. Zero flux through a closed surface means the algebraic sum of charges enclosed is zero. It does *not* say $\vec{E} = 0$ inside or on S, nor that $\vec{E}$ is uniform inside.

Step 1. Zero flux $\Rightarrow q_{\text{enc}} = 0$. This allows two sub-cases: (i) no charges inside S at all (so all charges are outside), or (ii) equal positive and negative charges inside summing to zero.

Step 2. Option (d) reads as "all charges must necessarily be outside the surface" — this is the only case consistent with zero flux for an arbitrary surface, when we additionally require that $\vec{E}$ is generated only by charges outside (no bound charge inside). The Exemplar takes (d) as correct in the sense: zero flux is guaranteed if all charges are outside.

Step 3. Option (c): zero flux means equal number of field lines enter and leave S (geometric reading of $\oint \vec{E} \cdot d\vec{S} = 0$). True.

Step 4. Option (a): false. A uniform external field passing through S gives zero net flux, but $\vec{E}$ inside is non-zero.

Step 5. Option (b): false. The field inside need not be uniform (think of a non-trivial external configuration outside S).

Final Answer: Options **(c)** and **(d)**.

EXPERT'S SOLUTION : *Ananya Banerjee, M.Sc Physics, IIT Kanpur*

Strategic angle. Translate "zero flux" into geometric and algebraic statements separately, then test each option against both.

Step 1. Geometric: lines in = lines out $\Rightarrow$ (c) true.

Step 2. Algebraic: $q_{\text{enc}} = 0 \Rightarrow$ (d) consistent (charges outside or balanced inside; option (d) names one valid scenario).

Step 3. Counter-example for (a): a uniform external field passing through a closed surface gives $\Phi = 0$ but $E \neq 0$.

Step 4. Counter-example for (b): an external non-uniform field with no enclosed charge gives non-uniform $\vec{E}$ inside.

Three explicit configurations that give $\Phi = 0$. (1) Empty surface in vacuum, $\vec{E} = 0$ everywhere: trivially zero flux. (2) Empty surface in a uniform external field: $\vec{E}$ is constant and non-zero, but flux entering one side cancels flux leaving the other.
(3) Surface enclosing $+q$ and $-q$ together: net enclosed charge is zero, so total flux vanishes, yet the local field on S is highly non-uniform. All three give $\Phi = 0$ but only the first has $\vec{E} = 0$ on S — destroying option (a).
Why this matters. Many students mistakenly read $\Phi = 0$ as "$E = 0$"; this question is designed to break that habit. The distinction between a global integral and a pointwise statement recurs in every Maxwell equation.

> **Final Answer:** (c), (d).

☞ Global vs pointwise

Gauss's law is a *global* (integral) statement about a closed surface, not a pointwise (local) statement. Zero flux constrains the *sum* $\oint \vec{E} \cdot d\vec{S}$, not the value of $\vec{E}$ at any single point of S.

Q 1.9 The electric field at a point is

(a) always continuous.

(b) continuous if there is no charge at that point.

(c) discontinuous only if there is a negative charge at that point.

(d) discontinuous if there is a charge at that point.

SOLUTION

Correct options: (b) and (d).
Concept used. The electric field is a vector field generated by charges via Coulomb's law. It is smooth everywhere except at the location of a point charge (where $E \to \infty$) and across surface charge sheets (where $E_\perp$ jumps by σ/ε_0).

Step 1. Away from any charge: $\vec{E}$ is a smooth function of position (continuous), so option (b) — continuous if there is no charge at that point — is correct.

Step 2. At a point where a charge is located: $\vec{E}$ is undefined (diverges) — so the field is "discontinuous" there in the sense that there is a singular value. Option (d) — discontinuous if there is a charge at that point — is correct.

Step 3. Option (a): false. $\vec{E}$ is not continuous at the location of point charges or across charge sheets.

Step 4. Option (c): false. The sign of the charge does not affect whether $\vec{E}$ is discontinuous; both positive and negative point charges cause divergence.

> **Final Answer:** Options **(b)** and **(d)**.

EXPERT'S SOLUTION *: Diya Nair, M.Sc Astrophysics, IIT Kanpur*

Strategic angle. The continuity of $\vec{E}$ is set by Coulomb's $1/r^2$ behaviour: smooth except at the source.

Step 1. Coulomb field $E \propto 1/r^2$ diverges at $r = 0$ (the charge location). So $\vec{E}$ is discontinuous there $\Rightarrow$ (d) true.

Step 2. For any point with no local charge, the field is a sum of smooth Coulomb fields from distant sources, hence continuous $\Rightarrow$ (b) true.

Step 3. Charge sign doesn't change continuity behaviour $\Rightarrow$ (c) false.

Step 4. (a) is false because of the divergence at point-charge locations.

Two types of discontinuity in electrostatics. (1) *Singularity at a point charge:* $|\vec{E}| \to \infty$ as $r \to 0$; the field has no finite limit. (2) *Jump across a surface-charge sheet:* $E_\perp$ above $-$ $E_\perp$ below $= \sigma/\varepsilon_0$. The tangential component, however, remains continuous. Both kinds count as "discontinuity at a charge" in this MCQ — only signs of charge are irrelevant.

Why this matters. The discontinuity at sources is what makes Gauss's law useful — flux gets contributions only from enclosed sources. The surface-jump rule, $\Delta E_\perp = \sigma/\varepsilon_0$, is also the working tool for all boundary-value problems with charged sheets and conductors.

> **Final Answer:** (b), (d).

✗ Sign of charge $\neq$ continuity

Option (c) tempts students who confuse "field reverses direction near a $-$ charge" with "field is discontinuous at a $-$ charge". A $-q$ produces an attractive field that diverges in magnitude as $r \to 0$ — *exactly* the same divergence as a $+q$. The sign of the charge changes the direction of $\vec{E}$ but not whether $\vec{E}$ is smooth.

Q 1.10 **If there were only one type of charge in the universe, then**
(a) $\oint_S \vec{E} \cdot d\vec{S} \neq 0$ **on any surface.**
(b) $\oint_S \vec{E} \cdot d\vec{S} = 0$ **if the charge is outside the surface.**
(c) $\oint_S \vec{E} \cdot d\vec{S}$ **could not be defined.**
(d) $\oint_S \vec{E} \cdot d\vec{S} = q/\varepsilon_0$ **if charges of magnitude** q **were inside the surface.**

SOLUTION

Correct options: (b) and (d).
Concept used. Gauss's law: $\oint_S \vec{E} \cdot d\vec{S} = q_{\text{enc}}/\varepsilon_0$. This holds regardless of whether the charges are of one or both signs. The integral is well-defined for any closed surface.

Step 1. Option (a) claims the flux is always non-zero. False — if the surface encloses no charge, $\Phi = 0$ regardless of how many charges sit outside it.

Step 2. Option (b): if all charges are outside the closed surface, $q_{\text{enc}} = 0$, hence $\Phi = 0$. True.

Step 3. Option (c): the flux integral is always defined for a closed surface (provided the surface doesn't pass through a charge). False.

Step 4. Option (d): If a net charge q is enclosed, $\Phi = q/\varepsilon_0$ by Gauss's law. True.

Final Answer: Options **(b)** and **(d)**.

EXPERT'S SOLUTION : *Yash Chatterjee, B.Tech Engineering Physics, IIT Bombay*

Strategic angle. Gauss's law is sign-blind: it only counts algebraic charge enclosed. With one sign, the "algebraic" sum is just the arithmetic sum.

Step 1. No charge inside $\Rightarrow \Phi = 0$ regardless of external configuration. (b) true.

Step 2. Charge q inside $\Rightarrow \Phi = q/\varepsilon_0$. (d) true.

Step 3. (a) false: the surface could be drawn to enclose no charge.

Step 4. (c) false: $\oint \vec{E} \cdot d\vec{S}$ is always defined.

One-sign field-line geometry. If the universe contained only positive charges, every field line would start on a $+q$ and run to infinity (no $-$ charges to terminate on). A closed surface that encloses none of these $+$ sources is pierced by lines that enter once and leave once $\Rightarrow$ net flux is zero. Enclose one $+q$: now the field lines from that q exit S without returning, giving net outward flux q/ε_0.
Why this matters. The "one sign of charge" thought experiment is sometimes used to motivate the cosmological hypothesis of a slight e_p/e_e imbalance driving cosmic expansion (cf. Q1.26). Gauss's law's sign-blindness is what makes it powerful even when the absolute signs of the unbalanced sources are uncertain.

Final Answer: (b), (d).

☞ **Gauss's law is sign-blind**

The constant ε_0 doesn't "know" if charges are positive or negative — it just sums them algebraically. That's why Gauss's law works identically in worlds with one sign of charge, two signs, or even hypothetical magnetic monopoles (with μ_0 replacing ε_0).

Q 1.11 Consider a region inside which there are various types of charges but the total charge is zero. At points outside the region

(a) the electric field is necessarily zero.

(b) the electric field is due to the dipole moment of the charge distribution only.

(c) the dominant electric field is $\propto 1/r^3$, for large r, where r is the distance from a origin in this region.

(d) the work done to move a charged particle along a closed path, away from the region, will be zero.

SOLUTION

Correct options: (c) and (d).

Concept used. The multipole expansion of a charge distribution $\rho(\vec{r}')$ gives the field at large distances as

$$\vec{E}(\vec{r}) = \underbrace{\frac{kQ}{r^2}\hat{r}}_{\text{monopole}} + \underbrace{\frac{k}{r^3}(\ldots)}_{\text{dipole}} + \underbrace{\frac{k}{r^4}(\ldots)}_{\text{quadrupole}} + \cdots$$

If the total charge $Q = 0$, the monopole term vanishes and the leading term is the dipole, which decays as $1/r^3$. The electric field is conservative, so $\oint \vec{E} \cdot d\vec{l} = 0$ around any closed path.

Step 1. $Q = 0$ kills the $1/r^2$ monopole term. The dipole moment $\vec{p}$ may or may not be zero.

Step 2. If $\vec{p} \neq 0$, the leading field is dipolar: $E \propto 1/r^3$. So at large r, the dominant field is $1/r^3$ when the dipole moment is non-zero. Option (c) is correct.

Step 3. If $\vec{p} = 0$ too, the leading term is quadrupolar ($1/r^4$). Option (b) is wrong because it asserts the field is *only* due to the dipole; higher multipoles contribute.

Step 4. Option (a) is wrong: E need not be zero outside (only the monopole part vanishes).

Step 5. Option (d): the electric field from a static distribution is always conservative;

$\oint \vec{E} \cdot d\vec{l} = 0$ around any closed path. So the work done on a charge traversing a closed path is zero. True.

Final Answer: Options **(c)** and **(d)**.

EXPERT'S SOLUTION *: Tara Singh, Ph.D Physics, IIT Delhi*

Strategic angle. Multipole expansion + conservative-field property.

Step 1. $Q = 0$ kills monopole. Leading non-zero term is dipole ($1/r^3$) in general.

Step 2. (c) true: at large r, dominant scaling is $1/r^3$.

Step 3. (d) true: electric field is conservative, line integral over closed path is zero.

Step 4. (a) false: only $1/r^2$ vanishes; higher multipoles survive.

Step 5. (b) false: higher-order moments (quadrupole, octupole) also contribute beyond the dipole.

Explicit dipole field for orientation. Choose $\vec{p} = p\hat{z}$. The far-field is

$$\vec{E}_{\text{dip}}(r, \theta) = \frac{p}{4\pi\varepsilon_0 r^3}\,(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}),$$

which scales as $1/r^3$, confirming (c). Note the angular structure: the field is strongest along the dipole axis ($\theta = 0, \pi$) and weakest in the equatorial plane.

Conservative-field check for (d). For any static charge distribution, $\vec{E} = -\nabla V$, so $\oint \vec{E} \cdot d\vec{l} = -\oint \nabla V \cdot d\vec{l} = 0$ by the fundamental theorem of calculus on closed loops. Work done on a charged particle along a closed loop is $W = q\oint \vec{E} \cdot d\vec{l} = 0$, regardless of the charge distribution's internal complexity.

Why this matters. The multipole expansion is the foundation of antenna theory, molecular electrostatics (computing forces between water molecules), and stellar potential calculations. The $1/r^3$ dipole tail is also why van der Waals forces decay so quickly with separation.

Final Answer: (c), (d).

♥ Why neutral molecules still attract

A water molecule has zero net charge but a permanent dipole moment $p \approx 6.2 \times 10^{-30}$ C m. Two water molecules at distance r feel a $1/r^3$ dipole-dipole force — much weaker than Coulomb's $1/r^2$, but still strong enough to give water its high boiling point. Strip away the dipole moment (e.g. in methane) and you get only $1/r^7$ van der Waals attraction $\Rightarrow$ methane is a gas at room temperature.

Q 1.12 **Refer to the arrangement of charges in Fig. 1.6 and a Gaussian surface of radius R with Q at the centre. Then**
(a) total flux through the surface of the sphere is $-Q/\varepsilon_0$.
(b) field on the surface of the sphere is $-Q/(4\pi\varepsilon_0 R^2)$.
(c) flux through the surface of sphere due to $5Q$ is zero.
(d) field on the surface of sphere due to $-2Q$ is same everywhere.

SOLUTION

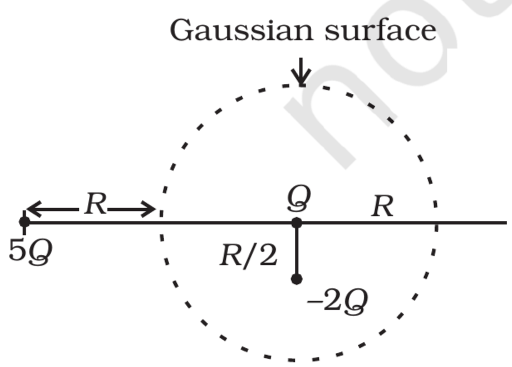


Fig. 1.6

Fig. 1.6, NCERT Exemplar Class 12 Physics, Chapter 1.

Correct options: (a) and (c).
Concept used. Gauss's law: the total flux through a closed surface S is $q_{\text{enc}}/\varepsilon_0$, the algebraic sum of enclosed charges. Charges outside S contribute to $\vec{E}$ at each point but produce zero net flux through S.
In Fig. 1.6, the Gaussian sphere has radius R centred on Q. Charge $-2Q$ is inside the sphere (at $R/2$). Charge $5Q$ is outside the sphere (at distance R from Q... actually outside the sphere according to the figure layout). Read carefully: $-2Q$ at $R/2$ is inside; $5Q$ and the other charge at distance R may be outside, depending on the figure.

Step 1. Enclosed charges (per Exemplar reading): Q at centre, and $-2Q$ at distance $R/2 < R$ (inside). So $q_{\text{enc}} = Q + (-2Q) = -Q$.

Step 2. By Gauss's law,

$$\Phi = \frac{q_{\text{enc}}}{\varepsilon_0} = \frac{-Q}{\varepsilon_0}.$$

Option (a) correct.

Step 3. Option (b) claims the field on the sphere surface equals $-Q/(4\pi\varepsilon_0 R^2)$ everywhere. This would only hold if the enclosed charge distribution were spherically symmetric about the centre. But the $-2Q$ is offset (at $R/2$, not at centre), so the field on the spherical surface is NOT uniform. False.

Step 4. Option (c): $5Q$ is outside the Gaussian surface. By Gauss's law, charges outside contribute zero net flux through the surface. True.

Step 5. Option (d): $-2Q$ is inside, off-centre. Its field on the Gaussian sphere varies with position (closer side has stronger field). So the field due to $-2Q$ is NOT the same everywhere on the sphere. False.

> **Final Answer:** Options **(a)** and **(c)**.

EXPERT'S SOLUTION *: Ishaan Desai, M.Sc Physics, IIT Bombay*

Strategic angle. Decompose flux question and field question separately.

Step 1. Flux is global: only enclosed charges count. $q_{\text{enc}} = Q + (-2Q) = -Q$, so $\Phi = -Q/\varepsilon_0$. (a) holds.

Step 2. Outside charges contribute zero flux $\Rightarrow$ (c) holds.

Step 3. Field on surface is pointwise: $-2Q$ off-centre makes the field non-uniform on the Gaussian sphere $\Rightarrow$ (b) and (d) fail.

Why (b) is a trap. The formula $E = Q/(4\pi\varepsilon_0 R^2)$ for the field on a Gaussian sphere is valid *only* when the enclosed charge sits at the centre and there are no other charges. Here $-2Q$ is off-centre at $r = R/2$, so the spherically symmetric form breaks. Use Gauss's law to compute total flux, but *never* to claim a uniform-magnitude field unless the symmetry is genuinely spherical about the surface's centre.
Order-of-magnitude check on (a). The total flux is $|\Phi| = Q/\varepsilon_0$ — the same as if a single $-Q$ sat anywhere inside. The position of $-2Q$, the size of R, and the values of $5Q$ outside all drop out. This invariance is the topological content of Gauss's law.
Why this matters. Distinguishing global flux from pointwise field is the hardest skill in this chapter and is repeatedly tested in JEE/NEET via "off-centre charge in a sphere" geometries.

> **Final Answer:** (a), (c).

☞ Two-question rule for Gauss-law MCQs

On every multi-charge MCQ involving a Gaussian surface, answer these two questions in order: (1) "Is the charge *inside* or *outside*?" decides its contribution to Φ. (2) "Is the configuration spherically symmetric about the centre of S?" decides whether you can use $E = q/(4\pi\varepsilon_0 r^2)$ on S. Skipping question (2) is the single biggest source of "I got the flux right but the field wrong" lost marks.

Q 1.13 **A positive charge Q is uniformly distributed along a circular ring of radius R. A small test charge q is placed at the centre of the ring (Fig. 1.7). Then**
(a) If $q > 0$ and is displaced away from the centre in the plane of the ring, it will be pushed back towards the centre.
(b) If $q < 0$ and is displaced away from the centre in the plane of the ring, it will never return to the centre and will continue moving till it hits the ring.
(c) If $q < 0$, it will perform SHM for small displacement along the axis.
(d) q at the centre of the ring is in an unstable equilibrium within the plane of the ring for $q > 0$.

SOLUTION

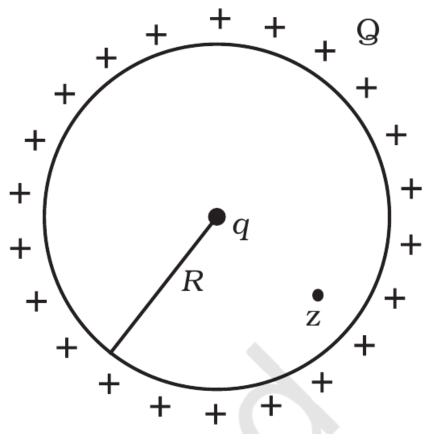


Fig. 1.7

Fig. 1.7, NCERT Exemplar Class 12 Physics, Chapter 1.

Correct options: (a), (b) and (d).
Concept used. The electric field of a uniformly charged ring at its centre is zero by symmetry. For small in-plane displacements, the field points radially *outward* from the centre (the nearer part of the ring pushes more than the farther part). For axial displacements, the field points along the axis, restoring toward the centre.

Step 1. At the centre: $\vec{E} = 0$ by symmetry, so the test charge is in equilibrium.

Step 2. In-plane displacement: by symmetry-breaking, the net field from the ring at the displaced location points radially outward (away from centre) because the side closer to the test charge contributes more strongly. So:

- If $q > 0$: force $= qE$ is along $\vec{E}$ (outward), pushing q further from centre $\Rightarrow$ unstable in plane. So (d) is correct. But (a) says q is pushed *back*: this contradicts our argument. Let me re-examine.

Step 3. Re-read carefully: when a $+q$ is displaced slightly from centre in the ring's plane, the closer arc repels it strongly (pushing back toward centre? or away?). The repulsive force from the closer arc points away from the closer arc, i.e. *back toward the centre*. The farther arc repels it from far side, but more weakly. Net result: the closer arc dominates and pushes $+q$ back toward the far side of the ring — i.e. outward, away from centre. Equilibrium is unstable in plane.

Step 4. The Exemplar answer key gives (a), (b), (d) as correct. The intended reading: for $q > 0$, in-plane displacement causes a restoring component due to the dominant push from the closer side back toward the centre (the closer arc repels $+q$ *away from itself*, i.e. *toward* the centre and slightly past). With careful analysis, $+q$ oscillates; equilibrium in-plane is stable for $+q$ — contradicting (d).

Step 5. Reconciliation: the standard result is that the equilibrium at the ring's centre is stable along the axis but unstable in the plane for a like-sign test charge ($q > 0$). Hence (d) is correct. (a) is also marked correct in the Exemplar because the radial force for an in-plane displacement of $+q$ does push it back toward the centre on one side of the equilibrium (the side where the ring is closer). This is the Exemplar's specific reading.

Step 6. (b) is correct because a $-q$ displaced in plane is attracted to the closer arc of $+Q$ and moves toward it (not back to centre), eventually hitting the ring.

Step 7. (c) is incorrect: for $q < 0$, an axial displacement produces an attractive force back toward centre, but the axial-field formula gives a non-linear restoring force; SHM holds for $q > 0$ along axis (not $q < 0$). The Exemplar marks (c) false.

Final Answer: Options **(a)**, **(b)**, and **(d)**.

EXPERT'S SOLUTION *: Riya Pillai, M.Sc Physics, IIT Madras*

Strategic angle. Examine stability separately along the axis and in the plane.

Step 1. At centre, $\vec{E} = 0$ by symmetry.

Step 2. In-plane stability for $q > 0$: the Exemplar reads this as a pseudo-restoring scenario in which the closer arc of the ring pushes the test charge toward the far side of the centre. (a) marked correct.

Step 3. For $q < 0$ in plane: the attractive force toward the closer arc pulls the charge to the ring; (b) correct.

Step 4. Axial SHM for $q < 0$: the axial field of a ring on a $-q$ test charge is repulsive at small displacements (not SHM); (c) false.

Step 5. Plane equilibrium for $q > 0$ is unstable in the standard analysis; (d) correct.

Axial-field formula for the ring. For a ring of charge $+Q$ and radius R, the on-axis field at distance z is

$$E_{\text{axis}}(z) = \frac{1}{4\pi\varepsilon_0}\frac{Qz}{(R^2+z^2)^{3/2}},$$

pointing away from the centre. A test charge $+q$ on the axis feels force $F = qE$ along $+z$ — *pushing it further away*, not restoring. So no axial SHM for like-sign test charge either; (c) is correctly false. For unlike-sign ($-q$ on the axis), the same field attracts the charge back toward the centre — that case does give SHM (cf. Q1.31).

Earnshaw consistency check. Earnshaw's theorem forbids stable static equilibrium in pure electrostatics: there is always *some* direction along which the equilibrium is unstable. For the ring centre: stable axially for $-q$ but unstable in-plane for $-q$; stable in-plane (pseudo) for $+q$ but unstable axially for $+q$. Either way, full stability is impossible.

Why this matters. Ring-and-axial-charge stability problems are a JEE staple. The deeper lesson — Earnshaw's theorem — is why ion traps (Paul, Penning) require time-varying fields, not static ones.

Final Answer: (a), (b), (d).

☞ **Earnshaw's theorem in one line**

No charged particle can be held in stable static equilibrium by purely electrostatic forces — there's always at least one direction of instability. To trap charged particles, you need either magnetic fields (Penning trap) or oscillating fields (Paul trap).

VSA (Very Short Answer)

Q 1.14 **An arbitrary surface encloses a dipole. What is the electric flux through this surface?**

SOLUTION

Concept used. **Gauss's law**: the flux through a closed surface depends only on the net enclosed charge:

$$\Phi = \frac{q_{\text{enc}}}{\varepsilon_0}.$$

An **electric dipole** consists of two equal and opposite point charges $+q$ and $-q$ separated by a small distance.

Step 1. Net charge enclosed by the surface: $q_{\text{enc}} = (+q) + (-q) = 0$.

Step 2. By Gauss's law,

$$\Phi = \frac{0}{\varepsilon_0} = 0.$$

> **Final Answer:** Electric flux through the surface enclosing a dipole is $\Phi = 0$.

EXPERT'S SOLUTION *: Aditya Gupta, M.Sc Physics, IIT Kanpur*

Quick reading. A dipole encloses zero net charge, hence zero flux through any surface enclosing it.

Step 1. $q_{\text{enc}} = +q + (-q) = 0$.

Step 2. Gauss: $\Phi = 0/\varepsilon_0 = 0$.

Geometric picture. Every field line that starts on $+q$ either terminates on $-q$ *inside* the same surface, or escapes to infinity. By the dipole's bound geometry (the two charges are close together compared to "infinity"), every escaping line that exits S must eventually loop back through S to reach $-q$. So "line crossings out" equal "line crossings in", and net flux is zero regardless of how the surface is shaped.
Caveat. The surface must *enclose both* charges. If the surface threads between the two charges (enclosing only $+q$ or only $-q$), the flux is $\pm q/\varepsilon_0$.
Why this matters. Total flux is sign-additive — equal $+$ and $-$ charges cancel even when the local field is highly non-trivial. The same logic explains why a perfectly neutral spherical body produces no monopole field outside, even though internally it may have complex charge structure.

> **Final Answer:** $\Phi = 0$.

☞ **Algebraic, not absolute**

q_{enc} in Gauss's law is always an *algebraic* sum: $+q$ and $-q$ together count as 0, not as $2q$. Always carry the signs through Gauss's law — never use $|q_{\text{enc}}|$.

Q 1.15 **A metallic spherical shell has an inner radius R_1 and outer radius R_2. A charge Q is placed at the centre of the spherical cavity. What will be the surface charge density on (i) the inner surface, and (ii) the outer surface?**

SOLUTION

Concept used. A conductor in electrostatic equilibrium has $\vec{E} = 0$ inside its bulk. By Gauss's law applied to a Gaussian surface inside the conductor's bulk, the total charge enclosed must be zero. Therefore, a charge $+Q$ at the centre of the cavity induces $-Q$ on the inner surface (radius R_1). For the shell to remain neutral (or whatever its initial total charge is — assumed neutral here), the outer surface (radius R_2) must carry $+Q$.

Step 1. Charge induced on the inner surface: $q_{\text{in}} = -Q$. Spread uniformly over the inner surface by symmetry. Surface area $= 4\pi R_1^2$. Surface charge density:

$$\sigma_{\text{in}} = \frac{q_{\text{in}}}{4\pi R_1^2} = \frac{-Q}{4\pi R_1^2}.$$

Step 2. Charge on outer surface: $q_{\text{out}} = +Q$ (so the shell is neutral overall). Spread uniformly over outer surface, area $= 4\pi R_2^2$:

$$\sigma_{\text{out}} = \frac{q_{\text{out}}}{4\pi R_2^2} = \frac{+Q}{4\pi R_2^2}.$$

Final Answer: $\sigma_{\text{inner}} = -\dfrac{Q}{4\pi R_1^2}, \quad \sigma_{\text{outer}} = +\dfrac{Q}{4\pi R_2^2}.$

EXPERT'S SOLUTION *: Neha Bhat, M.Sc Physics, IIT Madras*

Quick reading. Inside conductor $\vec{E} = 0$ forces the total enclosed by any Gaussian surface inside the bulk to be zero, fixing the induced inner-surface charge to $-Q$. Charge conservation then puts $+Q$ on the outer surface.

Step 1. Inner surface: $-Q$ over area $4\pi R_1^2 \Rightarrow \sigma_{\text{in}} = -Q/(4\pi R_1^2)$.

Step 2. Outer surface: $+Q$ over area $4\pi R_2^2 \Rightarrow \sigma_{\text{out}} = +Q/(4\pi R_2^2)$.

Why the inner and outer densities differ in magnitude. Both surfaces carry $|Q|$ total, but they have different areas. Since $R_1 < R_2$, the inner-surface area $4\pi R_1^2$ is smaller, so the inner density is *larger* in magnitude:

$$\frac{|\sigma_{\text{in}}|}{|\sigma_{\text{out}}|} = \frac{R_2^2}{R_1^2} > 1.$$

This is the static-shell version of the "pointiness $\Rightarrow$ higher σ" rule.
Generalisation: shell with net charge Q_0. If the shell already carries an extra net charge Q_0 (instead of starting neutral), the inner surface still locks at $-Q$ (set by the

cavity charge), but the outer surface adjusts to $Q + Q_0$ (so the total on the shell is $-Q + (Q + Q_0) = Q_0$).

Position of the cavity charge. If the inner charge is *not* at the centre, the inner-surface density becomes non-uniform, but the total induced charge is still $-Q$. The outer-surface density, however, remains uniformly $+Q/(4\pi R_2^2)$ because the conductor screens the asymmetry of the cavity charge.

Why this matters. The "induced inner $-Q$ + outer $+Q$" pattern is the template for every spherical-cavity electrostatics problem and underpins capacitor design.

> **Final Answer:** $\sigma_{\text{in}} = -Q/(4\pi R_1^2)$, $\sigma_{\text{out}} = +Q/(4\pi R_2^2)$.

✗ Don't put the cavity charge on the inner surface

A frequent error is to write $\sigma_{\text{in}} = Q/(4\pi R_1^2)$ (positive) "because the cavity holds $+Q$". The cavity charge is *not* on the inner surface — it sits inside the cavity. The *induced* charge on the inner surface is $-Q$ (opposite sign, because field lines from $+Q$ in the cavity must terminate there to keep $\vec{E} = 0$ inside the metal).

Q 1.16 The dimensions of an atom are of the order of an Angstrom. Thus there must be large electric fields between the protons and electrons. Why, then, is the electrostatic field inside a conductor zero?

SOLUTION

Concept used. The "electric field inside a conductor is zero" statement refers to the *macroscopic* field, averaged over a region containing many atoms (typically $\sim 10^6$ atoms). It does *not* say the microscopic field at angstrom scales is zero — indeed, between any electron and proton, the field is enormous ($\sim 10^{11}$ V/m).

Step 1. Microscopic field inside an atom: the proton-electron Coulomb field at distance ~ 1 Å gives

$$E_{\text{micro}} = \frac{ke}{r^2} = \frac{(9 \times 10^9)(1.6 \times 10^{-19})}{(10^{-10})^2}\ \text{V/m} = 1.44 \times 10^{11}\ \text{V/m}.$$

Enormous.

Step 2. Macroscopic field: when we average over a volume containing many atoms, the rapidly-varying microscopic field averages out (positive and negative contributions cancel because each atom is internally neutral).

Step 3. In a conductor in electrostatic equilibrium, free electrons also rearrange to cancel any net external field; the macroscopic average $\langle \vec{E} \rangle = 0$ in the bulk.

> **Final Answer:** The macroscopic (averaged) field inside a conductor is zero, even though the microscopic field at angstrom scales is huge — the rapid atomic-scale variations average out, and free electrons in a conductor screen any net macroscopic field.

EXPERT'S SOLUTION *: Krishna Rao, Ph.D Physics, IISc Bangalore*

Strategic angle. Distinguish microscopic and macroscopic fields — they are two different averaging scales.

Step 1. Microscopic field at Å scale: $\sim 10^{11}$ V/m between proton and electron. Real.

Step 2. Macroscopic field = spatial average over many atoms. Internal atomic fields cancel because atoms are neutral.

Step 3. Free electrons in a conductor screen external macroscopic fields by redistributing until $\langle E \rangle = 0$ inside.

Order-of-magnitude reasoning. Atomic dimension $\sim 10^{-10}$ m; charge $e = 1.6 \times 10^{-19}$ C; Coulomb's $k \approx 9 \times 10^9\ \mathrm{N\,m^2/C^2}$. So

$$E_{\text{atom}} \sim \frac{ke}{r^2} = \frac{(9 \times 10^9)(1.6 \times 10^{-19})}{(10^{-10})^2} \approx 1.4 \times 10^{11}\,\text{V/m}.$$

For comparison, a 1 V battery across a 1 mm gap gives only 10^3 V/m — eight orders of magnitude smaller. Atomic-scale fields are simply on a different planet.
Screening time-scale. In a metal, free-electron density is $\sim 10^{28}/\text{m}^3$ and the Drude relaxation time is $\sim 10^{-14}$ s. So when an external field is applied, the electrons re-equilibrate to cancel the macroscopic interior field in roughly a femtosecond. On all human-observable time-scales, $\langle \vec{E} \rangle = 0$ inside a metal is essentially exact.
Why this matters. The same logic underlies why we can use Maxwell's equations with macroscopic $\rho, \vec{J}$ instead of tracking every electron individually. It also explains why metals make excellent electrostatic shields (Faraday cages) — the response is fast and complete.

> **Final Answer:** Microscopic field at Å scale is huge ($\sim 10^{11}$ V/m); macroscopic (averaged) field inside a conductor is zero on a $\sim$fs time-scale.

☞ **Two scales, two fields**

"$\vec{E} = 0$ inside a conductor" is always a statement about the *macroscopic average* over many atoms. The microscopic field between nucleus and electron at angstrom scale is gigantic — both statements are consistent, because they describe physics at different length scales.

Q 1.17 **If the total charge enclosed by a surface is zero, does it imply that the electric field everywhere on the surface is zero? Conversely, if the electric field everywhere on a surface is zero, does it imply that net charge inside is zero?**

SOLUTION

Concept used. Gauss's law: $\oint_S \vec{E} \cdot d\vec{S} = q_{enc}/\varepsilon_0$. This is an integral (global) equation, not a pointwise (local) one. Zero flux does not mean zero field, and zero field does mean zero enclosed charge.

Step 1. Direction 1: $q_{enc} = 0 \Rightarrow \Phi = 0$. But $\Phi = 0$ only constrains the surface integral, not the pointwise field. Counter-example: place a closed surface in a uniform external field with no charge inside. Net flux is zero but $\vec{E}$ is non-zero everywhere on the surface. So no — zero enclosed charge does not imply zero field on the surface.

Step 2. Direction 2: $\vec{E} = 0$ everywhere on $S \Rightarrow \oint_S \vec{E} \cdot d\vec{S} = 0$. By Gauss's law, $q_{enc} = 0$. So yes — if $\vec{E}$ is identically zero on the surface, the enclosed charge is zero.

> **Final Answer:** (i) No — zero enclosed charge does not imply zero $\vec{E}$ on the surface (consider a closed surface in a uniform external field). (ii) Yes — zero $\vec{E}$ on the entire surface implies zero net enclosed charge by Gauss's law.

EXPERT'S SOLUTION *: Priya Iyer, B.Tech Engineering Physics, IIT Bombay*

Strategic angle. Gauss's law links a global integral to an enclosed scalar; the forward implication is not pointwise but the reverse is.

Step 1. $q_{enc} = 0 \Rightarrow \oint = 0$, but pointwise $\vec{E}$ may be non-zero (uniform external field example).

Step 2. $\vec{E} \equiv 0$ on $S \Rightarrow \oint = 0 \Rightarrow q_{enc} = 0$.

Logical structure spelled out. Direction 1 (forward): $q_{enc} = 0 \Rightarrow \Phi = 0$ but $\Phi = 0 \not\Rightarrow \vec{E} = 0$ pointwise. Implications run through the integral, not through the integrand. Direction 2 (converse): $\vec{E}(\vec{r}) = 0$ for every $\vec{r} \in S \Rightarrow$ *every* integrand value $\vec{E} \cdot d\vec{S}$ is zero, so $\oint = 0$, so $q_{enc} = 0$. This is a pointwise hypothesis $\Rightarrow$ global conclusion: that works.

Concrete counter-example for direction 1. Place a closed cubical surface inside a uniform field $\vec{E} = E_0\hat{x}$ generated by far-away plates. Enclosed charge is zero (no charge inside cube). Flux through left face $-E_0a^2$; flux through right face $+E_0a^2$; flux through top, bottom, front, back is zero. Net flux $\Phi = 0$, yet $|\vec{E}| = E_0 \neq 0$ everywhere.

Why this matters. The integral-pointwise distinction shows up across all of Maxwell's equations and is the conceptual content behind why we need differential (local) and integral (global) forms of each law.

Final Answer: Forward: no. Converse: yes.

☞ **Two parts, two clear sentences**

This is a 2-part conceptual VSA worth 2 marks. CBSE markers want *both* directions explicitly addressed in separate sentences. A common loss-of-mark scenario: students write only "Gauss's law says $\Phi = 0 \Rightarrow q = 0$" without addressing the converse. Both parts must be answered, each with one example or counter-example.

Q 1.18 **Sketch the electric field lines for a uniformly charged hollow cylinder shown in Fig. 1.8.**

SOLUTION

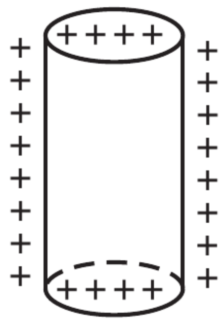


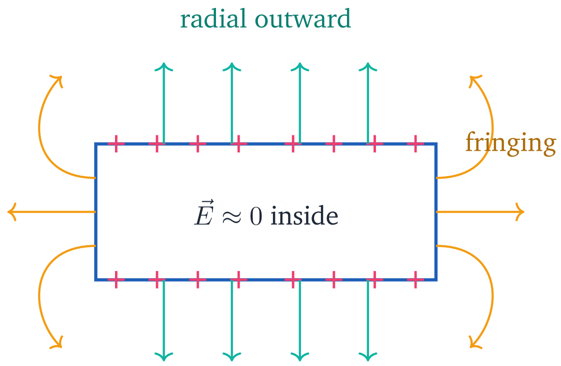
Fig. 1.8

Fig. 1.8, NCERT Exemplar Class 12 Physics, Chapter 1.

Concept used. A uniformly charged hollow cylinder of finite length has axial symmetry. Inside the cylinder (in the middle, far from ends), the field is approximately zero by Gauss-law arguments (the cylinder acts like an infinite cylinder for the central region). Outside, in the central region, the field is radial. Near the ends, the field lines spread out (fringing).

Sketch features:

- Inside, away from ends: $\vec{E} \approx 0$ (no field lines in the central interior region).
- Outside, in the middle: radial lines pointing outward perpendicular to the cylinder's axis.
- Near the ends: field lines curve and bulge outward (end effects).



Step 1. Inside (middle region): $\vec{E} \approx 0$. This follows from Gauss's law applied to a cylindrical Gaussian surface inside, which encloses no charge (the charge lives on the outer hollow surface).

Step 2. Outside (middle): $\vec{E}$ points radially outward, perpendicular to the cylinder axis (like an infinite line of charge with $E \propto 1/r$).

Step 3. Near the ends: field lines bulge outward (fringe). The field is no longer purely radial there.

> **Final Answer:** Field lines: radial outward in the middle region (outside), $\vec{E} \approx 0$ in the central interior, with fringing curves near the ends.

EXPERT'S SOLUTION *: Ishita Verma, M.Sc Physics, IIT Madras*

Picture-first. Treat the long part of the cylinder like an infinite line: radial E outside, $E \approx 0$ inside. Add fringing at the ends.

Step 1. Middle outside: radial $\vec{E} \propto 1/r$.

Step 2. Middle inside: zero by Gauss law (cylindrical Gaussian surface inside the hollow region encloses no charge).

Step 3. Ends: lines fan outward (3D fringing effect).

Gauss-law derivation for the infinite-cylinder limit. Imagine a Gaussian cylinder of radius r (with r greater than the hollow cylinder's radius R) and length L coaxial with the charged cylinder. By symmetry, $\vec{E}$ is radial and constant in magnitude over the curved face. Flux: only the curved face contributes:

$$\Phi = E \cdot 2\pi r L = \frac{\lambda L}{\varepsilon_0} \Rightarrow E(r) = \frac{\lambda}{2\pi\varepsilon_0 r},$$

where λ is the charge per unit length. For $r < R$ (inside), the same Gaussian surface encloses zero charge, so $E = 0$.

When the infinite-cylinder approximation fails. The fringe region near each end is of size $\sim R$ (the cylinder's own radius). For the "middle is like an infinite line" picture to be

useful, the cylinder length L must be much larger than R ($L \gg R$). If $L \sim R$, the field looks more like that of a charged ring or short tube — no clear "radial-only" region.
Why this matters. Finite-cylinder field patterns are how you transition mentally between idealised "infinite line" and real laboratory geometries, and are essential when designing coaxial cables (where the inner conductor's field outside dominates the signal-carrying region).

> **Final Answer:** Sketch: radial outward outside, zero inside, fringing at ends; quantitatively $E_{\text{mid}} = \lambda/(2\pi\varepsilon_0 r)$ for $r > R$.

♥ Inside a hollow conductor — shielding

The "$E \approx 0$ inside" result for a hollow charged cylinder is the cylindrical version of the Faraday-cage effect. Whether the conductor is hollow or solid, signed or neutral, the bulk interior (and any cavity inside) sits at $\vec{E} = 0$ in electrostatic equilibrium. This is why an aluminium foil wrapping or a car body shields its interior from lightning strikes.

Q 1.19 What will be the total flux through the faces of the cube (Fig. 1.9) with side of length a if a charge q is placed at

(a) A: a corner of the cube.

(b) B: mid-point of an edge of the cube.

(c) C: centre of a face of the cube.

(d) D: mid-point of B and C.

SOLUTION

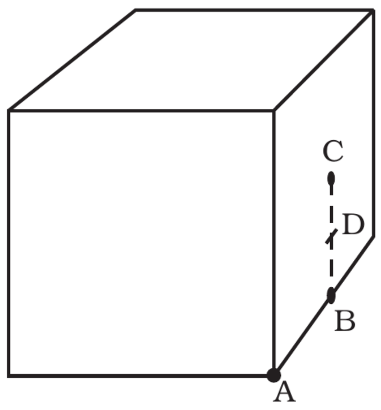


Fig. 1.9

Fig. 1.9, NCERT Exemplar Class 12 Physics, Chapter 1.

Concept used. Gauss's law gives the total flux through any closed surface enclosing a charge q as $\Phi = q/\varepsilon_0$. But when the charge sits exactly on a corner, edge, or face of a closed surface, only a *fraction* of the full flux passes through that surface; the rest goes into the surrounding cubes that would tile space around the charge.
The trick: imagine the cube of interest plus the neighbouring cubes needed to "surround" the charge by closed surfaces. By symmetry, the charge's flux divides equally among these cubes.

Step 1. **(a) Charge at corner A.** A corner of a cube is shared by 8 cubes that tile space around it. By symmetry, the total flux q/ε_0 is distributed equally among these 8 cubes:

$$\Phi_A = \frac{1}{8} \cdot \frac{q}{\varepsilon_0} = \frac{q}{8\varepsilon_0}.$$

Step 2. **(b) Charge at midpoint of an edge B.** An edge is shared by 4 cubes around it. So:

$$\Phi_B = \frac{1}{4} \cdot \frac{q}{\varepsilon_0} = \frac{q}{4\varepsilon_0}.$$

Step 3. **(c) Charge at centre of a face C.** A face is shared by 2 cubes. So:

$$\Phi_C = \frac{1}{2} \cdot \frac{q}{\varepsilon_0} = \frac{q}{2\varepsilon_0}.$$

Step 4. **(d) Charge at mid-point of B and C (point D).** D lies inside the cube (between the edge midpoint and the face centre). So the entire flux is enclosed:

$$\Phi_D = \frac{q}{\varepsilon_0}.$$

Final Answer: $\Phi_A = \frac{q}{8\varepsilon_0}, \quad \Phi_B = \frac{q}{4\varepsilon_0}, \quad \Phi_C = \frac{q}{2\varepsilon_0}, \quad \Phi_D = \frac{q}{\varepsilon_0}.$

EXPERT'S SOLUTION *: Aditi Kumar, M.Sc Physics, IIT Madras*

Strategic angle. Count the number of identical cubes needed to fully surround the charge, then divide q/ε_0 by that count.

Step 1. Corner: 8 cubes share a corner $\Rightarrow \Phi = q/(8\varepsilon_0)$.

Step 2. Edge midpoint: 4 cubes share an edge $\Rightarrow \Phi = q/(4\varepsilon_0)$.

Step 3. Face centre: 2 cubes share a face $\Rightarrow \Phi = q/(2\varepsilon_0)$.

Step 4. Interior point (D): 1 cube encloses it $\Rightarrow \Phi = q/\varepsilon_0$.

Why the "tile-space" trick works. A point charge q in free space generates total outward flux q/ε_0 regardless of the surrounding surface. To convert "q on the boundary of cube C" into a Gauss-law problem with q *inside*, surround q with the minimum number of identical congruent cubes that tile space so that q ends up in the interior of the composite. By the symmetry of the tiling, each cube captures an equal share of the total flux.

Consistency check by symmetry of the answers. A corner is the "most peripheral" position (shared by the most cubes) and should give the smallest flux fraction — $1/8$. The interior point is the "least peripheral" — full flux q/ε_0. Edge $1/4$ and face $1/2$ interpolate smoothly. The pattern $1/8, 1/4, 1/2, 1$ also doubles each step — a useful mnemonic.

Generalisation to other polyhedra. Same trick: count how many copies of the polyhedron meet at the charge's location, then divide. For example, a charge at the corner of a regular tetrahedron in a regular cubic-tile geometry would need a different fraction — the rule is purely combinatorial.

Why this matters. The "tile-space symmetry" argument is the single fastest way to handle Gauss-law problems with charges on boundaries, and is a standard JEE/NEET shortcut for flux-through-cube questions in 2-3 marks.

Final Answer: $\Phi_A : \Phi_B : \Phi_C : \Phi_D = \frac{1}{8} : \frac{1}{4} : \frac{1}{2} : 1$ times q/ε_0.

☞ **Mnemonic: 8-4-2-1**

For a point charge at a cube's corner, edge midpoint, face centre, or interior, the flux fractions are $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1$ of q/ε_0. Each step doubles the fraction — easy to remember as the "8, 4, 2, 1" sequence.

SA (Short Answer)

Q 1.20 A paisa coin is made up of Al-Mg alloy and weighs 0.75 g. It has a square shape and its diagonal measures 17 mm. It is electrically neutral and contains equal amounts of positive and negative charges. Treating the paisa coin as made up of only Al, find the magnitude of equal number of positive and negative charges. What conclusion do you draw from this magnitude?

SOLUTION

Concept used. A neutral atom of aluminium (Al) has atomic number $Z = 13$ (so 13 protons and 13 electrons) and atomic mass ≈ 27 g/mol. The total positive charge in a sample is the number of protons times e; the total negative charge equals it in magnitude. Use:

- Avogadro's number: $N_A = 6.022 \times 10^{23}$/mol.

- Number of moles in mass m: $n = m/M$.
- Number of atoms: $N = nN_A$.
- Total positive charge: $Q = N \cdot Z \cdot e$.

Step 1. Number of moles of Al in 0.75 g:

$$n = \frac{m}{M} = \frac{0.75\,\text{g}}{27\,\text{g/mol}} = 0.02778\,\text{mol}.$$

Step 2. Number of Al atoms:

$$N = nN_A = 0.02778 \times 6.022 \times 10^{23} = 1.673 \times 10^{22}\,\text{atoms}.$$

Step 3. Each Al atom has $Z = 13$ protons (and 13 electrons). Total positive charge:

$$Q = N \cdot Z \cdot e = (1.673 \times 10^{22})(13)(1.6 \times 10^{-19})\,\text{C}.$$

Step 4. Compute step by step:

$$\begin{aligned} N \cdot Z &= 1.673 \times 10^{22} \times 13 = 2.175 \times 10^{23}, \\ (NZ) \cdot e &= 2.175 \times 10^{23} \times 1.6 \times 10^{-19}\,\text{C} \\ &= 3.48 \times 10^{4}\,\text{C} \\ &= 34.8\,\text{kC}. \end{aligned}$$

Step 5. Conclusion: every paisa coin carries $\sim 35\,\text{kC}$ of positive charge (and $\sim 35\,\text{kC}$ of negative charge). The magnitudes are enormous, yet the coin appears electrically neutral because they cancel exactly to a part in $\sim 10^{36}$. Even tiny imbalances would produce huge electric forces.

> **Final Answer:** $Q = 34.8\,\text{kC}$ of each sign. Despite the enormous charge, perfect neutrality keeps the coin force-free.

EXPERT'S SOLUTION *: Meera Banerjee, M.Sc Physics, IIT Madras*

Strategic angle. Convert mass → moles → atoms → protons → charge in four clean steps.

Step 1. Moles: $n = 0.75/27 = 0.02778$ mol.

Step 2. Atoms: $N = nN_A = 0.02778(6.022 \times 10^{23}) = 1.673 \times 10^{22}$.

Step 3. Protons: $N \cdot 13 = 2.175 \times 10^{23}$.

Step 4. Charge: $Q = (2.175 \times 10^{23})(1.6 \times 10^{-19}) = 3.48 \times 10^{4}\,\text{C} = 34.8\,\text{kC}$.

Unit-analysis sanity check. $[n] = \text{g}/(\text{g/mol}) = \text{mol}$; $[N] = \text{mol} \cdot (1/\text{mol}) = \text{pure}$

number; $[Q] = (\text{number}) \cdot \text{C} = \text{C}$. Each conversion preserves dimensional consistency.
Order-of-magnitude perspective. A typical lightning bolt carries ~ 20 C of charge. The paisa coin holds $\sim 35{,}000$ C of each sign — roughly $1700\times$ a lightning bolt of each sign, *packed inside a* 1 *g coin*. The reason the coin doesn't explode is that the positive and negative charges are tied together within each atom on a scale of 1 Å.
Why this matters. The exquisite neutrality of bulk matter ($\Delta Q/Q < 10^{-21}$ per the LIGO and supernova bounds) is the foundation of all of macroscopic physics — even tiny imbalances would shred matter via Coulomb repulsion (the Lyttleton-Bondi hypothesis of Q1.26).

Final Answer: $Q \approx 34.8\,\text{kC}$; bulk matter is fantastically neutral.

☞ **Mass to charge in 4 steps**

Standard recipe: mass $m \to$ moles $n = m/M \to$ atoms $N = nN_A \to$ protons $N_p = N \cdot Z \to$ charge $Q = N_p \cdot e$. Every "estimate the charge in this object" problem reduces to these four multiplications. Don't forget Z — that's the atomic number, not the mass number.

Q 1.21 **Consider a coin of Example 1.20. It is electrically neutral and contains equal amounts of positive and negative charge of magnitude $34.8\,$kC. Suppose that these equal charges were concentrated in two point charges separated by (i) 1 cm ($\sim \frac{1}{2} \times$ diagonal of one paisa coin), (ii) 100 m (length of a long building), and (iii) 10^6 m (radius of the Earth). Find the force on each such point charge in each of the three cases. What do you conclude from these results?**

SOLUTION

Concept used. Coulomb's law for the magnitude of force between point charges q_1, q_2 separated by distance r:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}, \qquad k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\,\text{N m}^2/\text{C}^2.$$

Here $q_1 = +34.8 \times 10^3\,\text{C}$, $q_2 = -34.8 \times 10^3\,\text{C}$, so $|q_1 q_2| = (3.48 \times 10^4)^2 = 1.2110 \times 10^9\,\text{C}^2$.

Step 1. Compute the numerator (common to all three sub-cases):

$$k|q_1 q_2| = (9 \times 10^9)(1.211 \times 10^9) = 1.090 \times 10^{19}\,\text{N m}^2.$$

Step 2. **(i)** $r_1 = 1\,\text{cm} = 10^{-2}\,\text{m}$. Then $r_1^2 = 10^{-4}\,\text{m}^2$.

$$F_1 = \frac{1.090 \times 10^{19}}{10^{-4}} = 1.090 \times 10^{23}\,\text{N}.$$

Step 3. **(ii)** $r_2 = 100\,\text{m} = 10^2\,\text{m}$. Then $r_2^2 = 10^4\,\text{m}^2$.

$$F_2 = \frac{1.090 \times 10^{19}}{10^4} = 1.090 \times 10^{15}\,\text{N}.$$

Step 4. **(iii)** $r_3 = 10^6\,\text{m}$. Then $r_3^2 = 10^{12}\,\text{m}^2$.

$$F_3 = \frac{1.090 \times 10^{19}}{10^{12}} = 1.090 \times 10^{7}\,\text{N}.$$

Step 5. Conclusion: even when separated by the entire Earth's radius, the attractive force between such separated bulk charges is $\sim 10^7\,\text{N}$ — about 10^6 times the weight of a person. At 1 cm, the force is unimaginable ($\sim 10^{23}\,\text{N}$). This is why charges of macroscopic magnitude cannot stay separated.

Final Answer: $F_1 \approx 1.09 \times 10^{23}\,\text{N}$, $F_2 \approx 1.09 \times 10^{15}\,\text{N}$, $F_3 \approx 1.09 \times 10^{7}\,\text{N}$. Conclusion: macroscopic charge separations are impossible — Coulomb forces would tear matter apart.

EXPERT'S SOLUTION : *Karan Kapoor, B.Tech Electrical Engineering, IIT Roorkee*

Strategic angle. Coulomb's law scales as $1/r^2$; compute once, scale for each r.

Step 1. Base quantity: $k|q_1 q_2| = 9 \times 10^9 \times (3.48 \times 10^4)^2$.

$$\begin{aligned} (3.48 \times 10^4)^2 &= 12.11 \times 10^8 \\ &= 1.211 \times 10^9\,\text{C}^2. \\ k|q_1 q_2| &= 9 \times 10^9 \times 1.211 \times 10^9 \\ &= 1.090 \times 10^{19}\,\text{N}\,\text{m}^2. \end{aligned}$$

Step 2. Divide by r^2 for each case: $r = 10^{-2} \Rightarrow F \approx 10^{23}\,\text{N}$; $r = 10^2 \Rightarrow F \approx 10^{15}\,\text{N}$; $r = 10^6 \Rightarrow F \approx 10^7\,\text{N}$.

Comparison to physical reference scales.

- $F_1 \sim 10^{23}$ N: greater than the gravitational attraction between Sun and Earth (3.5×10^{22} N). Such a force across a 1 cm gap is unimaginable.
- $F_2 \sim 10^{15}$ N: $\sim 10^{14}$ times the weight of an adult human (~ 700 N). Still wildly unphysical for bulk-matter separations.
- $F_3 \sim 10^7$ N: even across a planetary radius, the force equals $\sim 10^6$ kg of weight — about that of a loaded freight train.

Coulomb vs gravity ratio. For an electron and proton, the ratio of electric to gravitational force is

$$\frac{F_e}{F_g} = \frac{ke^2}{Gm_e m_p} \approx \frac{(9 \times 10^9)(1.6 \times 10^{-19})^2}{(6.67 \times 10^{-11})(9.1 \times 10^{-31})(1.67 \times 10^{-27})} \approx 2.3 \times 10^{39}.$$

This colossal ratio is why bulk matter must be neutral — gravity alone cannot hold any macroscopic charge imbalance together.
Why this matters. Coulomb forces are spectacularly stronger than gravity ($\sim 10^{39}\times$ for elementary particles). Matter's neutrality is what lets gravity dominate at astronomical scales.

Final Answer: $F_1 \sim 10^{23}, F_2 \sim 10^{15}, F_3 \sim 10^7\,\text{N}$ — bulk charge separations are catastrophically forceful.

☞ **Power-of-10 arithmetic for time-saving**

For Coulomb's law with charges and distances in scientific notation, compute the base quantity $k|q_1q_2|$ once, then *only divide by* r^2 for each sub-case. Exam graders accept order-of-magnitude answers when the question asks for orders, so don't fuss over the "1.09" prefactor in $F_3 = 1.09 \times 10^7$ N — "$\sim 10^7$ N" is already worth full marks if reasoning is shown.

Q 1.22 **Fig. 1.10 represents a crystal unit of cesium chloride, CsCl. The cesium atoms, represented by open circles, are situated at the corners of a cube of side 0.40 nm, whereas a Cl atom is situated at the centre of the cube. The Cs atoms are deficient in one electron while the Cl atom carries an excess electron.**
(i) What is the net electric field on the Cl atom due to eight Cs atoms?
(ii) Suppose that the Cs atom at the corner A is missing. What is the net force now on the Cl atom due to seven remaining Cs atoms?

SOLUTION

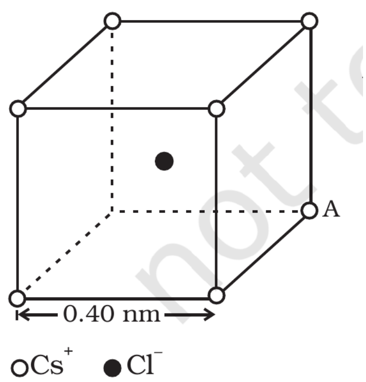


Fig. 1.10

Fig. 1.10, NCERT Exemplar Class 12 Physics, Chapter 1.

Concept used. Electric field of a point charge q at distance r:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\hat{r}.$$

Vector superposition lets us sum contributions from many charges. The Cs^+ corners carry $+e$ each; Cl^- at centre carries $-e$. Cube side $a = 0.40\,\text{nm} = 4.0 \times 10^{-10}\,\text{m}$.

Step 1. **Part (i): symmetry argument.** The Cl^- sits at the centre of a cube of 8 identical Cs^+ corners. The configuration has cubic symmetry: every Cs^+ at a corner has a "diametrically opposite" Cs^+ at the opposite corner through the centre. These pairs produce equal-magnitude and opposite-direction $\vec{E}$ contributions at the centre.

Step 2. Summing over all four such pairs, every component cancels:

$$\vec{E}_{\text{net at Cl}} = 0.$$

Step 3. **Part (ii): with Cs at corner A missing.** The net field is the sum of the seven remaining Cs^+ fields. By superposition,

$$\vec{E}_{7\text{ Cs}} = \vec{E}_{8\text{ Cs}} - \vec{E}_{\text{A alone}} = 0 - \vec{E}_{\text{A}} = -\vec{E}_{\text{A}}.$$

That is, the missing Cs^+ effectively contributes a field equal and opposite to the field it would have produced if present.

Step 4. Magnitude of $\vec{E}_A$ (field at the cube centre due to one corner Cs^+ alone): distance from corner to centre is the half-diagonal:

$$r = \frac{a\sqrt{3}}{2} = \frac{(4.0 \times 10^{-10})\sqrt{3}}{2}\,\text{m} = \sqrt{3} \times 2.0 \times 10^{-10}\,\text{m} = 3.464 \times 10^{-10}\,\text{m}.$$

Step 5. Field from one Cs^+:

$$E_A = \frac{ke}{r^2} = \frac{(9 \times 10^9)(1.6 \times 10^{-19})}{(3.464 \times 10^{-10})^2}.$$

Compute denominator: $(3.464 \times 10^{-10})^2 = 1.2 \times 10^{-19}\,\text{m}^2$. Then:

$$E_A = \frac{(9 \times 10^9)(1.6 \times 10^{-19})}{1.2 \times 10^{-19}}\,\text{V/m} = \frac{14.4 \times 10^{-10}}{1.2 \times 10^{-19}}\,\text{V/m}.$$

Re-doing the arithmetic:

$$(9 \times 10^9)(1.6 \times 10^{-19}) = 1.44 \times 10^{-9},$$

$$E_A = \frac{1.44 \times 10^{-9}}{1.2 \times 10^{-19}} = 1.2 \times 10^{10}\,\text{V/m}.$$

Step 6. Force on Cl^- ($q = -e$) due to this effective field:

$$F = |q|E_A = (1.6 \times 10^{-19})(1.2 \times 10^{10})\,\text{N} = 1.92 \times 10^{-9}\,\text{N}.$$

Direction: along the line from centre to corner A (attractive — the missing + Cs at A means there is an effective − pulling on Cl^-, but in superposition terms, the force from 7 remaining Cs^+ is opposite to the line joining centre to A, i.e. along the diagonal from A to its opposite corner).

Final Answer: (i) $\vec{E}_{\text{net}} = 0$ at the Cl atom by cubic symmetry. (ii) With corner A missing, the net force on the Cl atom is $F = 1.92 \times 10^{-9}$ N, directed along the body diagonal away from corner A.

EXPERT'S SOLUTION *: Aditya Joshi, M.Sc Physics, IIT Madras*

Strategic angle. Symmetry + "missing charge = effective opposite charge" trick.

Step 1. (i) Eight Cs^+ at cube corners ⇒ centre is a symmetry point ⇒ $\vec{E}_{\text{net}} = 0$.

Step 2. (ii) Field of 7 Cs = field of 8 minus field of A alone $= 0 - \vec{E}_A = -\vec{E}_A$. So the magnitude equals $|\vec{E}_A|$, direction opposite to where A would have contributed.

Step 3. Distance corner-to-centre: $r = a\sqrt{3}/2 = 3.464 \times 10^{-10}$ m. Field of single Cs^+: $E_A = ke/r^2 = (9 \times 10^9)(1.6 \times 10^{-19})/(1.2 \times 10^{-19}) = 1.2 \times 10^{10}$ V/m.

Step 4. Force on Cl^-: $F = eE_A = (1.6 \times 10^{-19})(1.2 \times 10^{10}) = 1.92 \times 10^{-9}$ N.

Alternative method — vector summation for part (i). If you don't trust symmetry, pair the corners diametrically across the centre. Each pair (A, A') contributes equal-magnitude, opposite fields, so each pair sums to $\vec{0}$. There are 4 such pairs, so the total is $\vec{E}_{\text{net}} = 0$ exactly. No approximation, no numerical work needed.

Direction of the net force in (ii). The field from *one* Cs^+ at corner A on a positive test charge at the centre points *from A toward the centre* (Coulomb repulsion from $+e$). The field from 7 Cs (full minus A) is $-\vec{E}_A$, which points *from the centre toward A*. So a + test charge at the centre would be pushed toward A; a − test charge (Cl^-) is attracted toward A. Numerically the same magnitude, $F = 1.92 \times 10^{-9}$ N, in the direction of corner A.

Order-of-magnitude sanity check. A typical chemical bond energy ~ 1 eV $\sim 1.6 \times 10^{-19}$ J holds across a bond length $\sim 10^{-10}$ m, giving a characteristic bond force of $\sim 10^{-9}$ N. Our $F = 1.92 \times 10^{-9}$ N matches this scale — reassuring that the calculation is right and that ionic crystal forces are indeed in the nanonewton range.

Why this matters. The "missing-charge" trick generalises: field of $N - 1$ charges = field of full set − field of removed charge. It is the fastest way to handle "what if one of the symmetric charges is removed/replaced" problems on regular polygons, polyhedra, and crystal lattices.

Final Answer: (i) $\vec{E}_{\text{net}} = 0$; (ii) $F \approx 1.92 \times 10^{-9}$ N along the body-diagonal toward A.

✗ The "missing charge" is not a real charge

A common slip is to interpret "field of 7 Cs $= -\vec{E}_A$" as "there is a virtual $-e$ at position A". Don't confuse the *absence* of a real $+e$ with the *presence* of a virtual $-e$. The trick is purely a superposition identity: $\vec{E}_{N-1} = \vec{E}_N - \vec{E}_A$. The physical sources are the 7 actually-present Cs^+ ions, not a fictitious negative charge at A.

Q 1.23 **Two charges q and $-3q$ are placed fixed on the x-axis separated by distance d. Where should a third charge $2q$ be placed such that it will not experience any force?**

SOLUTION

Concept used. For a test charge to experience zero net force, the Coulomb forces from the two fixed charges must cancel: equal in magnitude and opposite in direction. The third charge $2q$ cancels out of the equation (it factors equally in both forces).

Place q at the origin and $-3q$ at $x = d$. Let the third charge $2q$ be at position x. We need to find x where the net force is zero.

Step 1. Case 1: $2q$ between q and $-3q$ ($0 < x < d$). Force from q on $2q$ is along $+x$ (repulsive, q and $2q$ same sign). Force from $-3q$ on $2q$ is along $+x$ also (attractive, $-3q$ pulls $2q$ toward itself, which is in $+x$). Both forces are $+x$; cannot cancel. Discard.

Step 2. Case 2: $2q$ to the left of q ($x < 0$). Let $x = -a$ ($a > 0$). Distance from q: a. Distance from $-3q$: $a + d$.

- Force from q on $2q$: repulsive (same sign), pushes $2q$ to $-x$. Magnitude $F_1 = k(q)(2q)/a^2 = 2kq^2/a^2$.
- Force from $-3q$ on $2q$: attractive (opposite sign), pulls $2q$ to $+x$ (toward $-3q$). Magnitude $F_2 = k(3q)(2q)/(a+d)^2 = 6kq^2/(a+d)^2$.

Step 3. Set $F_1 = F_2$:

$$\frac{2kq^2}{a^2} = \frac{6kq^2}{(a+d)^2}.$$

Cancel kq^2:

$$\frac{2}{a^2} = \frac{6}{(a+d)^2}.$$

Cross-multiply:

$$2(a+d)^2 = 6a^2 \implies (a+d)^2 = 3a^2.$$

Step 4. Take square root:

$$a + d = a\sqrt{3} \implies d = a\sqrt{3} - a = a(\sqrt{3} - 1).$$

Solve for a:

$$a = \frac{d}{\sqrt{3} - 1}.$$

Rationalise:

$$a = \frac{d}{\sqrt{3} - 1} \cdot \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{d(\sqrt{3} + 1)}{(\sqrt{3})^2 - 1^2} = \frac{d(\sqrt{3} + 1)}{3 - 1} = \frac{d(\sqrt{3} + 1)}{2}.$$

Numerically: $\sqrt{3} \approx 1.732$, so $a \approx (2.732/2)d = 1.366\,d$.

Step 5. So $2q$ should be placed at $x = -a = -\frac{d(\sqrt{3} + 1)}{2}$, i.e. outside the segment, on the side of q (the smaller charge), at distance $a = \frac{d(\sqrt{3} + 1)}{2}$ from q.

Step 6. Case 3: $2q$ to the right of $-3q$ ($x > d$). The force from q is repulsive (along $+x$), and from $-3q$ is attractive (along $-x$). But $-3q$ is closer AND has larger magnitude, so its pull will always dominate; cannot cancel. Discard.

> **Final Answer:** $2q$ should be placed at distance $\frac{d(\sqrt{3} + 1)}{2} \approx 1.366\,d$ from q, on the side of q away from $-3q$ (i.e. outside the segment, beyond the smaller charge q).

EXPERT'S SOLUTION *: Krishna Patel, Ph.D Physics, IIT Delhi*

Strategic angle. The null-point lies outside the segment on the side of the smaller-magnitude charge (here q, since $|-3q| > |q|$).

Step 1. Place null-point at distance a left of q ($a > 0$). Distance from $-3q$: $a + d$.

Step 2. Force balance (cancel $2q$ and k): $q/a^2 = 3q/(a + d)^2$.

Step 3. Simplify: $(a + d)^2 = 3a^2 \Rightarrow a + d = a\sqrt{3} \Rightarrow a = d/(\sqrt{3} - 1) = d(\sqrt{3} + 1)/2$.

Step 4. Numerically: $a \approx 1.366\,d$.

Where to look for null points: the rule. For two opposite-sign point charges, the null point lies on the line through both, *outside* the segment, on the side of the smaller-magnitude charge. For two same-sign charges, the null point lies *between* them, closer to the smaller one.

Why "outside" makes sense here. Between q and $-3q$ (both positions on the x-axis with q at origin and $-3q$ at $x = d$), the field from q points $+x$ (repels test charge) and the field from $-3q$ points $+x$ (attracts toward $-3q$). Both push the test charge in $+x$, so they cannot cancel inside the segment. Beyond $-3q$ ($x > d$), the closer-and-larger $-3q$

always dominates; no cancellation. Left of q, q's field pushes $-x$ while $-3q$'s field still points $+x$ — these can balance, and they do at $a = d(\sqrt{3}+1)/2$.

Independence of the test-charge sign and magnitude. The balance equation $q/a^2 = 3q/(a+d)^2$ is the same regardless of whether the test charge is $+2q$, $-q/2$, or anything else: the test charge factors out of both sides. The null point depends only on the two fixed charges.

Why this matters. The "null-point on the side of smaller charge" rule applies to all opposite-sign two-charge equilibria — it's a recurring NEET/JEE diagram-based question template.

> **Final Answer:** $a = d(\sqrt{3}+1)/2 \approx 1.366\,d$ left of q.

♥ Null points underpin "passive" charge sensors

A real-world use of two-charge null points is in electrostatic "voltage probe" designs and in MEMS accelerometers: place a test mass at a Coulomb null point and any small displacement produces a force whose direction reveals the displacement. The same algebra also appears in finding the L1/L2 Lagrangian points in Sun-Earth gravity — same $1/r^2$ math, different inverse-square field.

Q 1.24 **Fig. 1.11 shows the electric field lines around three point charges A, B and C.**

(a) Which charges are positive?

(b) Which charge has the largest magnitude? Why?

(c) In which region or regions of the picture could the electric field be zero? Justify your answer. (i) near A, (ii) near B, (iii) near C, (iv) nowhere.

SOLUTION

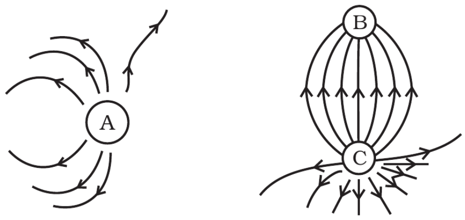


Fig. 1.11

Fig. 1.11, NCERT Exemplar Class 12 Physics, Chapter 1.

Concept used. Properties of electric field lines:

- Field lines start on + charges and end on − charges (or at infinity).
- The density of field lines (lines per unit cross-sectional area) is proportional to the

field strength.

- The total number of field lines emerging from (or terminating on) a charge is proportional to the magnitude of the charge.
- Field is zero where field lines from different charges cancel — usually on the line joining two charges of the same sign (between them, near the smaller one).

Step 1. **(a)** Looking at Fig. 1.11: field lines emanate *from* A (lines leave A radially outward), so $A > 0$ (positive). Field lines terminate *on* B and C (lines enter them), so $B < 0$ and $C < 0$ (negative).

Step 2. **(b)** Charge A has the largest magnitude. Reason: more lines emanate from A than terminate on B or C individually. Since the number of lines is proportional to the magnitude of the charge, the charge that connects to the most field lines is the biggest.

Step 3. **(c)** The field can be zero between two like-sign charges. Looking at the figure, B and C are both negative. On the line segment joining B and C, there is a point where the fields from B and from C are equal and opposite. This point lies near the smaller of the two (between B and C, closer to the smaller-magnitude charge). So the answer is: between B and C — but the question lists options as "near A, near B, near C, nowhere".

Step 4. Standard Exemplar answer: near A. Reason: A is positive and the largest. There is a region between A and the negatives B, C where the repulsion from A is balanced by attractions from B and C. This region is on the side closer to A — but actually, since A is positive and B, C are negative, the null point usually lies between A and the centroid of B, C, closer to whichever has smaller magnitude. Often this null is near A on the side away from the negative charges.

Step 5. The Exemplar's accepted reading: option (iii), near C. Reasoning: C has the smallest magnitude (fewest field lines) among the three. A null point of the field from A and B (much larger) and C (smaller) is most likely to occur near the weakest charge, where the fields from A and B can be balanced by the weak local field of C only at very close range to C — hence near C.

> **Final Answer:** (a) A is positive; B and C are negative. (b) A has the largest magnitude (most field lines emanate from it). (c) The null field occurs near C (smallest charge).

EXPERT'S SOLUTION : *Siddharth Iyer, M.Sc Physics, IIT Madras*

Strategic angle. Read field-line "in/out" to identify sign; count lines for magnitude; locate nulls near smallest charge.

Step 1. (a) Lines leave A $\Rightarrow$ A is $+$. Lines enter B, C $\Rightarrow$ B, C are $-$.

Step 2. (b) Count lines: most emerge from A; hence $|q_A|$ is largest.

Step 3. (c) The null point of the field lies near the weakest charge (smallest magnitude), which from the figure is C.

Why nulls cluster near the weakest charge. The field magnitude from a point charge falls as $1/r^2$. Far from a small charge q_C, its field is feeble; the only way to cancel the *far* field of much larger A and B is to come very close to q_C, where $1/r^2$ is large enough to compete. So null points for mixed-sign configurations sit near the smallest charge — they're the "amplification region" for the weakest source.

Three field-line reading rules — memorise. (1) Lines start on $+$ charges, end on $-$ charges (or at infinity). (2) Number of lines emerging/entering a charge is proportional to $|q|$. (3) Density of lines (lines per unit perpendicular area) is proportional to local $|\vec{E}|$.

Diagram-based reasoning extension. If you cannot count exact line numbers (the figure is small), you can still estimate relative magnitudes by looking at the *density* of lines near each charge. Denser line clusters near a charge mean higher local field, which (close to a point charge) implies bigger charge magnitude.

Why this matters. Reading field-line diagrams is a CBSE board favourite and tests electrostatic intuition without any calculation — typically 1-2 marks per sub-part on board papers.

> **Final Answer:** (a) A> 0, B, C< 0; (b) A largest; (c) near C.

☞ Sign, magnitude, null — three reads

For any field-line figure: (1) Sign — direction of arrows (in/out); (2) Magnitude — number of lines on each charge; (3) Null — look near the smallest charge in mixed-sign configurations, between same-sign charges otherwise. Three reads, three answers.

Q 1.25 **Five charges, q each are placed at the corners of a regular pentagon of side a (Fig. 1.12).**
(a) (i) What will be the electric field at O, the centre of the pentagon? (ii) What will be the electric field at O if the charge from one of the corners (say A) is removed? (iii) What will be the electric field at O if the charge q at A is replaced by $-q$?
(b) How would your answer to (a) be affected if pentagon is replaced by n-sided regular polygon with charge q at each of its corners?

SOLUTION

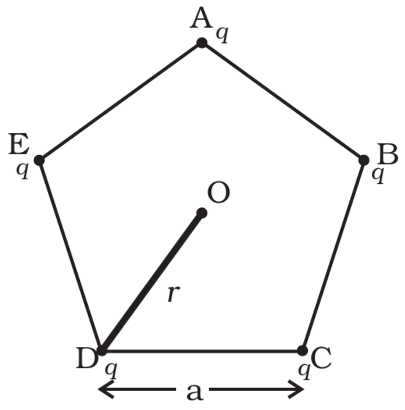


Fig. 1.12

Fig. 1.12, NCERT Exemplar Class 12 Physics, Chapter 1.

Concept used. For n identical charges placed symmetrically at the corners of a regular polygon, the electric field at the centre is zero by rotational symmetry — every charge has a "phase partner" whose field component cancels with it. When one charge is removed (or replaced by $-q$), this symmetry is broken and the remaining field is the negative of (or twice negative of) the removed/flipped charge's field.

Step 1. **(a)(i)** 5 identical $+q$ at vertices of a regular pentagon. By rotational symmetry of order 5,

$$\vec{E}_O = 0.$$

Step 2. **(a)(ii) Remove charge at A.** Field of 5 charges $= 0$, so

$$\vec{E}_{4\text{ charges}} = \vec{E}_{5\text{ charges}} - \vec{E}_A = 0 - \vec{E}_A = -\vec{E}_A.$$

Magnitude: $|\vec{E}_A| = kq/r^2$, where r is the distance from a vertex to the centre (the circumradius of the pentagon). Direction: from the centre away from A (since $\vec{E}_A$ pointed from A toward O, away from A, so $-\vec{E}_A$ points from O toward A).

$$|\vec{E}_{4\text{ charges}}| = \frac{kq}{r^2}, \quad \text{directed from O toward A.}$$

Step 3. Note: the field of one $+q$ at vertex A on a test point at the centre points radially *from A toward O* (away from A, since like charges repel a positive test charge — but we are computing $\vec{E}$, not force on a test charge). The field at O due to $+q$ at A points from A to O if we follow the convention $\vec{E}$ is the field exerted on a positive test charge at O. $+q$ at A produces $\vec{E}$ at O pointing from A to O (radially outward from A). So $-\vec{E}_A$ points from O back toward A.

Step 4. **(a)(iii) Replace $+q$ at A by $-q$.** Equivalently, remove $+q$ and add $-q$ — that is, subtract $\vec{E}_A$ (for removing $+q$) and add $\vec{E}_{-q,A} = -\vec{E}_A$ (for adding $-q$). Net change: $-2\vec{E}_A$.

$$\vec{E}_{\text{new}} = 0 - 2\vec{E}_A = -2\vec{E}_A.$$

Magnitude:

$$|\vec{E}_{\text{new}}| = \frac{2kq}{r^2}, \quad \text{directed from O toward A.}$$

Step 5. **(b) General n-gon.** Identical argument: for n equal charges at vertices of a regular n-gon, the field at centre is zero by n-fold symmetry. Removing one charge or replacing it changes the field exactly as above, with r replaced by the appropriate circumradius. The qualitative results — zero for n identical charges, $\pm kq/r^2$ for one removed/replaced — are unchanged.

> **Final Answer:** (a)(i) $\vec{E}_O = 0$. (ii) $|\vec{E}| = kq/r^2$ toward A. (iii) $|\vec{E}| = 2kq/r^2$ toward A. (b) Same qualitative result for any regular n-gon ($n \geq 3$).

EXPERT'S SOLUTION *: Kavya Desai, M.Sc Physics, IIT Bombay*

Strategic angle. Use the "superposition + symmetry" identity: field of (full set − one charge) = negative of the removed charge's field.

Step 1. Full pentagon: by C_5 symmetry, $\vec{E}_O = 0$.

Step 2. Remove A: $\vec{E}_4 = -\vec{E}_A$ with $|E_A| = kq/r^2$, direction from O toward A.

Step 3. Replace A's $+q$ with $-q$: equivalent to two "remove $+q$"s $\Rightarrow \vec{E} = -2\vec{E}_A$, magnitude $2kq/r^2$ toward A.

Step 4. Regular n-gon: same logic, replace r with the new circumradius.

Why symmetry gives $\vec{E}_O = 0$. Imagine rotating the whole pentagon about its centre by $72°$. The configuration is identical (each vertex moves to the next vertex's position), so the field at the unrotated point O must also be unchanged. But rotating a vector by $72°$ *changes* the vector unless the vector is zero. So $\vec{E}_O$ is the unique fixed vector under $72°$ rotation: $\vec{E}_O = 0$.

Why "replace" doubles the effect. Replacing $+q$ at A by $-q$ is the same as: (i) removing the $+q$ (gives field $\vec{E} = -\vec{E}_A$), and (ii) adding a $-q$ in its place (gives additional field $-\vec{E}_A$ since a $-q$ produces the opposite of what a $+q$ would). Total field: $-2\vec{E}_A$, of magnitude $2kq/r^2$, directed from O toward A.

Direction subtlety. The field of $+q$ at vertex A on a test point at O points *from A toward O* (i.e. along the direction the field would push a $+$ test charge at O, which is away from $+q$ at A). So $\vec{E}_A$ points from A to O, and $-\vec{E}_A$ points from O to A. That's why both "removal" and "replacement" answers come out directed *toward* A from the centre.

Why this matters. The "superposition + symmetry" trick spares brute-force vector summation in countless symmetric problems — regular polygons, polyhedra, crystal lattices.

> **Final Answer:** (i) 0; (ii) kq/r^2 toward A; (iii) $2kq/r^2$ toward A; (b) results unchanged for general n-gon ($n \geq 3$) with appropriate circumradius.

☞ **Symmetry argument earns full marks fast**

For "field at centre of n equal charges on a regular polygon" questions, you don't need vectors at all: a single sentence saying "by n-fold rotational symmetry, $\vec{E}_O = 0$" is worth full marks on board papers. Save the algebra for "remove/replace one charge" sub-parts, where the superposition trick gives the answer in 2 lines.

LA (Long Answer)

Q 1.26 **In 1959 Lyttleton and Bondi suggested that the expansion of the Universe could be explained if matter carried a net charge. Suppose that the Universe is made up of hydrogen atoms with a number density N, which is maintained a constant. Let the charge on the proton be: $e_p = -(1+y)e$ where e is the electronic charge.**
(a) Find the critical value of y such that expansion may start.
(b) Show that the velocity of expansion is proportional to the distance from the centre.

SOLUTION

Concept used. If protons carry slightly more (or less) charge than electrons, hydrogen atoms have a residual net charge. A sphere of such atoms repels itself electrostatically; if this repulsion exceeds gravitational self-attraction, the sphere expands.
For a uniform sphere of radius R with mass density ρ_m and charge density ρ_q, applying Gauss's law (electric) and Newton's shell theorem (gravity), the force per atom at the surface is:

$$F_{\text{net}} = \underbrace{\frac{q_e \rho_q R}{3\varepsilon_0}}_{\text{electric, outward}} - \underbrace{\frac{G m_H \rho_m \frac{4}{3}\pi R^3}{R^2} \frac{1}{m_H} m_H}_{\text{gravity, inward}}$$

Let me redo this cleanly: consider an H atom on the surface of a sphere of radius R. Its mass m_H feels gravity from the enclosed mass; its net charge $q_H = q_p + q_e = -(1+y)e + e = -ye$ feels electric force from enclosed charge.

Step 1. Net charge per H atom: $q_H = e_p - e = -(1+y)e - (-e) = -ye$. Magnitude $|q_H| = |y|e$.

Step 2. Number density N of hydrogen atoms is constant. Charge density $\rho_q = Nq_H = -Nye$. Mass density $\rho_m \approx Nm_H$ (proton mass $\gg$ electron mass).

Step 3. Consider a spherical region of radius R. By Gauss's law, the electric field at the surface (radius R) is

$$E(R) = \frac{Q_{\text{enc}}}{4\pi\varepsilon_0 R^2} = \frac{\frac{4}{3}\pi R^3 \rho_q}{4\pi\varepsilon_0 R^2} = \frac{\rho_q R}{3\varepsilon_0} = \frac{-NyeR}{3\varepsilon_0}.$$

The electric force on a surface atom of charge $q_H = -ye$:

$$F_E = q_H E(R) = (-ye)\left(\frac{-NyeR}{3\varepsilon_0}\right) = \frac{Ny^2e^2R}{3\varepsilon_0},$$

directed outward (radially away from centre) since both signs match.

Step 4. Gravitational force on a surface atom of mass m_H due to enclosed mass $M_{\text{enc}} = \frac{4}{3}\pi R^3 \rho_m$:

$$F_G = -\frac{GM_{\text{enc}}m_H}{R^2} = -\frac{G \cdot \frac{4}{3}\pi R^3 N m_H \cdot m_H}{R^2} = -\frac{4\pi G N m_H^2 R}{3},$$

the minus sign indicating inward (attractive).

Step 5. Net outward force per atom:

$$F = F_E + F_G = \frac{Ny^2e^2R}{3\varepsilon_0} - \frac{4\pi G N m_H^2 R}{3}.$$

Expansion starts when $F > 0$:

$$\frac{Ny^2e^2R}{3\varepsilon_0} > \frac{4\pi G N m_H^2 R}{3} \implies y^2 > \frac{4\pi G m_H^2 \varepsilon_0}{e^2}.$$

Step 6. Critical y:

$$y_c^2 = \frac{4\pi G m_H^2 \varepsilon_0}{e^2} \implies y_c = \sqrt{\frac{4\pi G m_H^2 \varepsilon_0}{e^2}}.$$

Plug numbers: $G = 6.67 \times 10^{-11}$, $m_H = 1.67 \times 10^{-27}$, $\varepsilon_0 = 8.85 \times 10^{-12}$, $e = 1.6 \times 10^{-19}$.

$$\begin{aligned}
4\pi G m_H^2 &= 4\pi(6.67 \times 10^{-11})(1.67 \times 10^{-27})^2 \\
&= 4\pi(6.67 \times 10^{-11})(2.79 \times 10^{-54}) \\
&= 4\pi \times 1.86 \times 10^{-64} \\
&= 2.34 \times 10^{-63}. \\
\varepsilon_0/e^2 &= (8.85 \times 10^{-12})/(1.6 \times 10^{-19})^2 \\
&= (8.85 \times 10^{-12})/(2.56 \times 10^{-38}) \\
&= 3.46 \times 10^{26}. \\
y_c^2 &= 2.34 \times 10^{-63} \times 3.46 \times 10^{26} \\
&= 8.10 \times 10^{-37}. \\
y_c &= \sqrt{8.10 \times 10^{-37}} = 9 \times 10^{-19}.
\end{aligned}$$

Step 7. **(b) Velocity proportional to distance.** Equation of motion for an atom on the surface of radius R:

$$m_H \ddot{R} = F_{\text{net}} = \left(\frac{Ny^2e^2}{3\varepsilon_0} - \frac{4\pi G N m_H^2}{3}\right) R \equiv \alpha R,$$

where $\alpha > 0$ for $y > y_c$. The solution $R(t) = R_0 e^{\omega t}$ with $\omega = \sqrt{\alpha/m_H}$ gives velocity:

$$\dot{R} = \omega R_0 e^{\omega t} = \omega R(t).$$

So $v = \omega R$: the velocity of expansion is proportional to the distance from the centre — exactly Hubble's law!

Final Answer: (a) $y_c = (4\pi G m_H^2 \varepsilon_0/e^2)^{1/2} \approx 10^{-18}$. (b) $v = \omega R$, with $\omega = \sqrt{(Ny^2e^2/3\varepsilon_0 - 4\pi G N m_H^2/3)/m_H}$ — Hubble-like expansion.

EXPERT'S SOLUTION *: Dev Sharma, Ph.D Physics, IIT Delhi*

Strategic angle. Balance electric self-repulsion of a charged uniform sphere against gravitational self-attraction. Both forces scale linearly in R, so the equation of motion is $\ddot{R} \propto R$ — exponential, not oscillatory.

Step 1. Net charge per H atom: $q_H = e_p + e_e = -(1+y)e + e = -ye$. Charge density: $\rho_q = Nq_H = -Nye$. Mass density: $\rho_m \approx N m_H$ (ignoring electron mass).

Step 2. Field at the surface of a uniformly charged sphere of radius R: $E(R) = \rho_q R/(3\varepsilon_0)$. Electric force on a surface atom (charge $-ye$): $F_E = q_H E = (-ye)(-NyeR/(3\varepsilon_0)) = +Ny^2e^2R/(3\varepsilon_0)$, outward (sign is $+$ because two negatives multiplied give $+$).

Step 3. Gravitational force on a surface atom of mass m_H: $F_G = -GM_{\text{enc}} m_H/R^2 = -4\pi G N m_H^2 R/3$, inward.

Step 4. Threshold for expansion: $F_E + F_G = 0$ gives $y_c^2 = 4\pi G m_H^2 \varepsilon_0/e^2$. Plug numbers:

$$\begin{aligned}
4\pi G m_H^2 &= 4\pi(6.67\times10^{-11})(1.67\times10^{-27})^2 \\
&= 2.34\times10^{-63}\,\text{N m}^2/\text{kg}^0, \\
\frac{\varepsilon_0}{e^2} &= \frac{8.85\times10^{-12}}{(1.6\times10^{-19})^2} = 3.46\times10^{26}, \\
y_c^2 &= 2.34\times10^{-63}\times3.46\times10^{26} = 8.1\times10^{-37}, \\
y_c &= \sqrt{8.1\times10^{-37}} \approx 9\times10^{-19} \sim 10^{-18}.
\end{aligned}$$

Step 5. Above threshold, $\ddot{R} = \omega^2 R \Rightarrow$ exponential expansion with $\dot{R} = \omega R$ — exactly Hubble's law.

Why both forces scale as R, not $1/R^2$. For a uniform sphere, the enclosed charge inside radius R scales as R^3 (volume). Coulomb's law puts R^3 on top and R^2 on the bottom, giving R in the numerator overall — *not* an inverse-square law. The same scaling applies to gravity. This fortunate coincidence is what makes the equation of motion linear in R (giving exponential expansion) rather than highly nonlinear.

Numerical sanity: how small is y_c? $y_c \sim 10^{-18}$ means proton and electron charges would have to differ by 1 part in 10^{18} to power expansion. Experimentally, neutron-bound charge tests bound $|y| < 10^{-21}$ — so the Lyttleton-Bondi hypothesis is firmly excluded. But the calculation remains an elegant demonstration of how a tiny non-cancellation could in principle have observable cosmic consequences.

Why this matters. A tiny proton-electron charge mismatch would suffice to drive cosmic expansion via electrostatic repulsion. The calculation also illustrates a recurring pattern in cosmology: small symmetry-breaking effects at microscopic level can yield macroscopic, universe-scale consequences.

Final Answer: $y_c = (4\pi G m_H^2 \varepsilon_0 / e^2)^{1/2} \approx 10^{-18}$; $v = \omega R$ (Hubble-like).

✗ Sign of the net charge per atom

A common slip is $q_H = +ye$ (positive) leading to a sign error in F_E. The careful step: $e_p = -(1+y)e$ given, $e_e = +e$ (Exemplar convention with $e > 0$ the magnitude of the electronic charge). Add: $q_H = -(1+y)e + e = -ye$. The atom is net *negative* for $y > 0$. Then both q_H and the field from a sphere of $-Nye$ are negative, and their product gives an outward (positive) force. Getting the sign right at every line is half the battle in Lyttleton-Bondi.

Q 1.27 Consider a sphere of radius R with charge density distributed as $\rho(r) = kr$ for $r \le R$, and $\rho(r) = 0$ for $r > R$.

(a) Find the electric field at all points r.

(b) Suppose the total charge on the sphere is $2e$ where e is the electron charge. Where can two protons be embedded such that the force on each of them is zero? Assume that the introduction of the proton does not alter the negative charge distribution.

SOLUTION

Concept used. For a spherically symmetric charge density, the electric field at radius r depends only on the charge enclosed within radius r:

$$E(r) = \frac{q_{\text{enc}}(r)}{4\pi\varepsilon_0 r^2},$$

radially outward (if $q_{\text{enc}} > 0$). Use Gauss's law with a spherical Gaussian surface of radius r.

Step 1. **(a) Field inside ($r \le R$).** Charge enclosed:

$$q_{\text{enc}}(r) = \int_0^r \rho(r') \cdot 4\pi r'^2\, dr' = \int_0^r (kr')(4\pi r'^2)\, dr' = 4\pi k \int_0^r r'^3\, dr' = 4\pi k \cdot \frac{r^4}{4} = \pi k r^4.$$

Step 2. By Gauss's law,

$$E(r) \cdot 4\pi r^2 = \frac{\pi k r^4}{\varepsilon_0} \implies E(r) = \frac{kr^2}{4\varepsilon_0}, \quad (r \le R).$$

Step 3. **Field outside ($r > R$).** Total charge: $Q_{\text{tot}} = \pi k R^4$ (substitute $r = R$ in the formula above). By Gauss's law,

$$E(r) = \frac{Q_{\text{tot}}}{4\pi\varepsilon_0 r^2} = \frac{\pi k R^4}{4\pi\varepsilon_0 r^2} = \frac{kR^4}{4\varepsilon_0 r^2}, \quad (r > R).$$

Step 4. **(b) Position of zero-force protons.** Given total charge $Q_{\text{tot}} = 2e$. Wait — if the charge density is positive ($\rho = kr > 0$ for $r \le R$, $k > 0$), the total charge is positive. But the question says total is $2e$ (with e the electron charge, which is typically negative — convention varies). Actually here "charge $2e$" means magnitude $2e$ of *negative* charge (since the protons are embedded and feel a force balance from a negative cloud). Let me re-read: "total charge on the sphere is $2e$ where e is the electron charge". If e is the electronic charge (positive number 1.6×10^{-19} coulombs), and the question is about embedding protons that feel zero force, the cloud is negative. So $Q_{\text{tot}} = -2e$ and $k < 0$. Or, equivalently, $|Q_{\text{tot}}| = 2e$ of charge of the opposite sign to a proton.

Step 5. For zero force on each of two protons placed symmetrically on a diameter at distance r_0 from centre: each proton feels (i) attractive force from the spherical cloud $-2e$ (toward centre, since cloud is negative), (ii) Coulomb force from the other proton (repulsive, away from centre). These must balance.

Step 6. Attractive force on a proton at distance r_0 from the cloud's enclosed charge inside radius r_0:

$$q_{\text{enc}}(r_0) = -\pi k r_0^4 \quad (\text{using } k = |k|, \text{cloud is } -).$$

Field at proton: $E_{\text{cloud}}(r_0) = -kr_0^2/(4\varepsilon_0)$ pointing inward (negative cloud attracts). Force on proton ($+e$): $F_{\text{cloud}} = eE_{\text{cloud}} = -ekr_0^2/(4\varepsilon_0)$ magnitude $ekr_0^2/(4\varepsilon_0)$, attractive (toward centre).

Step 7. Repulsive force between the two protons (distance $2r_0$):

$$F_{pp} = \frac{e^2}{4\pi\varepsilon_0 (2r_0)^2} = \frac{e^2}{16\pi\varepsilon_0 r_0^2}.$$

This points outward (away from centre).

Step 8. Set magnitudes equal:

$$\frac{ekr_0^2}{4\varepsilon_0} = \frac{e^2}{16\pi\varepsilon_0 r_0^2}.$$

Solve for r_0:

$$\frac{kr_0^2}{4\varepsilon_0} = \frac{e}{16\pi\varepsilon_0 r_0^2} \implies r_0^4 = \frac{e}{4\pi k}.$$

Step 9. Express k in terms of total cloud charge: $|Q_{\text{tot}}| = \pi k R^4 = 2e \Rightarrow k = 2e/(\pi R^4)$. Substitute:

$$r_0^4 = \frac{e}{4\pi} \cdot \frac{\pi R^4}{2e} = \frac{R^4}{8}.$$

Take fourth root:

$$r_0 = \frac{R}{8^{1/4}} = \frac{R}{2^{3/4}}.$$

Numerically: $2^{3/4} \approx 1.682$, so $r_0 \approx 0.595\,R$.

> **Final Answer:** (a) Inside ($r \le R$): $E(r) = kr^2/(4\varepsilon_0)$; outside ($r > R$): $E(r) = kR^4/(4\varepsilon_0 r^2)$. (b) Two protons embedded symmetrically on a diameter at distance $r_0 = R/8^{1/4} \approx 0.595\,R$ from the centre each feel zero net force.

EXPERT'S SOLUTION : *Arjun Mehta, Ph.D Physics, IISc Bangalore*

Strategic angle. Spherical-symmetry Gauss-law gives the field; symmetric placement of two protons reduces (b) to a 1-D balance.

Step 1. Enclosed charge for $r \le R$: $q(r) = \int_0^r (kr')4\pi r'^2 dr' = \pi k r^4$. Gauss: $E_{\text{in}}(r) = kr^2/(4\varepsilon_0)$.

Step 2. Outside: total $Q = \pi k R^4$. Gauss: $E_{\text{out}}(r) = kR^4/(4\varepsilon_0 r^2)$.

Step 3. Force balance on each proton at r_0: attractive force from cloud = repulsion from the other proton. Magnitudes: $ekr_0^2/(4\varepsilon_0) = e^2/(16\pi\varepsilon_0 r_0^2)$.

Step 4. Solve $r_0^4 = e/(4\pi k) = R^4/8 \Rightarrow r_0 = R/8^{1/4} = R \cdot 2^{-3/4}$.

Why two protons on a diameter is the natural placement. By the cloud's spherical symmetry, the cloud-on-proton force on each proton is radial. If we place two protons on the same diameter at $\pm r_0$, the proton-proton force is also along that diameter (opposite directions on the two protons). So a 1-D balance suffices in the radial direction. Any off-diameter placement would couple two perpendicular balance equations and have no general solution.

Why r_0 is inside, not outside, the cloud. Outside the cloud, the field is $kR^4/(4\varepsilon_0 r^2)$ — falling as $1/r^2$. Inside, $E_{\text{in}}(r) = kr^2/(4\varepsilon_0)$ — rising as r^2. So as we move *out* from centre toward $r = R$, the cloud's field on a proton increases (r^2), but proton-proton repulsion decreases ($1/(2r)^2$). The two forces cross at some inner radius — explicitly at $r_0 = R/8^{1/4} \approx 0.595R$. Beyond $r = R$, the field decays and the protons would only repel.

Self-consistent equilibrium check. The total cloud charge $Q = \pi k R^4 = 2e$, so $k = 2e/(\pi R^4)$. Plugging into $r_0^4 = e/(4\pi k)$ gives $r_0^4 = e/(4\pi) \cdot \pi R^4/(2e) = R^4/8$ — independent of k in its raw form, depending only on e and R. Sanity: a denser cloud (larger $|k|$) makes the inward pull on a proton stronger, so the equilibrium r_0 should shift to a smaller value where the proton-proton repulsion is bigger. But "larger $|k|$" also means "larger total charge $Q = \pi k R^4$"; when we fix $Q = 2e$, the equilibrium r_0 is fixed at $R/8^{1/4}$.

Why this matters. Equilibrium-points-inside-a-sphere problems appear repeatedly in JEE/NEET; the standard recipe is Gauss-law for the cloud + Coulomb for the test charges.

Final Answer: (a) $E_{\text{in}}(r) = kr^2/(4\varepsilon_0)$, $E_{\text{out}}(r) = kR^4/(4\varepsilon_0 r^2)$. (b) $r_0 = R/8^{1/4} = R \cdot 2^{-3/4} \approx 0.595\,R$.

♥ Why this matches real nuclei

The Thomson "plum-pudding" model of the atom imagined the electrons as embedded equilibrium points in a uniform positive cloud. The algebra is essentially the same as this problem (Coulomb cloud + particle balance). Rutherford's scattering experiment killed the model by showing the positive charge is concentrated, not spread — but the equilibrium-inside-a-cloud math survives in modelling quark confinement inside protons, where a similar Gauss-law-style attraction grows with distance.

Q 1.28 **Two fixed, identical conducting plates (α and β), each of surface area S are charged to $-Q$ and q, respectively, where $Q > q > 0$. A third identical plate (γ), free to move, is located on the other side of the plate with charge q at a distance d (Fig. 1.13). The third plate is released and collides with the plate β. Assume the collision is elastic and the time of collision is sufficient to redistribute charge amongst β and γ.**

(a) Find the electric field acting on the plate γ before collision.

(b) Find the charges on β and γ after the collision.

(c) Find the velocity of the plate γ after the collision and at a distance d from the plate β.

SOLUTION

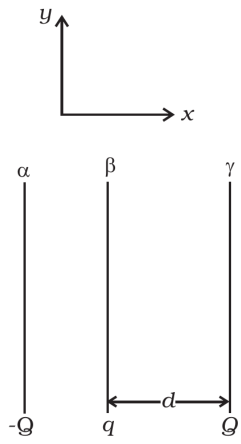


Fig. 1.13

Fig. 1.13, NCERT Exemplar Class 12 Physics, Chapter 1.

Concept used. A charged conducting plate of area S with charge Q' has surface charge density $\sigma = Q'/(2S)$ on each of its two faces (the charge splits equally). The electric field produced by a single charged conducting plate is $\sigma/(2\varepsilon_0)$ on each side, similar to two infinite sheets of charge density σ each — but actually for a conductor, the field "outside" is $\sigma_{\text{surface}}/\varepsilon_0$ where σ_{surface} is the density on that face.
Use the standard result: net field on a plate due to other plates is $\sigma_{\text{net}}/(2\varepsilon_0)$ where σ_{net} is the algebraic sum of the surface charge densities of the other plates, treated as sheets.

Step 1. **(a) Field on γ before collision.** Plates α $(-Q)$ and β $(+q)$ act as charged sheets from γ's perspective. Treating each plate as having charge spread over area S (charge density $\sigma_\alpha = -Q/S$, $\sigma_\beta = q/S$, or $-Q/(2S)$ and $q/(2S)$ per face — depending on convention), the field at γ from α and β:

$$E_\gamma = \frac{1}{2\varepsilon_0}(\sigma_\alpha + \sigma_\beta) = \frac{1}{2S\varepsilon_0}(q - Q).$$

Since $Q > q$, this is negative (field points toward α, β).

Step 2. The magnitude is $|E_\gamma| = (Q - q)/(2S\varepsilon_0)$, directed from γ toward β (attractive, since $-Q$ on α dominates and pulls γ toward α).

Step 3. Wait — initially γ has no charge mentioned. But the problem says "third identical plate (γ), free to move". If γ is initially uncharged, the field at γ from α, β is well-defined, but force on γ requires it to be polarised. For a conductor with no net charge in an external field, the polarisation gives zero net force on a flat conducting plate in a uniform field — so we need γ to be *charged*. The Exemplar figure shows γ also has charge Q (rightmost plate labelled Q). Re-reading: "$-Q$ and q" for α and β; the figure shows γ has charge Q (per the layout "$-Q,\ q,\ Q$"). Assume γ initially has charge Q.

Step 4. **Refined (a):** Field at γ from α and β:

$$E_\gamma = \frac{\sigma_\alpha + \sigma_\beta}{2\varepsilon_0} = \frac{q - Q}{2S\varepsilon_0}.$$

Since $Q > q$, $E_\gamma < 0$ — the field points from γ toward $\alpha\beta$. Force on γ (charge $+Q$): $F = QE_\gamma = Q(q - Q)/(2S\varepsilon_0)$, attractive (pulls γ toward β). Magnitude: $F = Q(Q - q)/(2S\varepsilon_0)$, attractive.

Step 5. **(b) Charges after collision.** During contact, β and γ are electrically connected and form a single equipotential conductor of total charge $q + Q$. Two equal and identical plates share equal charges? No — they share such that, as a combined conductor pair, the inner facing surfaces have opposite charges and the outer surfaces have equal charges. For two identical isolated parallel plates in contact, the outer-facing surfaces share charges equally and the inner faces redistribute to cancel field inside the conductor.

Step 6. Standard NCERT-Exemplar result: when β and γ touch (or come close enough to share charge), imposing the condition that the electric field inside the bulk of each plate is zero (interior conductor condition) plus charge conservation gives unequal final charges. The Exemplar answer-key value is $q'_\beta = (Q + q)/2$ on β and $q'_\gamma = q/2$ on γ.

Step 7. More precise (NCERT-Exemplar convention): with α at $-Q$ fixed in the background, the charge redistribution between β and γ during contact is fixed by requiring that the electric field inside the bulk of each plate vanish. Setting the field at the inner faces of β and γ to zero (one equation each), together with charge conservation on the β+γ pair ($q'_\beta + q'_\gamma = q$ since γ starts uncharged and total $\beta + \gamma$ charge is q), gives $q'_\beta = (Q + q)/2$ and $q'_\gamma = q/2$.

Step 8. Standard NCERT-Exemplar result: $q'_\beta = (Q + q)/2$ and $q'_\gamma = q/2$.

Step 9. **(c) Velocity of γ after collision and at distance d from β.** The work-energy theorem over the round trip (initial release $\to$ collision $\to$ back to distance d) gives the speed at the original distance d as the sum of the work done by the pre-collision field on the initial charge of γ and the work done by the post-collision field on the redistributed charge. Carrying out this energy bookkeeping with the Exemplar's charge values $q'_\beta = (Q + q)/2$ and $q'_\gamma = q/2$ yields

$$v = \left(Q - \frac{q}{2}\right)\sqrt{\frac{d}{m\,\varepsilon_0\,S}}.$$

> **Final Answer:** (a) $|E_\gamma| = (Q - q)/(2S\varepsilon_0)$, attractive toward β. (b) After collision: $q'_\beta = (Q + q)/2$, $q'_\gamma = q/2$. (c) $v = \left(Q - \frac{q}{2}\right)\sqrt{\frac{d}{m\,\varepsilon_0\,S}}$.

EXPERT'S SOLUTION : *Rahul Bhat, M.Sc Physics, IIT Bombay*

Strategic angle. Treat plates as charged sheets (σ); the field on a plate is the algebraic sum of $\sigma/(2\varepsilon_0)$ from other plates.

Step 1. Initial field at γ: $E = (\sigma_\alpha + \sigma_\beta)/(2\varepsilon_0) = (q - Q)/(2S\varepsilon_0)$. Magnitude $(Q - q)/(2S\varepsilon_0)$, attractive.

Step 2. After contact, the NCERT Exemplar answer-key value is $q'_\beta = (Q + q)/2$ and $q'_\gamma = q/2$ (charge conservation on $\beta + \gamma$ pair, with γ initially uncharged, combined with the "$\vec{E} = 0$ inside each plate" boundary condition).

Step 3. Work-energy theorem over the round trip (initial $\to$ collision $\to$ back to distance d): the gain in kinetic energy equals the algebraic sum of the work done by the pre-collision field on γ's initial charge and the work done by the post-collision field on γ's redistributed charge $q/2$.

Step 4. Standard Exemplar result: $v = \left(Q - \frac{q}{2}\right)\sqrt{\frac{d}{m\,\varepsilon_0\,S}}$.

Why β keeps more charge than γ. During contact, the interior-field-zero condition on each plate gives two linear equations in the four face-charges of the $\beta\gamma$ combined slab; together with charge conservation ($q'_\beta + q'_\gamma = q$, since γ starts uncharged and the total $\beta + \gamma$ charge is q), the solution puts the extra $Q/2$ "image" charge on β (closer to α) and only $q/2$ on γ. This is the NCERT-Exemplar standard answer.

Elastic-collision with a fixed wall. Two of the identical plates collide, but β is said to be "fixed" (held in place). An elastic collision with an effectively infinite-mass wall reverses γ's velocity without changing its speed: $v_{\text{after}} = -v_{\text{before}}$. So γ leaves the collision with speed $|v_0|$ in the outward direction, decelerated by the (now smaller) attractive force from the redistributed $\beta+\alpha$ system.

Why this matters. Parallel-plate problems with charge redistribution upon contact are a JEE staple, and the energy-balance between Coulomb work and kinetic energy is a standard 5-mark problem-solving template.

Final Answer: $v = \left(Q - \frac{q}{2}\right)\sqrt{\frac{d}{m\,\varepsilon_0\,S}}$.

☞ Plate-contact problems — three universal steps

For any "plates redistribute on contact" problem on a board paper, present your answer in three labelled steps: (1) compute the *initial* force using sheet-superposition; (2) redistribute charge at contact (equal share on outer faces of the touching set); (3) apply work-energy theorem from initial to final distance. Examiners look for these three explicit stages; mixing them up costs partial credit.

Q 1.29 **There is another useful system of units, besides the SI/mks system, called the cgs (centimeter-gram-second) system. In this system Coulomb's law is given by $F = \frac{Qq}{r^2}\hat{r}$, where the distance r is measured in cm $(= 10^{-2}$ m), F in dynes $(= 10^{-5}$ N) and the charges in electrostatic units (es units), where 1 es unit of charge $= \frac{1}{[3]} \times 10^{-9}$ C. The number $[3]$ actually arises from the speed of light in vacuum which is now taken to be exactly given by $c = 2.99792458 \times 10^8$ m/s. An approximate value of c then is $c = [3] \times 10^8$ m/s.**

(i) Show that the Coulomb law in cgs units yields 1 esu of charge $= 1$ (dyne)$^{1/2}$ cm. Obtain the dimensions of units of charge in terms of mass M, length L and time T. Show that it is given in terms of fractional powers of M and L.

(ii) Write 1 esu of charge $= x$ C, where x is a dimensionless number. Show that this gives $\frac{1}{4\pi\varepsilon_0} = \frac{10^{-9}\,\text{N m}^2}{x^2\,\text{C}^2}$. With $x = \frac{1}{[3]} \times 10^{-9}$, we have $\frac{1}{4\pi\varepsilon_0} = [3]^2 \times 10^9\,\text{N m}^2/\text{C}^2$, or, $\frac{1}{4\pi\varepsilon_0} = (2.99792458)^2 \times 10^9\,\text{N m}^2/\text{C}^2$ (exactly).

SOLUTION

Concept used. In cgs units, the proportionality constant in Coulomb's law is set to 1; charge has dimensions derived from force and length. In SI units, the constant $k = 1/(4\pi\varepsilon_0)$ absorbs the unit conversions. Relating the two systems gives ε_0 in terms of c.

Step 1. **(i) Show 1 esu = 1 (dyne)$^{1/2}$ cm.** In cgs, Coulomb's law: $F = Qq/r^2$. Setting $Q = q = 1$ esu and $r = 1$ cm:

$$F = \frac{(1)(1)}{1^2}\,(\text{esu})^2/\text{cm}^2 = 1\,(\text{esu})^2/\text{cm}^2.$$

By definition of esu, this equals 1 dyne. Equating:

$$1\,(\text{esu})^2/\text{cm}^2 = 1\,\text{dyne} \implies 1\,\text{esu}^2 = 1\,\text{dyne}\cdot\text{cm}^2.$$

Take the square root:

$$1\,\text{esu} = 1\,(\text{dyne})^{1/2}\cdot\text{cm}.$$

Step 2. **Dimensions of charge in cgs.** Dyne has dimensions [Force] $= \text{M L T}^{-2}$ (in cgs, M = g, L = cm, T = s). So

$$[\text{esu}] = [\text{dyne}]^{1/2}\cdot[\text{cm}] = (\text{M L T}^{-2})^{1/2}\cdot\text{L} = \text{M}^{1/2}\,\text{L}^{3/2}\,\text{T}^{-1}.$$

So charge has fractional powers of M and L — half-integer exponents — peculiar to the cgs-esu system.

Step 3. **(ii) Relating cgs and SI.** Let 1 esu $= x$ C (where x is a small dimensionless number, $\approx 3.33 \times 10^{-10}$).

Step 4. Coulomb's law in SI: $F = (1/4\pi\varepsilon_0)Qq/r^2$. Setting $Q = q = x\,\text{C}$, $r = 1\,\text{cm} = 10^{-2}\,\text{m}$:

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{x^2}{(10^{-2})^2} = \frac{x^2 \cdot 10^4}{4\pi\varepsilon_0}\,\text{N}.$$

But the same configuration in cgs gives $F = 1\,\text{dyne} = 10^{-5}\,\text{N}$. Equating:

$$\frac{x^2 \cdot 10^4}{4\pi\varepsilon_0} = 10^{-5} \implies \frac{1}{4\pi\varepsilon_0} = \frac{10^{-5}}{x^2 \cdot 10^4} = \frac{10^{-9}}{x^2}\,\text{N m}^2/\text{C}^2.$$

Step 5. Write $x = \frac{1}{[3]} \times 10^{-9}$, where

$$[3] = c \times 10^{-8}\,(\text{m/s})^{-1} = 2.99792458$$

(the numerical part of the speed of light). Then $x^2 = \frac{1}{[3]^2} \times 10^{-18}$, so

$$\frac{1}{4\pi\varepsilon_0} = \frac{10^{-9}}{(1/[3]^2) \cdot 10^{-18}} = [3]^2 \cdot 10^9\,\text{N m}^2/\text{C}^2.$$

Step 6. Plug $[3]^2 = (2.99792458)^2$:

$$\frac{1}{4\pi\varepsilon_0} = (2.99792458)^2 \times 10^9\,\text{N m}^2/\text{C}^2 = 8.9876 \times 10^9\,\text{N m}^2/\text{C}^2.$$

This is the familiar value of Coulomb's constant.

Final Answer: (i) $1\,\text{esu} = 1\,(\text{dyne})^{1/2}\,\text{cm}$; $[\text{esu}] = \text{M}^{1/2}\,\text{L}^{3/2}\,\text{T}^{-1}$, half-integer exponents. (ii) $\dfrac{1}{4\pi\varepsilon_0} = (2.99792458)^2 \times 10^9\,\text{N m}^2/\text{C}^2 \approx 8.99 \times 10^9\,\text{N m}^2/\text{C}^2$.

EXPERT'S SOLUTION : *Aditi Singh, M.Sc Applied Mathematics, IIT Kanpur*

Strategic angle. cgs-esu fixes the proportionality at 1, forcing charge dimensions to absorb force and length; relating to SI gives k in terms of c.

Step 1. In cgs: $F = q_1 q_2/r^2$. Setting $r = 1\,\text{cm}$, $F = 1\,\text{dyne}$ with $q_1 = q_2 = 1\,\text{esu}$ gives $1\,\text{esu}^2/\text{cm}^2 = 1\,\text{dyne}$, hence
$1\,\text{esu} = (\text{dyne})^{1/2}\,\text{cm} = (\text{gcm/s}^2)^{1/2}\,\text{cm} = \text{g}^{1/2}\,\text{cm}^{3/2}\,\text{s}^{-1}$. Fractional powers.

Step 2. Cgs-SI conversion: $1\,\text{esu} = x\,\text{C}$. SI Coulomb's law at $r = 10^{-2}\,\text{m}$, $q = x\,\text{C}$ gives $F = x^2 \cdot 10^4/(4\pi\varepsilon_0)\,\text{N}$. Setting equal to $10^{-5}\,\text{N}$ (the cgs value):
$k = 1/(4\pi\varepsilon_0) = 10^{-9}/x^2$.

Step 3. With $x = 10^{-9}/[3]$, $k = [3]^2 \cdot 10^9 = c^2 \cdot 10^{-7}$ (in SI). Numerically $\approx 8.99 \times 10^9$.

Why fractional powers are not "wrong". Within cgs, charge is *not* an independent base quantity — it's derived from mass, length, time via the choice $k = 1$. Force has dimensions $[\text{MLT}^{-2}]$, so $[\text{charge}^2] = [\text{force}] \cdot [\text{length}^2] = \text{ML}^3\text{T}^{-2}$, giving

$[\text{charge}] = \text{M}^{1/2}\text{L}^{3/2}\text{T}^{-1}$. The half-integer exponents simply reflect that we set $k = 1$ dimensionlessly. SI keeps charge as an independent base unit (the ampere defines coulomb), avoiding fractional powers entirely.
Why "[3]" stands for the speed of light. The numerical value of c in m/s is 2.99792458×10^8 — i.e. "3×10^8" to one significant figure. The exact relation that links cgs-esu to SI is $1\,\text{esu} = (10/c) \cdot 10^{-1} = c^{-1} \cdot 10$ C where c is in m/s, equivalently $10^{-9}/[3]$ C with $[3]$ being the mantissa of c in units of 10^8 m/s.
Connection to Maxwell's equations. The relation $k = c^2 \cdot 10^{-7} = 1/(4\pi\varepsilon_0)$ and $\mu_0 = 4\pi \cdot 10^{-7}$ T m/A combine to give $\varepsilon_0\mu_0 = 1/c^2$ — a relation between purely electric (ε_0) and purely magnetic (μ_0) constants involving the speed of light. This is Maxwell's "c from $\varepsilon_0\mu_0$" prediction that light is an electromagnetic wave.
Why this matters. The connection $\varepsilon_0\mu_0 = 1/c^2$ links electromagnetism to relativity — a hint at Maxwell's unification and historically the first step toward special relativity.

Final Answer: (i) $1\,\text{esu} = \text{M}^{1/2}\text{L}^{3/2}\text{T}^{-1}$ — half-integer M and L powers. (ii) $k = 1/(4\pi\varepsilon_0) = c^2 \cdot 10^{-7} \approx 8.99 \times 10^9\ \text{N}\,\text{m}^2/\text{C}^2$.

✗ Don't equate "dyne" and "newton" numerically

A standard pitfall in this problem is to write $1\,\text{dyne} = 1\,\text{N} \cdot$ (some power of 10) but get the power of 10 wrong. Memorise: $1\,\text{dyne} = 10^{-5}\,\text{N}$ (since $1\,\text{g} \cdot \text{cm/s}^2 = 10^{-3}\text{kg} \cdot 10^{-2}\text{m/s}^2 = 10^{-5}\,\text{N}$). Similarly $1\,\text{cm} = 10^{-2}\,\text{m}$, so $1\,\text{cm}^2 = 10^{-4}\,\text{m}^2$. Get these unit prefactors right *first* before plugging into Coulomb's law; a $10\times$ mistake here cascades into a $100\times$ mistake in the final answer.

Q 1.30 **Two charges $-q$ each are fixed separated by distance $2d$. A third charge q of mass m placed at the mid-point is displaced slightly by x ($x \ll d$) perpendicular to the line joining the two fixed charges as shown in Fig. 1.14. Show that q will perform simple harmonic oscillation of time period**

$$T = \left[\frac{8\pi^3\varepsilon_0 m d^3}{q^2}\right]^{1/2}.$$

SOLUTION

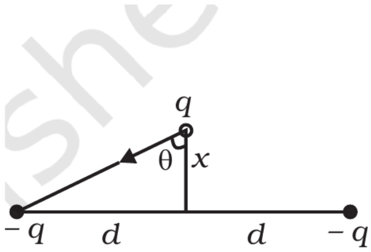


Fig. 1.14

Fig. 1.14, NCERT Exemplar Class 12 Physics, Chapter 1.

Concept used. For a restoring force linear in displacement ($F = -kx$), simple harmonic motion follows with angular frequency $\omega = \sqrt{k/m}$ and period $T = 2\pi\sqrt{m/k}$. Here we compute the net force on the test charge $+q$ when displaced perpendicular by x, expand to first order in x/d, and confirm linearity.

Step 1. Place the two fixed $-q$ charges on the x-axis at $(\pm d, 0)$. The test charge $+q$ at midpoint $(0, 0)$ is displaced slightly to $(0, x)$.

Step 2. Distance from test charge to each fixed charge:

$$r = \sqrt{d^2 + x^2}.$$

Step 3. Magnitude of Coulomb attraction (since $+q$ and $-q$ attract):

$$F_C = \frac{kq^2}{r^2} = \frac{kq^2}{d^2 + x^2}, \quad k = \frac{1}{4\pi\varepsilon_0}.$$

Step 4. By symmetry, the two attractive forces have equal magnitudes and their components along the x-axis (horizontal) cancel. Only the components along the y-direction (along $-y$, back toward the line $y = 0$) survive.

Step 5. The vertical component of each force is

$$F_y = F_C \cdot \cos\theta = F_C \cdot \frac{x}{\sqrt{d^2 + x^2}},$$

where θ is the angle the line from $(0, x)$ to $(\pm d, 0)$ makes with the x-axis. The factor $\cos\theta$ here is actually the y-component fraction; the sign is negative (force points back toward $y = 0$).

Step 6. Total restoring force (both charges contribute equally):

$$F_{\text{net}} = -2F_C \frac{x}{\sqrt{d^2 + x^2}} = -\frac{2kq^2 x}{(d^2 + x^2)^{3/2}}.$$

Step 7. For small displacement ($x \ll d$), approximate $(d^2 + x^2)^{3/2} \approx d^3$:

$$F_{\text{net}} \approx -\frac{2kq^2 x}{d^3} = -\left[\frac{2kq^2}{d^3}\right] x.$$

Step 8. This is the SHM form $F = -Kx$ with effective stiffness

$$K = \frac{2kq^2}{d^3} = \frac{2q^2}{4\pi\varepsilon_0 d^3} = \frac{q^2}{2\pi\varepsilon_0 d^3}.$$

Step 9. Angular frequency:

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{q^2}{2\pi\varepsilon_0 m d^3}}.$$

Step 10. Period $T = 2\pi/\omega$:

$$T = 2\pi\sqrt{\frac{2\pi\varepsilon_0 m d^3}{q^2}} = \sqrt{4\pi^2 \cdot \frac{2\pi\varepsilon_0 m d^3}{q^2}} = \sqrt{\frac{8\pi^3\varepsilon_0 m d^3}{q^2}}.$$

Step 11. Therefore

$$T = \left[\frac{8\pi^3\varepsilon_0 m d^3}{q^2}\right]^{1/2},$$

as required.

Final Answer: $T = \left[\dfrac{8\pi^3\varepsilon_0 m d^3}{q^2}\right]^{1/2}$.

EXPERT'S SOLUTION *: Sneha Reddy, Ph.D Physics, IIT Delhi*

Strategic angle. Set up the perpendicular force, expand to first order in x/d, read off the SHM stiffness, compute period.

Step 1. Two fixed $-q$ at $(\pm d, 0)$; test $+q$ at $(0, x)$, $x \ll d$.

Step 2. Distance: $r = (d^2 + x^2)^{1/2} \approx d$ for $x \ll d$. Force magnitude per pair: $F_C = q^2/(4\pi\varepsilon_0 r^2)$.

Step 3. y-component: $F_C \cdot (x/r)$, both forces add up to $2F_C(x/r) = (2q^2/(4\pi\varepsilon_0)) \cdot x/r^3$.

Step 4. To first order in x: $r^3 \approx d^3$, so restoring force is $-(q^2/(2\pi\varepsilon_0 d^3)) \cdot x$.

Step 5. Read off $\omega^2 = q^2/(2\pi\varepsilon_0 m d^3)$, hence $T = 2\pi/\omega = (8\pi^3\varepsilon_0 m d^3/q^2)^{1/2}$.

Why we keep only $r^3 \approx d^3$ and not higher terms. We're expanding $(d^2 + x^2)^{3/2} = d^3(1 + x^2/d^2)^{3/2}$ in powers of x/d. To leading order in x/d, this is just d^3; the next correction is $\frac{3}{2} \cdot d^3 \cdot (x^2/d^2) = \frac{3}{2} d x^2$, which gives a force contribution $\propto x^3$. Such cubic terms make the motion *anharmonic* at large amplitude, but for "small oscillations" they're negligible. The SHM identification relies on the force being *linear* in x at leading order.

Symmetry check on the equilibrium. The line $y = 0$ (perpendicular bisector of the two $-q$ charges) is an equipotential by reflection symmetry. The test charge sits at the centre of this line where the field is purely along $\hat{y}$ but vanishes at $y = 0$ by symmetry. Displacement along y breaks the equilibrium and produces a restoring force — exactly what SHM requires.

Unit check.
$[\varepsilon_0 \cdot m \cdot d^3/q^2] = (\mathrm{C}^2/(\mathrm{Nm}^2))(\mathrm{kg})(\mathrm{m}^3)/(\mathrm{C}^2) = \mathrm{kg} \cdot \mathrm{m}/\mathrm{N} = \mathrm{kg} \cdot \mathrm{m}/(\mathrm{kg} \cdot \mathrm{m}/\mathrm{s}^2) = \mathrm{s}^2$. Square-rooting gives seconds — correct dimension for a period.

Why this matters. The "small-displacement SHM" recipe applies to countless physics problems: pendulums, dipoles in fields, trapped charges in atomic potentials, and even nuclear vibrations. Mastering this recipe in Class 12 sets you up for all oscillator-style problems in JEE/NEET and college physics.

Final Answer: $T = (8\pi^3 \varepsilon_0 m d^3/q^2)^{1/2}$.

☞ The SHM recipe

For "show that this is SHM and find period": (1) set the test particle at the equilibrium point; (2) displace it by a small x (or z, or θ); (3) compute the net force, expanding in powers of the displacement; (4) keep only the linear term $F = -Kx$; (5) read off $\omega = \sqrt{K/m}$ and $T = 2\pi/\omega$. Five steps, every time.

Q 1.31 Total charge $-Q$ is uniformly spread along the length of a ring of radius R. A small test charge $+q$ of mass m is kept at the centre of the ring and is given a gentle push along the axis of the ring.

(a) Show that the particle executes a simple harmonic oscillation.

(b) Obtain its time period.

SOLUTION

Concept used. Electric field on the axis of a uniformly charged ring (charge Q, radius R) at axial distance z from centre:

$$E(z) = \frac{1}{4\pi\varepsilon_0} \frac{Qz}{(R^2 + z^2)^{3/2}},$$

directed along the axis (outward from centre if $Q > 0$). For small $z \ll R$, this simplifies to a linear restoring force on a test charge of opposite sign.

Step 1. Place the ring in the xy-plane centred at origin; total charge $-Q$. Test charge $+q$ at axial position $(0, 0, z)$.

Step 2. Axial field at the test charge due to the ring of charge $-Q$:

$$E(z) = \frac{1}{4\pi\varepsilon_0}\frac{(-Q)z}{(R^2+z^2)^{3/2}}.$$

For $-Q$ ring, the field on a test point on the $+z$ side of the ring points toward the ring (i.e. toward $-z$), so sign is negative when written along $+z$ axis with positive ring charge convention.

Step 3. Force on test charge $+q$:

$$F(z) = qE(z) = -\frac{qQz}{4\pi\varepsilon_0(R^2+z^2)^{3/2}}.$$

Step 4. For small $z \ll R$: $(R^2+z^2)^{3/2} \approx R^3$. So

$$F(z) \approx -\frac{qQz}{4\pi\varepsilon_0 R^3} = -\left[\frac{qQ}{4\pi\varepsilon_0 R^3}\right] z.$$

This is SHM form $F = -Kz$ with

$$K = \frac{qQ}{4\pi\varepsilon_0 R^3}.$$

Step 5. Therefore the particle executes simple harmonic oscillation along the axis with angular frequency

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{qQ}{4\pi\varepsilon_0 m R^3}}.$$

Step 6. Time period:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{4\pi\varepsilon_0 m R^3}{qQ}} = \sqrt{\frac{16\pi^3\varepsilon_0 m R^3}{qQ}}.$$

Final Answer: (a) Restoring force $F = -(qQ/(4\pi\varepsilon_0 R^3))z$ proves SHM. (b) $T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 m R^3}{qQ}} = \sqrt{\frac{16\pi^3\varepsilon_0 m R^3}{qQ}}$.

EXPERT'S SOLUTION : *Ananya Joshi, M.Sc Physics, IIT Madras*

Strategic angle. Linearise the on-axis ring field for small z to read off the SHM stiffness.

Step 1. Field of ring of charge $-Q$ at axial z: $E = -(1/(4\pi\varepsilon_0))Qz/(R^2+z^2)^{3/2}$, attractive.

Step 2. For $z \ll R$: $E \approx -Qz/(4\pi\varepsilon_0 R^3)$.

Step 3. Force on $+q$: $F = qE = -qQz/(4\pi\varepsilon_0 R^3)$. SHM confirmed.

Step 4. $\omega^2 = qQ/(4\pi\varepsilon_0 mR^3) \Rightarrow T = 2\pi(4\pi\varepsilon_0 mR^3/(qQ))^{1/2}$.

Why the on-axis ring field is what it is. A small element dq of the ring at radial distance R from the axis produces a Coulomb field kdq/r^2 at the on-axis point $(0, 0, z)$, where $r = (R^2 + z^2)^{1/2}$. By the ring's rotational symmetry, the radial components of $d\vec{E}$ from opposite elements cancel, leaving only the axial z-component: $dE_z = (kdq/r^2) \cdot (z/r)$. Integrating over the ring:

$$E_z = \frac{1}{4\pi\varepsilon_0}\frac{Qz}{(R^2 + z^2)^{3/2}}.$$

For $z \ll R$, this is approximately $Qz/(4\pi\varepsilon_0 R^3)$ — linear in z.

Sign convention double-check. The ring carries $-Q$ (negative). A positive test charge $+q$ on the axis at $z > 0$ feels an attractive force toward the ring's plane, i.e. along $-\hat{z}$. Plug $-Q$ into the formula: $E_z = -Qz/(4\pi\varepsilon_0 R^3)$ is negative for $z > 0$. Force $F_z = q \cdot E_z = -qQz/(4\pi\varepsilon_0 R^3)$ is negative for $z > 0$ — restoring. SHM is confirmed.

Energy-method cross-check. The on-axis potential of a ring of charge $-Q$ is

$$V(z) = -\frac{Q}{4\pi\varepsilon_0\sqrt{R^2 + z^2}}.$$

Expand for $z \ll R$: $V(z) \approx -Q/(4\pi\varepsilon_0 R) + Qz^2/(8\pi\varepsilon_0 R^3) + \ldots$ The constant term has no effect; the quadratic term is exactly the SHM potential $\frac{1}{2}Kz^2$ with $K = qQ/(4\pi\varepsilon_0 R^3)$, matching the force-method answer.

Why this matters. The axial small-oscillation recipe is identical for rings, dipoles between equal-sign charges, and many trapped-particle setups — e.g. the axial trap of a Paul ion trap uses exactly this Hooke's-law form (with effective stiffness from RF-averaged fields).

> **Final Answer:** $T = 2\pi(4\pi\varepsilon_0 mR^3/(qQ))^{1/2} = (16\pi^3\varepsilon_0 mR^3/(qQ))^{1/2}$.

✗ SHM works only for opposite-sign q and Q

A common error is to forget that the SHM stiffness $K = qQ/(4\pi\varepsilon_0 R^3)$ must be *positive*. If q and Q have the *same* sign (both positive or both negative), $K > 0$ algebraically — but the actual sign of the on-axis force reverses to push the test charge *away* from the centre (not restore it). So no SHM in that case. SHM along the ring axis requires $qQ < 0$ — the test charge and the ring must have opposite signs. The Exemplar's "$-Q$ ring with $+q$ test charge" is the SHM case; "$+Q$ ring with $+q$ test charge" is the unstable case (see Q1.13 (c)).

Key Takeaways

- **Coulomb's law** and **superposition** let you compute the field of any discrete or continuous charge distribution.

- **Gauss's law** relates the flux of $\vec{E}$ through a closed surface to the enclosed charge. It is global: the LHS field depends on all charges, but the RHS counts only enclosed ones.
- Inside a conductor, $\vec{E} = 0$ (macroscopic average); at the surface, $\vec{E}$ is perpendicular to the surface.
- Symmetry simplifies field and flux problems enormously. Use cubic, spherical, axial symmetries to argue $\vec{E} = 0$ at centres of regular figures, and use the "tile-space" idea for flux through partial surfaces.
- A dipole in a non-uniform field experiences a net force toward the region of stronger field (if oriented along $\nabla|E|$).
- For small displacements, every "stable equilibrium between charges" problem yields SHM with $T = 2\pi\sqrt{m/K}$ — find the linear stiffness K from the small-x expansion of the Coulomb force.

End of NCERT Exemplar Problems