Collegedunia NCERT Notes

The Ultimate NCERT Revision Guide for Class 12 Physics

Chapter 1: Electric Charges and Fields

1 Electric Charge and Its Properties

Electrostatics is the study of charges at rest and the forces, fields, and potentials they produce. The chapter opens with the fundamental object of the entire branch: **electric charge**. Most everyday phenomena — from the spark felt while removing a sweater to lightning during thunderstorms — are manifestations of charge in action.

1.1 Electric Charge

When two bodies are rubbed together, they acquire the property of attracting light objects like dust, paper bits, or hair. This property is called **electrification**, and the bodies are said to be *charged*. There are exactly two kinds of charge in nature:

- **Positive charge** — acquired by glass when rubbed with silk
- **Negative charge** — acquired by ebonite (or plastic) when rubbed with fur

Like charges repel; unlike charges attract. The naming convention (positive/negative) was Benjamin Franklin's choice and is purely conventional — nature does not prefer one sign over the other.

Origin of Charge

Charging by friction does not create new charge — it only **transfers electrons** from one body to the other. The body that gains electrons becomes negatively charged; the body that loses electrons becomes positively charged. **Total charge is conserved.**

1.2 Conductors and Insulators

Materials are classified by how easily charge moves through them:

- **Conductors** — metals, the human body, the Earth, and electrolytic solutions. They contain free charge carriers (electrons in metals, ions in solutions) that move freely through the bulk under any applied field.
- **Insulators** (dielectrics) — glass, plastic, wood, rubber. Charges placed on them stay where they are deposited; there are essentially no free carriers.

When a charged body is touched to a conductor, the charge *spreads* over the entire conductor. When it is touched to an insulator, the charge stays at the point of contact. This is why a charged comb attracts paper bits but does not lose its charge to the paper.

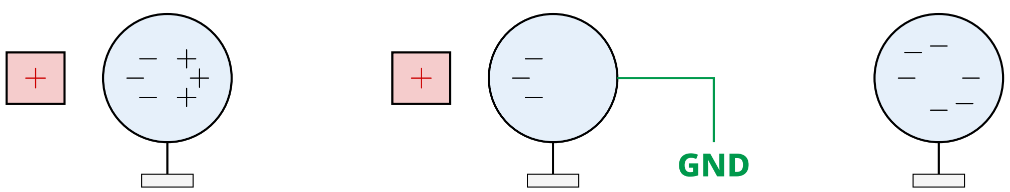
Real-World Application

Aircraft tyres are made of mildly conducting rubber to prevent the build-up of static charge during flight. Without this, friction with air could charge the body to dangerous voltages, causing a spark on landing. The same principle is used in fuel-tanker trucks — they drag a metal chain that grounds the body to the road.

1.3 Charging by Induction

A neutral conductor can be charged *without contact* by exploiting the redistribution of its free electrons. The procedure has three steps:

1. Bring a charged rod (say, positive) close to one end of a neutral metal sphere mounted on an insulating stand. Free electrons in the sphere drift toward the rod, leaving the far end positive and the near end negative.
2. While the rod is still nearby, connect the far end to the ground via a wire. Electrons flow from the ground into the sphere, neutralising the positive far end.
3. Disconnect the ground first, then remove the rod. The sphere is now left with a net **negative** charge — opposite in sign to the inducing rod.



(a) Bring rod near **(b) Earth, then remove rod** **(c) Net negative charge**

Quick Tip

The charge induced is always **opposite** in sign to the inducing charge. A useful sanity check: imagine where the electrons would flow under attraction to (or repulsion from) the rod.

1.4 Basic Properties of Electric Charge

Three foundational properties govern every problem in electrostatics:

Three Properties of Charge

1. **Additivity:** Total charge of a system is the algebraic sum of all charges:

$$q_{\text{total}} = q_1 + q_2 + q_3 + \cdots + q_n$$

2. **Conservation:** The net charge of an isolated system remains constant. Charge can be transferred but never created or destroyed.
3. **Quantisation:** Charge exists only in integer multiples of the elementary charge e:

$$q = ne, \quad n = 0, \pm 1, \pm 2, \ldots, \quad e = 1.6 \times 10^{-19}\ \text{C}$$

Quantisation was first hinted at by Faraday's laws of electrolysis and confirmed experimentally by R. A. Millikan's oil-drop experiment in 1909. Quarks — the constituents of protons and neutrons — carry fractional charges $\pm e/3$ or $\pm 2e/3$, but quarks never appear in isolation, so the smallest *free* charge observable is still e.

Common Mistake

At macroscopic scales (charges of microcoulombs or larger), the integer n is so enormous — of the order of 10^{12} — that quantisation can be ignored and charge treated as a continuous variable. Quantisation matters only at atomic and subatomic scales.

SI Unit of Charge

The SI unit is the **coulomb** (C). One coulomb is the charge that flows through a conductor when a current of one ampere flows for one second: 1 C $= 1$ A·s. A single coulomb is enormous in electrostatic terms — it equals the charge of about 6.25×10^{18} electrons. Common practical units are $1\ \mu$C $= 10^{-6}$ C and 1 nC $= 10^{-9}$ C.

2 Coulomb's Law and Forces between Charges

Once we accept that charges exist and have specific properties, the next question is quantitative: how strongly do two charges interact? The answer was settled experimentally by C. A. Coulomb in 1785 using a torsion balance.

2.1 Coulomb's Law

The electrostatic force between two stationary point charges q_1 and q_2 separated by a distance r in vacuum is directly proportional to the product of the charges and inversely proportional to the square of the distance between them. The force acts along the line joining the two charges.

Coulomb's Law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2} = k\frac{|q_1 q_2|}{r^2}$$

where

- $\varepsilon_0 = 8.854 \times 10^{-12}$ C^2 N^{-1} m^{-2} is the permittivity of free space
- $k = \dfrac{1}{4\pi\varepsilon_0} \approx 9 \times 10^9$ N m^2 C^{-2}

The force is **repulsive** for like charges and **attractive** for unlike charges.

In a medium of relative permittivity ε_r (also called the dielectric constant K), the force is reduced by a factor of ε_r:

$$F_{\text{medium}} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{|q_1 q_2|}{r^2} = \frac{F_{\text{vacuum}}}{\varepsilon_r}$$

For water, $\varepsilon_r \approx 80$, so the same charges in water feel only $1/80$th the vacuum force — which is why salt dissociates in water but not in air.

2.2 Vector Form of Coulomb's Law

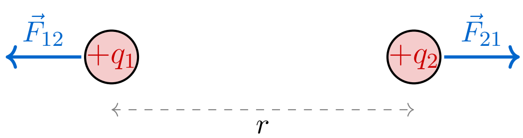
Direction matters. If $\hat{r}_{21}$ is the unit vector pointing *from* charge 2 *to* charge 1, then the force exerted on charge 1 by charge 2 is:

Vector Form

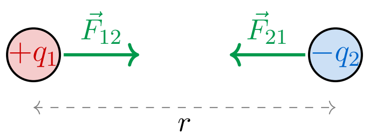
$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\,\hat{r}_{21}$$

Sign convention: substitute q_1 and q_2 *with their actual signs*. The result is positive (repulsion, along $\hat{r}_{21}$) for like charges, and negative (attraction, opposite to $\hat{r}_{21}$) for unlike charges.

By Newton's third law, $\vec{F}_{21} = -\vec{F}_{12}$.



Like charges: repulsion



Unlike charges: attraction

Coulomb's Law vs Newton's Law of Gravitation

Both follow an inverse-square law, but Coulomb force is roughly 10^{36} times stronger than the gravitational force between two protons. Coulomb forces can be attractive *or* repulsive, while gravity is always attractive. This explains why electrical forces dominate at the atomic scale, and gravity dominates at the astronomical scale (where bulk matter is electrically neutral).

Quick Tip

Always plug magnitudes only into the scalar form $F = kq_1q_2/r^2$, and decide the direction *separately* from the geometry. Mixing signs into a magnitude formula is the most common source of errors in three-charge problems.

2.3 Forces between Multiple Charges — Superposition

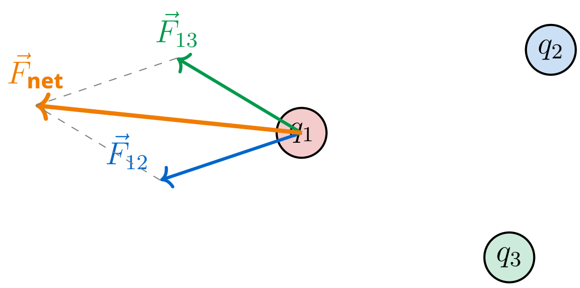
When more than two charges are present, the force on any one charge is simply the **vector sum** of the forces exerted on it by every other charge, taken *independently* of the others. The presence of charge 3 does not alter the force that charge 2 would exert on charge 1.

Superposition Principle

The net force on charge q_1 due to charges $q_2, q_3, \dots, q_n$ is

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \cdots + \vec{F}_{1n} = \frac{q_1}{4\pi\varepsilon_0} \sum_{i=2}^{n} \frac{q_i}{r_{1i}^2} \hat{r}_{i1}$$

This is what makes Coulomb's law a **linear** theory — forces simply add up, no cross-terms appear.



The diagram shows how the individual forces $\vec{F}_{12}$ (from q_2, here repulsive) and $\vec{F}_{13}$ (from q_3, here repulsive) on q_1 add as vectors via the parallelogram rule to give $\vec{F}_{\text{net}}$.

Memory Aid

For symmetric charge arrays: Look for symmetry first, draw vectors second. In a square or hexagonal arrangement of equal charges, oppositecorner forces often cancel pairwise — saving you the algebra entirely. *Symmetry then sum*.

3 Electric Field

Coulomb's law tells us the force one charge exerts on another, but introduces a conceptual puzzle: how does charge 1 "know" that charge 2 is there? The answer is the **electric field** — a region of altered space surrounding every charge that mediates the force.

3.1 Electric Field due to a Point Charge

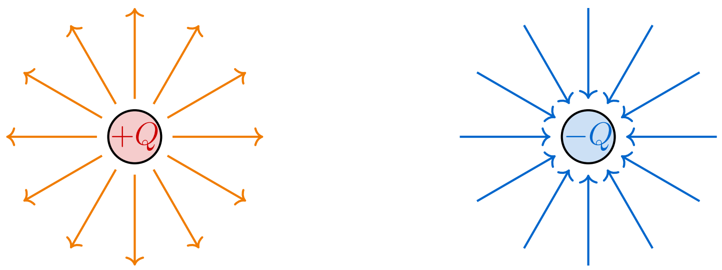
A charge Q produces a field $\vec{E}$ at every point in the surrounding space. The field at a point is defined as the force per unit positive test charge placed at that point (in the limit that the test charge is small enough not to disturb the source):

Electric Field of a Point Charge

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}\,\hat{r}$$

SI unit: $\mathrm{N\,C^{-1}}$ (also expressible as $\mathrm{V\,m^{-1}}$, used after Chapter 2).
The field of a positive charge points **radially outward**; that of a negative charge points **radially inward**.

- Once $\vec{E}$ is known at a point, the force on *any* test charge q placed there is simply $\vec{F} = q\vec{E}$.
- The field is a property of the source charge — it exists whether or not a test charge is present to "feel" it.
- $\vec{E}$ at a point due to many charges adds vectorially: $\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \cdots$ (superposition again).



Positive charge: outward **Negative charge: inward**

3.2 Field of a System of Charges

For a collection of point charges $q_1, q_2, \ldots, q_n$ at positions $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_n$, the field at point P (position $\vec{r}$) is:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i\,(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3}$$

The denominator carries an extra power of r because $(\vec{r} - \vec{r}_i)/|\vec{r} - \vec{r}_i|$ is the unit vector and $1/|\vec{r} - \vec{r}_i|^2$ is the magnitude factor.

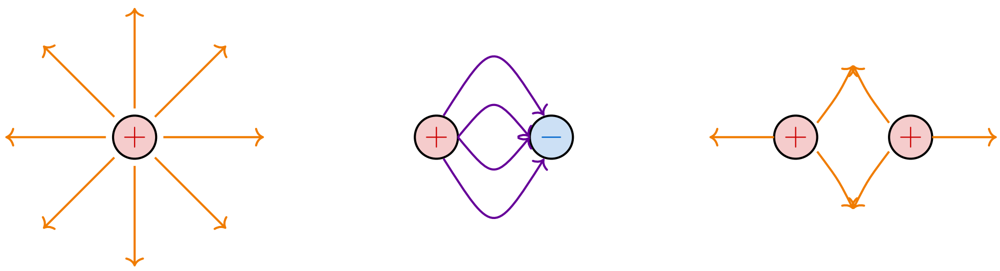
Why introduce a Field?

For static charges, the "field" may seem like a redundant bookkeeping tool — $\vec{F} = q\vec{E}$ is just $\vec{F} = kQq/r^2$ rewritten. The real value emerges with **time-varying** sources: when charges move, the field adjusts at the speed of light (not instantaneously), and the field itself carries energy and momentum. This is essential for understanding electromagnetic waves later. The field is the carrier of the interaction — not a mathematical convenience.

3.3 Electric Field Lines

Field lines are an imaginary, visual aid invented by Faraday to map the electric field. A field line is a curve drawn such that:

- The **tangent** at any point gives the direction of $\vec{E}$ at that point
- The **density** of lines (lines per unit cross-sectional area perpendicular to them) is proportional to the magnitude of $\vec{E}$



Single positive charge **Dipole** **Two like charges**

Properties of field lines:

- Originate on positive charges (or come from infinity) and terminate on negative charges (or go to infinity).
- **Never form closed loops** for static charges (a closed loop would imply work done in moving a charge around it, contradicting the conservative nature of electrostatic force).

- **Never intersect** each other — if they did, the field would have two directions at the intersection, which is impossible.
- Are continuous curves with no sudden breaks.
- Are **perpendicular** to the surface of any conductor in equilibrium.

Common Mistake

Field lines are a **visual aid**, not a physical entity. They have no mass, no momentum, and they don't "carry" the charge from one place to another. Different artists may draw different numbers of lines for the same field — only their relative density (and direction) carries information.

4 Electric Flux and Continuous Charge Distribution

So far we have dealt with discrete point charges. Real-world objects (a charged sphere, a long wire, a sheet of foil) have charge spread continuously over them. To handle these, we need two new tools: **flux** (a measure of how much field passes through a surface) and **charge densities** (how charge is distributed over a region).

4.1 Electric Flux

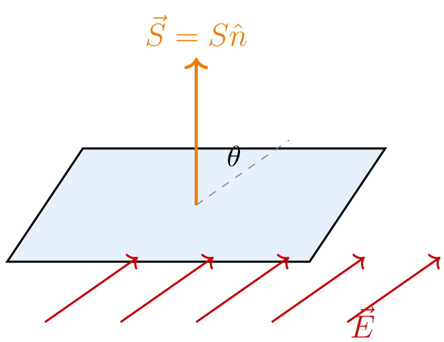
Imagine the electric field as a fluid flowing through space. The **electric flux** through a surface measures how much of this "flow" passes through the surface. Crucially, flux is a *scalar*: it has no direction, only a magnitude (and a sign).

For a uniform field $\vec{E}$ passing through a flat area $\vec{S}$:

Electric Flux — Uniform Field

$$\Phi_E = \vec{E} \cdot \vec{S} = ES\cos\theta$$

- θ is the angle between $\vec{E}$ and the area vector $\vec{S}$
- $\vec{S}$ points along the **outward normal** to the surface
- SI unit: $\mathrm{N\,m^2\,C^{-1}}$ (or V·m)



For a non-uniform field or a curved surface, divide the surface into infinitesimal patches $d\vec{S}$, each small enough that $\vec{E}$ is uniform over it, and integrate:

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{S}$$

The circle on the integral indicates a closed surface, which is the case relevant to Gauss's law.

Quick Tip

$\cos\theta$ is positive when field exits the surface ($\theta < 90°$) and negative when it enters ($\theta > 90°$). For a closed surface enclosing no source, the flux entering must equal the flux exiting — so the total is zero.

4.2 Continuous Charge Distribution

Real charged bodies are best described by *charge densities*, depending on the geometry:

Distribution Type	Density	Charge Element
Linear (1D, on a line/wire)	$\lambda = \dfrac{dq}{d\ell}$ (C m^{-1})	$dq = \lambda\, d\ell$
Surface (2D, on a sheet)	$\sigma = \dfrac{dq}{dA}$ (C m^{-2})	$dq = \sigma\, dA$
Volume (3D, in a solid)	$\rho = \dfrac{dq}{dV}$ (C m^{-3})	$dq = \rho\, dV$

The electric field at a point P due to a continuous distribution is found by treating each dq as a point charge and integrating:

Field of Continuous Distribution

$$\vec{E}(P) = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r^2}\,\hat{r}$$

where r is the distance from dq to P and $\hat{r}$ is the unit vector from dq towards P. Replace dq by $\lambda\, d\ell$, $\sigma\, dA$, or $\rho\, dV$ depending on the geometry.

In practice, the integral is rarely evaluated this way — Gauss's law (next section) gives a much faster route whenever symmetry permits.

5 Electric Dipole

A dipole is the simplest non-trivial charge configuration: two equal and opposite charges held a fixed distance apart. Although neutral overall, a dipole produces a non-zero electric field, and its behaviour models molecules like H_2O, HCl, and the polarisation of dielectrics.

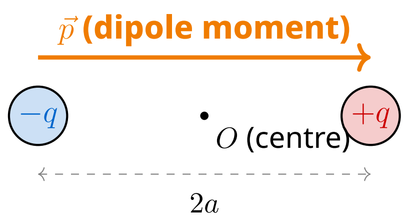
5.1 Definition and Dipole Moment

An electric dipole consists of two charges $+q$ and $-q$ separated by a small distance $2a$. The **dipole moment** is a vector defined as:

Electric Dipole Moment

$$\vec{p} = q \cdot 2\vec{a} \quad \text{(magnitude } p = q \cdot 2a\text{)}$$

- Direction: from $-q$ to $+q$ (by convention)
- SI unit: C · m
- Total charge of a dipole is zero, but $\vec{p}$ is non-zero — the geometric separation matters



5.2 Field due to a Dipole on Axial Line

The axial line is the line passing through both charges. For a point P on the axial line at distance r from the centre O (on the side of the $+q$ charge):

- Field due to $+q$: $E_+ = \frac{kq}{(r-a)^2}$, pointing *away* from $+q$ (along $+\hat{p}$)
- Field due to $-q$: $E_- = \frac{kq}{(r+a)^2}$, pointing *toward* $-q$ (along $-\hat{p}$)

The axial field is the difference $E_+ - E_-$ along $\hat{p}$:

$$E_{\text{axial}} = \frac{kq}{(r-a)^2} - \frac{kq}{(r+a)^2} = \frac{kq \cdot 4ra}{(r^2-a^2)^2}$$

For a **short dipole** ($r \gg a$), $(r^2-a^2)^2 \approx r^4$, and using $p = q \cdot 2a$:

Axial Field (Short Dipole, $r \gg a$)

$$\vec{E}_{\text{axial}} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$$

The field is along $\hat{p}$ (i.e. parallel to the dipole moment).

5.3 Field due to a Dipole on Equatorial Line

The equatorial line is the perpendicular bisector of the dipole. For a point P on this line at distance r from O:

- Both charges are equidistant from P, at distance $\sqrt{r^2 + a^2}$
- The magnitudes of $\vec{E}_+$ and $\vec{E}_-$ are equal
- Components perpendicular to the dipole axis cancel; components along $-\hat{p}$ add

After working out the geometry:

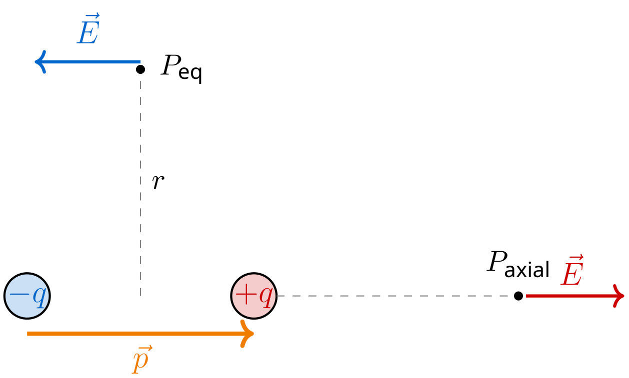
$$E_{\text{eq}} = 2 \cdot \frac{kq}{(r^2 + a^2)} \cdot \frac{a}{\sqrt{r^2 + a^2}} = \frac{kq \cdot 2a}{(r^2 + a^2)^{3/2}}$$

For $r \gg a$:

Equatorial Field (Short Dipole, $r \gg a$)

$$\vec{E}_{\text{eq}} = -\frac{1}{4\pi\varepsilon_0} \frac{\vec{p}}{r^3}$$

The field is **antiparallel** to $\hat{p}$, and **half** the magnitude of the axial field at the same distance.



Why the Dipole Field Falls as $1/r^3$

A point charge produces a field that falls off as $1/r^2$. A dipole's $+q$ and $-q$ *nearly* cancel at distant points, so the residual field falls off faster — as $1/r^3$. In general, multipoles of order n produce fields that fall as $1/r^{n+2}$ (charge: $1/r^2$; dipole: $1/r^3$; quadrupole: $1/r^4, \ldots$).

Quick Tip

The factor of **2** between axial and equatorial fields ($E_{\text{axial}} = 2E_{\text{eq}}$) is a frequent MCQ. Also note the **direction**: axial field is parallel to $\vec{p}$, equatorial is antiparallel.

5.4 Dipole in a Uniform External Field

Place a dipole in a uniform external field $\vec{E}$. The forces on the two charges are:

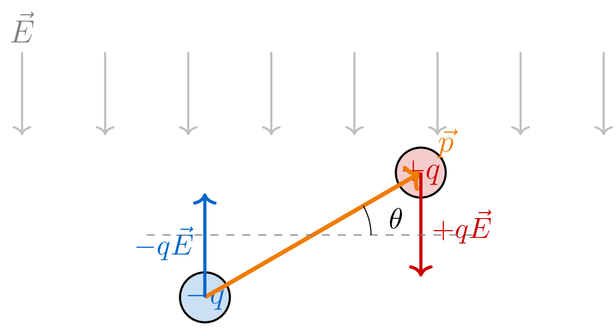
- On $+q$: $\vec{F}_+ = +q\vec{E}$
- On $-q$: $\vec{F}_- = -q\vec{E}$

These are equal in magnitude and opposite in direction — so the **net force is zero**. But because they act at different points, they form a couple, producing a non-zero **torque** that tries to align $\vec{p}$ with $\vec{E}$.

Torque on a Dipole

$$\vec{\tau} = \vec{p} \times \vec{E}, \qquad |\vec{\tau}| = pE \sin\theta$$

where θ is the angle between $\vec{p}$ and $\vec{E}$. The torque is perpendicular to the plane of $\vec{p}$ and $\vec{E}$.



Special cases:

- $\theta = 0°$: $\vec{p}$ aligned with $\vec{E}$, $\tau = 0$, **stable equilibrium**
- $\theta = 180°$: $\vec{p}$ antiparallel to $\vec{E}$, $\tau = 0$, **unstable equilibrium**
- $\theta = 90°$: torque is maximum, $\tau_{\text{max}} = pE$

In a **non-uniform** field, the two charges experience forces of different magnitudes, so the net force is non-zero — the dipole experiences both a torque *and* a translational force.

Real-World Application

A microwave oven exploits dipole rotation. Water molecules are permanent dipoles. The microwave field oscillates at 2.45 GHz, forcing the water dipoles to rotate back and forth. Friction between rotating molecules turns the rotational energy into heat — which is why food with high water content heats faster than dry food.

Memory Aid

Net force is Zero in a Uniform field; torque is Z*ero only at* $\theta = 0$ or $180°$. For everything in between, $\tau = pE \sin\theta$ tries to align $\vec{p}$ along $\vec{E}$ — nature minimises the orientation energy $U = -\vec{p} \cdot \vec{E}$.

6 Gauss's Law and Its Applications

Gauss's law is one of the four Maxwell equations and the most powerful tool in electrostatics. It relates the flux of $\vec{E}$ through a closed surface to the charge enclosed — enabling field calculations that would be impossible by direct Coulomb integration.

6.1 Gauss's Law

Gauss's Law

The total electric flux through any **closed surface** is equal to $1/\varepsilon_0$ times the net charge enclosed:

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

- The closed surface S is called a **Gaussian surface**; it is imaginary, chosen for convenience
- q_{enc} is the algebraic sum of all charges *inside* S — charges outside contribute nothing to the net flux
- Gauss's law is valid for *any* closed surface and *any* charge distribution

Why charges outside don't contribute: An external charge produces field lines that enter the closed surface and must leave it again — adding to the flux at entry and subtracting at exit, with a net of zero.

Strategy for using Gauss's Law

Gauss's law is always true, but only *useful* when the charge distribution has high symmetry (spherical, cylindrical, or planar). The trick is to choose a Gaussian surface where:

1. $\vec{E}$ is constant in magnitude over the surface (or a clearly identified part), and
2. $\vec{E}$ is either parallel or perpendicular to $d\vec{S}$ at every point.

This collapses the integral to $E \cdot A = q_{\text{enc}}/\varepsilon_0$, which can be solved for E instantly.

6.2 Field due to an Infinite Straight Charged Wire

Consider a long straight wire with uniform linear charge density λ. By symmetry, the field at a perpendicular distance r from the wire must point **radially outward** (or inward, if $\lambda < 0$) and depend only on r.

Choose a coaxial cylindrical Gaussian surface of radius r and length ℓ. The flux through:

- the two flat ends is zero ($\vec{E}$ is parallel to the ends, so $\vec{E} \cdot d\vec{S} = 0$)
- the curved side is $E \cdot (2\pi r\ell)$, since $\vec{E}$ is radial and constant in magnitude

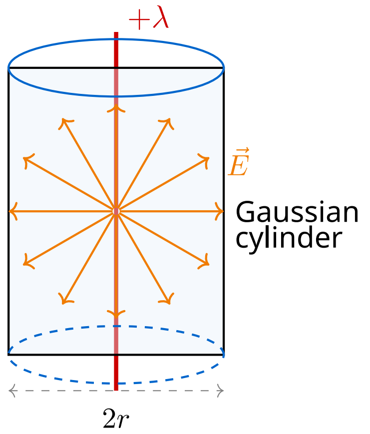
The enclosed charge is $\lambda\ell$. Applying Gauss's law:

$$E \cdot 2\pi r\ell = \frac{\lambda\ell}{\varepsilon_0}$$

Infinite Line of Charge

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\,\hat{r}$$

The field falls as $1/r$ (compare with $1/r^2$ for a point charge — the lower power reflects the lower symmetry).



6.3 Field due to a Uniformly Charged Infinite Plane Sheet

A thin infinite plane has uniform surface charge density σ. By symmetry, the field is perpendicular to the sheet and uniform in magnitude on both sides.

Choose a cylindrical Gaussian *pillbox* that pierces the sheet, with end caps of area A parallel to the sheet:

- Flux through curved side: zero ($\vec{E} \perp$ curved surface here, since $\vec{E}$ is parallel to the cylinder axis)
- Flux through each end cap: EA (field exits both ends)

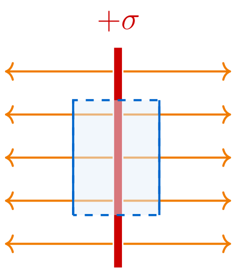
- Total flux: $2EA$
- Enclosed charge: σA

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

Infinite Plane Sheet of Charge

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

The field is **independent of distance** from the sheet — a uniform field on each side. Direction: away from the sheet for $\sigma > 0$, toward it for $\sigma < 0$.



Gaussian pillbox through sheet

Two parallel sheets with opposite charges ($+\sigma$ and $-\sigma$): fields add between the sheets, cancel outside. Between the sheets, $E = \sigma/\varepsilon_0$; outside, $E = 0$. This is the basis of the parallel-plate capacitor (Chapter 2).

Quick Tip

For a **thick conducting plate** with surface charge density σ on each side, the field just outside the surface is $E = \sigma/\varepsilon_0$, *not* $\sigma/2\varepsilon_0$. The factor differs because a conductor places all charge on its surface and the field inside is zero.

6.4 Field due to a Uniformly Charged Spherical Shell

A thin spherical shell of radius R carries total charge Q uniformly distributed over its surface.

Outside the shell ($r > R$): Choose a concentric spherical Gaussian surface of radius r. By symmetry, $\vec{E}$ is radial and constant in magnitude over this surface:

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \quad \Longrightarrow \quad E_{\text{out}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

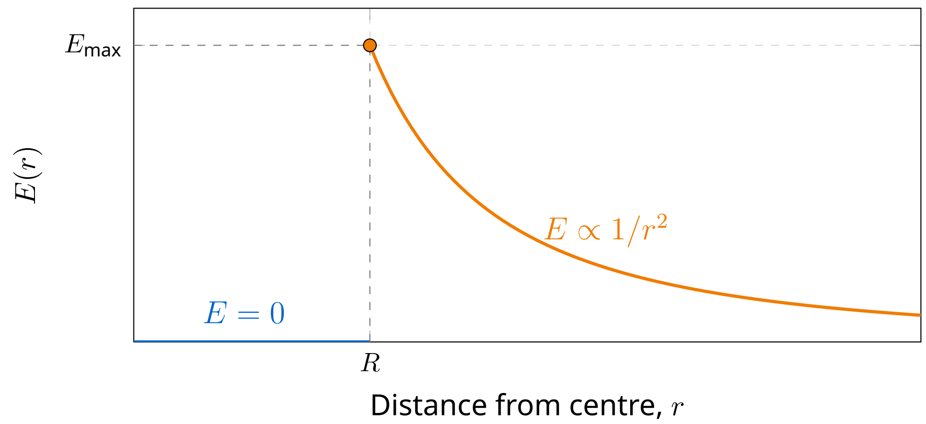
Inside the shell ($r < R$): The Gaussian surface encloses no charge, so:

$$E_{\text{in}} = 0$$

Uniformly Charged Spherical Shell

$$\vec{E}(r) = \begin{cases} 0 & \text{for } r < R \\ \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R^2}\hat{r} & \text{at } r = R^+ \\ \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}\hat{r} & \text{for } r > R \end{cases}$$

Outside, the shell behaves **exactly like a point charge Q at the centre.** Inside, the field is zero — a remarkable result, sometimes called the *shell theorem*.



The plot shows the discontinuity at $r = R$: E jumps from 0 inside to $\sigma/\varepsilon_0 = Q/(4\pi\varepsilon_0 R^2)$ on the surface, then falls as $1/r^2$.

Real-World Application

The shell theorem is why the Earth's gravitational field can be computed by treating Earth as a point mass at its centre (for points outside Earth). Newton actually delayed publishing his theory of gravitation for years until he could prove this result — which holds equally for any inverse-square central force, electric or gravitational.

Common Mistake

The result $E_{\text{in}} = 0$ holds only for a **uniformly charged shell**. For a uniformly charged *solid sphere*, the field inside is non-zero: $E_{\text{in}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qr}{R^3}$ (it grows linearly with r). *[JEE/NEET extension — not in NCERT but routinely tested.]*

6.5 [JEE/NEET extension] Solid Sphere with Uniform Volume Charge

For a uniformly charged solid sphere of radius R with total charge Q and volume density $\rho = \dfrac{3Q}{4\pi R^3}$:

- For $r > R$: same as a point charge: $E = \frac{kQ}{r^2}$
- For $r < R$: enclosed charge is $\frac{4\pi r^3}{3}\rho = Q\frac{r^3}{R^3}$, giving:

$$E_{\text{in}}(r) = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}\hat{r} \quad (r < R)$$

The field rises linearly inside, peaks at $r = R$, then falls as $1/r^2$ outside. Common in entrance-exam questions on charged dust grains, atomic-nucleus models, and uniformly charged spheres in dielectric problems.

7 Quick Reference Summary

7.1 Master Formula Sheet

Quantity / Configuration	**Formula**
Quantisation of charge	$q = ne, \quad e = 1.6 \times 10^{-19}$ C
Coulomb's law (vacuum)	$F = \frac{1}{4\pi\varepsilon_0}\frac{\lvert q_1 q_2\rvert}{r^2}$
Coulomb's law (medium)	$F = \frac{F_{\text{vac}}}{\varepsilon_r}$
$k = 1/(4\pi\varepsilon_0)$	9×10^9 N m^2 C^{-2}
ε_0	8.854×10^{-12} C^2 N^{-1} m^{-2}
Field of point charge	$E = \frac{kQ}{r^2}$
Force on charge in field	$\vec{F} = q\vec{E}$
Linear / Surface / Volume density	$\lambda = dq/d\ell, \;\; \sigma = dq/dA, \;\; \rho = dq/dV$
Electric flux (uniform field)	$\Phi = \vec{E}\cdot\vec{S} = ES\cos\theta$
Gauss's law	$\oint \vec{E}\cdot d\vec{S} = q_{\text{enc}}/\varepsilon_0$
Dipole moment	$\vec{p} = q \cdot 2\vec{a}$
Axial field of short dipole	$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$
Equatorial field of short dipole	$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}$
Torque on dipole in $\vec{E}$	$\vec{\tau} = \vec{p}\times\vec{E}, \;\; \lvert\tau\rvert = pE\sin\theta$
Infinite line of charge	$E = \frac{\lambda}{2\pi\varepsilon_0 r}$
Infinite plane sheet	$E = \frac{\sigma}{2\varepsilon_0}$
Conductor surface (just outside)	$E = \frac{\sigma}{\varepsilon_0}$
Spherical shell, $r > R$	$E = \frac{kQ}{r^2}$
Spherical shell, $r < R$	$E = 0$
Solid sphere, $r < R$ *[JEE]*	$E = \frac{kQr}{R^3}$

7.2 Field Comparison — A Glance

Source	Field magnitude	Distance dependence
Point charge	kQ/r^2	$1/r^2$
Electric dipole (short, axial)	$2kp/r^3$	$1/r^3$
Electric dipole (short, equatorial)	kp/r^3	$1/r^3$
Infinite line of charge	$\lambda/2\pi\varepsilon_0 r$	$1/r$
Infinite plane sheet	$\sigma/2\varepsilon_0$	independent of r
Spherical shell (outside)	kQ/r^2	$1/r^2$
Spherical shell (inside)	0	N/A

The Big Picture

Every result in this chapter ultimately rests on two laws: **Coulomb's law** (the inverse-square force between two charges) and the **principle of superposition** (forces and fields add as vectors). Gauss's law is mathematically equivalent to Coulomb's law plus superposition, but is far easier to apply when symmetry is present. Master the symmetry-detection skill: spherical $\rightarrow$ Gaussian sphere; cylindrical $\rightarrow$ Gaussian cylinder; planar $\rightarrow$ Gaussian pillbox.

7.3 Common Constants

- Elementary charge: $e = 1.6 \times 10^{-19}$ C
- Permittivity of free space: $\varepsilon_0 = 8.854 \times 10^{-12}$ C^2 N^{-1} m^{-2}
- Coulomb constant: $k = 9 \times 10^9$ N m^2 C^{-2}
- Mass of electron: $m_e = 9.11 \times 10^{-31}$ kg
- Mass of proton: $m_p = 1.67 \times 10^{-27}$ kg
- Avogadro's number: $N_A = 6.022 \times 10^{23}$ mol^{-1}

Quick Tip

Last-minute revision checklist for board exam:

- Coulomb's law in scalar and vector form (must be able to write both)
- Definition of $\vec{E}$ and superposition (1-mark question staple)
- Properties of field lines (5 properties — often a 2-marker)
- Derivations: axial field of dipole, equatorial field of dipole, torque on

dipole
- Statement and proof of Gauss's law
- Three Gauss's law applications: line, sheet, shell — with the diagram