NCERT SOLUTIONS

Class 12 Physics

Chapter 1 — Electric Charges and Fields

Detailed Step-by-Step Exercise Solutions

Q1 What is the force between two small charged spheres having charges of 2×10^{-7} C and 3×10^{-7} C placed 30 cm apart in air?

Solution

Given Data:

- Charge on first sphere, $q_1 = 2 \times 10^{-7}$ C
- Charge on second sphere, $q_2 = 3 \times 10^{-7}$ C
- Distance between the spheres, $r = 30$ cm $= 0.30$ m
- Medium: Air (dielectric constant $\varepsilon_r \approx 1$)

Concept: Coulomb's Law
The electrostatic force between two point charges is given by **Coulomb's law**:

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

where:

- $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N m^2 C^{-2} (in SI units)
- The force is repulsive if charges have the same sign, attractive if opposite.

Here, both charges are positive, so the force is **repulsive**.

Step 1: Substitute the Values

$$F = 9 \times 10^9 \times \frac{(2 \times 10^{-7}) \times (3 \times 10^{-7})}{(0.30)^2}$$

Step 2: Simplify the Numerator

$$q_1 q_2 = (2 \times 10^{-7}) \times (3 \times 10^{-7}) = 6 \times 10^{-14} \text{ C}^2$$

Step 3: Simplify the Denominator

$$r^2 = (0.30)^2 = 0.09 \text{ m}^2$$

Step 4: Calculate the Force

$$F = 9 \times 10^9 \times \frac{6 \times 10^{-14}}{0.09}$$

$$F = 9 \times 10^9 \times \frac{6 \times 10^{-14}}{9 \times 10^{-2}}$$

$$F = 9 \times 10^9 \times \frac{6}{9} \times \frac{10^{-14}}{10^{-2}}$$

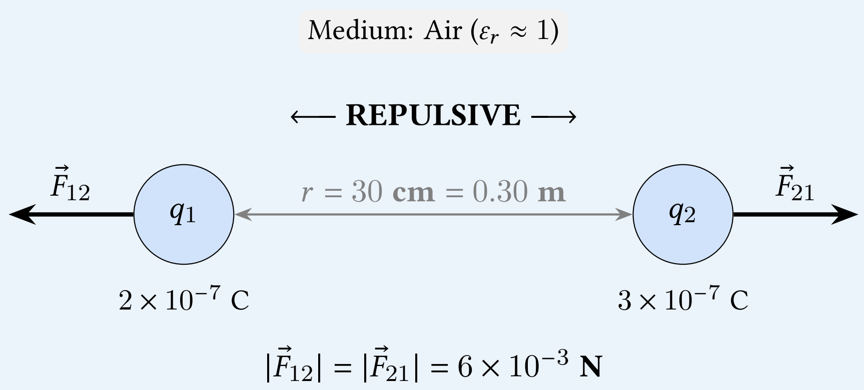
$$F = 9 \times 10^9 \times \frac{2}{3} \times 10^{-12}$$

$$F = 6 \times 10^{-3} \text{ N}$$

Final Answer:

$$\boxed{F = 6 \times 10^{-3} \text{ N} \quad \text{(Repulsive)}}$$

Visual Representation: Electrostatic Force Between Two Charges



Both charges are positive, so they repel each other with equal and opposite forces as per Newton's third law.

Expert's Solution — Dr. Rajesh Kumar, Ph.D. Physics, IIT Delhi

Key Points to Remember:

- **Coulomb's Law in Vector Form:**

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \cdot \hat{r}_{21}$$

  where $\hat{r}_{21}$ is the unit vector from q_2 to q_1.

- **Nature of Force:**
  - Like charges (++ or −−) → Repulsive
  - Unlike charges (+−) → Attractive

- **Superposition Principle:** If multiple charges are present, the net force on any charge is the vector sum of individual forces.

Did You Know?

The value of $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N m^{2}C^{-2} is valid only for vacuum (and approximately for air). In any other medium with dielectric constant ε_r, the force reduces by a factor of ε_r:

$$F_{\text{medium}} = \frac{F_{\text{vacuum}}}{\varepsilon_r}$$

Water ($\varepsilon_r \approx 80$) reduces electrostatic forces drastically — that's why ionic compounds dissolve easily in water!

Q2 The electrostatic force on a small sphere of charge $0.4\ \mu$C due to another small sphere of charge $-0.8\ \mu$C in air is 0.2 N.

(a) What is the distance between the two spheres?

(b) What is the force on the second sphere due to the first?

Solution

Given Data:

- Charge on first sphere, $q_1 = 0.4\ \mu\text{C} = 0.4 \times 10^{-6}\ \text{C} = 4 \times 10^{-7}\ \text{C}$

- Charge on second sphere, $q_2 = -0.8\ \mu C = -0.8 \times 10^{-6}\ C = -8 \times 10^{-7}\ C$
- Magnitude of electrostatic force, $F = 0.2\ N$
- Medium: Air ($\varepsilon_r \approx 1$)

Since the charges are of opposite signs (q_1 positive, q_2 negative), the force is **attractive**.

Part (a): Distance Between the Two Spheres

Concept: Using Coulomb's law:

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|q_1||q_2|}{r^2}$$

We take absolute values since we are solving for the magnitude of force.

Step 1: Rearrange to Find r^2

$$r^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|q_1||q_2|}{F}$$

Step 2: Substitute the Values

$$|q_1| = 4 \times 10^{-7}\ C, \quad |q_2| = 8 \times 10^{-7}\ C$$

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ N\ m^2C^{-2}$$

$$r^2 = 9 \times 10^9 \times \frac{(4 \times 10^{-7}) \times (8 \times 10^{-7})}{0.2}$$

Step 3: Simplify the Numerator

$$|q_1||q_2| = 4 \times 8 \times 10^{-14} = 32 \times 10^{-14}\ C^2$$

$$r^2 = 9 \times 10^9 \times \frac{32 \times 10^{-14}}{0.2}$$

$$r^2 = 9 \times 10^9 \times 160 \times 10^{-14}$$

$$r^2 = 1440 \times 10^{-5} = 144 \times 10^{-4}$$

$$r^2 = 144 \times 10^{-4}\ m^2$$

Step 4: Calculate r

$$r = \sqrt{144 \times 10^{-4}} = 12 \times 10^{-2}\ m$$

$$r = 0.12\ m = 12\ cm$$

Answer (a):

$$\boxed{r = 0.12\ m = 12\ cm}$$

Part (b): Force on the Second Sphere Due to the First

Concept: According to **Newton's Third Law of Motion**, the force exerted by the first sphere on the second sphere is equal in magnitude and opposite in direction to the force exerted by the second sphere on the first sphere.

Mathematically:

$$\vec{F}_{21} = -\vec{F}_{12}$$

Given that the force on the first sphere due to the second is 0.2 N (attractive, directed towards the second sphere), the force on the second sphere due to the first is:

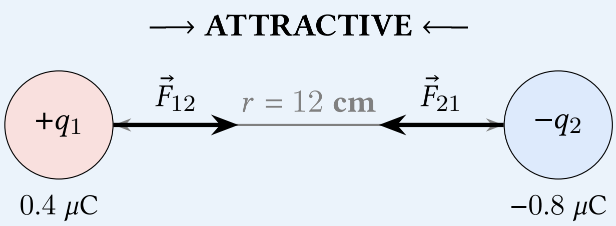
- **Magnitude:** 0.2 N (same as before)
- **Direction:** Towards the first sphere (attractive)

Answer (b):

$$\boxed{F_{21} = 0.2\text{ N} \quad \text{(Attractive, directed towards } q_1\text{)}}$$

Visual Representation: Attractive Force Between Opposite Charges

Newton's Third Law: $\vec{F}_{12} = -\vec{F}_{21}$



$$|\vec{F}_{12}| = |\vec{F}_{21}| = 0.2\ \mathbf{N}$$

Unlike charges attract each other. The forces are equal in magnitude and opposite in direction, obeying Newton's third law.

Expert's Solution — Dr. Anita Sharma, Ph.D. Physics, University of Delhi

Key Insights from This Problem:

- **Coulomb's Law is symmetric:** The magnitude of force depends on the product $|q_1||q_2|$, so swapping the charges gives the same magnitude. The force on each charge is equal in strength.
- **Vector Nature:** While both charges experience forces of equal magnitude, the directions are

opposite:

- q_1 (positive) is pulled towards q_2 (negative) $\rightarrow \vec{F}_{12}$ points right.
- q_2 (negative) is pulled towards q_1 (positive) $\rightarrow \vec{F}_{21}$ points left.

- **Common Mistake Alert:** Students often think the larger charge experiences a greater force. This is incorrect! Newton's third law guarantees equal forces. The difference lies in **acceleration** ($a = F/m$), not force.

Did You Know?

Quick Check Formula for Distance: When solving for r, you can use:

$$r = \sqrt{\frac{9 \times 10^9 \times |q_1||q_2|}{F}}$$

With practice, you can estimate: For charges in μC range with F in Newtons, r typically comes out in the cm to m range. Always convert to SI units first — μC to C is the most common source of error!

Q3 Check that the ratio $\dfrac{ke^2}{G\, m_e m_p}$ is dimensionless. Look up a Table of Physical Constants and determine the value of this ratio. What does the ratio signify?

Solution

Given Ratio:

$$\frac{ke^2}{G\, m_e\, m_p}$$

where the symbols have their usual meanings.

Part I: Dimensional Analysis — Checking if the Ratio is Dimensionless

Step 1: Identify the physical quantities and their dimensions.

Symbol	Physical Quantity	SI Unit	Dimensional Formula
k	Coulomb's constant $\left(\frac{1}{4\pi\varepsilon_0}\right)$	$\mathrm{N\,m^2C^{-2}}$	$[\mathrm{M\,L^3T^{-4}A^{-2}}]$
e	Elementary charge	C	$[\mathrm{A\,T}]$
G	Universal gravitational constant	$\mathrm{N\,m^2kg^{-2}}$	$[\mathrm{M^{-1}L^3T^{-2}}]$
m_e	Mass of electron	kg	$[\mathrm{M}]$
m_p	Mass of proton	kg	$[\mathrm{M}]$

Step 2: Write the dimensional formula for the numerator and denominator.
Numerator: ke^2

$$[k] = [\mathrm{M\,L^3T^{-4}A^{-2}}]$$

$$[e] = [\mathrm{A\,T}] \quad \Rightarrow \quad [e^2] = [\mathrm{A^2T^2}]$$

$$\text{Dimension of numerator} = [\mathrm{M\,L^3T^{-4}A^{-2}}] \times [\mathrm{A^2T^2}] = [\mathrm{M\,L^3T^{-2}}]$$

Denominator: $G\,m_e\,m_p$

$$[G] = [\mathrm{M^{-1}L^3T^{-2}}]$$

$$[m_e] = [\mathrm{M}], \quad [m_p] = [\mathrm{M}] \quad \Rightarrow \quad [m_e m_p] = [\mathrm{M^2}]$$

$$\text{Dimension of denominator} = [\mathrm{M^{-1}L^3T^{-2}}] \times [\mathrm{M^2}] = [\mathrm{M\,L^3T^{-2}}]$$

Step 3: Find the dimension of the ratio.

$$\left[\frac{ke^2}{G\,m_e m_p}\right] = \frac{[\mathrm{M\,L^3T^{-2}}]}{[\mathrm{M\,L^3T^{-2}}]} = [\mathrm{M^0L^0T^0A^0}] = \text{Dimensionless}$$

Conclusion: The ratio is indeed **dimensionless**.

Part II: Numerical Value of the Ratio

Step 1: Substitute the known values of physical constants.

Constant	Symbol	Value
Coulomb's constant	$k = \frac{1}{4\pi\varepsilon_0}$	$8.99 \times 10^9\ \mathrm{N\,m^2C^{-2}}$
Elementary charge	e	$1.602 \times 10^{-19}\ \mathrm{C}$
Gravitational constant	G	$6.674 \times 10^{-11}\ \mathrm{N\,m^2kg^{-2}}$
Mass of electron	m_e	$9.109 \times 10^{-31}\ \mathrm{kg}$
Mass of proton	m_p	$1.673 \times 10^{-27}\ \mathrm{kg}$

Step 2: Calculate ke^2 (Numerator).

$$ke^2 = (8.99 \times 10^9) \times (1.602 \times 10^{-19})^2$$

$$ke^2 = 8.99 \times 10^9 \times 2.566 \times 10^{-38}$$

$$ke^2 = 2.307 \times 10^{-28} \text{ N m}^2$$

Step 3: Calculate $G\, m_e m_p$ (Denominator).

$$G\, m_e m_p = (6.674 \times 10^{-11}) \times (9.109 \times 10^{-31}) \times (1.673 \times 10^{-27})$$

$$G\, m_e m_p = 6.674 \times 10^{-11} \times 1.524 \times 10^{-57}$$

$$G\, m_e m_p = 1.017 \times 10^{-67} \text{ N m}^2$$

Step 4: Compute the ratio.

$$\frac{ke^2}{G\, m_e m_p} = \frac{2.307 \times 10^{-28}}{1.017 \times 10^{-67}}$$

$$\frac{ke^2}{G\, m_e m_p} = 2.27 \times 10^{39}$$

Final Value:

$$\boxed{\frac{ke^2}{G\, m_e m_p} \approx 2.27 \times 10^{39}}$$

Part III: Physical Significance of the Ratio

This ratio represents the **relative strength of the electrostatic force to the gravitational force** between an electron and a proton.

Interpretation:
Consider an electron and a proton separated by a distance r:

- **Electrostatic force** between them: $F_e = \dfrac{ke^2}{r^2}$
- **Gravitational force** between them: $F_g = \dfrac{G\, m_e m_p}{r^2}$

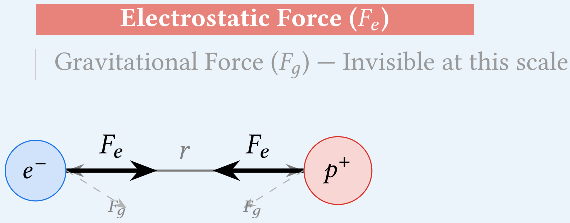
$$\frac{F_e}{F_g} = \frac{ke^2/r^2}{G\, m_e m_p/r^2} = \frac{ke^2}{G\, m_e m_p} \approx 2.27 \times 10^{39}$$

Conclusion: The electrostatic force between an electron and a proton is approximately 10^{39} **times stronger** than the gravitational force between them! This explains why:

- Atomic and molecular structures are governed by electrostatic forces.
- Gravitational forces are negligible at the atomic and subatomic scales.

- Despite being much weaker, gravity dominates at astronomical scales because large bodies are electrically neutral overall.

Visual Representation: Comparison of Forces



$$\frac{F_e}{F_g} \approx 2.27 \times 10^{39}$$

Electrostatic force is 10^{39} times stronger!

The diagram illustrates the enormous disparity between electrostatic and gravitational forces at the atomic scale.

Expert's Solution — Dr. Suresh Iyer, Ph.D. Theoretical Physics, TIFR Mumbai

Why This Ratio Matters in Physics:

- This dimensionless number ($\sim 10^{39}$) is one of the **fundamental large numbers** in physics. Dirac first pointed out its significance in his Large Numbers Hypothesis.
- **Connection to Cosmology:** Interestingly, the ratio of the radius of the observable universe to the classical electron radius is also $\sim 10^{40}$, hinting at deep connections between micro-physics and cosmology.
- **Teaching Point:** This problem beautifully illustrates **dimensional analysis** as a verification tool. If the dimensions don't cancel, you've made an error somewhere!

★ Did You Know?

Quick Memory Trick: The ratio $ke^2/G\, m_e m_p \approx 10^{39}$ means that if the electrostatic force between an electron and proton were the size of a mountain ($\sim 10^4$ m), the gravitational force would be smaller than an atomic nucleus ($\sim 10^{-14}$ m). That's how incredibly weak gravity is at small scales!

Q4

(a) Explain the meaning of the statement 'electric charge of a body is quantised'.

(b) Why can one ignore quantisation of electric charge when dealing with macroscopic i.e., large scale charges?

Solution

Part (a): Meaning of Quantisation of Electric Charge

Statement: "Electric charge of a body is quantised."

Meaning: The quantisation of electric charge means that the total charge on any body is always an **integral multiple** of a fundamental unit of charge, denoted by e. In other words, electric charge cannot take any arbitrary value; it exists only in discrete packets or quanta.

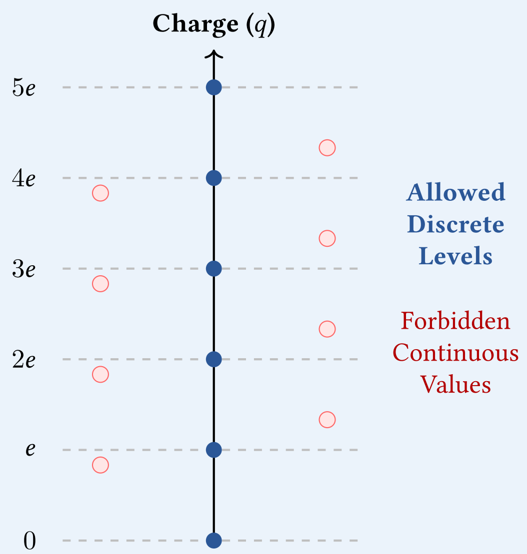
Mathematically, if q is the total charge on a body, then:

$$q = \pm ne$$

where:

- $n = 1, 2, 3, \ldots$ (positive integer)
- $e = 1.602 \times 10^{-19}$ C (elementary charge — the charge of one proton or the magnitude of charge of one electron)

Visualising Charge Quantisation:



Charge exists only in discrete packets of e

$q = \pm ne \quad (n = 1, 2, 3, \ldots)$

The diagram shows that charge can only take discrete values ($0, \pm e, \pm 2e, \pm 3e, \ldots$). Values between these allowed levels (shown as red dots) do not exist in nature.

Key Points:

- The fundamental unit e is the smallest free charge observed in nature.
- Quarks have fractional charges ($\pm e/3$ or $\pm 2e/3$), but they are never observed in isolation (quark confinement). Hence, for all observable free charges, the smallest unit is e.
- This quantisation was first established experimentally by **R.A. Millikan** through his famous **oil drop experiment** (1909).

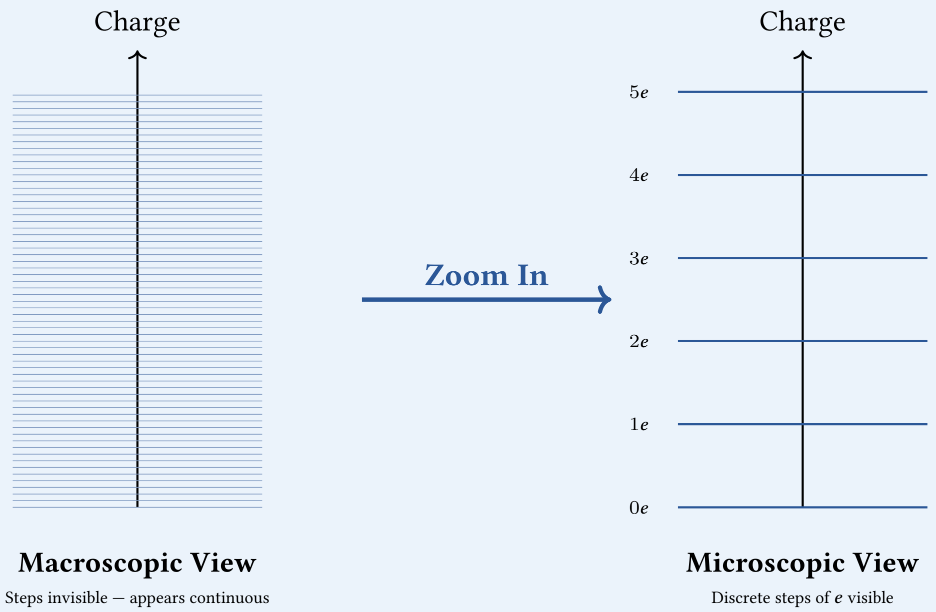
Part (b): Why Quantisation Can Be Ignored at Macroscopic Scales

Answer:
At the macroscopic level, the charges involved are enormously large compared to the elementary charge e.

Reasoning:

- In everyday charged objects, the total charge q is typically of the order of microcoulombs (μC), i.e., 10^{-6} C.
- The elementary charge $e = 1.602 \times 10^{-19}$ C is extremely small.
- The number of elementary charges (n) that make up a macroscopic charge is astronomically large.

Quantisation at Different Scales:



Example Calculation: Consider a body with charge $q = 1\ \mu C = 10^{-6}$ C.

$$n = \frac{q}{e} = \frac{10^{-6}}{1.602 \times 10^{-19}} \approx 6.24 \times 10^{12}$$

This means the charge is made up of approximately **6.24 trillion** elementary charges!

Conclusion:

- When n is so large ($\sim 10^{12}$ or more), adding or removing one e changes the total charge by only ~ 1 part in 10^{12}.
- Such an infinitesimally small change is **undetectable** by any macroscopic measuring instrument.
- Therefore, charge at the macroscopic level can be treated as a **continuous quantity** (analogous to treating a liquid as continuous despite being made of discrete molecules).

Final Answer:

Quantisation of charge is ignored at macroscopic scales because $n = q/e \sim 10^{12}$ or larger, making discrete steps practically undetectable.

This is similar to how we treat distance as continuous in our daily lives even though it is quantised at the Planck scale ($\sim 10^{-35}$ m).

Expert's Solution — Prof. Meera Nair, Ph.D. Experimental Physics, IISc Bangalore

Deeper Understanding of Charge Quantisation:

- **Millikan's Oil Drop Experiment:** Millikan observed that the charge on oil droplets was always an integral multiple of 1.6×10^{-19} C. This was the first direct evidence of charge quantisation and earned him the Nobel Prize in Physics (1923).
- **Quarks and Fractional Charges:** Protons and neutrons are made of quarks with charges $\pm e/3$ and $\pm 2e/3$. However, quarks are confined within hadrons — they cannot be isolated. Hence, **all observable free charges are integral multiples of** e.
- **Analogy for Students:** Think of charge like money in a country where the smallest currency note is ₹1. You can have ₹1, ₹2, ₹100, but never ₹1.50. Similarly, charge comes only in multiples of e.

★ **Did You Know?**

When to Consider Quantisation vs When to Ignore It:

Scale	Typical n	Quantisation?
Few electrons	1–100	**Must consider**
Nanocoulombs (nC)	$\sim 10^{10}$	Negligible effect
Microcoulombs (μC)	$\sim 10^{13}$	**Ignore**
Coulombs (C)	$\sim 10^{19}$	Completely continuous

Q5 When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge.

Solution

Concept: Charging by Friction (Triboelectric Effect)
When two different materials are rubbed together, one loses electrons and the other gains electrons. This is called **charging by friction** or the **triboelectric effect**. No new charges are created — electrons are simply transferred from one body to the other.

The Observation:

- When a glass rod is rubbed with silk, the **glass rod becomes positively charged**.
- The **silk cloth becomes negatively charged**.
- The *magnitudes* of positive and negative charges produced are exactly **equal**.
- Similar behaviour is seen with ebonite & fur, plastic & wool, etc.

Explanation — Law of Conservation of Charge:
The **Law of Conservation of Charge** states:

The total electric charge of an isolated system remains constant.

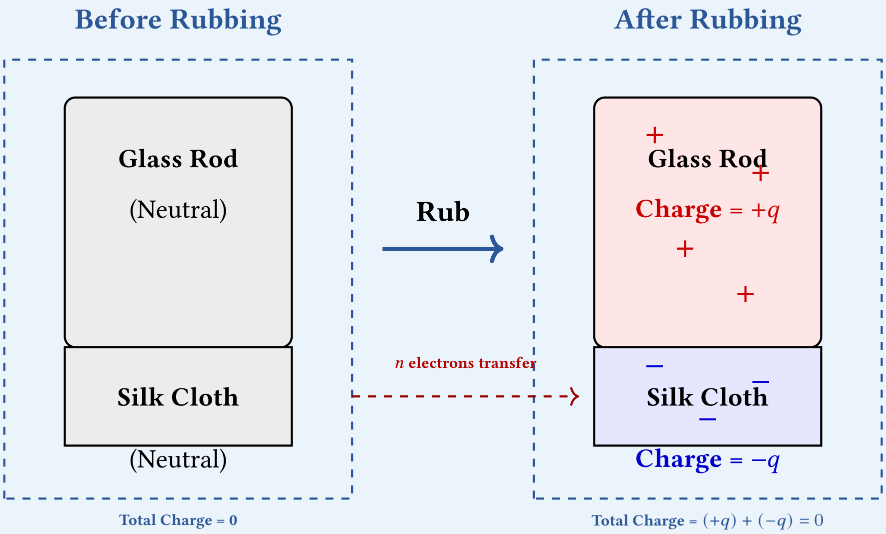
Charge can neither be created nor destroyed; it can only be **transferred** from one body to another.

How Rubbing Demonstrates This Law:

1. **Before Rubbing:**
   - Both glass rod and silk cloth are electrically neutral.
   - Total charge of the system = 0.
2. **During Rubbing:**
   - Due to difference in electron affinity, **electrons transfer** from glass to silk.
   - Glass loses n electrons $\longrightarrow$ acquires charge $+ne$.
   - Silk gains those same n electrons $\longrightarrow$ acquires charge $-ne$.
3. **After Rubbing:**
   - Charge on glass $= +q \quad (q = ne)$
   - Charge on silk $= -q$
   - Total charge $= (+q) + (-q) = 0$ — **unchanged!**

Conclusion: The appearance of equal and opposite charges on the two bodies is **direct evidence** for charge conservation. No charge is created or destroyed — only **separated**.

Visual Representation:



Before rubbing, total charge is 0. After rubbing, glass becomes +q and silk becomes −q. The net charge of the isolated system remains 0, verifying the law of conservation of charge.

Final Answer:

Equal and opposite charges appear — charge is neither created nor destroyed, only transferred.

Expert's Solution — Dr. Vikram Rao, Ph.D. Condensed Matter Physics, IIT Bombay

The Triboelectric Series — Predicting Charge Transfer:

The **triboelectric series** ranks materials by their tendency to gain or lose electrons:

Loses Electrons (becomes +ve)	Gains Electrons (becomes −ve)
Rabbit Fur	Teflon
Glass	Silicon
Human Hair	PVC
Nylon	**Silk**
Wool	**Ebonite**
Lead	Polythene

- **Glass** is higher than **Silk** → Glass loses electrons, Silk gains them.
- The **farther apart** in the series, the **greater** the charge transfer.

Did You Know?

Quick Mnemonic: "Glass Gives, Silk Steals!"

- Glass + Silk → Glass (+), Silk (−)
- Ebonite + Fur → Ebonite (−), Fur (+)

Always remember: Only **electrons move** — protons stay inside the nucleus!

Q6 **Four point charges $q_A = 2\ \mu$C, $q_B = -5\ \mu$C, $q_C = 2\ \mu$C, and $q_D = -5\ \mu$C are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of $1\ \mu$C placed at the centre of the square?**

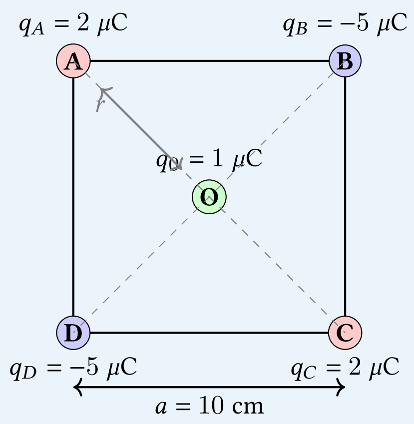
Solution

Given Data:

- Charge at corner A: $q_A = 2\ \mu\text{C} = 2 \times 10^{-6}$ C
- Charge at corner B: $q_B = -5\ \mu\text{C} = -5 \times 10^{-6}$ C
- Charge at corner C: $q_C = 2\ \mu\text{C} = 2 \times 10^{-6}$ C

- Charge at corner D: $q_D = -5\ \mu\text{C} = -5 \times 10^{-6}$ C
- Charge at centre O: $q_0 = 1\ \mu\text{C} = 1 \times 10^{-6}$ C
- Side of square: $a = 10$ cm $= 0.10$ m

Step 1: Geometry — Distance from Centre to Each Corner



The distance from the centre O to any corner of the square is half the length of the diagonal:

$$\text{Diagonal} = a\sqrt{2} = 0.10\sqrt{2}\ \text{m}$$

$$r = \text{OA} = \text{OB} = \text{OC} = \text{OD} = \frac{a\sqrt{2}}{2} = \frac{0.10\sqrt{2}}{2} = 0.05\sqrt{2}\ \text{m}$$

$$r^2 = (0.05\sqrt{2})^2 = 0.0025 \times 2 = 0.005\ \text{m}^2 = 5 \times 10^{-3}\ \text{m}^2$$

Step 2: Calculate Individual Forces

Using Coulomb's law: $F = \dfrac{k|q_1 q_2|}{r^2}$, where $k = 9 \times 10^9$ N m^{2}C^{-2}.

Force due to q_A (attractive as signs are same: + and +? No — $q_A = +2\ \mu$C, $q_0 = +1\ \mu$C — REPULSIVE, directed away from A):

$$F_A = \frac{9 \times 10^9 \times (2 \times 10^{-6}) \times (1 \times 10^{-6})}{5 \times 10^{-3}}$$

$$F_A = \frac{9 \times 10^9 \times 2 \times 10^{-12}}{5 \times 10^{-3}} = \frac{18 \times 10^{-3}}{5 \times 10^{-3}} = 3.6\ \text{N}$$

Direction: Along OA, away from A (down-right).

Force due to q_C ($q_C = +2\ \mu$C, $q_0 = +1\ \mu$C — REPULSIVE):

$$F_C = 3.6\ \text{N}$$

Direction: Along OC, away from C (up-left).

Notice: F_A and F_C are **equal in magnitude and opposite in direction**. They cancel each other!

$$\vec{F}_A + \vec{F}_C = \vec{0}$$

Force due to q_B ($q_B = -5\ \mu$C, $q_0 = +1\ \mu$C — ATTRACTIVE, directed towards B):

$$F_B = \frac{9 \times 10^9 \times (5 \times 10^{-6}) \times (1 \times 10^{-6})}{5 \times 10^{-3}}$$

$$F_B = \frac{9 \times 10^9 \times 5 \times 10^{-12}}{5 \times 10^{-3}} = \frac{45 \times 10^{-3}}{5 \times 10^{-3}} = 9.0 \text{ N}$$

Direction: Along BO, towards B (up-right).

Force due to q_D ($q_D = -5\ \mu$C, $q_0 = +1\ \mu$C — ATTRACTIVE):

$$F_D = 9.0 \text{ N}$$

Direction: Along DO, towards D (down-left).

Notice: F_B and F_D are **equal in magnitude and opposite in direction**. They also cancel each other!

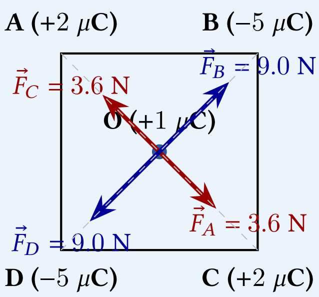
$$\vec{F}_B + \vec{F}_D = \vec{0}$$

Step 3: Net Force

$$\vec{F}_{\text{net}} = \vec{F}_A + \vec{F}_B + \vec{F}_C + \vec{F}_D$$

$$\vec{F}_{\text{net}} = (\vec{F}_A + \vec{F}_C) + (\vec{F}_B + \vec{F}_D) = \vec{0} + \vec{0} = \vec{0}$$

Visual Representation: Force Vectors at Centre



Net Force = 0

The four forces pair up along the two diagonals. Each pair consists of equal and opposite forces, resulting in perfect cancellation.

Final Answer:

$$\boxed{\vec{F}_{\text{net}} = 0 \text{ N}}$$

Why This Happens: The symmetric arrangement of charges causes pairwise cancellation:

- Equal positive charges at A and C → equal repulsive forces in opposite directions → cancel.
- Equal negative charges at B and D → equal attractive forces in opposite directions → cancel.

Expert's Solution — Dr. Ananya Gupta, Ph.D. Electrodynamics, NIT Kurukshetra

Key Insight — Symmetry Simplifies Problems:
This problem is a beautiful example of how **symmetry** can reduce complex calculations to almost zero work:

- The square has **reflection symmetry** across both diagonals.
- Positive charges at A and C are at **opposite ends of a diagonal** — their forces on the centre cancel.
- Negative charges at B and D are also at opposite ends — their forces also cancel.
- **General Rule:** In any symmetric arrangement, if charges are equal in magnitude and sign at opposite points, their forces on the centre cancel pairwise.

What If Charges Were Different? If $q_A \neq q_C$ or $q_B \neq q_D$, the cancellation wouldn't happen, and we'd need to compute the vector sum. The net force would then point along some direction determined by the imbalance.

Did You Know?

Pro Tip: Before diving into calculations, always **sketch the configuration** and check for symmetry. Many competitive exam problems (JEE, NEET) have zero net force at the centre due to symmetric charge arrangements. This saves precious time!
Also note: The distance of each corner from the centre is $a/\sqrt{2}$, not $a\sqrt{2}$. A common mistake is confusing the full diagonal with the half-diagonal.

Q7

(a) An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not?

(b) Explain why two field lines never cross each other at any point?

Solution

Part (a): Why Electric Field Lines Are Continuous

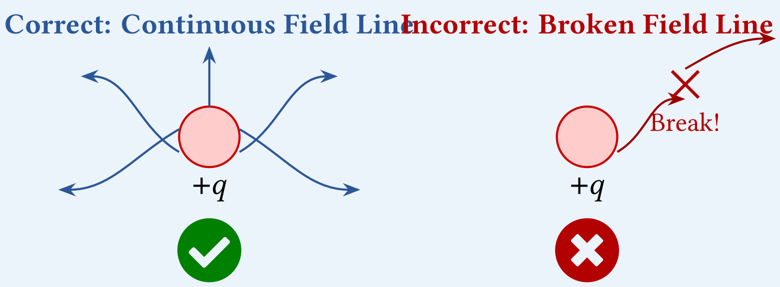
Statement: An electrostatic field line is a continuous curve — it cannot have sudden breaks.

Reason:
A field line represents the path along which a small positive test charge would move if placed in the electric field. At every point on the field line, the tangent gives the direction of the electric field $\vec{E}$.

- The electric field $\vec{E}$ is **defined at every point** in space (except at point charges themselves).
- Since $\vec{E}$ exists **everywhere** in the region, a field line must extend continuously through space.
- A sudden break would imply that $\vec{E}$ suddenly **vanishes** at some point and reappears elsewhere — which is not possible for an electrostatic field.
- The only places where field lines **start** or **end** are on charges (positive charges for start, negative charges for end).

Physical Explanation:
If a field line had a break, it would mean that at some point P, there is no electric field, but at an infinitesimally close point Q, there is a field. This would require $\vec{E}$ to be **discontinuous**, which contradicts the fact that the electric field of point charges varies smoothly ($\propto 1/r^2$) — there are no abrupt jumps.

Visualising Field Line Continuity:



Field lines must be smooth, continuous curves starting at positive charges and ending at negative charges.

Part (b): Why Two Field Lines Never Cross Each Other

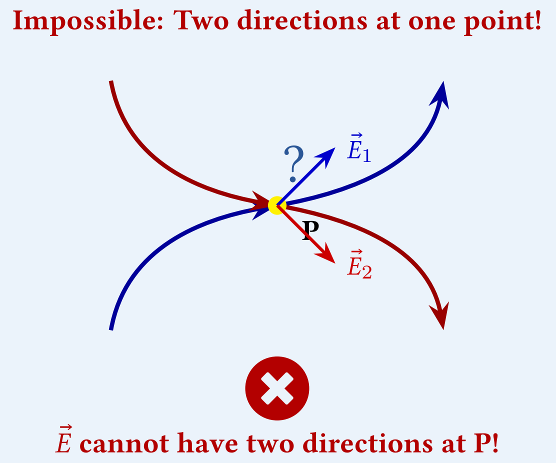
Statement: Two electric field lines can never intersect or cross each other at any point.

Reason — Uniqueness of Electric Field Direction:
At any given point in space, the electric field $\vec{E}$ has a **unique direction**. It cannot point in two different directions simultaneously.

- If two field lines crossed at a point P, then at that point there would be **two tangents** to the two lines.
- Each tangent represents the direction of $\vec{E}$ at P.

- This would mean $\vec{E}$ has **two different directions** at the same point — which is impossible.
- Therefore, field lines **cannot** cross.

Visualising Why Field Lines Cannot Cross:



If field lines crossed, the electric field at the intersection would have two different directions — a physical impossibility.

Additional Notes:

- Field lines **can** appear to merge at a point where $\vec{E} = 0$ (null point), but even there, they don't actually cross — the field simply vanishes.
- In the case of a **null point** (like between two equal positive charges), the field lines approach the point asymptotically but do not cross.
- The density of field lines represents the **strength** of the electric field — closer lines mean stronger field.

Final Answer:

(a) Field lines are continuous because $\vec{E}$ exists everywhere — no sudden breaks.

(b) Field lines never cross because $\vec{E}$ has a unique direction at every point.

Expert's Solution — Dr. Priya Sharma, Ph.D. Physics, St. Stephen's College, Delhi University

Properties of Electric Field Lines — Complete Summary:

1. **Origin and Termination:** Field lines start from positive charges and end at negative charges (or extend to infinity if no negative charge is present).

2. **Continuity:** Field lines are continuous curves without any sudden breaks (except at charges where they start/end).
3. **Non-intersection:** Two field lines never intersect. If they did, the electric field would have two directions at the intersection point.
4. **Tangency:** The tangent to a field line at any point gives the direction of $\vec{E}$ at that point.
5. **Density $\propto$ Field Strength:** Where field lines are closer, the electric field is stronger. In a uniform field, lines are parallel and equally spaced.
6. **No Closed Loops:** Electrostatic field lines do not form closed loops (unlike magnetic field lines). They always go from higher potential to lower potential.

★ **Did You Know?**

Think of Field Lines Like River Streams:
Imagine electric field lines as water flowing down a hill:

- Water flows **continuously** — no sudden jumps (analogy for part a).
- Two streams can merge, but at any point, water flows in **only one direction** — it never flows in two directions at once (analogy for part b).

This analogy also helps remember: Field lines go from **higher** potential (positive charge) to **lower** potential (negative charge), just like water flows downhill!

Q8 Two point charges $q_A = 3\ \mu$C and $q_B = -3\ \mu$C are located 20 cm apart in vacuum.

(a) What is the electric field at the midpoint O of the line AB joining the two charges?

(b) If a negative test charge of magnitude 1.5×10^{-9} C is placed at this point, what is the force experienced by the test charge?

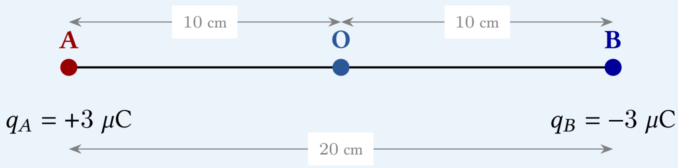
Solution

Given Data:

- Charge at A: $q_A = 3\ \mu\text{C} = 3 \times 10^{-6}$ C
- Charge at B: $q_B = -3\ \mu\text{C} = -3 \times 10^{-6}$ C

- Distance between A and B: $AB = 20$ cm $= 0.20$ m
- Midpoint O: AO = OB = $\frac{0.20}{2} = 0.10$ m
- Test charge (part b): $q_0 = -1.5 \times 10^{-9}$ C

Configuration:



Part (a): Electric Field at Midpoint O

Concept: The electric field at a point due to a point charge is:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \cdot \hat{r}$$

where $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N m^{2}C^{-2}.
The direction of $\vec{E}$ is:

- **Away** from the charge if q is positive.
- **Towards** the charge if q is negative.

Step 1: Electric Field at O due to q_A ($\vec{E}_A$)
Magnitude:

$$E_A = \frac{k|q_A|}{(\text{AO})^2} = \frac{9 \times 10^9 \times 3 \times 10^{-6}}{(0.10)^2}$$

$$E_A = \frac{27 \times 10^3}{0.01} = 27 \times 10^5 = 2.7 \times 10^6 \text{ N C}^{-1}$$

Direction: q_A is positive, so $\vec{E}_A$ points **away** from A, i.e., from A towards O towards B — **Rightwards** ($\longrightarrow$).

Step 2: Electric Field at O due to q_B ($\vec{E}_B$)
Magnitude:

$$E_B = \frac{k|q_B|}{(\text{OB})^2} = \frac{9 \times 10^9 \times 3 \times 10^{-6}}{(0.10)^2}$$

$$E_B = 2.7 \times 10^6 \text{ N C}^{-1}$$

Direction: q_B is negative, so $\vec{E}_B$ points **towards** B, i.e., from O towards B — **Rightwards** ($\longrightarrow$).

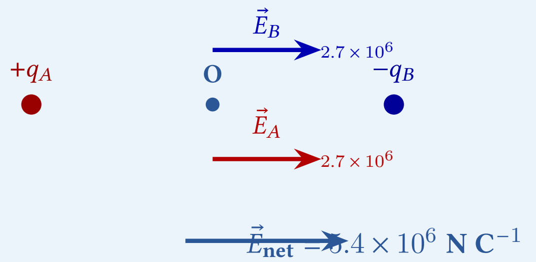
Step 3: Net Electric Field at O
Both $\vec{E}_A$ and $\vec{E}_B$ point in the **same direction** (towards B). Therefore, they add up:

$$E_{\text{net}} = E_A + E_B = 2.7 \times 10^6 + 2.7 \times 10^6$$

$$E_{\text{net}} = 5.4 \times 10^6 \text{ N C}^{-1}$$

Direction: Along the line AB, pointing from A towards B (Rightwards $\longrightarrow$).

Electric Field Vectors at Midpoint:



$\longrightarrow$ **Both fields add in the same direction**

Answer (a):

$$\boxed{\vec{E}_{\text{net}} = 5.4 \times 10^6 \text{ N C}^{-1} \quad \text{(from A towards B)}}$$

Part (b): Force on a Negative Test Charge Placed at O

Concept: The force on a charge q_0 placed in an electric field $\vec{E}$ is:

$$\vec{F} = q_0\vec{E}$$

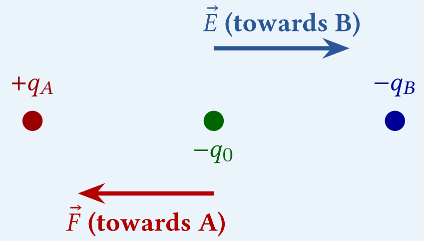
Step 1: Substitute Values

$$q_0 = -1.5 \times 10^{-9} \text{ C}$$
$$E_{\text{net}} = 5.4 \times 10^6 \text{ N C}^{-1} \quad \text{(towards B)}$$
$$F = |q_0 E_{\text{net}}| = (1.5 \times 10^{-9}) \times (5.4 \times 10^6)$$
$$F = 8.1 \times 10^{-3} \text{ N} = 0.0081 \text{ N}$$

Step 2: Direction of Force
Since q_0 is **negative**, the force $\vec{F} = q_0\vec{E}$ acts in the direction **opposite** to $\vec{E}$.

- $\vec{E}_{\text{net}}$ points from A towards B (rightwards $\longrightarrow$).
- Therefore, $\vec{F}$ points from B towards A (leftwards $\longleftarrow$), i.e., **towards the positive charge** q_A.

Force on the Test Charge:



$\vec{F} = q_0\vec{E}$

Since q_0 is negative, $\vec{F} \uparrow\downarrow \vec{E}$

Answer (b):

$$\boxed{\vec{F} = 8.1 \times 10^{-3}\ \text{N} \quad \text{(directed from O towards A, i.e., towards the positive charge)}}$$

Summary:

- At the midpoint of an electric dipole, the fields due to both charges **add constructively**.
- The electric field points from the positive charge towards the negative charge.
- A negative test charge experiences a force **opposite** to the field direction, i.e., towards the positive charge.

Expert's Solution — Dr. Arjun Mehta, Ph.D. Electrostatics, IIT Roorkee

Key Insight — Electric Field of a Dipole at Midpoint:
The arrangement of $+q$ and $-q$ separated by distance $2a$ forms an **electric dipole**. The midpoint O is on the **axial line** of the dipole.

- **Field at midpoint (axial):**
$$E = \frac{2kp}{r^3} \quad \text{(for } r \gg a\text{)}$$
But here $r = a$, so we use exact calculation as done above.
- **Important Observation:** At the midpoint of a dipole, both fields point in the **same direction** — from $+q$ to $-q$. This is why they add up, unlike the equatorial point where they partially cancel.
- **Direction Rule:**
  - $\vec{E}$ always points from positive to negative.
  - $\vec{F} = q\vec{E}$ — for negative q, force is opposite to field.

Did You Know?

Quick Check — Does Your Answer Make Sense?

- The negative test charge is **attracted** to the positive charge q_A and **repelled** by the negative charge q_B.
- Both effects push it **towards A**. So the force should point towards A — which matches our answer!

This physical intuition is faster than vector math. Always do a quick intuition check after solving.

Q9 **A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A: (0, 0, -15 cm) and B: (0, 0, $+15$ cm), respectively. What are the total charge and electric dipole moment of the system?**

Solution

Given Data:

- Charge at A: $q_A = 2.5 \times 10^{-7}$ C
- Charge at B: $q_B = -2.5 \times 10^{-7}$ C
- Position of A: $\vec{r}_A = (0, 0, -15 \text{ cm}) = (0, 0, -0.15 \text{ m})$
- Position of B: $\vec{r}_B = (0, 0, +15 \text{ cm}) = (0, 0, +0.15 \text{ m})$

Part I: Total Charge of the System

The total charge Q of a system is the algebraic sum of all individual charges:

$$Q = q_A + q_B$$

$$Q = 2.5 \times 10^{-7} + (-2.5 \times 10^{-7}) = 0$$

Answer:

$$\boxed{Q = 0 \text{ C}}$$

The system is **electrically neutral** — it consists of equal and opposite charges.

Part II: Electric Dipole Moment

Concept: An electric dipole consists of two equal and opposite charges $\pm q$ separated by a distance $2a$. The **electric dipole moment** $\vec{p}$ is a vector quantity defined as:

$$\vec{p} = q \times 2\vec{a}$$

where $2\vec{a}$ is the vector from the **negative charge** to the **positive charge**.

Alternatively:

$$\vec{p} = \sum_i q_i \vec{r}_i$$

(taking the origin as reference, valid for neutral systems).

Step 1: Determine the Displacement Vector from $-q$ to $+q$

Negative charge q_B is at B: $(0, 0, +0.15\text{ m})$.
Positive charge q_A is at A: $(0, 0, -0.15\text{ m})$.
The vector from the negative charge (B) to the positive charge (A) is:

$$2\vec{a} = \vec{r}_A - \vec{r}_B = (0, 0, -0.15) - (0, 0, +0.15)$$

$$2\vec{a} = (0, 0, -0.30)\text{ m}$$

The magnitude of separation is:

$$2a = |2\vec{a}| = 0.30\text{ m} = 30\text{ cm}$$

Step 2: Calculate Dipole Moment

$$\vec{p} = q \times 2\vec{a}$$

where $q = |q_A| = |q_B| = 2.5 \times 10^{-7}$ C (magnitude of either charge).

$$\vec{p} = 2.5 \times 10^{-7} \times (0, 0, -0.30)$$
$$\vec{p} = (0, 0, -7.5 \times 10^{-8})\text{ C m}$$

Magnitude:

$$p = 7.5 \times 10^{-8}\text{ C m}$$

Alternative Method — Using $\vec{p} = \sum q_i \vec{r}_i$:

$$\vec{p} = q_A \vec{r}_A + q_B \vec{r}_B$$

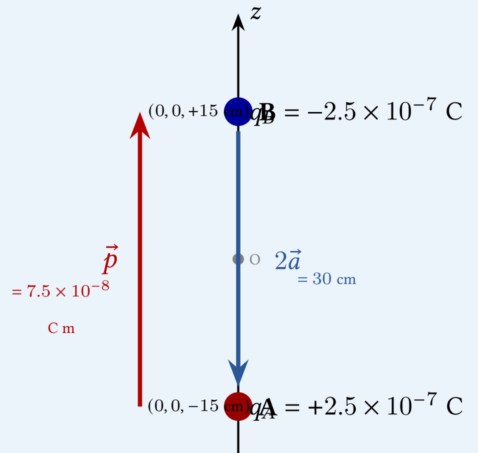
$$\vec{p} = (2.5 \times 10^{-7})(0, 0, -0.15) + (-2.5 \times 10^{-7})(0, 0, +0.15)$$
$$\vec{p} = (0, 0, -3.75 \times 10^{-8}) + (0, 0, -3.75 \times 10^{-8})$$
$$\vec{p} = (0, 0, -7.5 \times 10^{-8})\text{ C m}$$

Both methods give the same result.

Answer:

$$\boxed{\vec{p} = -7.5 \times 10^{-8}\,\hat{k}\text{ C m} \quad \text{or} \quad p = 7.5 \times 10^{-8}\text{ C m (along } -z\text{ direction)}}$$

Visual Representation: Electric Dipole



Dipole moment $\vec{p}$ points from $-q$ to $+q$ (along $-z$ direction)

The electric dipole moment vector points from the negative charge towards the positive charge.

Expert's Solution — Dr. Kavya Iyer, Ph.D. Electromagnetic Theory, IISc Bangalore

Dipole Moment — Key Concepts and Tricks:

Trick 1: Total Charge First! Always check the total charge first. If $Q_{\text{total}} \neq 0$, the dipole moment depends on the choice of origin. But if $Q_{\text{total}} = 0$ (as here), the dipole moment is **independent of origin** — use any method freely!

Trick 2: Quick Formula for Two-Charge Dipole For two equal and opposite charges $\pm q$:

$$\vec{p} = q \times (\vec{r}_+ - \vec{r}_-)$$

where $\vec{r}_+$ is the position of $+q$ and $\vec{r}_-$ is the position of $-q$.
Just remember: $\vec{p}$ **always points from negative to positive!**

Trick 3: Check Units Dipole moment unit: C m. Here:

$$q \sim 10^{-7}\text{ C}, \quad \text{distance} \sim 10^{-1}\text{ m} \quad \Rightarrow \quad p \sim 10^{-8}\text{ C m}$$

This quick estimate catches calculation errors.

Did You Know?

Memorise This Forever:

$$\boxed{\vec{p} = q \times (\vec{r}_+ - \vec{r}_-)}$$

$$\boxed{\vec{p}\ \textbf{ALWAYS goes from } - \textbf{ to } +}$$

Think: The arrow of $\vec{p}$ is like an arrow pointing to the "positive" side of the dipole — just like electric field lines go from + to −, but $\vec{p}$ goes the other way (from − to +) by convention!

Q10 **An electric dipole with dipole moment 4×10^{-9} C m is aligned at $30°$ with the direction of a uniform electric field of magnitude 5×10^4 N C^{-1}. Calculate the magnitude of the torque acting on the dipole.**

Solution

Given Data:

- Dipole moment, $p = 4 \times 10^{-9}$ C m
- Electric field magnitude, $E = 5 \times 10^4$ N C^{-1}
- Angle between $\vec{p}$ and $\vec{E}$: $\theta = 30°$

Concept: Torque on an Electric Dipole in a Uniform Field

When an electric dipole is placed in a uniform electric field, the forces on the two charges are **equal and opposite** ($+q\vec{E}$ and $-q\vec{E}$). These two forces form a **couple**, producing a net torque that tends to align the dipole with the field.

The torque $\vec{\tau}$ on a dipole in a uniform electric field is given by:

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

The **magnitude** of torque is:

$$\boxed{\tau = pE \sin\theta}$$

where θ is the angle between $\vec{p}$ and $\vec{E}$.

Step 1: Substitute the Values

$$\tau = pE \sin\theta$$

$$\tau = (4 \times 10^{-9}) \times (5 \times 10^4) \times \sin 30°$$

Step 2: Simplify

$$pE = 4 \times 10^{-9} \times 5 \times 10^{4} = 20 \times 10^{-5} = 2 \times 10^{-4}$$

$$\sin 30^\circ = \frac{1}{2}$$

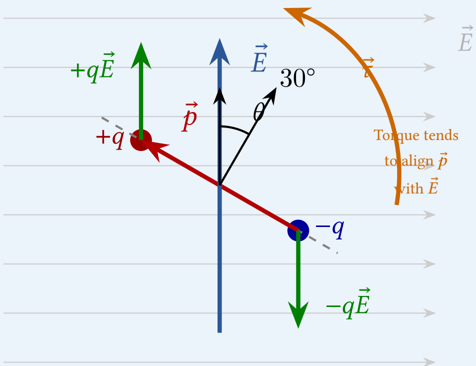
$$\tau = 2 \times 10^{-4} \times \frac{1}{2}$$

$$\tau = 1 \times 10^{-4} \text{ N m}$$

Final Answer:

$$\boxed{\tau = 10^{-4} \text{ N m}}$$

Visual Representation: Dipole in Uniform Electric Field



$\tau = pE \sin\theta = 10^{-4}$ **N m**

The dipole experiences a torque that tends to rotate it into alignment with the electric field. The torque is maximum at $\theta = 90^\circ$ and zero at $\theta = 0^\circ$.

Important Notes:

- Torque $\vec{\tau} = \vec{p} \times \vec{E}$ — its direction is given by the **right-hand rule** (perpendicular to both $\vec{p}$ and $\vec{E}$).
- The torque tends to **align the dipole** with the electric field (minimum potential energy position).
- At $\theta = 0^\circ$ (dipole aligned with field): $\tau = 0$ (stable equilibrium).
- At $\theta = 90^\circ$ (dipole perpendicular to field): $\tau = pE$ (maximum).
- At $\theta = 180^\circ$ (dipole anti-aligned): $\tau = 0$ (unstable equilibrium).

Expert's Solution — Dr. Rajesh Menon, Ph.D. Physics, University of Hyderabad

Torque on a Dipole — Quick Problem-Solving Tricks:

Trick 1: Direct Formula There's only one formula to remember:

$$\tau = pE \sin\theta$$

Make sure θ is the angle between $\vec{p}$ and $\vec{E}$, not between the dipole axis and some other direction.

Trick 2: Special Angles

θ	$\sin\theta$	τ
$0°$	0	0 (stable equilibrium)
$30°$	$\frac{1}{2}$	$\frac{pE}{2}$
$45°$	$\frac{1}{\sqrt{2}}$	$\frac{pE}{\sqrt{2}}$
$60°$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}pE$
$90°$	1	pE (maximum)
$180°$	0	0 (unstable equilibrium)

Trick 3: Physical Intuition

- If $\vec{p}$ is **aligned** with $\vec{E}$ $\longrightarrow$ no torque (happiest state).
- If $\vec{p}$ is **perpendicular** to $\vec{E}$ $\longrightarrow$ maximum torque (most stressed).
- Torque always tries to **reduce** θ (rotate $\vec{p}$ towards $\vec{E}$).

★ **Did You Know?**

Right-Hand Rule for Torque Direction:

Point fingers of right hand in direction of $\vec{p}$,
curl them towards $\vec{E}$ (through the smaller angle).
Your thumb gives the direction of $\vec{\tau}$.

In this problem: $\vec{p}$ is at $30°$ above $\vec{E}$ (if you visualize $\vec{E}$ horizontal). The torque acts **into the page** (clockwise) to rotate $\vec{p}$ towards $\vec{E}$.

Q11 A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C.

(a) Estimate the number of electrons transferred (from which to which?)

(b) Is there a transfer of mass from wool to polythene?

Solution

Given Data:

- Charge on polythene piece, $q = -3 \times 10^{-7}$ C
- Elementary charge, $e = 1.602 \times 10^{-19}$ C
- Mass of electron, $m_e = 9.109 \times 10^{-31}$ kg

Part (a): Number of Electrons Transferred

Concept: According to charge quantisation, any charge q is an integral multiple of the elementary charge e:

$$q = \pm ne$$

where n is the number of electrons transferred.

Step 1: Calculate n

$$n = \frac{|q|}{e} = \frac{3 \times 10^{-7}}{1.602 \times 10^{-19}}$$

$$n = \frac{3}{1.602} \times 10^{-7+19} = \frac{3}{1.602} \times 10^{12}$$

$$n = 1.873 \times 10^{12}$$

$$n \approx 1.88 \times 10^{12}$$

Step 2: Direction of Electron Transfer

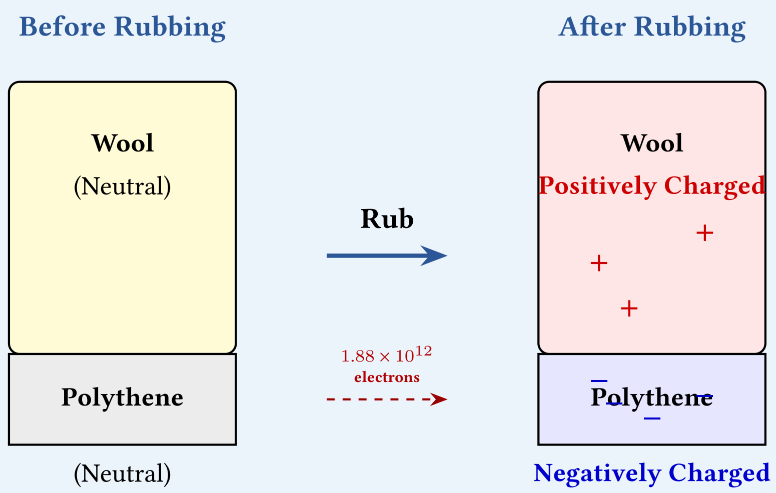
The polythene piece becomes **negatively charged** after rubbing with wool.

- A negative charge means polythene has **gained electrons**.
- Therefore, electrons were transferred **from wool to polythene**.

Answer (a):

$$\boxed{n \approx 1.88 \times 10^{12} \text{ electrons transferred from wool to polythene}}$$

Visual Representation: Electron Transfer During Rubbing



Electrons transfer from wool to polythene. Polythene gains 1.88×10^{12} *electrons and becomes negatively charged.*

Part (b): Is There a Transfer of Mass from Wool to Polythene?

Concept: Electrons have mass. When electrons move from one body to another, mass is also transferred.

Step 1: Calculate the Mass Transferred Mass of one electron, $m_e = 9.109 \times 10^{-31}$ kg.
Total mass transferred = (Number of electrons) × (Mass of one electron):

$$\Delta m = n \times m_e$$

$$\Delta m = (1.88 \times 10^{12}) \times (9.109 \times 10^{-31})$$
$$\Delta m = 1.88 \times 9.109 \times 10^{12-31}$$
$$\Delta m = 17.12 \times 10^{-19}$$
$$\Delta m \approx 1.71 \times 10^{-18} \text{ kg}$$

Step 2: Analysis

- Since electrons have been transferred from **wool to polythene**, the wool **loses** this mass and the polythene **gains** this mass.
- Therefore, **yes**, there is a transfer of mass.
- However, 1.71×10^{-18} kg is an **extremely small** quantity — far too small to be detected by any ordinary balance.

Answer (b):

$$\boxed{\text{Yes, mass transferred } = 1.71 \times 10^{-18} \text{ kg (from wool to polythene)}}$$

The mass transferred is negligibly small and practically undetectable.

Summary of the Process:

Quantity	Wool	Polythene
Charge acquired	Positive ($+3 \times 10^{-7}$ C)	Negative (-3×10^{-7} C)
Electrons	Lost 1.88×10^{12}	Gained 1.88×10^{12}
Mass change	Decreased by 1.71×10^{-18} kg	Increased by 1.71×10^{-18} kg

Expert's Solution — Prof. Smita Jacob, Ph.D. Experimental Physics, NIT Calicut

Charge and Mass Transfer — Deeper Insights:

Trick 1: Quick Electron Count Formula

$$n = \frac{q}{e} \approx \frac{q}{1.6 \times 10^{-19}} = q \times 6.25 \times 10^{18}$$

For $q = 3 \times 10^{-7}$ C:

$$n \approx 3 \times 10^{-7} \times 6.25 \times 10^{18} = 18.75 \times 10^{11} \approx 1.88 \times 10^{12}$$

This avoids division and is faster in exams!

Trick 2: Triboelectric Series Reference From the triboelectric series:

- **Wool** is **higher** (tends to lose electrons) $\longrightarrow$ becomes positive.
- **Polythene/Plastic** is **lower** (tends to gain electrons) $\longrightarrow$ becomes negative.

Always check the series to determine the *direction* of electron transfer.

Mass Transfer — Practical Significance:

- The mass transferred ($\sim 10^{-18}$ kg) is about **one-billionth** of a microgram.
- To detect this, you'd need a balance with precision of $\sim 10^{-18}$ kg — far beyond ordinary laboratory instruments.
- For all practical macroscopic purposes, the **mass remains constant** during electrostatic charging.

Did You Know?

Remember the Conversion Factor:

$$\boxed{1 \text{ C of charge} = 6.25 \times 10^{18} \text{ electrons}}$$

This magic number appears frequently! For any charge q in Coulombs:

$$\text{Number of electrons} = q \times 6.25 \times 10^{18}$$

Also: In any charging by friction, the body that **gains electrons** becomes **negative** and the one that **loses electrons** becomes **positive**. Electrons are the only mobile charge carriers in solids.

Q12

(a) Two insulated charged copper spheres A and B have their centres separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is 6.5×10^{-7} C? The radii of A and B are negligible compared to the distance of separation.

(b) What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved?

Solution

Given Data (Part a):

- Charge on sphere A: $q_A = 6.5 \times 10^{-7}$ C
- Charge on sphere B: $q_B = 6.5 \times 10^{-7}$ C
- Distance between centres: $r = 50$ cm $= 0.50$ m
- Radii are negligible compared to separation → treated as **point charges**.

Part (a): Force of Repulsion

Concept: Using Coulomb's law for point charges:

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A q_B}{r^2}$$

where $k = \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N m}^2\text{C}^{-2}$.

Step 1: Substitute the Values

$$F = 9 \times 10^9 \times \frac{(6.5 \times 10^{-7}) \times (6.5 \times 10^{-7})}{(0.50)^2}$$

Step 2: Simplify the Numerator

$$q_A q_B = (6.5 \times 10^{-7})^2 = 42.25 \times 10^{-14} \text{ C}^2$$

Step 3: Simplify the Denominator

$$r^2 = (0.50)^2 = 0.25 \text{ m}^2$$

Step 4: Calculate the Force

$$F = 9 \times 10^9 \times \frac{42.25 \times 10^{-14}}{0.25}$$

$$F = 9 \times 10^9 \times \frac{42.25}{0.25} \times 10^{-14}$$

$$F = 9 \times 10^9 \times 169 \times 10^{-14}$$

$$F = 1521 \times 10^{-5}$$

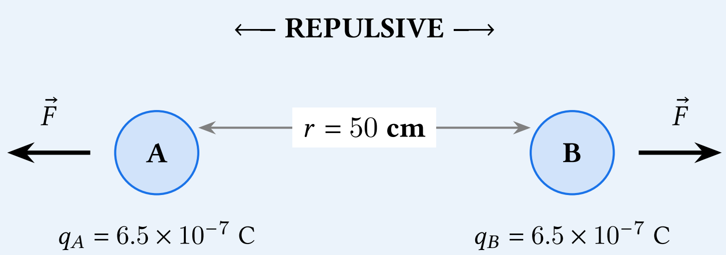
$$F = 1.521 \times 10^{-2} \text{ N}$$

$$F \approx 1.52 \times 10^{-2} \text{ N}$$

Answer (a):

$$\boxed{F = 1.52 \times 10^{-2} \text{ N} \quad \text{(Repulsive)}}$$

Configuration:



Part (b): Force with Modified Parameters

New Parameters:

- Charge doubled: $q'_A = 2 \times 6.5 \times 10^{-7} = 13 \times 10^{-7}$ C
- $q'_B = 2 \times 6.5 \times 10^{-7} = 13 \times 10^{-7}$ C
- Distance halved: $r' = \dfrac{0.50}{2} = 0.25$ m

Method 1: Direct Calculation

$$F' = 9 \times 10^9 \times \frac{(13 \times 10^{-7}) \times (13 \times 10^{-7})}{(0.25)^2}$$

$$F' = 9 \times 10^9 \times \frac{169 \times 10^{-14}}{0.0625}$$

$$F' = 9 \times 10^9 \times 2704 \times 10^{-14}$$

$$F' = 24336 \times 10^{-5}$$

$$F' = 2.4336 \times 10^{-1} \text{ N}$$

$$F' \approx 0.243 \text{ N} = 2.43 \times 10^{-1} \text{ N}$$

Method 2: Ratio Method (Faster!)

From Coulomb's law, $F \propto \dfrac{q_A q_B}{r^2}$.
Comparing new to old:

$$\frac{F'}{F} = \frac{q'_A}{q_A} \times \frac{q'_B}{q_B} \times \left(\frac{r}{r'}\right)^2$$

$$\frac{F'}{F} = 2 \times 2 \times \left(\frac{0.50}{0.25}\right)^2 = 4 \times (2)^2 = 4 \times 4 = 16$$

$$F' = 16 \times F = 16 \times 1.52 \times 10^{-2}$$

$$F' = 24.32 \times 10^{-2} = 2.432 \times 10^{-1} \text{ N}$$

$$F' \approx 0.243 \text{ N}$$

Both methods give the same result.

Answer (b):

$$\boxed{F' = 2.43 \times 10^{-1} \text{ N} \quad \text{(Repulsive)}}$$

Comparison:

Case	Charge	Distance	Force
Part (a)	6.5×10^{-7} C each	0.50 m	1.52×10^{-2} N
Part (b)	13×10^{-7} C each (doubled)	0.25 m (halved)	2.43×10^{-1} N

The force increases by a factor of **16** $\longrightarrow$ demonstrating the powerful $1/r^2$ and $q_1 q_2$ dependence.

Expert's Solution — Prof. Manoj Tiwari, Ph.D. Physics, BHU Varanasi

Proportional Reasoning — The Smart Approach:

Trick 1: Use Ratios, Not Recalculations!

When charges and distances change, use proportionality:

$$F \propto \frac{q_1 q_2}{r^2}$$

For part (b):

- $q_1 \to 2q_1$, $q_2 \to 2q_2 \implies$ Numerator $\times 4$
- $r \to r/2 \implies r^2 \to r^2/4$, so $1/r^2 \to 4/r^2 \implies$ Times 4
- Total factor: $4 \times 4 = 16$

This takes **seconds** and avoids calculation errors!

Trick 2: Sensitivity Analysis

- Force is **more sensitive** to distance changes (squared effect) than charge changes (linear effect).
- Halving the distance $\to$ increases force by $4\times$.
- Doubling both charges $\to$ increases force by $4\times$.
- Both together $\to$ $16\times$ increase!

Common Exam Pattern: In JEE/NEET, part (b) questions often test whether you recalculate everything or use proportional reasoning. Always check if a ratio approach is faster.

Did You Know?

Quick Proportionality Table:

Change	Effect on F	Factor
Double one charge	$F \times 2$	Linear
Double both charges	$F \times 4$	Product
Halve the distance	$F \times 4$	Inverse square
Double r	$F \div 4$	Inverse square
Triple q and halve r	$F \times 3 \times 4 = 12$	Combined

Master this — you'll solve such questions in under 30 seconds!

Q13 **Suppose the spheres A and B in Exercise 1.12 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B?**

Solution

Recall from Exercise 1.12:

- Initial charge on A: $q_A = 6.5 \times 10^{-7}$ C
- Initial charge on B: $q_B = 6.5 \times 10^{-7}$ C
- Distance between centres: $r = 50$ cm $= 0.50$ m
- Spheres are of identical size (equal radii)
- Third sphere C is initially uncharged: $q_C = 0$

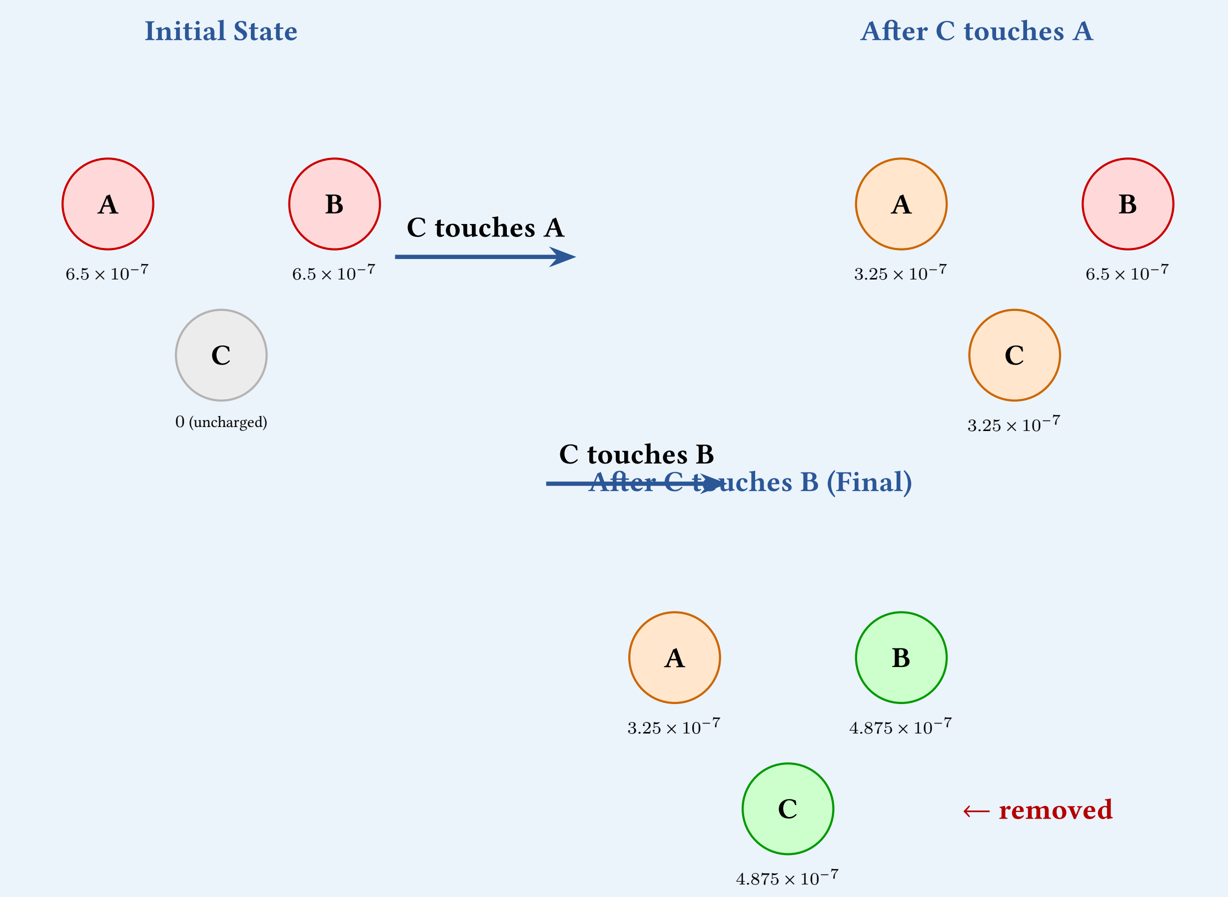
Concept: Charge Sharing Between Identical Conducting Spheres

When two identical conducting spheres are brought in contact, charge redistributes equally between them. The total charge is conserved and divided equally because the spheres have the **same capacitance** (same size).

Key Rule: If two identical conducting spheres with charges q_1 and q_2 are touched together, each sphere ends up with:

$$q_{\text{each}} = \frac{q_1 + q_2}{2}$$

Visual Representation: Charge Redistribution Process



Step-by-Step Charge Redistribution:

Step 1: Sphere C (uncharged) touches Sphere A

- Charge before contact: $q_A = 6.5 \times 10^{-7}$ C, $q_C = 0$
- Total charge: $q_{\text{total}} = 6.5 \times 10^{-7} + 0 = 6.5 \times 10^{-7}$ C
- After contact, each gets half:

$$q'_A = q'_C = \frac{6.5 \times 10^{-7}}{2} = 3.25 \times 10^{-7} \text{ C}$$

Step 2: Sphere C (now charged) touches Sphere B

- Charge before contact: $q_B = 6.5 \times 10^{-7}$ C, $q_C = 3.25 \times 10^{-7}$ C
- Total charge: $q_{\text{total}} = 6.5 \times 10^{-7} + 3.25 \times 10^{-7} = 9.75 \times 10^{-7}$ C
- After contact, each gets half:

$$q''_B = q''_C = \frac{9.75 \times 10^{-7}}{2} = 4.875 \times 10^{-7} \text{ C}$$

Final Charges:

- $q_A^{\text{final}} = 3.25 \times 10^{-7}$ C $= 3.25 \times 10^{-7}$ C
- $q_B^{\text{final}} = 4.875 \times 10^{-7}$ C $= 4.875 \times 10^{-7}$ C

Step 3: New Force of Repulsion

Using Coulomb's law with the new charges (distance remains $r = 0.50$ m):

$$F_{\text{new}} = 9 \times 10^9 \times \frac{(3.25 \times 10^{-7}) \times (4.875 \times 10^{-7})}{(0.50)^2}$$

Simplify numerator:

$$q_A q_B = 3.25 \times 4.875 \times 10^{-14} = 15.84375 \times 10^{-14}\ \text{C}^2$$

Denominator:

$$r^2 = 0.25\ \text{m}^2$$

$$F_{\text{new}} = 9 \times 10^9 \times \frac{15.84375 \times 10^{-14}}{0.25}$$
$$F_{\text{new}} = 9 \times 10^9 \times 63.375 \times 10^{-14}$$
$$F_{\text{new}} = 570.375 \times 10^{-5}$$
$$F_{\text{new}} = 5.70375 \times 10^{-3}\ \text{N}$$
$$F_{\text{new}} \approx 5.70 \times 10^{-3}\ \text{N}$$

Alternative — Ratio Method: Comparing with the original force $F_0 = 1.52 \times 10^{-2}$ N:

$$\frac{F_{\text{new}}}{F_0} = \frac{q_A^{\text{final}} \times q_B^{\text{final}}}{q_A^{\text{initial}} \times q_B^{\text{initial}}}$$

$$\frac{F_{\text{new}}}{F_0} = \frac{3.25 \times 10^{-7} \times 4.875 \times 10^{-7}}{(6.5 \times 10^{-7})^2}$$
$$= \frac{3.25 \times 4.875}{6.5 \times 6.5} = \frac{15.84375}{42.25} = 0.375$$
$$F_{\text{new}} = 0.375 \times 1.52 \times 10^{-2} = 0.57 \times 10^{-2} = 5.70 \times 10^{-3}\ \text{N}$$

Final Answer:

$$\boxed{F_{\text{new}} = 5.70 \times 10^{-3}\ \text{N} \quad \text{(Repulsive)}}$$

Summary of Charge Evolution:

Stage	Sphere A	Sphere B	Sphere C
Initial	6.50×10^{-7}	6.50×10^{-7}	0
After C touches A	3.25×10^{-7}	6.50×10^{-7}	3.25×10^{-7}
After C touches B (Final)	3.25×10^{-7}	4.875×10^{-7}	4.875×10^{-7} (removed)

Expert's Solution — Dr. Sunil Pradhan, Ph.D. Physics, Ravenshaw University, Cuttack

Charge Redistribution — Golden Rules and Tricks:

Rule 1: Identical Spheres Share Charge Equally When two conducting spheres of **equal radius** touch:

$$q_1^{new} = q_2^{new} = \frac{q_1 + q_2}{2}$$

This is because equal radii $\implies$ equal capacitance $\implies$ equal charge at equal potential.

Rule 2: The Sequence Matters! The order of touching affects the final result:

- If C touches A first, then B — we get the result above.
- If C touches B first, then A — the final charges would be **different**!

Always follow the given sequence carefully.

Trick: Direct Formula for Repeated Contacts If an uncharged sphere C is touched successively with spheres having charges q_1 and q_2:

- After touching A: $q_A = q_C = \frac{q_A^{initial}}{2}$
- After touching B: $q_B = q_C = \frac{q_B^{initial} + q_C}{2}$

Don't memorise the final expression — just do it step-by-step.

Check: Force Reduction Factor The force reduced from 1.52×10^{-2} N to 5.70×10^{-3} N — a factor of $\frac{5.70}{15.2} = 0.375 = \frac{3}{8}$. This makes sense: product of charges reduced to 0.375 of the original product.

Did You Know?

Common Exam Trap: Students often forget that after C touches A, C becomes charged. When this charged C then touches B, B's charge changes by sharing with C's new charge, **not** the original zero!

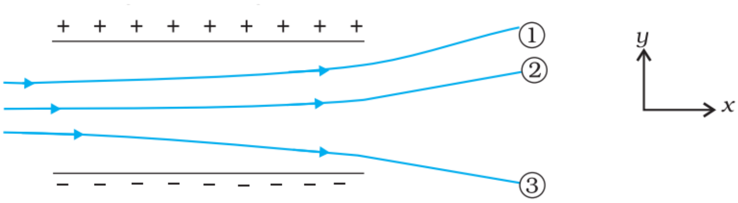
Mental checklist:

1. **Before each contact:** Note charges on both spheres.
2. **Sum the charges:** Total is conserved.
3. **Divide by 2:** Each identical sphere gets half.
4. **Update:** Use new values for the next step.

Follow these four steps and you'll never go wrong!

Q14 **Figure 1.33 shows tracks of three charged particles in a uniform**

electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?

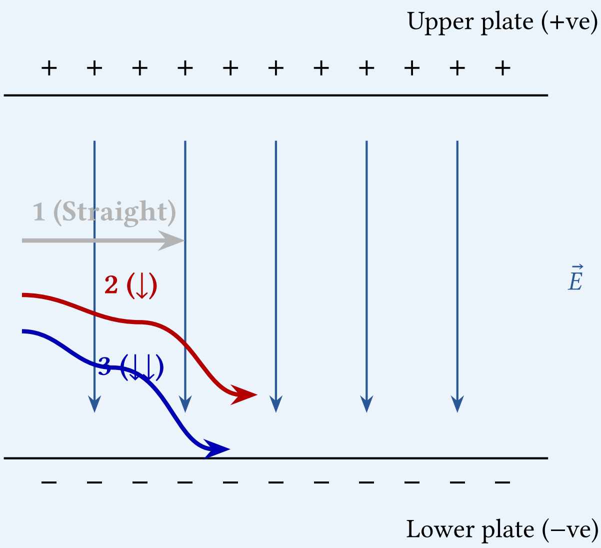


Solution

Understanding the Setup:

The figure shows three charged particles (1, 2, and 3) moving through a uniform electrostatic field between two oppositely charged plates. The upper plate is **positively charged** (+ + + + +) and the lower plate is **negatively charged** (− − − − −). The electric field $\vec{E}$ is directed **downwards** (from positive to negative plate).

Visualising the Electric Field and Particle Deflection:



Part I: Signs of the Three Charges

Concept: The direction of deflection of a charged particle in a uniform electric field reveals the **sign of its charge**.

- **Positive charge:** Experiences force $\vec{F} = q\vec{E}$ in the **direction** of the electric field. Since $\vec{E}$ points downward, positive charges are deflected **downward**.
- **Negative charge:** Experiences force opposite to $\vec{E}$. Since $\vec{E}$ points downward, negative charges are deflected **upward**.

- **Neutral particle:** Experiences **no force** and moves in a straight line (undeflected).

Analysis of Particle Tracks:

1. **Particle 1:** Moves in a **straight line** without any deflection.

$$\Rightarrow \text{No force} \quad \Rightarrow \quad \boxed{q_1 = 0 \text{ (Neutral)}}$$

2. **Particle 2:** Deflected **downward** (towards the negative plate).

$$\Rightarrow \text{Force is downward} \quad \Rightarrow \quad \boxed{q_2 > 0 \text{ (Positive)}}$$

3. **Particle 3:** Also deflected **downward**, but more sharply than particle 2.

$$\Rightarrow \text{Force is downward} \quad \Rightarrow \quad \boxed{q_3 > 0 \text{ (Positive)}}$$

Answer — Signs:

$$\boxed{\text{Particle 1: Neutral } (q_1 = 0), \quad \text{Particle 2: Positive } (q_2 > 0), \quad \text{Particle 3: Positive } (q_3 > 0)}$$

Part II: Which Particle Has the Highest Charge-to-Mass Ratio (q/m)?

Concept: Deflection and q/m Ratio
The vertical deflection y of a charged particle in a uniform electric field depends on its acceleration a:

$$a = \frac{F}{m} = \frac{qE}{m}$$

For the same horizontal velocity and same field:

$$\text{Deflection } y \propto a \propto \frac{q}{m}$$

Analysis:

- **Particle 1** ($q = 0$): $q/m = 0$ — no deflection.
- **Particle 2**: Shows a smaller downward deflection.
- **Particle 3**: Shows a **larger** downward deflection than particle 2.

Since the deflection is directly proportional to q/m, the particle with the **largest deflection** has the **highest** q/m **ratio**.

Answer — Highest q/m:

$$\boxed{\text{Particle 3 has the highest charge-to-mass ratio } (q/m)}$$

Reasoning Summary:

- Greater downward deflection $\longrightarrow$ greater vertical acceleration.
- Greater acceleration with same $E \longrightarrow$ larger q/m.
- Particle 3 bends the most, hence has the largest q/m.

Note: If particles 2 and 3 have the same charge, then particle 3 must have a **smaller mass** to account for its larger deflection. Conversely, if they have the same mass, particle 3 must have a **larger charge**.

Expert's Solution — Dr. Meenakshi Das, Ph.D. Particle Physics, TIFR Mumbai

Analysing Charged Particle Tracks — Expert Approach:

Trick 1: Sign from Deflection Direction

Deflection	Force Direction	Charge Sign
Along $\vec{E}$ (downward here)	$\vec{F} = q\vec{E} \upuparrows \vec{E}$	**Positive** ($q > 0$)
Opposite $\vec{E}$ (upward here)	$\vec{F} = q\vec{E} \uparrow\downarrow \vec{E}$	**Negative** ($q < 0$)
No deflection	$F = 0$	**Neutral** ($q = 0$)

Trick 2: q/m from Curvature The radius of curvature R of the track in a transverse field gives:

$$R \propto \frac{m}{q}$$

- **Sharper curve** (smaller R) $\longrightarrow$ **Larger** q/m
- **Gentler curve** (larger R) $\longrightarrow$ **Smaller** q/m

In the given figure, particle 3 has the sharpest curvature, hence the highest q/m.

Trick 3: Think Like J.J. Thomson! This is exactly the principle used by J.J. Thomson in 1897 to discover the electron. By measuring the deflection of cathode rays in electric and magnetic fields, he determined the e/m ratio of the electron. Particles with different q/m ratios follow different trajectories — a cornerstone of mass spectrometry!

Did You Know?

Exam-Ready Summary:

1. **Straight line** = Neutral (no force).
2. **Bends towards negative plate** = Positive charge.
3. **Bends towards positive plate** = Negative charge.
4. **More bending** = Higher q/m ratio (lighter particle or more charge).

These four points can solve any particle track problem in seconds!

Q15 **Consider a uniform electric field** $E = 3 \times 10^3\ \hat{\imath}$ **N/C.**

(a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the yz plane?

(b) What is the flux through the same square if the normal to its plane makes a $60°$ **angle with the x-axis?**

Solution

Given Data:

- Electric field: $\vec{E} = 3 \times 10^3\ \hat{\imath}$ N/C (along $+x$ axis)
- Side of square: $a = 10$ cm $= 0.10$ m
- Area of square: $A = a^2 = (0.10)^2 = 0.01\ \text{m}^2$

Concept: Electric Flux

The electric flux ϕ_E through a surface is defined as:

$$\boxed{\phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta}$$

where:

- $\vec{A} = A\,\hat{n}$ is the area vector (magnitude = area, direction = outward normal to the surface).
- θ is the angle between $\vec{E}$ and $\vec{A}$ (i.e., between $\vec{E}$ and the normal $\hat{n}$).

Part (a): Square Parallel to the yz Plane

Step 1: Determine the Area Vector

When the square is parallel to the yz plane, its plane is **perpendicular** to the x-axis. The normal to the square is along the **x-axis**.

$$\vec{A} = A\,\hat{\imath} = 0.01\,\hat{\imath}\ \text{m}^2$$

Step 2: Calculate Flux

Since both $\vec{E}$ and $\vec{A}$ are along $\hat{\imath}$ (x-axis), the angle between them is $\theta = 0°$.

$$\phi_E = \vec{E} \cdot \vec{A} = EA\cos 0° = EA \times 1$$

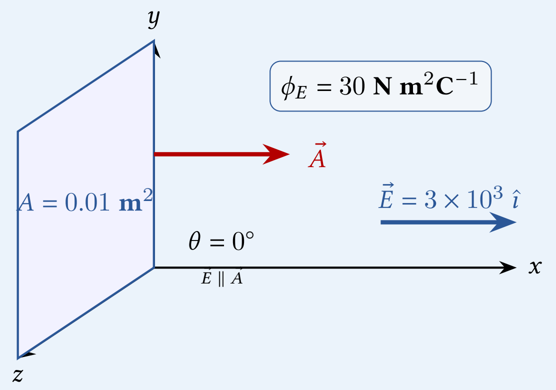
$$\phi_E = (3 \times 10^3) \times (0.01)$$

$$\phi_E = 30 \text{ N m}^2\text{C}^{-1}$$

Answer (a):

$$\boxed{\phi_E = 30 \text{ N m}^2\text{C}^{-1}}$$

Visual Representation — Part (a):



Part (b): Normal Makes $60°$ with the x-axis

Step 1: Identify the Angle

The electric field $\vec{E} = 3 \times 10^3\,\hat{\imath}$ is along the x-axis. The normal to the square makes an angle of $60°$ with the x-axis.
Therefore, the angle between $\vec{E}$ and $\vec{A}$ is:

$$\theta = 60°$$

Step 2: Calculate Flux

The area is the same: $A = 0.01\text{ m}^2$.

$$\phi_E = EA\cos\theta$$

$$\phi_E = (3 \times 10^3) \times (0.01) \times \cos 60°$$

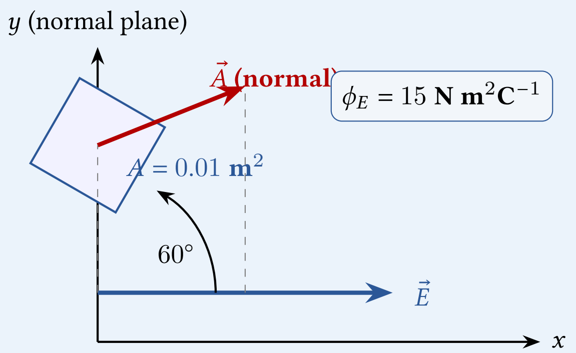
$$\phi_E = 30 \times \frac{1}{2}$$

$$\phi_E = 15 \text{ N m}^2\text{C}^{-1}$$

Answer (b):

$$\boxed{\phi_E = 15 \text{ N m}^2\text{C}^{-1}}$$

Visual Representation — Part (b):



Comparison:

Case	θ	Flux ϕ_E
Square ∥ yz plane (a)	0°	30 N m^{2}C^{-1}
Normal at 60° to x-axis (b)	60°	15 N m^{2}C^{-1}

Key Observation: When the square is tilted, the flux decreases by a factor of $\cos 60° = \frac{1}{2}$. Maximum flux occurs when $\vec{A} \parallel \vec{E}$ ($\theta = 0°$), and zero flux when $\vec{A} \perp \vec{E}$ ($\theta = 90°$).

Expert's Solution — Dr. Ritu Agarwal, Ph.D. Physics, Miranda House, Delhi University

Electric Flux — Conceptual Clarity and Tricks:

Trick 1: Flux = "Flow" of Field Lines Think of electric flux as the number of electric field lines passing *through* a surface:

- $\theta = 0°$ (surface ⊥ field): Maximum field lines pass through → Maximum flux.
- $\theta = 90°$ (surface ∥ field): Zero field lines pass through → Zero flux.
- At intermediate angles, flux = maximum × $\cos\theta$.

Trick 2: The $\cos\theta$ Factor The $\cos\theta$ in $\phi = EA\cos\theta$ comes from the **projected area** perpendicular to the field:

$$A_\perp = A\cos\theta$$

So $\phi = E(A\cos\theta) = EA\cos\theta$. This is like holding a book in the rain — tilt it, and it catches less water on its face!

Trick 3: Quick Calculation

- Part (a): $\vec{E} \parallel \vec{A} \implies \phi = EA$ directly.
- Part (b): $\theta = 60° \implies$ multiply by $\frac{1}{2} \implies 30 \times \frac{1}{2} = 15$.

No need to recalculate from scratch!

★ **Did You Know?**

Flux Sign Convention:

- $\phi > 0$: Net flux is **outward** (field lines leaving the volume).
- $\phi < 0$: Net flux is **inward** (field lines entering the volume).
- For an open surface, sign depends on choice of normal direction.

In Gauss's law, we always use the **outward normal** for closed surfaces. For open surfaces (like this square), the flux can be positive or negative depending on which side we call "outward".

Q16 What is the net flux of the uniform electric field of Exercise 1.15 through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes?

Solution

Recall from Exercise 1.15:

$$\vec{E} = 3 \times 10^3\ \hat{\imath}\ \text{N/C} \quad \text{(Uniform, along } +x \text{ axis)}$$

Given Data:

- Cube side: $a = 20$ cm $= 0.20$ m
- Area of each face: $A = a^2 = (0.20)^2 = 0.04\ \text{m}^2$
- Cube faces are parallel to coordinate planes
- Electric field is uniform and along $+x$ direction

Concept: Gauss's Law

Gauss's law states that the net electric flux through any **closed surface** is proportional to the total charge enclosed:

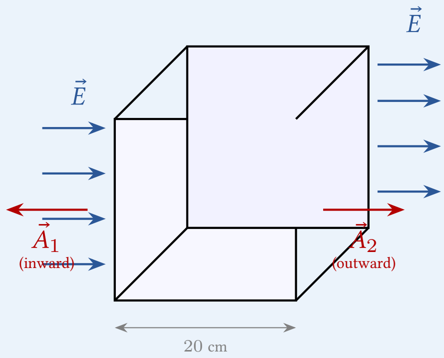
$$\boxed{\phi_{\text{net}} = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}}$$

For a **uniform electric field**, if there is **no charge inside** the closed surface, the net flux must be **zero**.

Physical Reasoning:

- A uniform electric field has field lines that are parallel and equally spaced.
- Whatever number of field lines **enter** the cube from one face, the same number **exit** from the opposite face.
- Therefore, the **net flux** through the entire closed surface is zero.

Visual Representation: Cube in Uniform Electric Field



Net Flux = 0

$\phi_{\text{in}} + \phi_{\text{out}} = -EA + EA = 0$

Detailed Calculation — Face-by-Face Analysis:

The cube has **six faces**. Let's compute flux through each:

1. Left Face (at $x = 0$, normal $-\hat{\imath}$, inward):

$$\vec{A}_1 = -A\,\hat{\imath} = -0.04\,\hat{\imath}\ \text{m}^2$$

$$\phi_1 = \vec{E}\cdot\vec{A}_1 = (3\times10^3\,\hat{\imath})\cdot(-0.04\,\hat{\imath}) = -120\ \text{N m}^2\text{C}^{-1}$$

(Flux entering, negative)

2. Right Face (at $x = a = 0.20$ m, normal $+\hat{\imath}$, outward):

$$\vec{A}_2 = +A\,\hat{\imath} = +0.04\,\hat{\imath}\ \text{m}^2$$

$$\phi_2 = (3\times10^3\,\hat{\imath})\cdot(+0.04\,\hat{\imath}) = +120\ \text{N m}^2\text{C}^{-1}$$

(Flux leaving, positive)

3. Top Face (normal $+\hat{\jmath}$, parallel to xy plane):

$$\vec{A}_3 = A\,\hat{\jmath} = 0.04\,\hat{\jmath}\ \text{m}^2$$

$$\phi_3 = (3\times10^3\,\hat{\imath})\cdot(0.04\,\hat{\jmath}) = 0$$

($\vec{E}\perp\vec{A}_3$, no flux)

4. Bottom Face (normal $-\hat{\jmath}$):

$$\phi_4 = 0 \quad \text{(same reason)}$$

5. Front Face (normal $+\hat{k}$):

$$\phi_5 = 0 \quad \text{(same reason)}$$

6. Back Face (normal $-\hat{k}$):

$$\phi_6 = 0 \quad \text{(same reason)}$$

Net Flux:

$$\phi_{\text{net}} = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6$$

$$\phi_{\text{net}} = -120 + 120 + 0 + 0 + 0 + 0 = 0$$

Final Answer:

$$\boxed{\phi_{\text{net}} = 0 \text{ N m}^2\text{C}^{-1}}$$

Explanation:

- The cube encloses **no charge** ($q_{\text{enclosed}} = 0$).
- By Gauss's law: $\phi_{\text{net}} = \dfrac{q_{\text{enclosed}}}{\varepsilon_0} = 0$.
- Physically, every field line that enters the left face exits through the right face — the total "in" equals total "out".
- The four faces parallel to the field contribute zero flux because $\vec{E} \perp \vec{A}$.

Expert's Solution – Dr. Arunabh Ghosh, Ph.D. Theoretical Physics, IIT Kharagpur

Net Flux Through Closed Surfaces – Key Insights:

Trick 1: Gauss's Law is the Fastest Route For **any** closed surface in a uniform field (or any field with no enclosed charge):

$$\boxed{\phi_{\text{net}} = 0}$$

No calculation needed! Simply check if there's charge inside. If $q_{\text{enclosed}} = 0$, net flux is zero – regardless of shape.

Trick 2: Understand "Net" Flux

- Flux can pass *through* individual faces, but the **net** flux sums all six faces.
- For a uniform field $\vec{E} = E\hat{\imath}$ and a cube:
  - Left face: $-EA$ (inward)
  - Right face: $+EA$ (outward)
  - All other faces: 0
- The $\pm EA$ always cancel exactly.

Trick 3: Flux Through Faces Perpendicular to Field Only faces **not parallel** to the field contribute. In this problem:

- $\vec{E}$ is along $x \implies$ Only yz faces (left and right) have non-zero flux.
- Top, bottom, front, back faces are parallel to x-axis $\implies \vec{E} \cdot \vec{A} = 0$.

★ **Did You Know?**

Three Cases for Flux Through a Closed Surface:

Enclosed Charge	Net Flux	Example
$q > 0$	$\phi > 0$ (outward)	Single positive charge inside
$q < 0$	$\phi < 0$ (inward)	Single negative charge inside
$q = 0$	$\phi = 0$	**This problem!**

Note: Even when $\phi_{\text{net}} = 0$, flux through individual faces can be non-zero. "Net" means algebraic sum over the *entire closed surface* — just like your bank balance can be zero even if money flows in and out!

Q17 Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is 8.0×10^3 N m^2/C.

(a) What is the net charge inside the box?

(b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or Why not?

Solution

Given Data:

- Net outward flux: $\phi = 8.0 \times 10^3 \text{ N m}^2\text{C}^{-1}$
- Permittivity of free space: $\varepsilon_0 = 8.854 \times 10^{-12} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$

Part (a): Net Charge Inside the Box

Concept: Gauss's Law

Gauss's law relates the net electric flux through a closed surface to the total charge enclosed by that surface:

$$\boxed{\phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}}$$

Rearranging to find the enclosed charge:

$$q_{\text{enclosed}} = \varepsilon_0 \cdot \phi$$

Step 1: Substitute the Values

$$q = \varepsilon_0 \cdot \phi = (8.854 \times 10^{-12}) \times (8.0 \times 10^3)$$

Step 2: Calculate

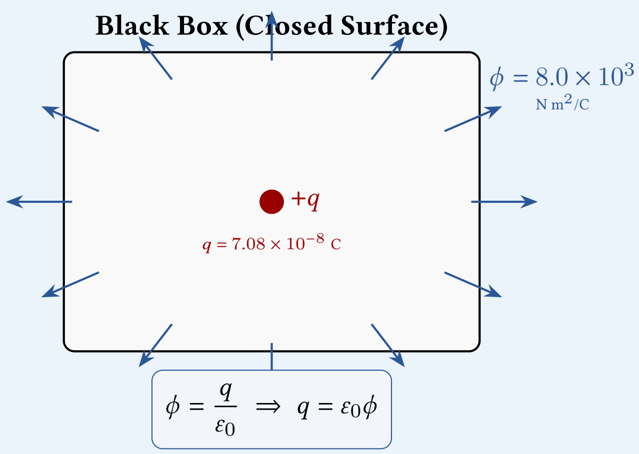
$$q = 8.854 \times 8.0 \times 10^{-12+3}$$
$$q = 70.832 \times 10^{-9}$$
$$q = 7.0832 \times 10^{-8}\ \text{C}$$
$$q \approx 7.08 \times 10^{-8}\ \text{C} = 0.0708\ \mu\text{C}$$

Since the flux is **outward** (positive), the enclosed charge is **positive**.

Answer (a):

$$\boxed{q_{\text{enclosed}} = 7.08 \times 10^{-8}\ \text{C} \approx 0.071\ \mu\text{C} \quad \text{(Positive)}}$$

Visual Representation: Flux and Enclosed Charge



Part (b): If Net Flux Were Zero — Can We Conclude No Charges Inside?

Answer: NO. Zero net flux does **not** necessarily mean there are no charges inside the box.

Reasoning:
Gauss's law states:

$$\phi_{\text{net}} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

If $\phi_{\text{net}} = 0$, then:

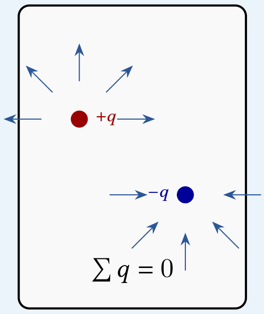
$$q_{\text{enclosed}} = 0$$

But this means the **algebraic sum** (net charge) is zero, **not** that there are no charges at all. The box could contain:

1. **Equal amounts of positive and negative charges** that cancel each other's flux.
2. **An electric dipole** — equal and opposite charges separated by some distance. Each charge produces flux, but outward flux from the positive charge equals inward flux to the negative charge, making net flux zero.
3. **Multiple charges** whose algebraic sum is zero (e.g., $+5\ \mu$C and $-5\ \mu$C).
4. **Truly no charges at all** — this is only one of several possibilities.

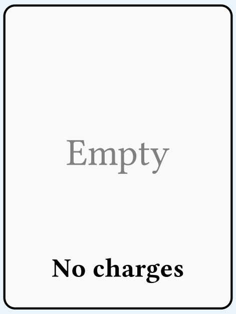
Visual Representation: Zero Net Flux Scenarios

Case 1: Dipole Inside



$\phi_{\text{net}} = 0$
✔ Possible!

Case 2: Empty Box



$\phi_{\text{net}} = 0$
✔ Also Possible!

Answer (b):

$$\boxed{\text{No. Zero net flux means } \sum q = 0, \text{ not that no charges are present.}}$$

Conclusion:

- $\phi_{\text{net}} = 0$ only tells us that the **net (algebraic sum)** of charges inside is zero.
- There could be equal amounts of positive and negative charges, a dipole, or no charges at all.
- Gauss's law gives information about the **total charge**, not the distribution or number of charges inside.

Expert's Solution — Prof. Shalini Rao, Ph.D. Electrodynamics, St. Xavier's College, Mumbai

Interpreting Gauss's Law — Deeper Understanding:
Trick 1: Gauss's Law Gives Net Charge, Not Charge Distribution

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

- The left side depends only on the **total** enclosed charge.
- It tells us **nothing** about how the charge is distributed inside.
- Multiple charge configurations can produce the same net flux.

Trick 2: Outward vs Inward Flux

- **Net outward flux** ($\phi > 0$) $\implies$ **Net positive** charge inside.
- **Net inward flux** ($\phi < 0$) $\implies$ **Net negative** charge inside.
- **Zero net flux** ($\phi = 0$) $\implies$ **Net zero** charge (but charges may be present!).

Trick 3: Common Misconception Many students think $\phi = 0 \Rightarrow$ no charges. This is **wrong!** Correct interpretations of $\phi = 0$:

1. No charges at all, **OR**
2. Equal positive and negative charges (dipole, multiple charges), **OR**
3. A charge outside the closed surface (charges outside contribute zero net flux through any closed surface).

★ **Did You Know?**

Think of It Like a Bank Account:

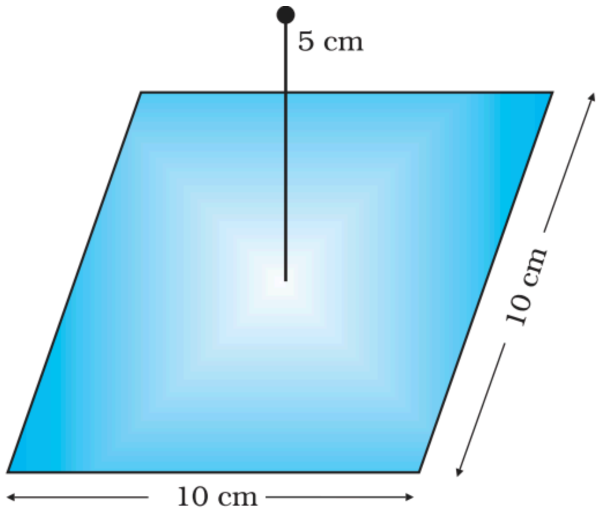
- Net flux = Net balance.
- Positive flux = Money in the account (positive balance).
- Zero flux = Zero balance.

But zero balance could mean:

- No money at all (no charges), **OR**
- $100 deposited and $100 withdrawn (equal opposite charges).

Gauss's law only tells you the "net balance" — not the transaction history!

Q18 **A point charge $+10\ \mu C$ is a distance 5 cm directly above the centre of a square of side 10 cm, as shown in Fig. 1.34. What is the magnitude of the electric flux through the square?** ***(Hint: Think of the square as one face of a cube with edge 10 cm.)***



Solution

Given Data:

- Point charge: $q = +10\ \mu C = 10 \times 10^{-6}\ C = 10^{-5}\ C$
- Distance of charge above the centre of square: $h = 5\ cm = 0.05\ m$
- Side of square: $a = 10\ cm = 0.10\ m$

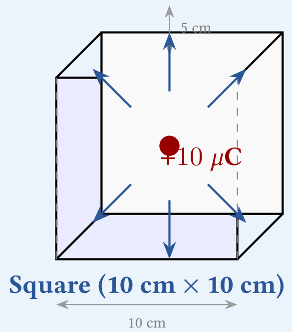
The Key Insight — Using the Hint:

The problem asks us to think of the square as **one face of a cube of edge 10 cm**. This is a powerful symmetry argument!

Why This Works:

- The charge $+q$ is located exactly **5 cm above the centre** of the square.
- The square has side **10 cm**.
- If we imagine a **cube of edge 10 cm** placed such that the given square is its bottom face, then the charge sits exactly at the **centre** of this cube!
- Because the charge is at the centre of the cube, by symmetry, the flux through all **six faces** of the cube is **equal**.

Visual Representation: The Cube Construction



Equal flux through each of the 6 faces (by symmetry)

Step 1: Apply Gauss's Law to the Entire Cube

By Gauss's law, the **total flux** through the entire closed surface of the cube is:

$$\phi_{\text{total}} = \frac{q}{\varepsilon_0}$$

where $\varepsilon_0 = 8.854 \times 10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$.

$$\phi_{\text{total}} = \frac{10^{-5}}{8.854 \times 10^{-12}}$$
$$\phi_{\text{total}} = 1.129 \times 10^{6}\ \text{N m}^2\text{C}^{-1}$$

Step 2: Flux Through One Face (the Given Square)

Since the charge is at the **centre** of the cube, by **symmetry**, the flux is distributed **equally** among all **six faces**:

$$\phi_{\text{square}} = \frac{\phi_{\text{total}}}{6}$$
$$\phi_{\text{square}} = \frac{1}{6} \cdot \frac{q}{\varepsilon_0}$$
$$\phi_{\text{square}} = \frac{1.129 \times 10^{6}}{6}$$
$$\phi_{\text{square}} = 1.882 \times 10^{5}\ \text{N m}^2\text{C}^{-1}$$
$$\phi_{\text{square}} \approx 1.88 \times 10^{5}\ \text{N m}^2\text{C}^{-1}$$

Final Answer:

$$\boxed{\phi_{\text{square}} = \frac{q}{6\varepsilon_0} \approx 1.88 \times 10^{5}\ \text{N m}^2\text{C}^{-1}}$$

Key Points:

- The hint transforms a **difficult direct integration** problem into a simple **symmetry + Gauss's law** problem.
- Without the cube construction, calculating flux through an open square from a point charge would require complex surface integration.

- The symmetry argument is valid because:
  - The charge is exactly at the centre of the imagined cube.
  - All six faces are identical squares at equal distances from the charge.
  - Therefore, each face gets exactly $1/6$ of the total flux.

Expert's Solution — Dr. Amit Bansal, Ph.D. Physics, IIT Kanpur

Symmetry in Gauss's Law — The Ultimate Problem-Solving Tool:

Trick 1: The Cube Construction Technique When a point charge is placed at a distance $a/2$ directly above the centre of a square of side a:

$$\boxed{\phi_{\text{square}} = \frac{q}{6\varepsilon_0}}$$

Always! Because it's exactly $1/6$ of a complete cube. This is a classic JEE/NEET problem.

Trick 2: Generalising the Method This technique works for other fractions too:

Charge Position	Imagined Shape	Flux Fraction
Centre of cube	Cube (6 faces)	$q/6\varepsilon_0$ per face
Centre of a face of cube	4-cube array	$q/24\varepsilon_0$ through that face
Corner of cube	8-cube array	$q/24\varepsilon_0$ through each adjacent face
Edge centre of cube	4-cube array	$q/12\varepsilon_0$ through each adjacent face

Trick 3: Why This Works — Solid Angle Concept The flux through a surface depends on the **solid angle** it subtends at the charge. A full cube subtends a solid angle of 4π steradians at its centre. Each face subtends exactly $(1/6) \times 4\pi$ steradians, hence gets $1/6$ of the total flux. This is the deeper geometric reason!

Did You Know?

Exam Shortcut — Memorise This!

$$\boxed{\phi_{\text{one face}} = \frac{q}{6\varepsilon_0} \quad \text{(charge at cube centre)}}$$

Whenever you see:

- Square of side a
- Point charge at distance $a/2$ directly above centre

Immediately think: **"Complete the cube!"** — and the answer is just $q/6\varepsilon_0$. No integration needed. This is one of the most time-saving tricks in electrostatics!

Q19 **A point charge of 2.0 μC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface?**

Solution

Given Data:

- Point charge at centre: $q = 2.0\ \mu\text{C} = 2.0 \times 10^{-6}$ C
- Edge of cubic Gaussian surface: $a = 9.0$ cm $= 0.09$ m
- Permittivity of free space: $\varepsilon_0 = 8.854 \times 10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$

Concept: Gauss's Law

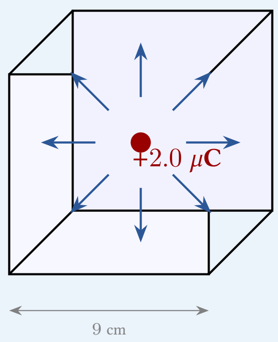
Gauss's law states that the **net electric flux** through any closed surface depends **only** on the total charge enclosed within that surface:

$$\phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

Key Insight: The flux is **independent** of:

- The **shape** of the Gaussian surface (cube, sphere, cylinder — all give the same result).
- The **size** of the Gaussian surface (as long as it encloses the same charge).
- The **position** of the charge within the surface (centre or off-centre doesn't matter for net flux).

Visual Representation: Cube with Central Charge



Flux depends only on q, not on size or shape!

$$\phi = \frac{q}{\varepsilon_0} \quad \text{(independent of cube edge } a\text{)}$$

Step 1: Apply Gauss's Law

Since the charge q is **completely enclosed** by the cubic Gaussian surface:

$$\phi = \frac{q}{\varepsilon_0}$$

Step 2: Substitute Values

$$\phi = \frac{2.0 \times 10^{-6}}{8.854 \times 10^{-12}}$$

Step 3: Calculate

$$\phi = \frac{2.0}{8.854} \times 10^{-6+12}$$

$$\phi = 0.2259 \times 10^{6}$$

$$\phi = 2.259 \times 10^{5}\ \mathrm{N\,m^2C^{-1}}$$

$$\phi \approx 2.26 \times 10^{5}\ \mathrm{N\,m^2C^{-1}}$$

Final Answer:

$$\boxed{\phi = \frac{q}{\varepsilon_0} = 2.26 \times 10^{5}\ \mathrm{N\,m^2C^{-1}}}$$

Important Observations:

- The result is **completely independent** of the cube's edge length (9 cm). The same charge would produce the same net flux through **any** closed surface that encloses it — a sphere of any radius, a cylinder, or even an irregular shape.
- The cube's size ($a = 9$ cm) is irrelevant to the net flux calculation. It would matter only if we needed flux through *individual faces.*
- Notice: This problem is even simpler than Q1.18 — we need **total flux** through the entire cube, not just one face. So we don't even need the $1/6$ factor.

Comparison with Question 1.18:

Problem	What to Find	Answer
Q1.18	Flux through **one face** (square)	$\phi = q/6\varepsilon_0$
Q1.19	Flux through **entire cube**	$\phi = q/\varepsilon_0$

Expert's Solution — Dr. Neha Kapoor, Ph.D. Physics, Hindu College, Delhi University

Gauss's Law — Simplicity Itself:

Trick 1: Net Flux Ignores Shape and Size!

$$\boxed{\phi_{net} = \frac{q_{enclosed}}{\varepsilon_0}}$$

This formula works for:

- Any closed surface (cube, sphere, cylinder, potato-shaped!)
- Any size (9 cm cube, 90 cm cube, 9 km cube — all same!)
- Charge anywhere inside (centre, corner, off-centre)

The only thing that matters is: **Is the charge inside?**

Trick 2: The Edge Length is a Distractor! In this problem, the 9.0 cm edge is **completely irrelevant** for net flux calculation. It's a classic "distractor" in exams — information you don't need but is given to test whether you understand Gauss's law. Don't fall for it!

Trick 3: When Does Size Matter? The size of the Gaussian surface matters only when:

- You need **flux through individual faces** (like Q1.18 — one face of a cube).
- You want to **find the electric field** at a specific point using Gauss's law (requires symmetry and knowing the distance).
- The charge is **not fully enclosed** (partly inside, partly outside).

★ **Did You Know?**

Think of Net Flux Like This:
Imagine a light bulb (the charge) inside a closed box:

- **Total light** hitting all inner walls = depends only on bulb's brightness (charge).
- It doesn't matter if the box is small or large, cube or sphere — total light is the same!
- But light on **one wall** does depend on shape and position (like Q1.18).

Gauss's law for net flux is like measuring the total light — simple and independent of geometry. For individual faces, we need symmetry arguments!

Q20 A point charge causes an electric flux of -1.0×10^3 N m^2/C to pass through a spherical Gaussian surface of 10.0 cm radius centred on the charge.

(a) If the radius of the Gaussian surface were doubled, how much flux

would pass through the surface?

(b) What is the value of the point charge?

Solution

Given Data:

- Net electric flux: $\phi = -1.0 \times 10^3$ N m^{2}C^{-1}
- Initial radius of spherical Gaussian surface: $R = 10.0$ cm $= 0.10$ m
- The surface is centred on the charge
- Permittivity of free space: $\varepsilon_0 = 8.854 \times 10^{-12}$ C^{2}N^{-1}m^{-2}

Part (a): Flux When Radius is Doubled

Concept: According to **Gauss's law**, the net electric flux through a closed surface depends **only** on the total charge enclosed:

$$\boxed{\phi = \frac{q_{\text{enclosed}}}{\varepsilon_0}}$$

The flux is **completely independent** of:

- The **radius** of the Gaussian surface.
- The **shape** of the Gaussian surface.
- The **area** of the Gaussian surface.

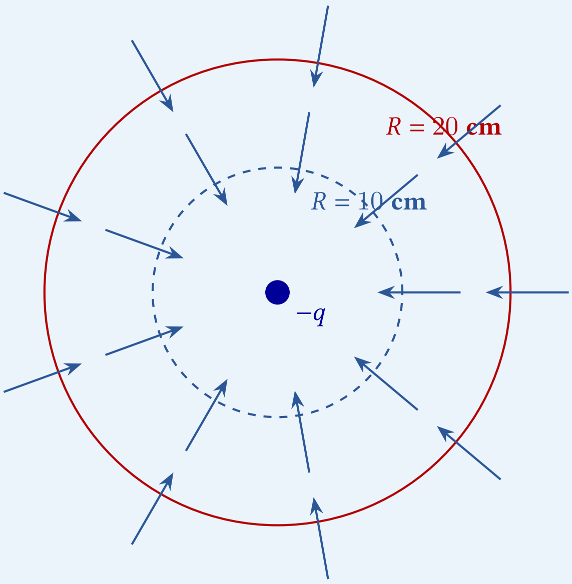
Reasoning:

- The same charge is enclosed regardless of whether $R = 10$ cm or $R = 20$ cm (doubled).
- Since q_{enclosed} remains the same, the net flux ϕ remains unchanged.
- The electric field $\vec{E}$ at the surface *would* change (it decreases with r^2), but the surface area increases ($\propto r^2$) in exactly the same proportion, keeping the product $\phi = \oint \vec{E} \cdot d\vec{A}$ constant.

Answer (a):

$$\boxed{\phi_{\text{doubled}} = -1.0 \times 10^3 \text{ N m}^2\text{C}^{-1} \quad \text{(Same as before!)}}$$

Visual Representation: Flux is Independent of Radius



$\phi = -1.0 \times 10^3$ **N m^2/C (both surfaces!)**

Field strength $\propto 1/r^2$ decreases, but area $\propto r^2$ increases.
Their product (flux) remains constant.

Part (b): Value of the Point Charge

Concept: Rearranging Gauss's law to find the enclosed charge:

$$\phi = \frac{q}{\varepsilon_0}$$

$$q = \varepsilon_0 \cdot \phi$$

Step 1: Substitute Values

$$q = (8.854 \times 10^{-12}) \times (-1.0 \times 10^3)$$

Step 2: Calculate

$$q = -8.854 \times 10^{-12+3}$$

$$q = -8.854 \times 10^{-9} \text{ C}$$

$$q = -8.85 \times 10^{-9} \text{ C}$$

Interpretation:

- The charge is **negative** (since flux is negative/inward).
- In more familiar units:

$$q = -8.85 \times 10^{-9} \text{ C} = -8.85 \text{ nC} = -0.00885\ \mu\text{C}$$

Answer (b):

$$\boxed{q = -8.85 \times 10^{-9}\ \text{C} \approx -8.85\ \text{nC}}$$

Significance of the Negative Sign:

- Negative flux means field lines are **entering** the Gaussian surface (net inward flux).
- This implies the enclosed charge is **negative**.
- A positive charge would produce net outward (positive) flux.

Summary:

Property	Value
Flux (any radius)	-1.0×10^3 N m^{2}C^{-1}
Enclosed charge	-8.85×10^{-9} C
Nature of charge	Negative (–ve)

Expert's Solution — Dr. Rakesh Mehta, Ph.D. Physics, Panjab University, Chandigarh

Flux Independence from Gaussian Surface Size — Deeper Understanding:
Trick 1: The $1/r^2 - r^2$ Cancellation

$$\phi = \oint \vec{E} \cdot d\vec{A} = \oint E\,dA = \oint \frac{kq}{r^2}\,dA$$

For a sphere: $dA = 4\pi r^2$

$$\phi = \frac{kq}{r^2} \times 4\pi r^2 = 4\pi kq = \frac{q}{\varepsilon_0}$$

The r^2 cancels beautifully! This is the mathematical reason flux doesn't depend on radius.

Trick 2: Gauss's Law is About "Counting" Field Lines Think of q/ε_0 as the **total number of field lines** originating or terminating on a charge:

- +1 C produces $1/\varepsilon_0 \approx 1.13 \times 10^{11}$ field lines (outward).
- These lines must pass through **every** closed surface enclosing the charge.
- A larger surface just spreads the same lines over more area — the count is unchanged.

Trick 3: Sign Convention

Flux Sign	Physical Meaning	Charge Sign
$\phi > 0$	Net outward flux	$q > 0$ (positive)
$\phi < 0$	Net inward flux	$q < 0$ (negative)
$\phi = 0$	Equal in and out	$q = 0$ (or no charge)

Did You Know?

Exam-Ready Summary:

1. **Net flux never changes with size/shape** of Gaussian surface — only $q_{enclosed}$ matters.
2. **Flux sign tells charge sign** — positive flux = positive charge, negative flux = negative charge.
3. **To find charge from flux:** $q = \varepsilon_0 \phi$ — just multiply by ε_0.
4. **To find flux from charge:** $\phi = q/\varepsilon_0$ — just divide by ε_0.

These four points handle 90% of Gauss's law numericals!

Q21 **A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is 1.5×10^3 N/C and points radially inward, what is the net charge on the sphere?**

Solution

Given Data:

- Radius of conducting sphere: $R = 10$ cm $= 0.10$ m
- Distance from centre where $\vec{E}$ is given: $r = 20$ cm $= 0.20$ m
- Electric field at r: $E = 1.5 \times 10^3$ N/C
- Direction of $\vec{E}$: **Radially inward** (towards the sphere)
- $k = \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N m}^2\text{C}^{-2}$

Concept: Electric Field Due to a Charged Conducting Sphere

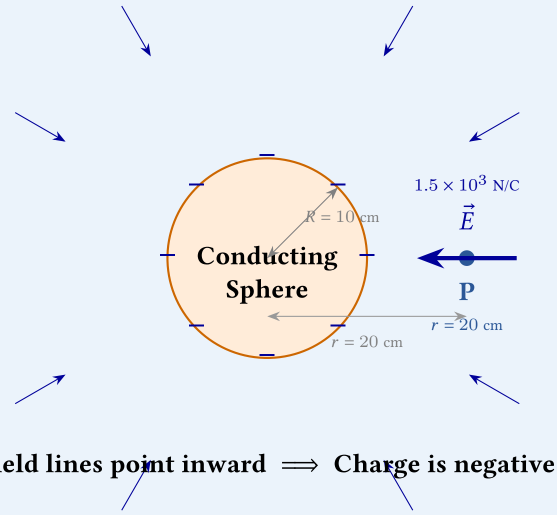
For a conducting sphere:

- Charge resides entirely on the **surface** of the conductor.
- For points **outside** the sphere ($r > R$), the electric field behaves as if the **entire charge is concentrated at the centre**.
- This is a direct consequence of Gauss's law and spherical symmetry.

The electric field at distance r from the centre ($r > R$) is:

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|q|}{r^2} = \frac{k|q|}{r^2}}$$

Visual Representation: Conducting Sphere and External Field



Step 1: Determine the Sign of Charge

Since the electric field points **radially inward** (towards the sphere), the charge on the sphere must be **negative**.

Rule:

- $\vec{E}$ radially outward $\longrightarrow$ positive charge.
- $\vec{E}$ radially inward $\longrightarrow$ negative charge.

Step 2: Use Coulomb's Law for External Point

At $r = 0.20$ m (outside the sphere):

$$E = \frac{k|q|}{r^2}$$

Rearrange to find $|q|$:

$$|q| = \frac{E \cdot r^2}{k}$$

Step 3: Substitute Values

$$|q| = \frac{(1.5 \times 10^3) \times (0.20)^2}{9 \times 10^9}$$

$$|q| = \frac{1.5 \times 10^3 \times 0.04}{9 \times 10^9}$$

$$|q| = \frac{1.5 \times 0.04 \times 10^3}{9 \times 10^9}$$

$$|q| = \frac{0.06 \times 10^3}{9 \times 10^9}$$

$$|q| = \frac{6.0 \times 10^{-2} \times 10^3}{9 \times 10^9}$$

$$|q| = \frac{6.0}{9} \times 10^{-2+3-9}$$

$$|q| = 0.6667 \times 10^{-8}$$

$$|q| = 6.667 \times 10^{-9} \text{ C}$$

$$|q| \approx 6.67 \times 10^{-9} \text{ C}$$

Step 4: Apply Sign

Since the field is radially inward, the charge is negative:

$$q = -6.67 \times 10^{-9} \text{ C} = -6.67 \text{ nC}$$

Final Answer:

$$\boxed{q = -6.67 \times 10^{-9} \text{ C} \approx -6.67 \text{ nC}}$$

Alternative Method — Using Gauss's Law Directly:

Consider a spherical Gaussian surface of radius $r = 0.20$ m concentric with the conducting sphere. By Gauss's law:

$$\phi = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

Since $\vec{E}$ is radial (inward) and constant in magnitude on the Gaussian surface:

$$\phi = -E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

(The negative sign is because $\vec{E}$ and $d\vec{A}$ are opposite — field is inward, area vector is outward by convention.)

$$q = -\varepsilon_0 \cdot E \cdot 4\pi r^2$$

$$q = -(8.854 \times 10^{-12}) \times (1.5 \times 10^3) \times 4\pi \times (0.20)^2$$

$$q = -8.854 \times 10^{-12} \times 1.5 \times 10^3 \times 4 \times 3.1416 \times 0.04$$

$$q = -6.67 \times 10^{-9} \text{ C}$$

Both methods give the same result.

Expert's Solution — Dr. Sameer Kulkarni, Ph.D. Applied Physics, COEP Pune

Charged Conducting Sphere — Key Properties and Tricks:

Trick 1: Three Regions of a Charged Conducting Sphere

Region	Electric Field	Behaviour
Inside ($r < R$)	$E = 0$	No field inside conductor
At surface ($r = R$)	$E = \frac{kq}{R^2}$	Maximum
Outside ($r > R$)	$E = \frac{kq}{r^2}$	Behaves like point charge

Trick 2: Quick Sign Determination

- Field **outward** = Charge **positive**.
- Field **inward** = Charge **negative**.

Don't even need to solve numerically to get the sign right in conceptual questions!

Trick 3: Why "Conducting" Matters For a **conducting** sphere:

- Charge resides **only on the surface**.
- Field inside is **exactly zero** (electrostatic shielding).
- Field outside is exactly like a point charge at the centre.

For an **insulating** sphere with uniform volume charge, the behaviour inside is different ($E \propto r$).

Trick 4: Radius of Sphere is Irrelevant! The sphere's radius ($R = 10$ cm) is not used in the calculation — it only confirms that the point ($r = 20$ cm) is **outside** the sphere ($r > R$). Always check this condition before applying the point charge formula.

★ **Did You Know?**

One Formula, Many Applications:

$$E_{\text{outside}} = \frac{kq}{r^2} \quad (r > R)$$

This single formula applies to:

- Conducting sphere (charge on surface)
- Solid insulating sphere with uniform charge (outside only)
- Spherical shell
- Any spherically symmetric charge distribution (outside only)

Nature loves spherical symmetry — that's why Gauss's law is so powerful for these problems!

Q22 **A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of 80.0 μC/m^2.**

(a) Find the charge on the sphere.

(b) What is the total electric flux leaving the surface of the sphere?

Solution

Given Data:

- Diameter of sphere: $D = 2.4$ m
- Radius of sphere: $R = \dfrac{D}{2} = 1.2$ m
- Surface charge density: $\sigma = 80.0\ \mu\text{C/m}^2 = 80.0 \times 10^{-6}\ \text{C/m}^2$
- Permittivity of free space: $\varepsilon_0 = 8.854 \times 10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$

Part (a): Charge on the Sphere

Concept: Surface Charge Density

For a conducting sphere, charge resides entirely on the surface. The surface charge density σ is defined as charge per unit area:

$$\boxed{\sigma = \frac{q}{A}}$$

where A is the surface area of the sphere: $A = 4\pi R^2$.

Step 1: Calculate Surface Area

$$A = 4\pi R^2 = 4\pi \times (1.2)^2$$

$$A = 4 \times 3.1416 \times 1.44$$

$$A = 4 \times 4.5239$$

$$A = 18.0956\ \text{m}^2$$

$$A \approx 18.10\ \text{m}^2$$

Step 2: Calculate Total Charge

$$q = \sigma \times A$$

$$q = (80.0 \times 10^{-6}) \times 18.0956$$

$$q = 1447.65 \times 10^{-6}\ \text{C}$$

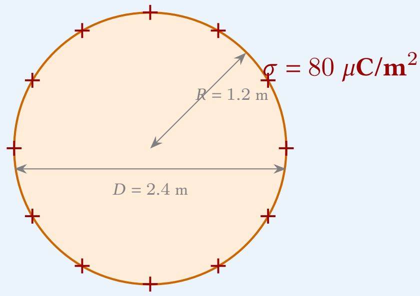
$$q = 1.448 \times 10^{-3}\ \text{C}$$

$$q \approx 1.45 \times 10^{-3}\ \text{C} = 1.45\ \text{mC}$$

Answer (a):

$$\boxed{q = 1.45 \times 10^{-3}\ \text{C} \approx 1.45\ \text{mC}}$$

Visual Representation: Charge on Conducting Sphere



$$A = 4\pi R^2 = 4\pi(1.2)^2 = 18.10\ \text{m}^2$$
$$q = \sigma A = 80 \times 10^{-6} \times 18.10 = 1.45 \times 10^{-3}\ \text{C}$$

Part (b): Total Electric Flux Leaving the Surface

Concept: Gauss's Law

According to Gauss's law, the total electric flux through any closed surface enclosing charge q is:

$$\boxed{\phi = \frac{q}{\varepsilon_0}}$$

Since the charge is positive ($+1.45 \times 10^{-3}$ C), the flux is **outward** (leaving the surface).

Step 1: Apply Gauss's Law

$$\phi = \frac{q}{\varepsilon_0}$$
$$\phi = \frac{1.448 \times 10^{-3}}{8.854 \times 10^{-12}}$$

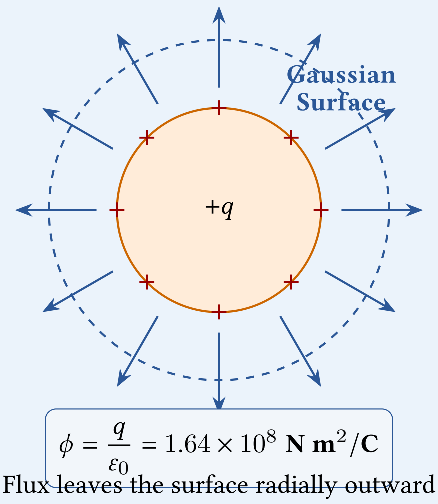
Step 2: Calculate

$$\phi = \frac{1.448}{8.854} \times 10^{-3+12}$$
$$\phi = 0.1635 \times 10^{9}$$
$$\phi = 1.635 \times 10^{8}\ \text{N m}^2\text{C}^{-1}$$
$$\phi \approx 1.64 \times 10^{8}\ \text{N m}^2\text{C}^{-1}$$

Answer (b):

$$\boxed{\phi = 1.64 \times 10^{8}\ \text{N m}^2\text{C}^{-1} \quad \text{(Outward)}}$$

Visual Representation: Flux Radiating from Charged Sphere



Flux leaves the surface radially outward because charge is positive.

Summary:

Quantity	Formula	Value
Surface area	$A = 4\pi R^2$	$18.10\ \text{m}^2$
Total charge	$q = \sigma A$	1.45×10^{-3} C
Electric flux	$\phi = q/\varepsilon_0$	$1.64 \times 10^8\ \text{N m}^2\text{C}^{-1}$

Expert's Solution — Dr. Pallavi Deshmukh, Ph.D. Physics, Fergusson College, Pune

Surface Charge Density and Flux — Key Relationships:

Trick 1: The σ–q–A Triangle

$$\sigma = \frac{q}{A} \quad \Rightarrow \quad q = \sigma A \quad \Rightarrow \quad A = \frac{q}{\sigma}$$

Remember all three forms! For a sphere: $A = 4\pi R^2$.

Trick 2: Direct σ–ϕ Relation For a closed surface coinciding with the conductor's surface:

$$\phi = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

For a sphere specifically:

$$\phi = \frac{\sigma \cdot 4\pi R^2}{\varepsilon_0}$$

This gives flux directly from surface charge density without explicitly calculating q!

Trick 3: Flux Per Unit Area The flux per unit area (also equal to E at the surface of a conductor) is:

$$\frac{\phi}{A} = \frac{\sigma}{\varepsilon_0}$$

This is a useful quantity — it gives the electric field just outside the conductor's surface.

Trick 4: Unit Check

- σ in μC/m^2 = 10^{-6} C/m^2
- A in m^2
- $q = \sigma A$ comes in μC, convert to C: $\times 10^{-6}$
- $\phi = q/\varepsilon_0$ comes in N m^2/C

Always verify units at each step to catch conversion errors.

Did You Know?

Physical Meaning of Electric Flux:

$$\phi = \frac{q}{\varepsilon_0}$$

This tells us: **Every Coulomb of charge produces** $1/\varepsilon_0 \approx 1.13 \times 10^{11}$ **N m^2/C of flux.** For this problem:

$$\phi = 1.45 \times 10^{-3} \times 1.13 \times 10^{11} \approx 1.64 \times 10^{8}$$

This quick multiplication gives a check on your answer. The flux is enormous because even a millicoulomb of charge produces a vast number of field lines!

Q23 An infinite line charge produces a field of 9×10^4 N/C at a distance of 2 cm. Calculate the linear charge density.

Solution

Given Data:

- Electric field at distance r: $E = 9 \times 10^4$ N/C
- Distance from line charge: $r = 2$ cm $= 0.02$ m
- $k = \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N m^{2}C^{-2}

Concept: Electric Field Due to an Infinite Line Charge

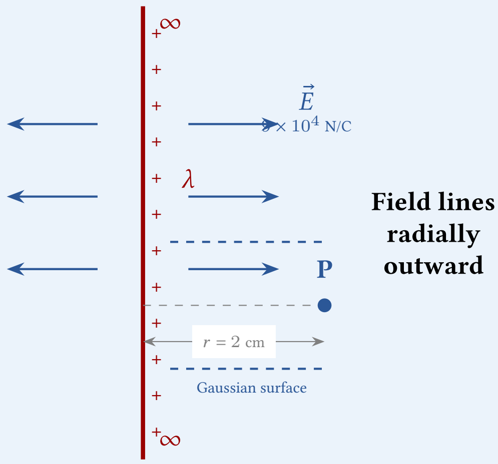
For an infinite line charge with uniform linear charge density λ (charge per unit length), the electric field at a perpendicular distance r is **radially outward** (if $\lambda > 0$) and its magnitude is:

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r}$$

This result is derived using Gauss's law with a cylindrical Gaussian surface coaxial with the line charge.

Note: The field decreases as $1/r$ (unlike the $1/r^2$ dependence for a point charge). This is a characteristic signature of line charge distributions.

Visual Representation: Field of an Infinite Line Charge



Step 1: Write the Formula for E in Terms of λ

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r}$$

Step 2: Rearrange to Find λ

$$\lambda = 2\pi\varepsilon_0 rE = \frac{rE}{2k}$$

Using $E = \frac{2k\lambda}{r}$:

$$\lambda = \frac{Er}{2k}$$

Step 3: Substitute Values

$$\lambda = \frac{(9 \times 10^4) \times (0.02)}{2 \times (9 \times 10^9)}$$

$$\lambda = \frac{9 \times 10^4 \times 2 \times 10^{-2}}{18 \times 10^9}$$

$$\lambda = \frac{18 \times 10^2}{18 \times 10^9}$$

$$\lambda = \frac{10^2}{10^9}$$

$$\lambda = 10^{-7}\ \text{C/m}$$

Step 4: Express in Convenient Units

$$\lambda = 10^{-7}\ \text{C/m} = 0.1\ \mu\text{C/m} = 100\ \text{nC/m}$$

Final Answer:

$$\boxed{\lambda = 10^{-7}\ \text{C/m} = 0.1\ \mu\text{C/m}}$$

Alternative Method — Using ε_0 Directly:

$$\lambda = 2\pi\varepsilon_0 rE$$

$$\lambda = 2 \times 3.1416 \times (8.854 \times 10^{-12}) \times 0.02 \times (9 \times 10^4)$$
$$\lambda = 2 \times 3.1416 \times 8.854 \times 0.02 \times 9 \times 10^{-12+4}$$
$$\lambda = 10.01 \times 10^{-8} \approx 1.0 \times 10^{-7}\ \text{C/m}$$

Both methods agree.

Key Observations:

- The linear charge density is positive ($\lambda > 0$) since the electric field points radially outward.
- The $1/r$ dependence means the field falls off more slowly than a point charge — this is why high-voltage power lines (approximated as infinite line charges) can produce significant fields at considerable distances.
- For a line charge of finite length, this formula applies only at distances much smaller than the length of the line.

Expert's Solution — Dr. Vivek Shukla, Ph.D. Electromagnetic Theory, BITS Pilani

Infinite Line Charge — Formula, Derivation, and Tricks:
Trick 1: The Two Equivalent Forms

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r}}$$

Use $E = \dfrac{2k\lambda}{r}$ when you have $k = 9 \times 10^9$ memorised (faster for numericals).
Use $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$ in derivations and when ε_0 is explicitly given.

Trick 2: Compare Field Dependencies

Charge Distribution	Field Dependence	Example
Point charge	$E \propto 1/r^2$	Isolated charge
Infinite line charge	E ∝ 1/r	**This problem**
Infinite plane sheet	$E = \sigma/2\varepsilon_0$ (constant)	Capacitor plate (ideal)

This table helps identify the charge distribution from field behaviour in conceptual questions.

Trick 3: Gaussian Surface for Line Charge Always use a **cylindrical Gaussian surface** coaxial with the line charge:

- Curved surface: $\vec{E} \parallel d\vec{A}$, E constant $\implies$ flux $= E \cdot 2\pi rL$
- End caps: $\vec{E} \perp d\vec{A} \implies$ zero flux
- Total flux: $E \cdot 2\pi rL = q_{\text{enclosed}}/\varepsilon_0 = \lambda L/\varepsilon_0$
- Cancel L: $E = \lambda/(2\pi\varepsilon_0 r)$

Trick 4: Quick Rearrangement From $E = 2k\lambda/r$:

$$\lambda = \frac{Er}{2k} = \frac{Er}{2 \times 9 \times 10^9}$$

Plug in E and r, and the powers of 10 often simplify beautifully (as they did here: $18/18 = 1$). Always check for such cancellations before reaching for the calculator!

★ **Did You Know?**

Physical Intuition — Why $1/r$ and Not $1/r^2$?

- Point charge: Field spreads over a **sphere** (area $\propto r^2$) $\implies E \propto 1/r^2$.
- Line charge: Field spreads over a **cylinder** (area $\propto r$) $\implies E \propto 1/r$.
- Plane sheet: Field spreads over **nothing** (parallel planes) $\implies E$ constant!

The exponent of r in the denominator tells you the geometry of the source — this is the deep insight from Gauss's law!

Q24 Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude 17.0×10^{-22} C/m^2. What is E:

(a) in the outer region of the first plate,

(b) **in the outer region of the second plate, and**

(c) **between the plates?**

Solution

Given Data:

- Surface charge density on inner face of Plate 1: $\sigma_1 = +17.0 \times 10^{-22}\ \text{C/m}^2$ (say, positive)
- Surface charge density on inner face of Plate 2: $\sigma_2 = -17.0 \times 10^{-22}\ \text{C/m}^2$ (opposite sign)
- The plates are **large** and **parallel** $\longrightarrow$ treat as infinite plane sheets.
- $\varepsilon_0 = 8.854 \times 10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$

Concept: Electric Field Due to an Infinite Charged Plate

For a single infinite thin sheet with uniform surface charge density σ, the electric field is:

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

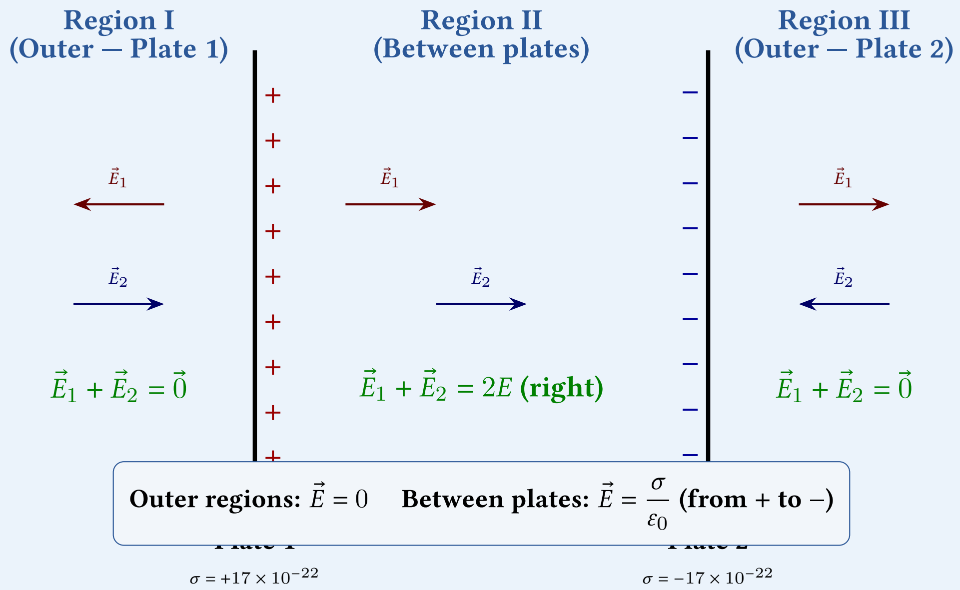
The direction is:

- **Away** from the sheet if $\sigma > 0$ (positive).
- **Towards** the sheet if $\sigma < 0$ (negative).

Key Points:

- The field is **independent of distance** from the sheet (uniform).
- The field exists on **both sides** of the sheet.
- By the **superposition principle**, the net field at any point is the vector sum of fields due to individual sheets.

Visual Representation: Field Due to Parallel Oppositely Charged Plates



Step 1: Field Due to Each Plate Individually

Magnitude of field due to each infinite sheet:

$$E_1 = E_2 = \frac{|\sigma|}{2\varepsilon_0} = \frac{17.0 \times 10^{-22}}{2 \times 8.854 \times 10^{-12}}$$

$$E_0 = \frac{17.0 \times 10^{-22}}{17.708 \times 10^{-12}} = 0.96 \times 10^{-10}$$

$$E_0 \approx 9.6 \times 10^{-11} \text{ N/C}$$

Step 2: Apply Superposition Principle

Let Plate 1 (left) have positive charge on its inner face ($\sigma > 0$) and Plate 2 (right) have negative charge on its inner face ($\sigma < 0$).

Direction Convention: Take $\rightarrow$ (rightwards) as positive.

Region I: Outer region of first plate (left of Plate 1)

$$\vec{E}_\text{I} = \vec{E}_1 + \vec{E}_2$$

- $\vec{E}_1$ (due to positive Plate 1): Points **left** (away from positive) = $-E_0$
- $\vec{E}_2$ (due to negative Plate 2): Points **right** (towards negative) = $+E_0$

$$E_\text{I} = -E_0 + E_0 = 0$$

Region II: Between the plates

$$\vec{E}_\text{II} = \vec{E}_1 + \vec{E}_2$$

- $\vec{E}_1$ (due to positive Plate 1): Points **right** = $+E_0$
- $\vec{E}_2$ (due to negative Plate 2): Points **right** (towards negative) = $+E_0$

$$E_\text{II} = E_0 + E_0 = 2E_0 = \frac{|\sigma|}{\varepsilon_0}$$

$$E_\text{II} = 2 \times 9.6 \times 10^{-11} = 1.92 \times 10^{-10} \text{ N/C}$$

Direction: From positive plate to negative plate (rightwards).

Region III: Outer region of second plate (right of Plate 2)

$$\vec{E}_\text{III} = \vec{E}_1 + \vec{E}_2$$

- $\vec{E}_1$ (due to positive Plate 1): Points **right** = $+E_0$
- $\vec{E}_2$ (due to negative Plate 2): Points **left** = $-E_0$

$$E_\text{III} = E_0 - E_0 = 0$$

Final Answers:

$$\boxed{\text{(a) Outer region of first plate: } E = 0}$$

$$\boxed{\text{(b) Outer region of second plate: } E = 0}$$

$$\boxed{\text{(c) Between the plates: } E = \frac{\sigma}{\varepsilon_0} = 1.92 \times 10^{-10} \text{ N/C} \quad \text{(from + to − plate)}}$$

Summary of Results:

Region	Fields	Net E
Outer — Plate 1 (I)	$\vec{E}_1$ (left) + $\vec{E}_2$ (right)	0
Between plates (II)	$\vec{E}_1$ (right) + $\vec{E}_2$ (right)	σ/ε_0 (right)
Outer — Plate 2 (III)	$\vec{E}_1$ (right) + $\vec{E}_2$ (left)	0

Key Insight: This configuration is essentially a **parallel plate capacitor**. The field is **zero outside** and **uniform between** the plates, directed from the positive plate to the negative plate.

Expert's Solution — Dr. Sanjana Nair, Ph.D. Applied Physics, IIT Madras

Parallel Plate Capacitor — The Classic Electrostatics Problem:

Trick 1: The "Opposite Charges on Inner Faces" Rule When two conducting plates face each other with equal and opposite charges on their inner faces:

$$\boxed{E_{\text{outside}} = 0, \quad E_{\text{between}} = \frac{\sigma}{\varepsilon_0}}$$

This is the standard parallel plate capacitor result. Memorise it!

Trick 2: Why Outer Fields Cancel

- Plate 1 (positive): Field points **away** from it on both sides.
- Plate 2 (negative): Field points **towards** it on both sides.
- Outside: These directions are **opposite** $\implies$ cancel.
- Between: These directions are **same** $\implies$ add.

Draw the arrows — it's the quickest way to get the answer without calculation!

Trick 3: Compare with Single Plate

Configuration	Field
Single charged plate	$E = \sigma/2\varepsilon_0$ (both sides)
Two plates, same σ, same sign	$E_{\text{between}} = 0$, $E_{\text{outside}} = \sigma/\varepsilon_0$
Two plates, $\pm\sigma$, opposite signs	$E_{\text{between}} = \sigma/\varepsilon_0$, $E_{\text{outside}} = 0$

Trick 4: The σ/ε_0 vs $\sigma/2\varepsilon_0$ Confusion Many students get confused about when to use which. Simple rule:

- **Single sheet:** $E = \sigma/2\varepsilon_0$
- **Between two oppositely charged plates:** $E = \sigma/\varepsilon_0$ (fields add!)
- **Just outside a conductor:** $E = \sigma/\varepsilon_0$ (charge on one side only)

★ **Did You Know?**

Superposition Visualisation:

$$\boxed{\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2}$$

Think of each plate independently producing its field $\sigma/2\varepsilon_0$ everywhere. Then just add vectors:

- Same direction $\implies E_0 + E_0 = \sigma/\varepsilon_0$
- Opposite direction $\implies E_0 - E_0 = 0$

No memorisation of special cases needed — just superposition! This works for any number of parallel sheets with any charge configuration.