Chapter 10

Wave Optics

Light is a wave (Huygens 1678, Young 1801);
ray optics is only the short - wavelength limit.

Why a wave theory ?

Ray optics treats light as straight rays.
It cannot explain :
* (i) interference (alternate bright / dark)
(ii) diffraction (bending around obstacles)
(iii) polarisation (transverse nature)

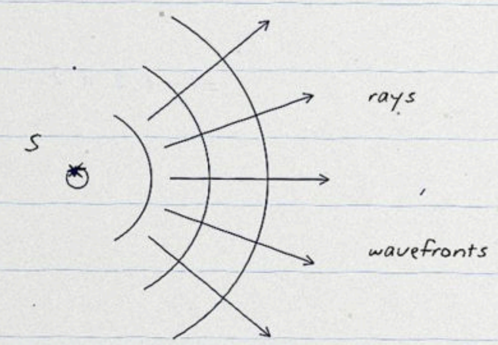


Fig. rays are perpendicular to wavefronts.

Topics ahead

Huygens ; YDSE ; diffraction ; polarisation.

Wavefront

A wavefront is a surface (locus) on which all points oscillate in the same phase.
Light travels perpendicular to the wavefront.

Types of wavefront

(1) Spherical : from a ~~flat~~ point source.
radius grows with time ; $r = c.t.$

(2) Cylindrical : from a line source
(e.g. a slit illuminated by lamp).

(3) Plane : far away from any small source
(spherical of huge r looks flat locally).

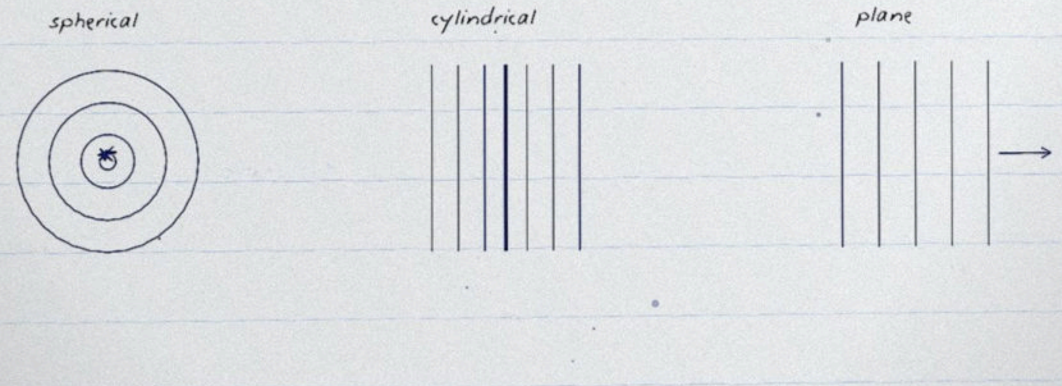


Fig. spherical , cylindrical , plane wavefronts

Energy flows in direction of ray = normal.

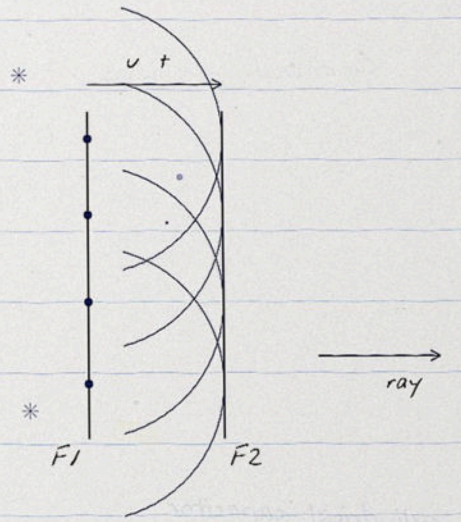
Huygens' Principle

Statement

(i) Every point on a wavefront acts as a source of secondary spherical wavelets.

(ii) The wavelets travel forward with speed v.

(iii) After time t, the new wavefront is the forward tangent (envelope) to all these secondary wavelets.



(modern : zero amp.).
are ignored
Backward wavelets

after time t.

F2 : new wavefront

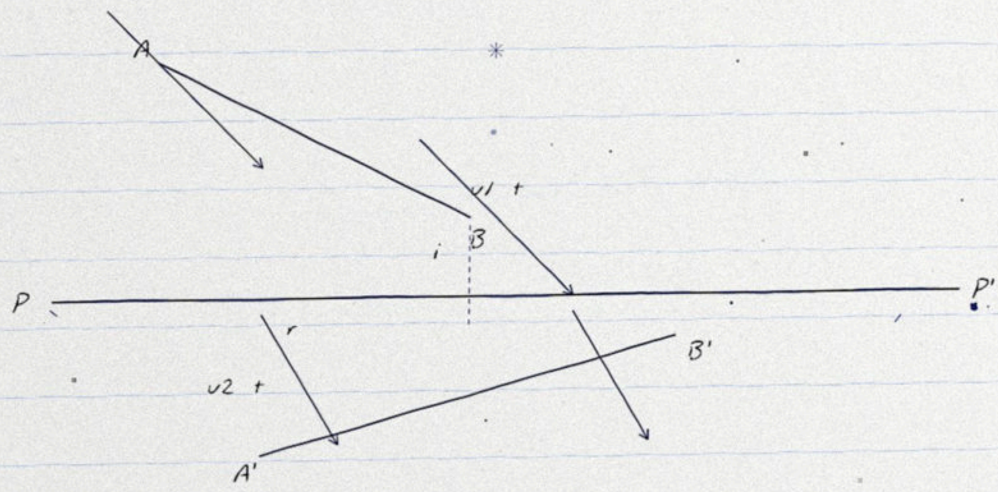
F1 : original

Use : derive laws of reflection, refraction, and explain diffraction.

Limitation : backward wavelets needed Fresnel fix.

Refraction by Huygens' Principle

Plane wave AB hits a plane surface PP' ;
wave speed : $v1$ in medium 1 , $v2$ in medium 2.



$BC = v1\, t$; $AE = v2\, t$ (where C , E = foot)

From right triangles :

$\sin i = BC/AC$; $\sin r = AE/AC$

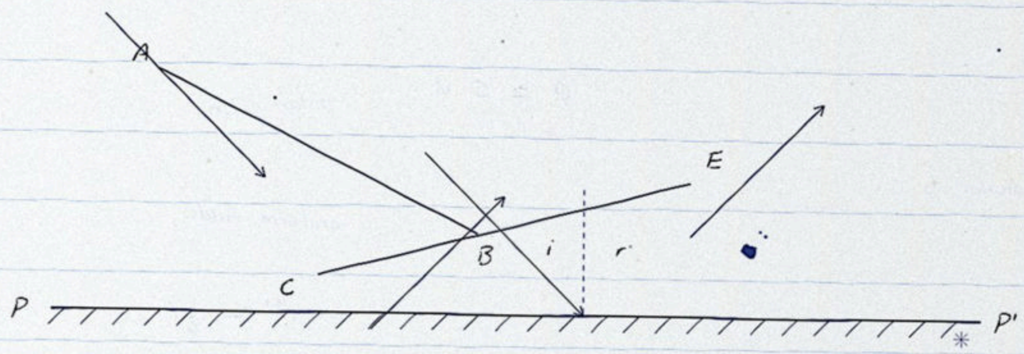
$\boxed{\sin i \,/\, \sin r = v1 \,/\, v2 = n2 \,/\, n1}$ <- Snell's law
<- derived

Frequency stays same ; $\lambda\_2 = \lambda\_1 \; v2 \,/\, v1$.

Denser medium -> smaller speed , smaller lambda.

Reflection by Huygens' Principle

Plane wave AB hits a plane mirror ;
wave speed same on both sides (v).



BC = v t in medium 1 ; AE = v t reflected.
Triangles ABC and AEC are congruent (SSS).

$\boxed{\text{angle } i = \text{angle } r}$ <- law of reflection

Also : incident ray , normal , reflected ray
all lie in one plane (geometry of triangles).
Wave theory recovers ray laws : consistent !

Doppler Effect for Light

When source and observer move relative to each other, the observed frequency changes.

Non - relativistic case ($v \ll c$)

$$\boxed{d\nu / \nu = - v_{radial} / c}$$

<- fractional
<- change

v_{radial} : component of source velocity along line of sight ; positive if moving away.

Observations

(1) Source receding $\rightarrow$ λ increases
called RED SHIFT.

(2) Source approaching $\rightarrow$ λ decreases
called BLUE SHIFT.

Applications

(i) Galaxies $\rightarrow$ red shift $\rightarrow$ universe expands.
(ii) Speed of stars from spectral line shifts.
(iii) Radar speed guns, Doppler weather radar.

Hubble : $v = H_0 d$; inferred from $\Delta\lambda / \lambda$

Exact relativistic form needs special relativity.

Coherent Sources

Two sources are coherent if they emit waves with a constant phase difference in time.

Conditions for sustained interference

1. Sources must be coherent (constant phase difference).
2. Same frequency -> same wavelength.
3. Equal (or nearly equal) amplitudes so dark fringes are truly dark.
4. Same state of polarisation.
5. Sources close enough (small d).

Incoherent = no fixed phase relation

Two ordinary bulbs -> random phase -> no pattern.
Intensities just add : $I = I_1 + I_2$.

Practical trick

Use ONE source split into two by slits / mirror / biprism. Both daughter sources stay locked in phase.

Superposition Principle

At a point reached by two waves, the total displacement is the vector sum of individual displacements (linear superposition).

$y1 = a \cos w t$

$y2 = a \cos (w t + \phi)$

Resultant :

$y = y1 + y2 = 2 a \cos (\phi/2) \cos (w t + \phi/2)$

Amplitude $R = 2 a \cos (phi / 2)$

Intensity I R 2

$\boxed{I = 4 I\_0 \cos 2 (phi / 2}$) <- I_0 = single <- slit intensity

Special cases

phi = 0 , 2 Π , 4 Π ... -> I_max = 4 I_0

phi = Π , 3 Π ... -> I_min = 0

∴

Path difference & phase difference :

$\phi = (2 \Pi / \lambda) \Delta x$

Energy is not destroyed -> redistributed.

Young's Double Slit Experiment

Setup

Monochromatic source S -> narrow slit SO.
-> two close slits S1, S2 (separation d)
-> screen at distance D from the slits.

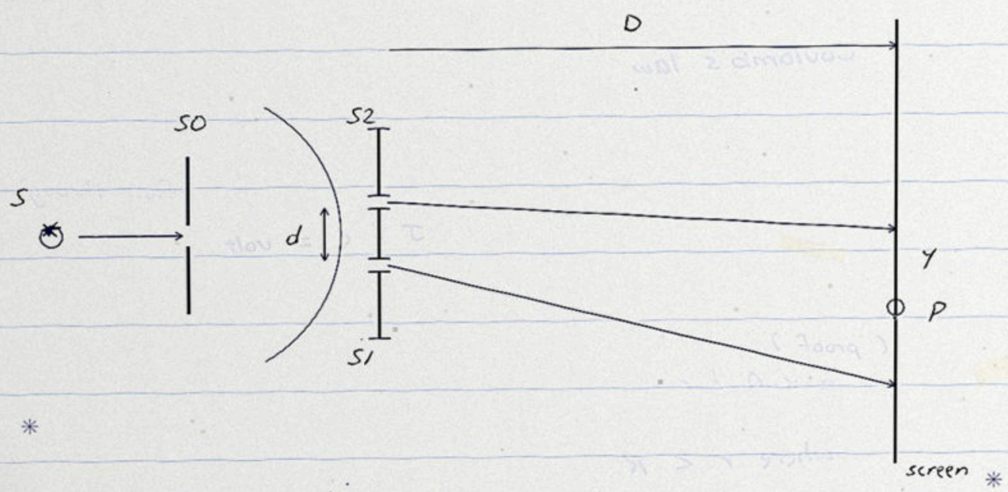


Fig. Young's double - slit arrangement

Path difference

Light from S1 and S2 reaches P with
path difference delta = S2 P - S1 P.

YDSE : Path Difference

Let O be the foot of perpendicular from the slits ; P on screen at distance y from O.

$S1\ P\ \ 2 = D\ \ 2 + (y - d/2)\ \ 2$

$S2\ P\ \ 2 = D\ \ 2 + (y + d/2)\ \ 2$

Subtract :

$S2\ P\ \ 2 - S1\ P\ \ 2 = 2\ y\ d$

$(S2\ P + S1\ P)(S2\ P - S1\ P) = 2\ y\ d$

For $d << D$, both D , so

$S2\ P + S1\ P = 2\ D$

$\boxed{\Delta = S2\ P - S1\ P = y\ d}$ / D <- main result

Equivalent angular form :

$\Delta = d \sin \Theta = d\ \Theta$ (small angle)

where theta $= y / D$ is the angular position of P from central axis.

Phase difference : $\phi = (2\ \pi / \lambda)\ \Delta$

All interference results follow from this.

YDSE : Bright & Dark Fringes

Constructive -> Bright

Waves arrive in phase if path difference is an integer multiple of lambda.

$$\boxed{\Delta = n\lambda \quad , \quad n = 0, \cancel{\pm} 1, 2 \ldots}$$ <- bright

Position of n - th bright fringe :

$y_n = n \lambda D / d$

Destructive -> Dark

Waves arrive out of phase (pi rad) if path difference is half - integer of lambda.

$$\boxed{\Delta = (n + 1/2) \lambda}$$ <- dark

Position of n - th dark fringe :

$y_n' = (n + 1/2) \lambda D / d$

Central fringe

At 0 , delta = 0 -> always bright (white center in white-light fringes ; coloured edges on sides).

Fringe Width

Fringe width beta = spacing between two consecutive bright (or two dark) fringes.

From y_n = n lambda D / d and
y_(n+1) = (n+1) lambda D / d :

$\beta = y_{n+1} - y_n$

$$\boxed{\beta = \lambda D / d}$$

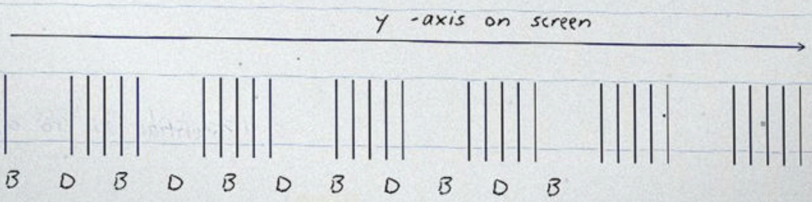
<- fringe width
<- (bright = dark)

Observations

① beta proportional to lambda
red > blue -> red fringes wider.

② beta proportional to D ; inverse to d.

③ Bright and dark widths are equal.



Typical lab : lambda 590nm , D 1m , d 1mm -> beta 0.6mm.

Intensity Distribution in YDSE

If both slits send wave of amplitude a , individual intensity I_O a 2.

Resultant amplitude $R = 2a\cos(\phi/2)$

Resultant intensity $I = 4I_0\cos^2(\phi/2)$

Maxima , minima , average

$I_{max} = 4I_0$ at $\phi = 0, 2\pi \ldots$

$I_{min} = 0$ at $\phi = \pi, 3\pi \ldots$

$I_{avg} = 2I_0$ (energy conserved)

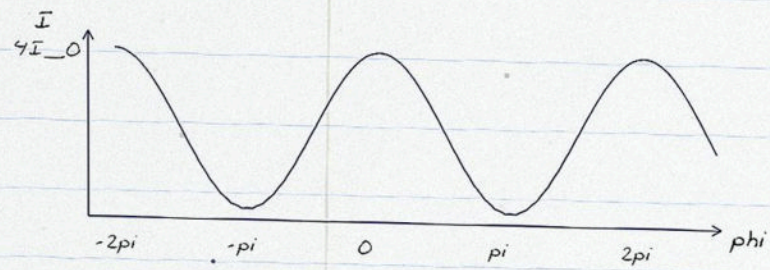


Fig. intensity vs phase difference

Ratio I_max : I_min

For unequal amps : (a1+a2) 2 : (a1-a2) 2.

Worked Example - Fringe Width

Problem

In a YDSE : $d = 0.28$ mm , $D = 1.4$ m , distance between 4th bright fringes on either side of centre = 1.2 cm. Find lambda.

Solution

Distance between 4th order on both sides = 8 beta.

So $8\beta = 1.2$ cm $\rightarrow$ $\beta = 0.15$ cm

From $\beta = \lambda D / d$:

$$\lambda = \beta d / D$$

$$\lambda = (0.15 \times 10^{-2})(0.28 \times 10^{-3}) / 1.4$$

$$\lambda = 3.0 \times 10^{-7} \text{ m} = 300 \text{ nm}$$

Hmm , 300 nm is near UV ; ~~orange~~ ultraviolet.

Sanity check

units : $m \cdot m / m = m$ ~~kg~~ ok.

If we instead used 5th $\rightarrow$ 5th (10 beta = 1.2 cm) , we'd get beta = 0.12 cm , lambda = 240 nm.

Moral : read 'either side' carefully.

Diffraction at a Single Slit

Bending of light around edges : visible only when slit width a is comparable to lambda.

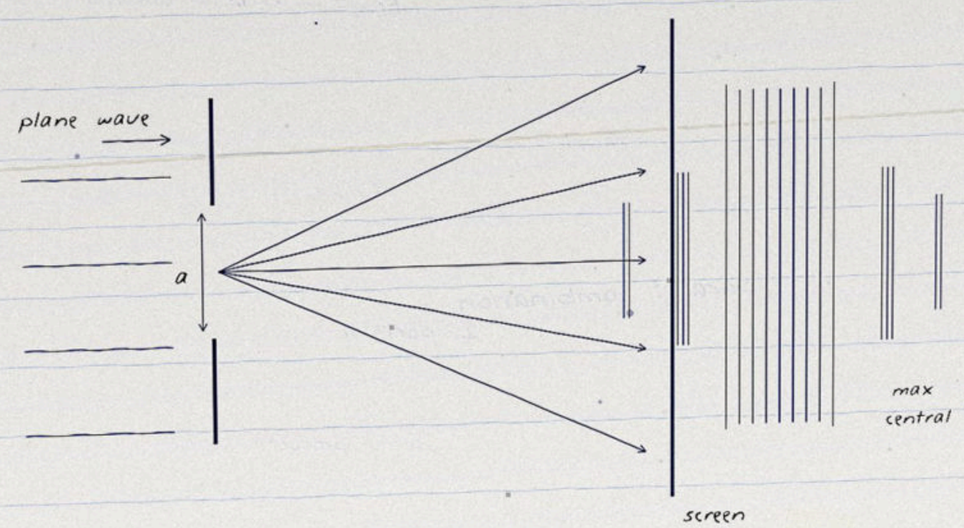


Fig. central bright max + weak side bands

Geometry

Slit of width a ; path difference between extreme rays = a sin theta.

Divide slit into 2 halves , pair points at spacing a/2 cancelling pairwise gives minima.

Single Slit : Minima & Maxima

Condition for minima

Path diff. of halves cancels in pairs when

$$a \sin \theta = n \lambda , \quad n = +1 , +2 , \ldots$$

$$\boxed{a \sin \theta = n \lambda}$$ <- DARK at theta

$n = 0$ is excluded ($n = 0$ -> central max).

Condition for secondary maxima

$$a \sin \theta = (n + 1/2) \lambda$$

These are much fainter than the central max ;

intensities approx $1 : 1/22 : 1/61 : \ldots$

Central maximum width

Between first minima on either side :

$$\sin \theta = \lambda / a \; ; \; y = D \theta$$

$$\boxed{\text{Width} = 2 \lambda D / a}$$ <- central max
<- = 2 x side max

Width grows if a shrinks ; bright region

spreads more -> more diffraction.

Interference vs Diffraction

Property	Interference	Diffraction
Sources	2 coherent	Same wavefront
Fringe sp.	uniform beta	decreasing
Intensity	all bright eq	central biggest
Width	narrow	central = 2x
Min I	exactly zero	not zero
Pattern	B/D/B/D ...	central + bands

Common feature

Both arise from SUPERPOSITION of waves ;
energy is redistributed , not lost.

Ray vs wave validity

Ray optics works while lambda << obstacle.
Fresnel distance z_F = a 2 / lambda :

beyond z_F , beam spreads (Fraunhofer)
before z_F , beam still ray - like

Example : a = 4 mm , lambda = 500 nm
-> z_F = (4e-3) 2 / 5e-7 = 32 m.

Worked Example - Single Slit

Problem

Light of lambda = 600 nm passes through a slit of width 0.12 mm. The screen is 1 m away.

Find : (a) width of central max

(b) distance to 2nd dark fringe.

Solution (a)

Width $= 2 \lambda D / a$

$= 2 (600 \times 10^{-9})(1) / (0.12 \times 10^{-3})$

$= 1.0 \times 10^{-2}$ m = 10 mm

Solution (b)

2nd minimum : a sin theta = 2 lambda

$y_2 = 2 \lambda D / a$

$= 2 (600 \times 10^{-9})(1)/(0.12 \times 10^{-3})$

$= 1.0 \times 10^{-2}$ m = 10 mm

Comment

Central maximum is bright over 10 mm ;

first minimum sits at 5 mm , second at 10 mm.

units check : m . m / m = m ∴ ok.

Compare with double slit : beta would be 5mm.

Resolving Power

Two close objects diffract through any aperture and form overlapping Airy patterns.

Rayleigh criterion : central max of one image falls on first minimum of the other.

Telescope

Smallest angle two stars subtend so they are still resolved :

$$\boxed{\Delta\theta = 1.22 \lambda / D}$$

<- D = aperture
<- diameter

Microscope

Smallest separation between two object points :

$$\boxed{\Delta d = 1.22 \lambda / (2 n \sin \beta)}$$

<- n sin b =
<- numerical ap.

Improving R.P.

(i) Larger aperture D -> smaller delta-theta.
(ii) Shorter lambda (UV / electron) -> finer detail.
(iii) Higher refractive index oil (oil immersion).

Electron microscope : pm wavelengths

Polarisation

Light is a TRANSVERSE EM wave : E and B vibrate perpendicular to direction of travel.

Natural light : E vibrates in all directions perpendicular to propagation -> unpolarised.

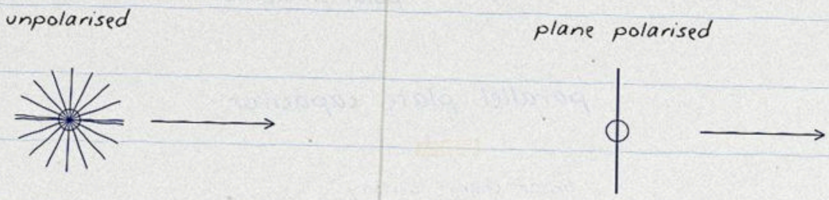


Fig. cross - section seen looking along ray

Linear (plane) polarised

E vibrates in only one fixed plane (plane of polarisation). Sound , being longitudinal , cannot be polarised -> proves light is transverse.

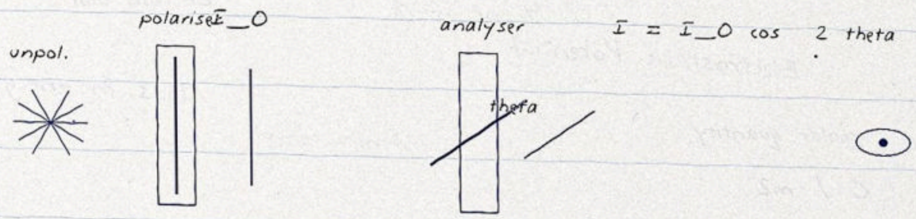
Methods

(1) polariser ; (2) reflection ; (3) scattering.

Polariser & Malus' Law

A POLAROID transmits E - vibrations only along its pass axis ; blocks the perpendicular part.

Unpolarised light -> polaroid -> plane polarised with intensity I_0 / 2.



$$\boxed{I = I_0 \cos^2 \theta .}$$ <- Malus' law

Special cases

theta = 0 -> I = I_0 (axes parallel)

theta = 90 -> I = 0 (axes crossed)

Rotate analyser -> intensity varies as $\cos^2$.

Average over 360 deg : <I> = I_0 / 2.

Polarisation by Reflection

When unpolarised light is reflected from a transparent surface, the reflected light is partially polarised in general.

At one special angle i_B (BREWSTER's angle) the reflected ray is FULLY polarised.

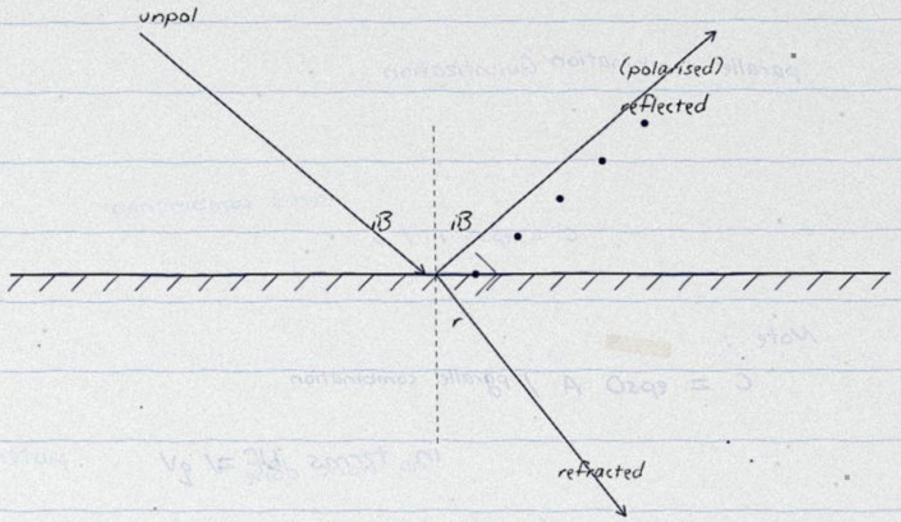


Fig. at $i = i_B$ reflected perpendicular to refracted

$$\boxed{\tan i_B = n}$$ <- Brewster's law

Also : $i_B + r = 90$ deg. For glass i_B 57 deg.

Worked Example - Brewster

Problem

Unpolarised light is incident on water ($n = 1.33$).
Find Brewster's angle and the corresponding angle of refraction.

Solution

$\tan i_B = n = 1.33$

$i_B = \tan^{-1}(1.33)$

$i_B = 53.06$ deg

Using $i_B + r = 90$ deg :

$r = 90 - 53.06 = 36.94$ deg

Cross - check via Snell

$\sin i_B / \sin r = n$

$\sin 53.06 / \sin 36.94$

$= 0.7990 / 0.6010$

$= 1.329$ 1.33 (matches)

Quick facts

Glass ($n=1.5$) : i_B 56.3 deg

Water ($n=1.33$) : i_B 53.1 deg

Diamond ($n=2.4$) : i_B 67.4 deg

Polaroid sunglasses cut horizontal glare !

Polarisation by Scattering

Sunlight scattered by air molecules at 90 deg to the incident direction is plane polarised.

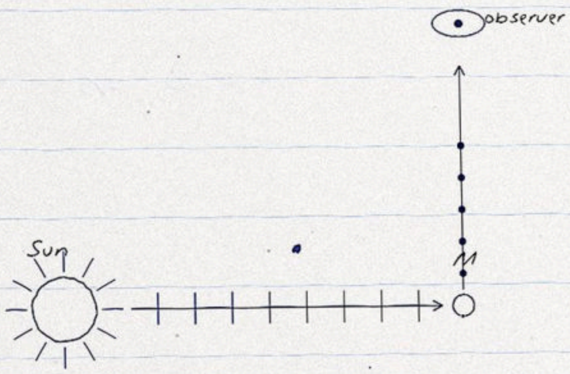


Fig. scattered light at 90 deg is plane - polarised

Why ?

An electron in air molecule oscillates parallel to E of incoming light. It re - radiates only perpendicular to its motion ; hence at 90 deg the scattered light has only one E direction.

Polarisation : Uses

Polaroid sunglasses

Reflected glare off roads / water is mostly horizontally polarised -> vertical pass axis in lens blocks it.

LCD displays

Two crossed polarisers with a twisted nematic liquid crystal sandwich ; voltage controls alignment -> each pixel switches dark / bright.

Photoelasticity

Strain in transparent plastics -> birefringent , shows coloured fringes between crossed polars.

Optical activity

Some substances (sugar , quartz) rotate the plane of polarisation. Used in polarimeter to measure sugar concentration.

Sky colour & polarisation

Rayleigh scattering : I 1 / lambda 4 .

Blue scatters more -> blue sky.

Sunset : long path -> blue gone -> red / orange.

Sky polarisation max at 90 deg from sun.

Wave Optics - Summary

Wavefront & Huygens

Wavefront : surface of constant phase.

Every point -> secondary wavelet -> envelope.

Snell & Reflection

sin i / sin r = v1 / v2 = n2 / n1

angle i = angle r (mirror)

YDSE

$\Delta = d \sin\theta = y d / D$

bright : $\Delta = n\lambda$; dark : $(n+1/2)\lambda$

$\beta = \lambda D / d$; $I = 4 I\_0 \cos 2(\phi/2)$

Single slit

minima : $a \sin\theta = n\lambda$

central max width $= 2 \lambda D / a$

Resolving power

telescope : $\Delta\theta = 1.22 \lambda/D$

microscope : $\Delta d = 1.22 \lambda / (2 n \sin\beta)$

Polarisation

Malus : $I = I\_0 \cos^2\theta$; Brewster : tan i_B

Transverse : light can be polarised, sound cannot.