NCERT SOLUTIONS

Class 12 Physics

Chapter 10: Wave Optics

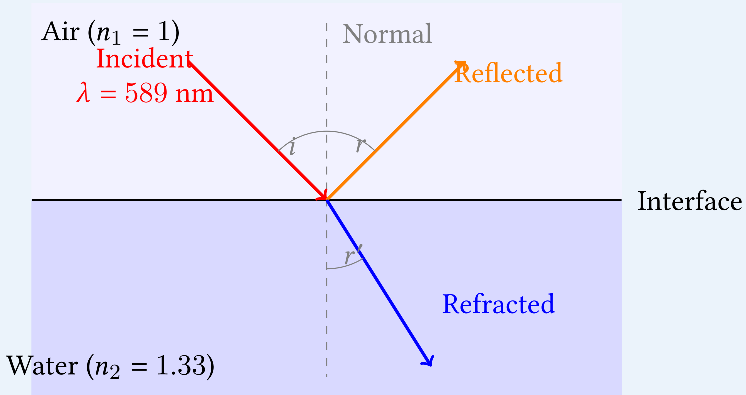
Detailed Step-by-Step Exercise Solutions

Q1 **Monochromatic light of wavelength 589 nm is incident from air on a water surface. What are the wavelength, frequency and speed of (a) reflected, and (b) refracted light? Refractive index of water is 1.33.**

Solution

Understanding the Physical Situation

When light travels from one medium to another, its interaction at the boundary gives rise to both a reflected ray and a refracted (transmitted) ray. The fundamental principle governing this behavior is that the frequency of light remains unchanged regardless of the medium — it depends only on the source. However, the speed and wavelength change upon refraction due to the different optical properties of the medium.



Key Physical Constants and Given Data

- Wavelength in air (vacuum): $\lambda_{air} = 589 \text{ nm} = 5.89 \times 10^{-7}$ m
- Speed of light in vacuum/air: $c = 3.0 \times 10^8$ m/s
- Refractive index of air: $n_1 \approx 1$
- Refractive index of water: $n_2 = 1.33$

Part (a): Reflected Light
The reflected ray remains in the **same medium** (air) as the incident ray. Therefore, all its wave characteristics remain unchanged:

For reflection at a boundary:

- **Speed:** $v_{reflected} = c$ (same medium)
- **Frequency:** $\nu_{reflected} = \nu_{incident}$
- **Wavelength:** $\lambda_{reflected} = \lambda_{incident}$

Step 1: Frequency of Incident Light
First, let's calculate the frequency of the incident light using the relation $c = \nu\lambda$:

$$\nu = \frac{c}{\lambda_{air}}$$

$$\nu = \frac{3.0 \times 10^8 \text{ m/s}}{5.89 \times 10^{-7} \text{ m}}$$

$$\nu = \frac{3.0}{5.89} \times 10^{8+7} = \frac{3.0}{5.89} \times 10^{15} \text{ Hz}$$

$$\nu = 0.5093 \times 10^{15} \text{ Hz} = 5.093 \times 10^{14} \text{ Hz}$$

Step 2: Characteristics of Reflected Light
Since reflection doesn't change the medium:

- **Frequency:** $\nu_{reflected} = 5.09 \times 10^{14}$ Hz (unchanged)
- **Speed:** $v_{reflected} = c = 3.0 \times 10^8$ m/s (still in air)
- **Wavelength:** $\lambda_{reflected} = 589$ nm (unchanged)

Answer (a) — Reflected Light:

$$\boxed{\nu_{reflected} = 5.09 \times 10^{14} \text{ Hz} \quad ; \quad v_{reflected} = 3.0 \times 10^8 \text{ m/s} \quad ; \quad \lambda_{reflected} = 589 \text{ nm}}$$

Part (b): Refracted Light
When light enters water, its speed changes due to the different optical density. The refractive index n relates the speed of light in vacuum to the speed in the medium:

Refractive Index Relation:

$$n = \frac{c}{v} \quad \Rightarrow \quad v = \frac{c}{n}$$

The **frequency remains constant** across any boundary:

$$\nu_{\text{water}} = \nu_{\text{air}} = \nu$$

Using $v = \nu\lambda$, the wavelength in the medium becomes:

$$\lambda_{\text{water}} = \frac{v}{\nu} = \frac{c}{n\nu} = \frac{\lambda_{\text{air}}}{n}$$

Step 1: Speed of Light in Water

$$v_{\text{water}} = \frac{c}{n_2} = \frac{3.0 \times 10^8 \text{ m/s}}{1.33}$$

$$v_{\text{water}} = 2.256 \times 10^8 \text{ m/s} \approx 2.26 \times 10^8 \text{ m/s}$$

Step 2: Frequency in Water

The frequency remains the same as in air:

$$\nu_{\text{water}} = 5.093 \times 10^{14} \text{ Hz} \approx 5.09 \times 10^{14} \text{ Hz}$$

Step 3: Wavelength in Water

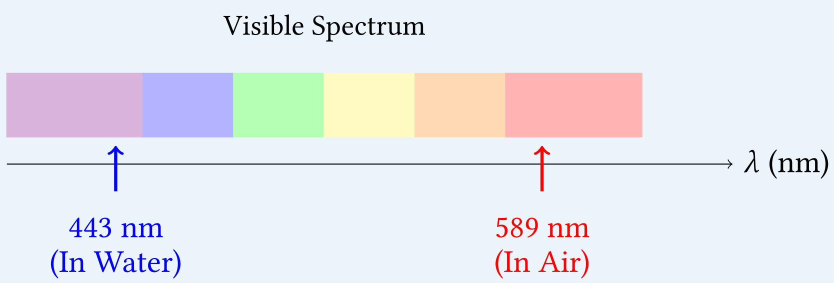
Using the relation $\lambda_{\text{water}} = \lambda_{\text{air}}/n$:

$$\lambda_{\text{water}} = \frac{589 \text{ nm}}{1.33}$$

$$\lambda_{\text{water}} = 442.86 \text{ nm} \approx 443 \text{ nm}$$

Alternatively, verifying with $v = \nu\lambda$:

$$\lambda_{\text{water}} = \frac{v_{\text{water}}}{\nu} = \frac{2.256 \times 10^8}{5.093 \times 10^{14}} = 4.429 \times 10^{-7} \text{ m} = 443 \text{ nm}$$



Interesting Observation: The wavelength has decreased from 589 nm (yellow-orange in air) to about 443 nm (violet-blue in water). However, the **color perceived** is determined by frequency, not wavelength. Since frequency remains unchanged, the light still appears the same color to an observer underwater!

Answer (b) — Refracted Light:

$$\boxed{\nu_{\text{water}} = 5.09 \times 10^{14} \text{ Hz} \quad ; \quad v_{\text{water}} \approx 2.26 \times 10^{8} \text{ m/s} \quad ; \quad \lambda_{\text{water}} \approx 443 \text{ nm}}$$

Summary Table:

Quantity	Incident (Air)	Reflected (Air)	Refracted (Water)
Speed	3.00×10^{8} m/s	3.00×10^{8} m/s	2.26×10^{8} m/s
Frequency	5.09×10^{14} Hz	5.09×10^{14} Hz	5.09×10^{14} Hz
Wavelength	589 nm	589 nm	443 nm

Fundamental Principle:

$$\boxed{\nu_{\text{air}} = \nu_{\text{water}} = \nu_{\text{source}}}$$

Frequency is invariant across boundaries.
Speed decreases in denser medium: $v = c/n$
Wavelength adjusts to keep $\lambda = v/\nu$

Expert's Solution — Aryan Shukla, B.Tech Electrical Engineering, IIT Roorkee

The Invariance of Frequency: A Deep Insight

The fact that frequency remains unchanged when light passes from one medium to another is not just a mathematical convenience — it's a fundamental consequence of the **boundary conditions** of electromagnetic waves. At the interface, the tangential component of the electric field must be continuous. This forces the time-dependence ($\propto e^{i\omega t}$) to match on both sides, requiring ω (and hence ν) to be identical.

Why Color Depends on Frequency, Not Wavelength

This is a common conceptual trap. Our eyes detect color based on the **energy of photons** ($E = h\nu$) interacting with retinal pigments. The photochemical reactions in cone cells are triggered by photon energy, which is determined by frequency, not wavelength. So when light enters water:

In Air: $\lambda = 589$ nm (Yellow light)
In Water: $\lambda = 443$ nm
(Wavelength corresponds to blue)
BUT: The light still **looks yellow!**
Because $\nu = 5.09 \times 10^{14}$ Hz remains unchanged.

Did You Know?

Exam Memory Aid:

- **Frequency** (ν) — **Never changes** across refraction. It's the fundamental property of the source.
- **Speed** (v) — **Decreases** in denser medium ($n > 1$).
- **Wavelength** (λ) — **Decreases** by factor of $1/n$ in denser medium.
- **Mnemonic:** *"Frequent Visitors Stay"* — Frequency is the Visitor, it Stays constant.

Numerical Verification Trick:
Always check consistency using $v = \nu\lambda$. For water:

$$v_{water} \stackrel{?}{=} \nu_{water} \times \lambda_{water}$$

$$\text{LHS: } 2.26 \times 10^8 \text{ m/s} \quad ; \quad \text{RHS: } (5.09 \times 10^{14}) \times (4.43 \times 10^{-7}) \approx 2.25 \times 10^8 \text{ m/s}$$

Matches within rounding — the physics is satisfied! ✓

Q2 **What is the shape of the wavefront in each of the following cases:**

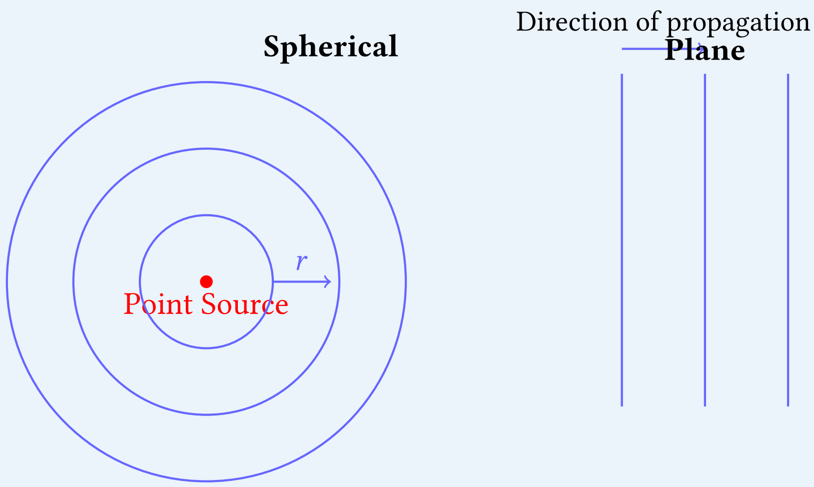
(a) Light diverging from a point source.

(b) Light emerging out of a convex lens when a point source is placed at its focus.

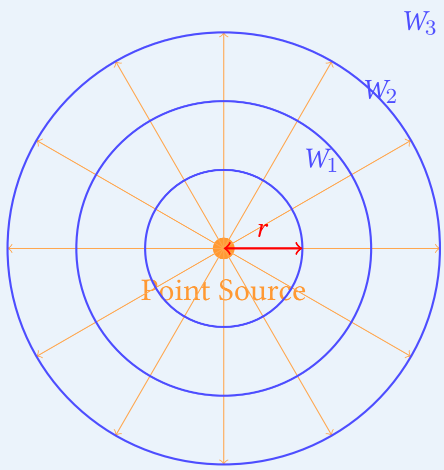
(c) The portion of the wavefront of light from a distant star intercepted by the Earth.

Solution

Understanding Wavefronts
A **wavefront** is defined as the locus of all points in a wave that are in the same phase of oscillation. In simpler terms, it is a surface connecting all points of a wave that have travelled from the source in exactly the same time. The shape of a wavefront depends on the nature of the source and the optical elements encountered.



Part (a): Light Diverging from a Point Source
A point source emits light uniformly in all directions. Since the speed of light is constant in a homogeneous medium, all points at a given distance r from the source receive the wave at exactly the same time. These points form the surface of a sphere centered at the source.



Each wavefront is a sphere centered at the source

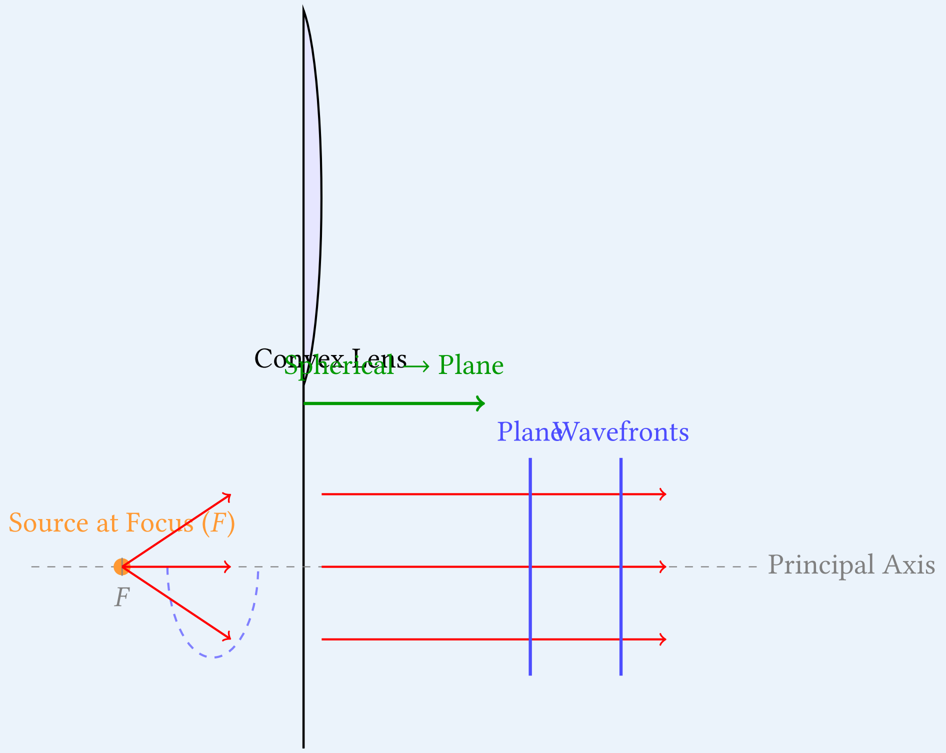
The wavefronts are **spherical** and concentric with the source. As the wave propagates outward, the radius of each spherical wavefront increases with time: $r = vt$, where v is the speed of light.

Answer (a):

Spherical wavefront

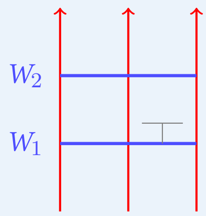
The wavefronts are concentric spheres centered at the point source, expanding radially outward with the speed of light.

Part (b): Light Emerging from a Convex Lens with Point Source at Focus
When a point source is placed at the focus of a convex lens, an important optical transformation occurs. Recall the fundamental property of a convex lens: rays diverging from its focus emerge **parallel** to the principal axis after refraction.



Physical Explanation:

- Before the lens, the wavefronts are **spherical** (diverging from the focus).
- The lens refracts the rays such that all rays from the focus emerge parallel to the principal axis.
- For the rays to be parallel, the corresponding wavefronts must be **perpendicular** to these rays.
- A surface perpendicular to a set of parallel lines is a **plane**.



Parallel rays $\perp$ Plane wavefronts

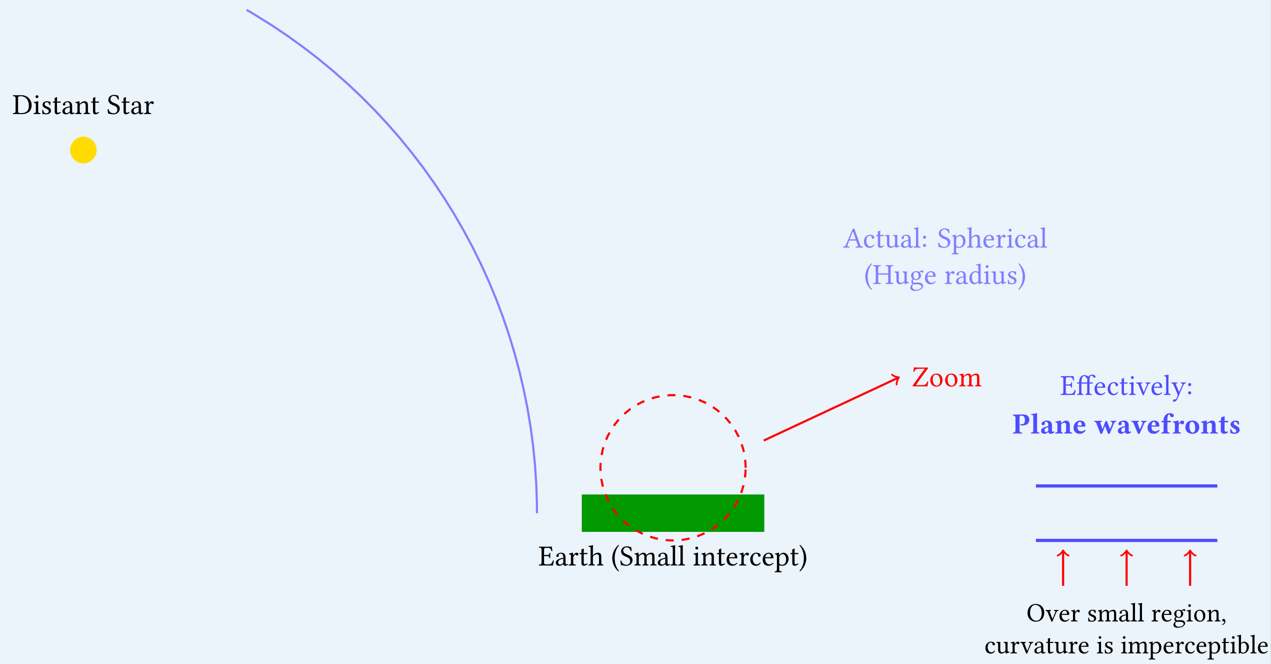
Thus, after the lens, the emerging light has **plane wavefronts**.

✔ **Answer (b):**

Plane wavefront

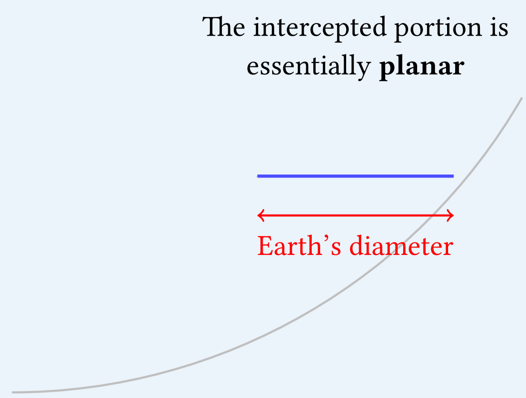
The convex lens converts the diverging spherical wavefronts from the focal point into plane wavefronts, as all rays emerge parallel to the principal axis.

Part (c): Light from a Distant Star Intercepted by Earth
Stars are at such enormous distances from Earth that the curvature of their wavefronts becomes practically undetectable over the relatively small aperture of the Earth.



Reasoning:

- A star emits spherical wavefronts, just like any point source.
- However, stars are light-years away. The radius of curvature of the wavefront by the time it reaches Earth is **enormous**.
- The Earth's diameter ($\sim 12,742$ km) is a **tiny** fraction of this radius.
- Over such a small intercept, a sphere of enormous radius is indistinguishable from a plane — just as a small patch on the surface of a large ball appears flat.



Answer (c):

Plane wavefront

Although the star emits spherical wavefronts, the enormous distance makes the radius of curvature so large that the small portion intercepted by Earth is essentially planar.

Conceptual Summary:

Case	Source Type	Optical Element	Wavefront Shape
(a)	Point source (diverging)	None	**Spherical**
(b)	Point source at focus	Convex lens	**Plane**
(c)	Very distant point source	None (large distance)	**Plane** (effectively)

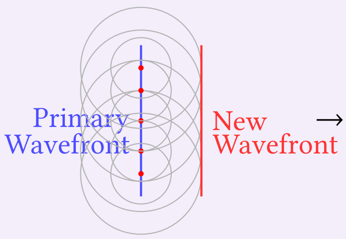
Golden Rule for Wavefronts:

The wavefront is always ***perpendicular*** *to the direction of ray propagation.*

→ If rays are radial from a point → **Spherical** wavefronts
→ If rays are parallel → **Plane** wavefronts
→ If rays are from a line source → **Cylindrical** wavefronts

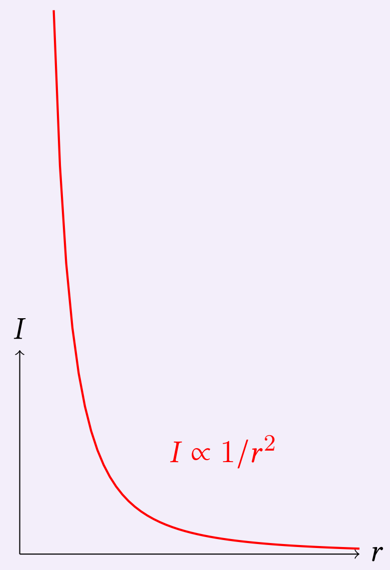
Expert's Solution — Priya Malhotra, B.Tech Engineering Physics, NIT Calicut

Wavefronts and Huygens' Principle: The Big Picture
The concept of wavefronts is central to **Huygens' Principle**, which states that every point on a wavefront acts as a source of secondary spherical wavelets. The envelope of these wavelets gives the new wavefront at a later time.



Detailed Analysis of Each Case:
(a) Point Source — Spherical Wavefront:

- A true point source in a homogeneous, isotropic medium produces **perfectly spherical** wavefronts.
- The intensity falls off as $1/r^2$ (inverse square law) because the same energy spreads over an ever-increasing spherical surface area ($4\pi r^2$).
- *Real-world example:* A small incandescent bulb, a star (at close range), or sound from a small speaker.



(b) Convex Lens — Plane Wavefront:

- This is the principle behind **collimators** in optical instruments.
- By placing a source at the focal point, the lens creates a beam of parallel rays — essential for spectrometers, autocollimators, and laser beam expanders.
- *Conversely:* A plane wave incident on a convex lens converges to a point at its focus — the basis of **focusing** in cameras and telescopes.

(c) Distant Star — Effectively Plane Wavefront:

- Consider a star 4 light-years away (Proxima Centauri). The wavefront radius at Earth $\approx 3.8 \times 10^{16}$ m.
- Over the Earth's diameter ($\sim 1.3 \times 10^{7}$ m), the sagitta (deviation from a plane) is:

$$\delta \approx \frac{d^2}{8R} = \frac{(1.3 \times 10^7)^2}{8 \times 3.8 \times 10^{16}} \approx 5.5 \times 10^{-4} \text{ m} = 0.55 \text{ mm}$$

- This deviation of ~ 0.5 mm over the entire Earth is negligible for most purposes!

Key Insight: The Plane Wave Approximation

In geometrical optics, we often treat wavefronts from distant sources as **plane waves**. This is a mathematical idealization that works beautifully when the source distance is much larger than the dimensions of our optical system.

Analogy: A basketball court appears perfectly flat to an ant, even though it's part of the curved Earth!

Did You Know?

Quick Revision — Wavefront Shapes:

Source	Medium/Optics	Wavefront
Point source (near)	Homogeneous	Spherical
Point source (at ∞)	Homogeneous	Plane
Point source at focus	After convex lens	Plane
Line source	Homogeneous	Cylindrical
Plane wave	After concave lens	Spherical (diverging)

Mental shortcut: Think of the direction of energy flow (rays). Draw a surface perpendicular to all rays — that's your wavefront!

Q3

(a) The refractive index of glass is 1.5. What is the speed of light in glass? (Speed of light in vacuum is 3.0×10^8 m s^{-1})

(b) Is the speed of light in glass independent of the colour of light? If not, which of the two colours red and violet travels slower in a glass prism?

Solution

Part (a): Speed of Light in Glass

The refractive index n of a medium is fundamentally defined as the ratio of the speed of light in vacuum to the speed of light in that medium. This definition directly gives us the method to calculate the speed.

Refractive Index Definition:

$$n = \frac{c}{v}$$

where:

- n = refractive index of the medium
- $c = 3.0 \times 10^8$ m/s (speed of light in vacuum)
- v = speed of light in the medium

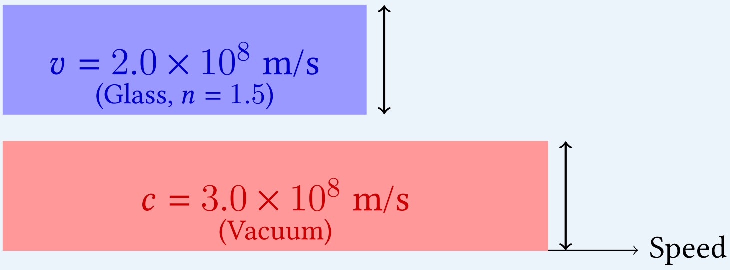
Rearranging to solve for the speed in glass:

$$v = \frac{c}{n}$$

Substituting the given values:

$$v_{\text{glass}} = \frac{3.0 \times 10^8 \text{ m/s}}{1.5}$$

$$v_{\text{glass}} = 2.0 \times 10^8 \text{ m/s}$$



Light travels **slower** in glass by a factor of n

Answer (a):

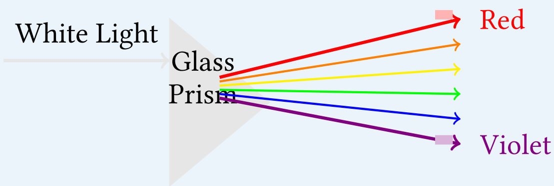
$$\boxed{v_{\text{glass}} = \frac{c}{n} = \frac{3.0 \times 10^8}{1.5} = 2.0 \times 10^8 \text{ m/s}}$$

The speed of light in glass is **two-thirds** of its speed in vacuum.

Part (b): Dependence of Speed on Colour

Is the speed independent of colour?

No, the speed of light in glass is **not independent** of colour. This phenomenon is known as **dispersion** — the refractive index of a material varies with the wavelength (or frequency) of light.



Different colours bend by different amounts

Which travels slower — Red or Violet?

To determine this, we need to understand the relationship between refractive index and wavelength. In most transparent materials (including glass), the refractive index follows **Cauchy's equation**:

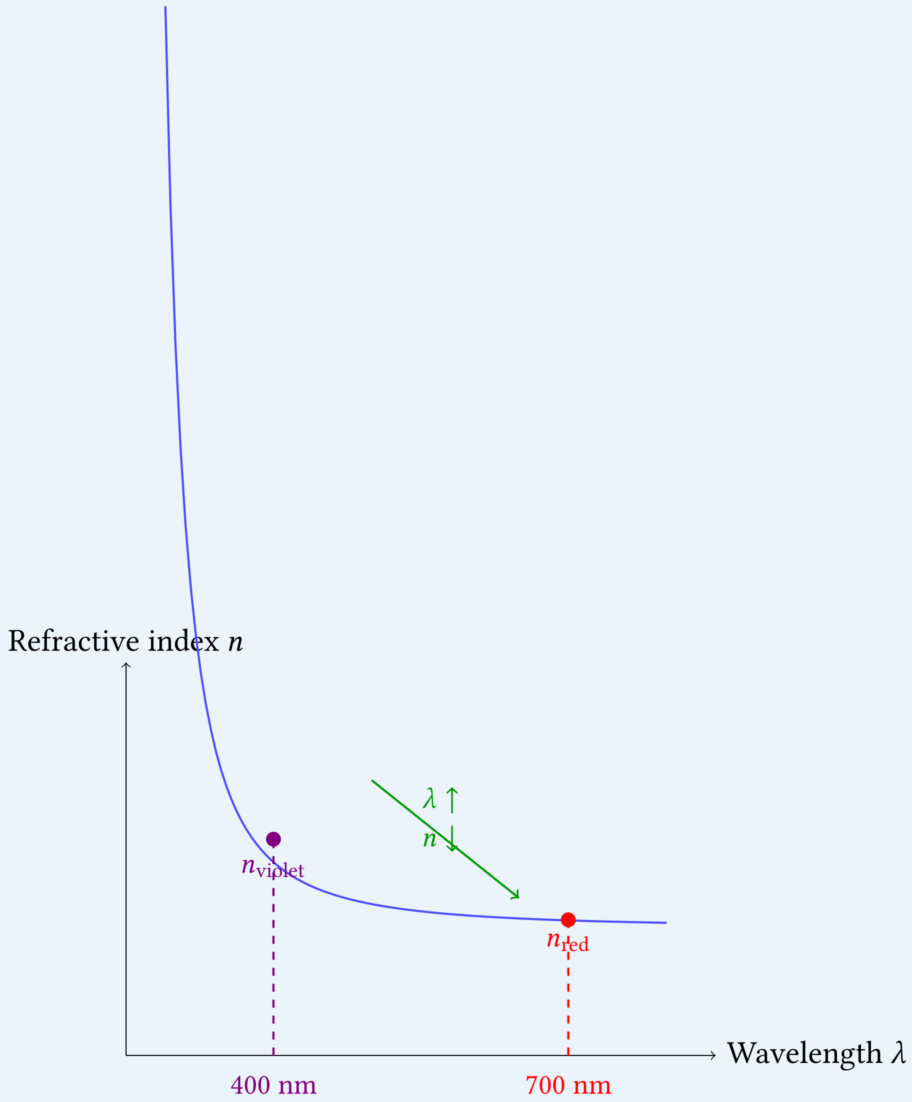
Cauchy's Equation (Approximate):

$$n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \ldots$$

where A, B, C are material constants.

From this equation, we can see that:

- **Shorter wavelength** (λ) $\longrightarrow$ **Larger** refractive index (n)
- **Larger refractive index** $\longrightarrow$ **Slower** speed ($v = c/n$)

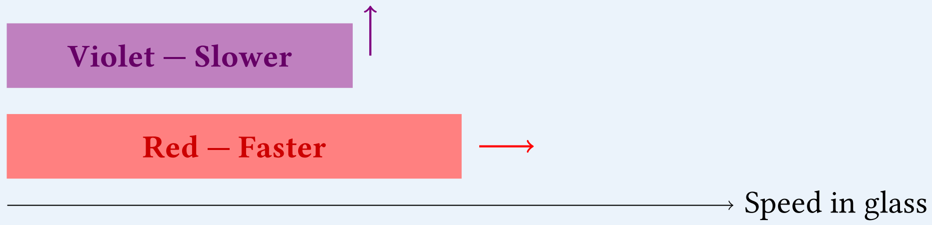


Specific Comparison — Red vs. Violet:

Colour	Wavelength (approx.)	Refractive Index	Speed in Glass
Red	$\sim$ 700 nm (Longer)	Lower (n_r)	**Faster** ($v_{\text{red}} = c/n_r$)
Violet	$\sim$ 400 nm (Shorter)	Higher (n_v)	**Slower** ($v_{\text{violet}} = c/n_v$)

Since $n_{\text{violet}} > n_{\text{red}}$, it follows that:

$$v_{\text{violet}} = \frac{c}{n_{\text{violet}}} < \frac{c}{n_{\text{red}}} = v_{\text{red}}$$



$v_{\text{violet}} < v_{\text{red}}$

✔ **Answer (b):**

No, the speed of light in glass is **not independent** of colour.

Violet light travels slower than red light in a glass prism.

Reason: Violet light has a shorter wavelength, experiences a higher refractive index ($n_{\text{violet}} > n_{\text{red}}$), and consequently travels slower ($v = c/n$).

Physical Significance — Why Dispersion Matters:
This wavelength-dependent speed is the fundamental reason behind the formation of a **spectrum** when white light passes through a glass prism. Each colour travels at a slightly different speed in glass, leading to different angles of refraction according to Snell's law:

$$n_1 \sin i = n_2 \sin r$$

Since n varies with colour, the angle of refraction r varies with colour, physically separating the colours spatially.

Summary — The Chain of Causality:

$$\begin{array}{ccc} \lambda_{\text{violet}} & < & \lambda_{\text{red}} \\ & \Downarrow & \\ n_{\text{violet}} & > & n_{\text{red}} \\ & \Downarrow & \\ v_{\text{violet}} & < & v_{\text{red}} \end{array}$$

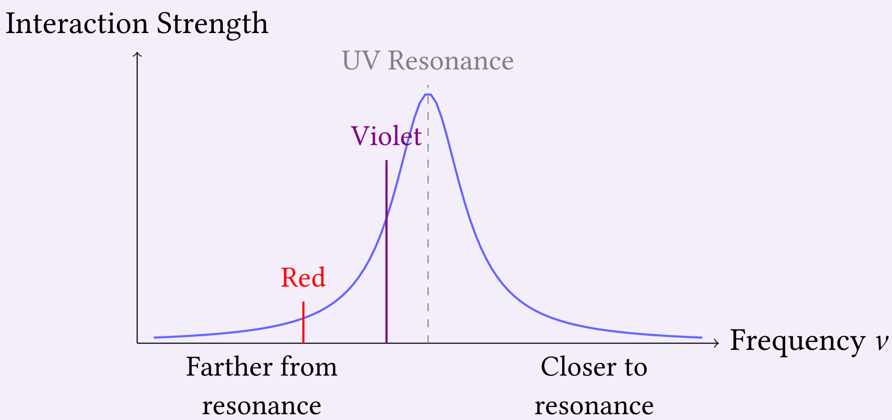
(Hence, violet deviates more and travels slower)

Expert's Solution — Karan Verma, B.Tech Electronics & Communication, NIT Kurukshetra

Dispersion: The Physics of Colour Separation
The variation of refractive index with wavelength is called **dispersion**, and it's one of the most important concepts in optics. Let's dive deeper into why this happens.
Microscopic Origin of Dispersion:

When light enters a medium, its oscillating electric field causes the electrons in the atoms to oscillate. These oscillating electrons re-radiate electromagnetic waves that interfere with the original wave, effectively slowing it down. The key insight is:

- Electrons have **natural resonant frequencies** (typically in the ultraviolet for glass).
- **Violet light** ($\sim 7.5 \times 10^{14}$ Hz) is closer to these resonant frequencies than **red light** ($\sim 4.3 \times 10^{14}$ Hz).
- The closer the driving frequency is to resonance, the stronger the interaction, and the **more the light is slowed down**.



Typical Refractive Indices for Crown Glass:

Colour	Wavelength (nm)	Refractive Index
Red	656.3 (H-α line)	1.515
Yellow	589.3 (Na D-line)	1.517
Green	546.1 (Hg line)	1.520
Blue	486.1 (H-β line)	1.523
Violet	404.7 (Hg line)	1.530

Notice how n increases steadily from red to violet. The difference $\Delta n = n_{\text{violet}} - n_{\text{red}}$ is called the **mean dispersion** of the material.

Calculating Speeds for Red and Violet:

Using the above data:

$$v_{\text{red}} = \frac{3.0 \times 10^8}{1.515} \approx 1.980 \times 10^8 \text{ m/s}$$

$$v_{\text{violet}} = \frac{3.0 \times 10^8}{1.530} \approx 1.961 \times 10^8 \text{ m/s}$$

The difference is about 1.9×10^6 m/s — small but significant enough to produce observable dispersion!

Real-World Applications of Dispersion:

Phenomenon/Device	Role of Dispersion
Rainbow formation	Water droplets disperse sunlight
Prism spectrometer	Separates wavelengths for analysis
Chromatic aberration	Unwanted dispersion in lenses
Optical fiber communication	Limits data rate (pulse broadening)
Gemstone brilliance	Dispersion creates "fire" in diamonds

★ **Did You Know?**

Exam Mnemonic — Which colour bends more?
"Violet Bends Better, Red Refracts Rudely"
Or simply remember: **VIBGYOR** from bottom to top in a spectrum formed on a screen.

- **Violet** — Most deviated (highest n, slowest v, shortest λ)
- **Red** — Least deviated (lowest n, fastest v, longest λ)

Key Formula to Remember:

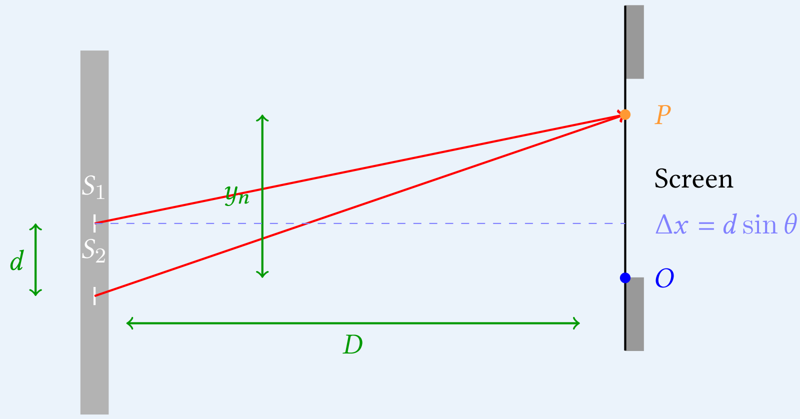
$$n(\lambda) \propto \frac{1}{\lambda^2} \quad \text{(Cauchy's approximation)}$$

$$\Rightarrow \quad n_{\text{violet}} > n_{\text{red}} \quad \Rightarrow \quad v_{\text{violet}} < v_{\text{red}}$$

Q4 **In a Young's double-slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2 cm. Determine the wavelength of light used in the experiment.**

Solution

Understanding Young's Double-Slit Experiment (YDSE)
Young's double-slit experiment is one of the most elegant demonstrations of the wave nature of light. When coherent light passes through two narrow, closely spaced slits, the emerging waves interfere to produce a pattern of alternating bright and dark fringes on a screen.



The Fringe Position Formula
For **bright fringes** (constructive interference), the path difference between waves from the two slits must be an integer multiple of the wavelength:

Condition for Bright Fringes:

$$d\sin\theta = n\lambda \quad \text{where } n = 0, 1, 2, 3, \ldots$$

For small angles (θ is small), $\sin\theta \approx \tan\theta \approx \theta$:

$$\sin\theta \approx \tan\theta = \frac{y_n}{D}$$

Therefore, the position of the n^{th} bright fringe from the central maximum is:

$$\boxed{y_n = \frac{n\lambda D}{d}}$$

where:

- y_n = distance of n^{th} bright fringe from central bright fringe
- n = order of the fringe ($n = 0$ for central, $n = 4$ for fourth bright)
- λ = wavelength of light
- D = distance between slits and screen
- d = separation between the two slits

Given Data:

Symbol	Value	Description
d	0.28 mm	Slit separation
	$= 2.8 \times 10^{-4}$ m	(Convert to SI)
D	1.4 m	Screen distance
y_4	1.2 cm	Distance to 4^{th} bright fringe
	$= 1.2 \times 10^{-2}$ m	(Convert to SI)
n	4	Order of the fringe

Step 1: Apply the Fringe Position Formula
For the 4^{th} bright fringe ($n = 4$):

$$y_4 = \frac{4\lambda D}{d}$$

Step 2: Rearrange to Solve for Wavelength λ

$$\lambda = \frac{y_4 \cdot d}{4 \cdot D}$$

Step 3: Substitute the Values

$$\lambda = \frac{(1.2 \times 10^{-2}\ \text{m}) \times (2.8 \times 10^{-4}\ \text{m})}{4 \times 1.4\ \text{m}}$$

Step 4: Calculate Carefully
First, compute the numerator:

$$\text{Numerator} = 1.2 \times 10^{-2} \times 2.8 \times 10^{-4}$$

$$= 1.2 \times 2.8 \times 10^{-6}$$

$$= 3.36 \times 10^{-6}\ \text{m}^2$$

Now, compute the denominator:

$$\text{Denominator} = 4 \times 1.4 = 5.6\ \text{m}$$

Therefore:

$$\lambda = \frac{3.36 \times 10^{-6}}{5.6}\ \text{m}$$

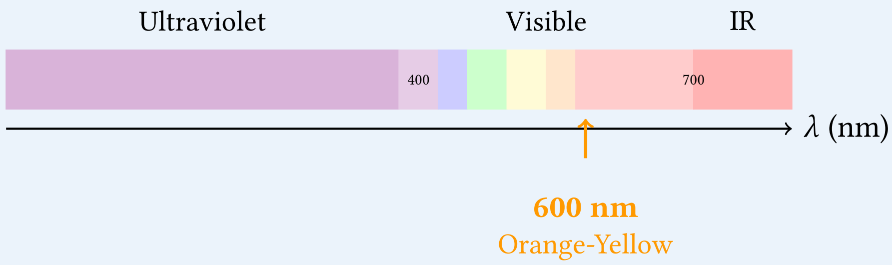
$$\lambda = 0.6 \times 10^{-6}\ \text{m}$$

$$\lambda = 6.0 \times 10^{-7}\ \text{m}$$

Step 5: Express in Nanometers
Converting to nanometers (1 nm = 10^{-9} m):

$$\lambda = 6.0 \times 10^{-7}\ \text{m} = 6.0 \times 10^{-7} \times 10^{9}\ \text{nm}$$

$$\lambda = 600\ \text{nm}$$

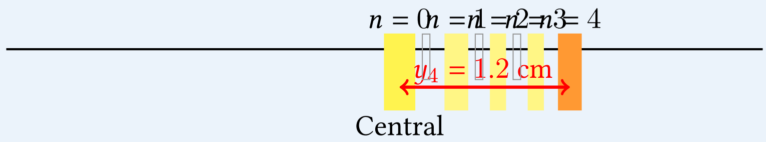


Verification:
Let's check if the small angle approximation is valid. The angular position of the 4^{th} fringe:

$$\tan\theta = \frac{y_4}{D} = \frac{1.2 \times 10^{-2}}{1.4} \approx 8.57 \times 10^{-3}$$

$$\theta \approx \tan^{-1}(8.57 \times 10^{-3}) \approx 0.49^\circ$$

Since $\theta \approx 0.49^\circ$ is very small, the approximation $\sin\theta \approx \tan\theta$ is excellent, and our formula is well-justified.



Answer:

$$\boxed{\lambda = 6.0 \times 10^{-7} \text{ m} = 600 \text{ nm}}$$

The wavelength of light used in the experiment is **600 nm**, which corresponds to **orange-yellow** light in the visible spectrum.

Fringe Width (Additional Insight):
The fringe width β — the separation between two consecutive bright (or dark) fringes — can also be calculated:

$$\beta = \frac{\lambda D}{d} = \frac{6.0 \times 10^{-7} \times 1.4}{2.8 \times 10^{-4}}$$

$$\beta = \frac{8.4 \times 10^{-7}}{2.8 \times 10^{-4}} = 3.0 \times 10^{-3} \text{ m} = 3.0 \text{ mm}$$

Indeed, checking with our result: $y_4 = 4\beta = 4 \times 3.0 \text{ mm} = 12 \text{ mm} = 1.2 \text{ cm}$ ✓

Expert's Solution — Nandini Gupta, B.Tech Engineering Physics, IIT BHU Varanasi

Young's Double-Slit Experiment: The Heart of Wave Optics
YDSE is arguably the most important experiment in wave optics, providing the first conclusive evidence for the wave nature of light. Let's understand the deeper aspects of this beautiful experiment.

Why the Small Angle Approximation?
In most YDSE problems, $D \gg d$ and $D \gg y_n$. This ensures that θ is very small, making $\sin\theta \approx \tan\theta$ a highly accurate approximation. The exact expression for bright fringe position is:

$$y_n = D\tan\left[\sin^{-1}\left(\frac{n\lambda}{d}\right)\right]$$

For our values:

$$\sin\theta = \frac{4\times 6.0\times 10^{-7}}{2.8\times 10^{-4}} \approx 8.57\times 10^{-3}$$
$$\theta \approx 0.491^\circ$$

The exact $y_4 = 1.4\times\tan(0.491^\circ) \approx 1.20012$ cm — the error from approximation is **less than 0.01%**!

When is the small angle approximation valid?

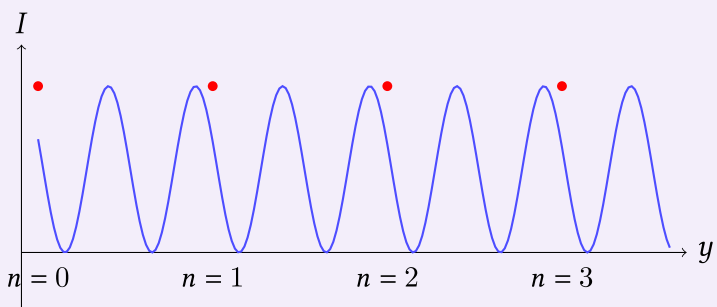
Condition	Error in y_n
$\theta < 5^\circ$	Error $< 0.4\%$
$\theta < 10^\circ$	Error $< 1.5\%$
$\theta < 15^\circ$	Error $< 3.5\%$

For most NCERT problems, θ is well below 5°, so the linear formula $y_n = n\lambda D/d$ is perfectly adequate.

The Complete Intensity Distribution:
The intensity at any point on the screen is given by:

$$I = I_0\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right) \approx I_0\cos^2\left(\frac{\pi dy}{\lambda D}\right)$$

where I_0 is the intensity at the central maximum. This produces the characteristic equally spaced bright and dark fringes.



$\beta = 3.0$ mm (constant fringe width)

Key Observations from Our Result:

- **Wavelength:** 600 nm is in the orange-yellow region of the visible spectrum. This is close to the sodium D-line (589.3 nm), commonly used in laboratory YDSE setups.

- **Fringe width:** 3.0 mm is comfortably visible to the naked eye, which is why YDSE can be demonstrated in classrooms.
- **Path difference for 4^{th} bright:** $\Delta x = 4\lambda = 2400$ nm $= 2.4$ μm

Did You Know?

Formula Cheat Sheet for YDSE:

Quantity	Formula	Notes
Position of n^{th} bright fringe	$y_n = \frac{n\lambda D}{d}$	$n = 0, \pm1, \pm2, \ldots$
Position of n^{th} dark fringe	$y_n = \frac{(2n-1)\lambda D}{2d}$	$n = \pm1, \pm2, \ldots$
Fringe width	$\beta = \frac{\lambda D}{d}$	Constant for all fringes
Angular fringe width	$\theta_\beta = \frac{\beta}{D} = \frac{\lambda}{d}$	In radians

Common Pitfall Alert: Always convert all units to SI (metres) before calculation! Here, mm $\rightarrow$ m, cm $\rightarrow$ m. A single unit conversion error can give a wavelength off by a factor of 10 or 100.

Q5 In Young's double-slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen where path difference is λ, is K units. What is the intensity of light at a point where path difference is $\lambda/3$?

Solution

Understanding Intensity in Young's Double-Slit Experiment
In YDSE, the intensity at any point on the screen depends on the **phase difference** between the waves arriving from the two slits. The phase difference ϕ is directly related to the path difference Δx by the fundamental relation:

Relation Between Path Difference and Phase Difference:

$$\boxed{\phi = \frac{2\pi}{\lambda} \cdot \Delta x}$$

where:

- ϕ = phase difference (in radians)
- Δx = path difference between waves from S_1 and S_2
- λ = wavelength of light

The Intensity Formula
When two coherent waves of equal amplitude interfere, the resultant intensity is given by:

Resultant Intensity in YDSE:

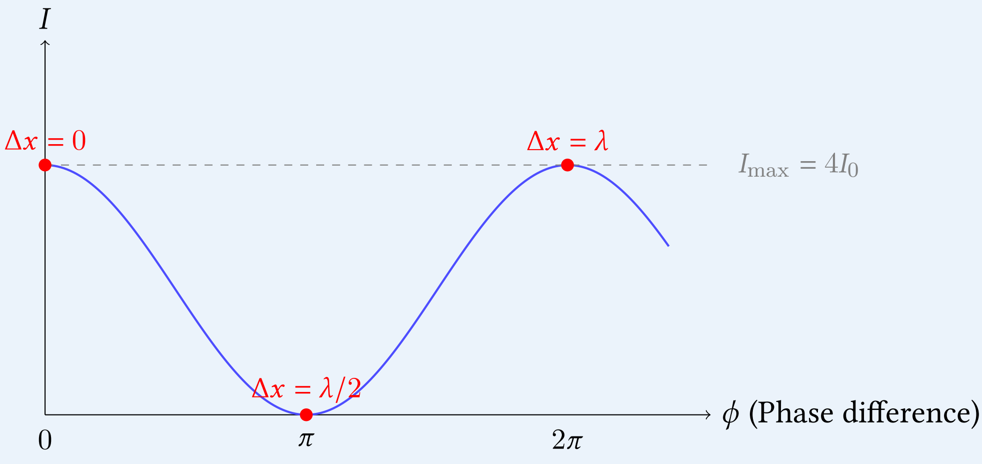
$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi}$$

For identical slits, $I_1 = I_2 = I_0$ (intensity due to each slit individually):

$$I = I_0 + I_0 + 2\sqrt{I_0 \cdot I_0}\cos\phi$$

$$I = 2I_0 + 2I_0\cos\phi$$

$$\boxed{I = 2I_0(1 + \cos\phi) = 4I_0\cos^2\left(\frac{\phi}{2}\right)}$$



Step 1: Determine I_0 Using the Given Information
We are given that when path difference $\Delta x = \lambda$, the intensity is K units.
First, find the phase difference for $\Delta x = \lambda$:

$$\phi_1 = \frac{2\pi}{\lambda} \cdot \lambda = 2\pi \text{ radians}$$

Now, substitute into the intensity formula:

$$I = 4I_0 \cos^2\left(\frac{\phi_1}{2}\right)$$

$$K = 4I_0 \cos^2\left(\frac{2\pi}{2}\right)$$

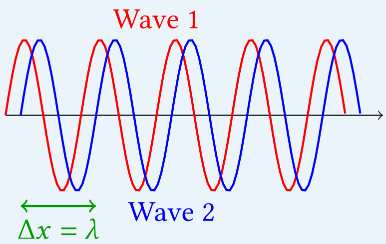
$$K = 4I_0 \cos^2(\pi)$$

Since $\cos(\pi) = -1$, we have $\cos^2(\pi) = 1$:

$$K = 4I_0 \times 1$$

$$\boxed{I_0 = \frac{K}{4}}$$

This makes physical sense: at path difference λ, the waves are **in phase** (phase difference 2π), producing **constructive interference** with maximum intensity $4I_0$.

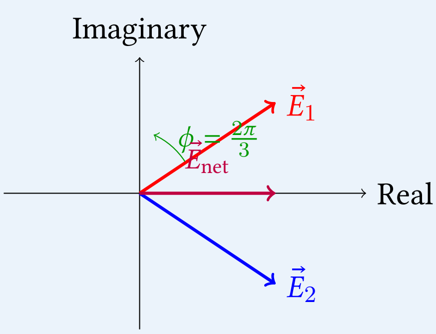


Waves in Phase
Constructive Interference

Step 2: Calculate Intensity for Path Difference $\lambda/3$
Now, for $\Delta x = \lambda/3$, compute the phase difference:

$$\phi_2 = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{3}$$

$$\phi_2 = \frac{2\pi}{3} \text{ radians} = 120^\circ$$



Phasor Addition of
Two Electric Fields

Now apply the intensity formula:

$$I = 4I_0 \cos^2\left(\frac{\phi_2}{2}\right)$$

Substitute $\phi_2 = 2\pi/3$:

$$I = 4I_0 \cos^2\left(\frac{2\pi/3}{2}\right)$$

$$I = 4I_0 \cos^2\left(\frac{\pi}{3}\right)$$

We know that $\cos(\pi/3) = \cos(60°) = 1/2$:

$$I = 4I_0 \times \left(\frac{1}{2}\right)^2$$

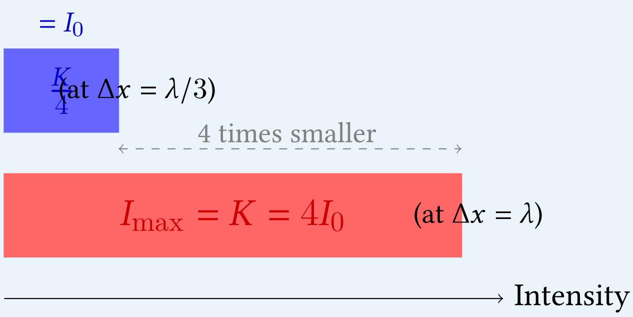
$$I = 4I_0 \times \frac{1}{4}$$

$$I = I_0$$

Step 3: Express the Answer in Terms of K

Since we found $I_0 = K/4$ from Step 1:

$$I = I_0 = \frac{K}{4}$$



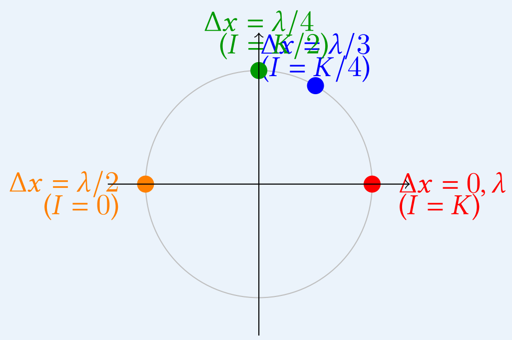
Answer:

$$\boxed{I = \frac{K}{4}}$$

The intensity at a point where the path difference is $\lambda/3$ is **one-fourth** of the intensity at a point where the path difference is λ.

Physical Explanation:

- When $\Delta x = \lambda$, the phase difference is 2π, and the two waves are **perfectly in phase**. This produces the **maximum possible intensity**: $I_{max} = 4I_0 = K$.
- When $\Delta x = \lambda/3$, the phase difference is $2\pi/3 = 120°$. The waves are **partially out of phase**, resulting in reduced constructive interference.
- At $\phi = 120°$, $\cos(60°) = 0.5$, giving $I = 4I_0 \times 0.25 = I_0 = K/4$.



Expert's Solution — Vikram Rathore, B.Tech CSE, IIIT Allahabad

Intensity Distribution in YDSE: A Deeper Understanding
The intensity variation in Young's double-slit experiment follows a $\cos^2$ pattern, which is one of the most elegant results in wave optics. Let's explore this comprehensively.
The Complete Intensity Formula:
In terms of path difference Δx, the intensity can be written directly as:

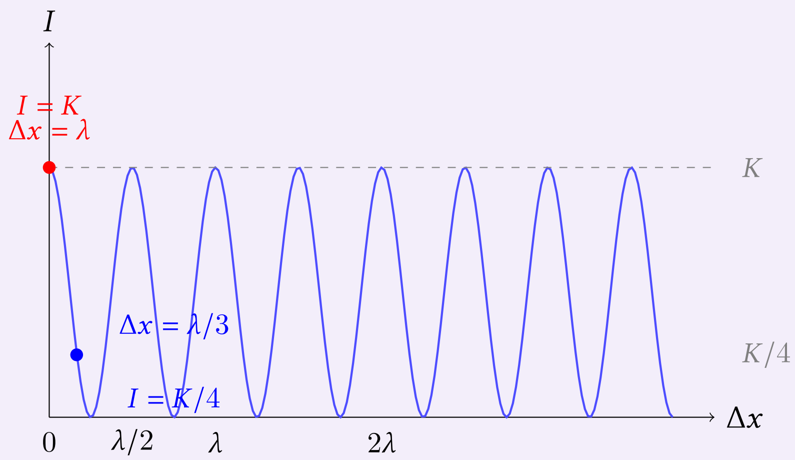
$$I = 4I_0 \cos^2\left(\frac{\pi \Delta x}{\lambda}\right)$$

This is a powerful form because it eliminates the intermediate step of calculating phase difference.
Verification for Our Problem:

- For $\Delta x = \lambda$:
$$I = 4I_0 \cos^2\left(\frac{\pi \cdot \lambda}{\lambda}\right) = 4I_0 \cos^2(\pi) = 4I_0 = K \quad \checkmark$$
- For $\Delta x = \lambda/3$:
$$I = 4I_0 \cos^2\left(\frac{\pi \cdot \lambda/3}{\lambda}\right) = 4I_0 \cos^2\left(\frac{\pi}{3}\right) = 4I_0 \times \frac{1}{4} = I_0 = \frac{K}{4} \quad \checkmark$$

Important Intensity Values to Memorize:

Path Difference Δx	Phase Difference ϕ	Intensity I	In Terms of K
0	0	$4I_0$	K (Maximum)
$\lambda/6$	$\pi/3 = 60°$	$3I_0$	$3K/4$
$\lambda/4$	$\pi/2 = 90°$	$2I_0$	$K/2$
$\lambda/3$	$2\pi/3 = 120°$	I_0	$\boxed{K/4}$
$\lambda/2$	$\pi = 180°$	0	0 (Minimum)
λ	$2\pi = 360°$	$4I_0$	K (Maximum)



Phasor Addition — The Visual Approach:
At $\Delta x = \lambda/3$, the two electric field vectors are separated by 120°. The resultant amplitude is:

$$E_{\text{net}} = \sqrt{E_0^2 + E_0^2 + 2E_0^2\cos(120°)} = \sqrt{2E_0^2 - E_0^2} = E_0$$

Since intensity is proportional to amplitude squared:

$$I \propto E_{\text{net}}^2 = E_0^2 = I_0$$

And since $K = 4I_0$, we get $I = I_0 = K/4$.

Key Insight:

The intensity in YDSE varies **sinusoidally** with position, but the relationship between intensity and path difference follows $\cos^2$, not a linear relationship.

This is why the fringes appear equally spaced (linear position relationship with n), but the intensity fall-off from the center is gradual and symmetric.

Did You Know?

Quick Formula for YDSE Intensity:
Remember this direct relationship:

$$I(\Delta x) = I_{max}\cos^2\left(\frac{\pi\Delta x}{\lambda}\right)$$

where $I_{max} = K$ (intensity at $\Delta x = n\lambda$).
Common Values at a Glance:

- $\Delta x = 0, \lambda, 2\lambda, \ldots$: $I = K$ (Bright fringes)
- $\Delta x = \lambda/2, 3\lambda/2, \ldots$: $I = 0$ (Dark fringes)
- $\Delta x = \lambda/3$: $I = K/4$ (This problem!)
- $\Delta x = \lambda/4$: $I = K/2$
- $\Delta x = \lambda/6$: $I = 3K/4$

Mental Check: At $\Delta x = \lambda/3$, $\cos(\pi/3) = 1/2$, so $I = K \times (1/2)^2 = K/4$ ✓

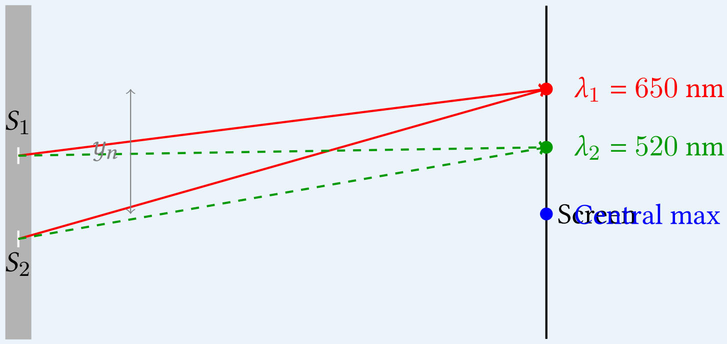
Q6 A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes in a Young's double-slit experiment.

(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.

(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

Solution

Understanding the Problem
This problem involves Young's double-slit experiment with **two different wavelengths** of light simultaneously illuminating the slits. Each wavelength produces its own independent interference pattern on the screen. The fringe positions depend on wavelength, so the patterns are slightly shifted relative to each other.



Given Data:

- Wavelength 1: $\lambda_1 = 650 \text{ nm} = 6.5 \times 10^{-7}$ m
- Wavelength 2: $\lambda_2 = 520 \text{ nm} = 5.2 \times 10^{-7}$ m
- The slit separation d and screen distance D are not explicitly given — but we'll discover an interesting insight!

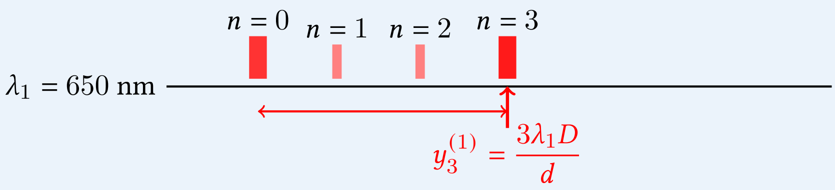
Part (a): Position of the Third Bright Fringe for $\lambda_1 = 650$ nm
The position of the n^{th} bright fringe from the central maximum is given by:

Position of n^{th} Bright Fringe:

$$y_n = \frac{n\lambda D}{d}$$

For the 3^{rd} bright fringe ($n = 3$) with wavelength λ_1:

$$y_3^{(1)} = \frac{3\lambda_1 D}{d}$$



Since the values of d and D are not provided, the answer must be expressed in terms of these parameters:

$$y_3^{(1)} = \frac{3 \times (6.5 \times 10^{-7}) \times D}{d}$$

$$y_3^{(1)} = \frac{1.95 \times 10^{-6} \times D}{d} \text{ m}$$

Answer (a):

$$\boxed{y_3^{(1)} = \frac{3\lambda_1 D}{d} = \frac{1.95 \times 10^{-6} \cdot D}{d} \text{ m}}$$

The distance depends on the specific values of D and d used in the experimental setup. Once these are known, the expression $3\lambda_1 D/d$ gives the exact position.

Part (b): Least Distance Where Bright Fringes Coincide
For the bright fringes of the two wavelengths to coincide at a point on the screen, that point must satisfy the bright fringe condition for **both** wavelengths simultaneously.

Condition for Coincidence of Bright Fringes:

$$y_n^{(1)} = y_m^{(2)}$$

$$\frac{n\lambda_1 D}{d} = \frac{m\lambda_2 D}{d}$$

Canceling D/d (which are common to both):

$$\boxed{n\lambda_1 = m\lambda_2}$$

where n and m are integers representing the order of bright fringes for λ_1 and λ_2 respectively.

Step 1: Find the Ratio of Integers
From the coincidence condition:

$$\frac{n}{m} = \frac{\lambda_2}{\lambda_1}$$

Substituting the given wavelengths:

$$\frac{n}{m} = \frac{520 \text{ nm}}{650 \text{ nm}}$$

$$\frac{n}{m} = \frac{520}{650}$$

Simplify the fraction:

$$\frac{n}{m} = \frac{52}{65} = \frac{4 \times 13}{5 \times 13} = \frac{4}{5}$$

Simplifying the Ratio:

$$\frac{\lambda_2}{\lambda_1} = \frac{520}{650} = \frac{52}{65} = \frac{4 \times 13}{5 \times 13} = \frac{4}{5}$$

Therefore: $\boxed{\frac{n}{m} = \frac{4}{5}}$

Step 2: Interpret the Ratio
The ratio $n/m = 4/5$ means:

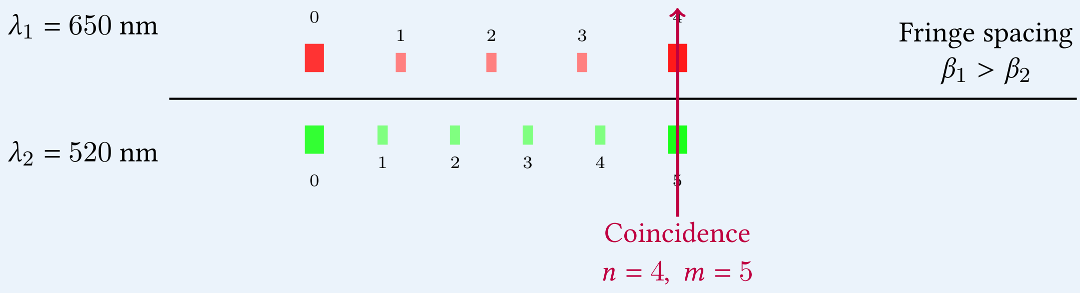
$$n = 4k \quad \text{and} \quad m = 5k \quad \text{where } k = 1, 2, 3, \ldots$$

For the **least distance** (first coincidence), we take the smallest possible integers, i.e., $k = 1$:

$$n = 4 \quad \text{and} \quad m = 5$$

This means:

- The 4^{th} bright fringe of $\lambda_1 = 650$ nm coincides with
- The 5^{th} bright fringe of $\lambda_2 = 520$ nm



Step 3: Calculate the Distance of Coincidence
Using either wavelength's bright fringe formula:
Using λ_1 $(n = 4)$:

$$y_{\text{coincidence}} = \frac{4\lambda_1 D}{d}$$

$$y_{\text{coincidence}} = \frac{4 \times (6.5 \times 10^{-7}) \times D}{d}$$

$$y_{\text{coincidence}} = \frac{2.6 \times 10^{-6} \cdot D}{d} \text{ m}$$

Verification using λ_2 $(m = 5)$:

$$y_{\text{coincidence}} = \frac{5\lambda_2 D}{d} = \frac{5 \times (5.2 \times 10^{-7}) \times D}{d} = \frac{2.6 \times 10^{-6} \cdot D}{d} \text{ m}$$

Both methods give the same result — a beautiful consistency check! ✓

Verification:

$4 \times 650 = 2600$ nm and $5 \times 520 = 2600$ nm

Both products equal **2600 nm**, confirming that at this point, both wavelengths satisfy the constructive interference condition.

✔ **Answer (b):**

$$\boxed{y_{\text{coincidence}} = \frac{4\lambda_1 D}{d} = \frac{5\lambda_2 D}{d} = \frac{2.6 \times 10^{-6} \cdot D}{d} \text{ m}}$$

The **least distance** from the central maximum where bright fringes of both wavelengths coincide corresponds to:

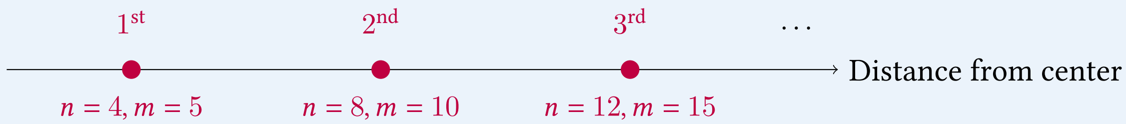
$$\boxed{n = 4 \text{ (for } \lambda_1 = 650 \text{ nm)} \quad \text{and} \quad m = 5 \text{ (for } \lambda_2 = 520 \text{ nm)}}$$

At this position, the path difference is 2600 nm $= 4\lambda_1 = 5\lambda_2$.

General Pattern of Coincidences:
The coincidences occur periodically at:

$$n = 4, 8, 12, 16, \dots \quad (\text{multiples of 4 for } \lambda_1)$$

$$m = 5, 10, 15, 20, \dots \quad (\text{multiples of 5 for } \lambda_2)$$



Expert's Solution — Sanjana Reddy, B.Tech Applied Physics, NIT Warangal

The LCM Principle in Multi-Wavelength Interference
This problem illustrates a beautiful mathematical principle: the coincidence of fringes for different wavelengths occurs at positions where the **optical path difference** is a **common multiple** of both wavelengths.
The Fundamental Condition:
For bright fringes of two wavelengths to coincide:

$$\Delta x = n\lambda_1 = m\lambda_2$$

This means Δx must be a **common multiple** of λ_1 and λ_2. The **least** such path difference is the **LCM (Least Common Multiple)** of the two wavelengths.

Finding LCM of Wavelengths:

$$\lambda_1 = 650 = 2 \times 5^2 \times 13$$

$$\lambda_2 = 520 = 2^3 \times 5 \times 13$$

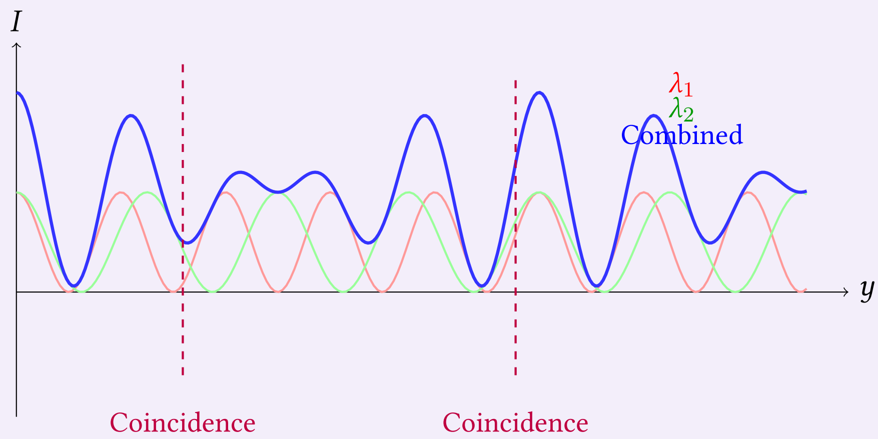
$$\text{LCM}(650, 520) = 2^3 \times 5^2 \times 13 = 8 \times 25 \times 13 = 2600 \text{ nm}$$

$$\text{Now: } n = \frac{\text{LCM}}{\lambda_1} = \frac{2600}{650} = 4$$

$$m = \frac{\text{LCM}}{\lambda_2} = \frac{2600}{520} = 5$$

Visualizing the Superposition of Two Patterns:
When both wavelengths illuminate the slits simultaneously, the total intensity pattern is the **sum** of the individual intensity patterns:

$$I_{\text{total}}(y) = I_1(y) + I_2(y) = 4I_0^{(1)} \cos^2\left(\frac{\pi d y}{\lambda_1 D}\right) + 4I_0^{(2)} \cos^2\left(\frac{\pi d y}{\lambda_2 D}\right)$$



General Formula for Multi-Wavelength YDSE:
If a beam contains wavelengths $\lambda_1, \lambda_2, \lambda_3, \ldots$, the bright fringes coincide at positions where:

$$y_{\text{coincidence}} = \frac{p \cdot \text{LCM}(\lambda_1, \lambda_2, \lambda_3, \ldots) \cdot D}{d}$$

where $p = 1, 2, 3, \ldots$ for successive coincidences.
Real-World Application — White Light YDSE:
This problem is closely related to what happens with white light in YDSE:

- The central fringe ($n = 0$) is **white** because all wavelengths have their maxima there.
- For higher orders, different colours spread out, creating rainbow-like patterns.
- At very high orders, fringes of different orders for different colours overlap, eventually washing out into uniform illumination.

Did You Know?

The technique of using two known wavelengths to find the ratio n/m is a powerful method in optical metrology. By measuring the coincidence positions, one can determine an unknown wavelength if the other is known — a principle used in **interferometric wavelength measurement**.

Did You Know?

Step-by-Step Algorithm for Coincidence Problems:

1. **Set up the equality:** $n\lambda_1 = m\lambda_2$
2. **Find the ratio:** $\frac{n}{m} = \frac{\lambda_2}{\lambda_1}$
3. **Simplify to lowest terms:** Reduce the fraction completely
4. **Identify smallest integers:** The numerator gives n (for λ_1), denominator gives m (for λ_2)
5. **Calculate distance:** $y = \frac{n\lambda_1 D}{d}$ or $y = \frac{m\lambda_2 D}{d}$

Numerical Check:

- Always verify: $n\lambda_1 \stackrel{?}{=} m\lambda_2$
- For this problem: $4 \times 650 = 2600$ and $5 \times 520 = 2600$ ✓

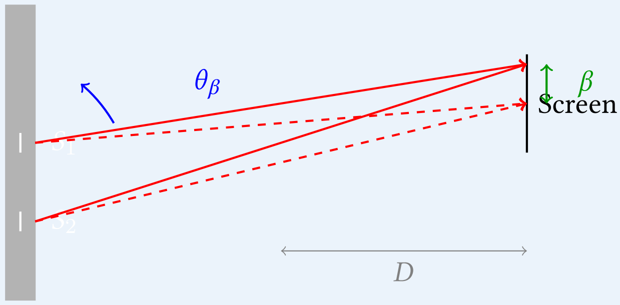
Common Mistake: Students often write $n/m = \lambda_1/\lambda_2$ (inverted). Remember: larger λ means fewer fringes in the same distance, so $n < m$ when $\lambda_1 > \lambda_2$.

Q7 In a double-slit experiment the angular width of a fringe is found to be $0.2°$ on a screen placed 1 m away. The wavelength of light used is 600 nm. What will be the angular width of the fringe if the entire experimental apparatus is immersed in water? Take refractive index of water to be $4/3$.

Solution

Understanding Angular Fringe Width

The **angular fringe width** (θ_β) is the angle subtended by one fringe width (β) at the plane of the slits. It represents the angular separation between two consecutive bright (or dark) fringes as seen from the slits.



Key Relationships:

Fringe Width and Angular Fringe Width:

$$\beta = \frac{\lambda D}{d} \quad \text{(Linear fringe width)}$$

$$\theta_\beta = \frac{\beta}{D} = \frac{\lambda}{d} \quad \text{(Angular fringe width in radians)}$$

where:

- β = linear fringe width (separation between consecutive bright/dark fringes)
- θ_β = angular fringe width (angle subtended by one fringe at the slits)
- λ = wavelength of light in the medium
- d = slit separation
- D = distance between slits and screen

Crucial Insight: The angular fringe width $\theta_\beta = \lambda/d$ is **independent of** D! It depends only on the wavelength in the medium and the slit separation. This means the distance to the screen (1 m in this problem) is actually not needed for the calculation.

Step 1: Angular Fringe Width in Air
Given in air:

- $\theta_\beta^{(\text{air})} = 0.2°$
- $\lambda_{\text{air}} = 600 \text{ nm} = 6.0 \times 10^{-7} \text{ m}$

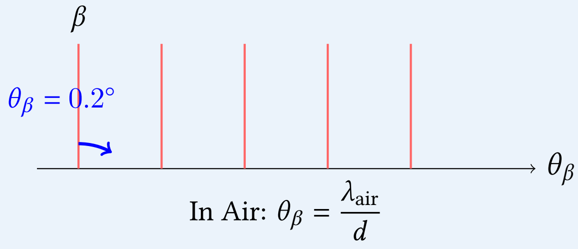
From the formula $\theta_\beta = \lambda/d$, we can find the slit separation d. First, convert the angle to radians:

$$\theta_\beta^{(\text{air})} = 0.2° = 0.2 \times \frac{\pi}{180} \text{ radians}$$

$$\theta_\beta^{(\text{air})} = \frac{0.2 \times 3.1416}{180} \approx 3.49 \times 10^{-3} \text{ rad}$$

Now, the slit separation:

$$d = \frac{\lambda_{\text{air}}}{\theta_\beta^{(\text{air})}} = \frac{6.0 \times 10^{-7}}{3.49 \times 10^{-3}} \approx 1.72 \times 10^{-4} \text{ m} = 0.172 \text{ mm}$$



Step 2: Effect of Immersing in Water
When the entire apparatus is immersed in water, **two things change**:

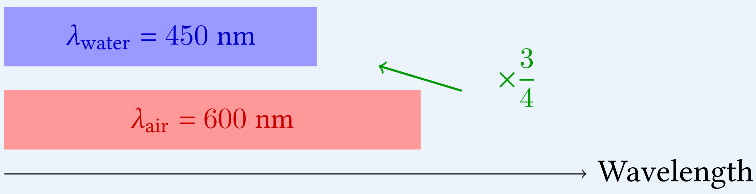
1. **The wavelength of light decreases** because the speed of light decreases in water.
2. **The slit separation** d **and screen distance** D **remain unchanged** — these are geometrical parameters of the setup.

The wavelength in water is related to the wavelength in air by the refractive index:

$$\lambda_{water} = \frac{\lambda_{air}}{n_{water}}$$

Given $n_{water} = 4/3$:

$$\lambda_{water} = \frac{600 \text{ nm}}{4/3} = 600 \times \frac{3}{4} = 450 \text{ nm}$$



Step 3: Calculate Angular Fringe Width in Water
Since the angular fringe width depends only on λ and d:

$$\theta_\beta^{(water)} = \frac{\lambda_{water}}{d}$$

The slit separation d is unchanged. Therefore, we can use the ratio:

$$\frac{\theta_\beta^{(water)}}{\theta_\beta^{(air)}} = \frac{\lambda_{water}/d}{\lambda_{air}/d} = \frac{\lambda_{water}}{\lambda_{air}}$$

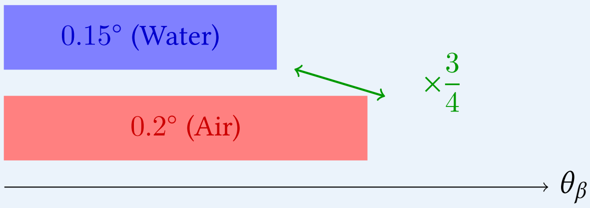
Substituting $\lambda_{water} = \lambda_{air}/n$:

$$\frac{\theta_\beta^{(water)}}{\theta_\beta^{(air)}} = \frac{1}{n}$$

$$\theta_\beta^{(\text{water})} = \frac{\theta_\beta^{(\text{air})}}{n}$$

Given $n_{\text{water}} = 4/3$:

$$\theta_\beta^{(\text{water})} = \frac{0.2^\circ}{4/3} = 0.2^\circ \times \frac{3}{4} = 0.15^\circ$$



Angular width decreases by factor of n

Note: Interestingly, the distance $D = 1$ m given in the problem serves as a distractor! The angular fringe width is independent of D, so this information is not required for the calculation. However, it confirms that the small angle approximation ($\theta_\beta \approx \tan\theta_\beta$) is valid.

Verification of Small Angle Approximation:

$$\theta_\beta = 0.2^\circ = 3.49 \times 10^{-3} \text{ rad}$$
$$\sin(0.2^\circ) \approx 3.49 \times 10^{-3}$$
$$\tan(0.2^\circ) \approx 3.49 \times 10^{-3}$$

Since θ_β is very small, $\theta_\beta \approx \sin\theta_\beta \approx \tan\theta_\beta$, so the approximation used in deriving $\theta_\beta = \lambda/d$ is excellent.

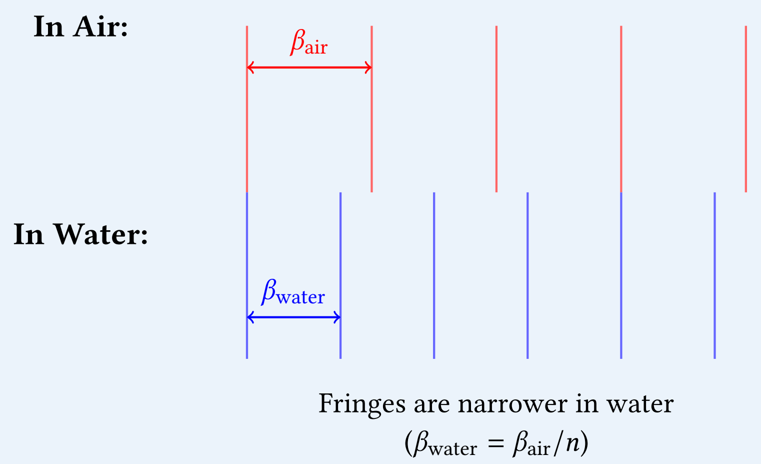
Answer:

$$\boxed{\theta_\beta^{(\text{water})} = 0.15^\circ}$$

When the apparatus is immersed in water, the angular fringe width decreases from 0.2° to 0.15°, i.e., it reduces by a factor equal to the refractive index of water ($n = 4/3$).

Physical Reasoning:

- The angular fringe width is $\theta_\beta = \lambda/d$.
- Immersing in water reduces the wavelength: $\lambda_{\text{water}} = \lambda_{\text{air}}/n$.
- Since d is unchanged, θ_β decreases proportionally.
- The fringes become **narrower** (both linearly and angularly) in water because the reduced wavelength means a smaller path difference is needed to produce the same phase difference.



Expert's Solution — Rahul Joshi, B.Tech Physics, IIT Indore

The Angular Fringe Width: A Geometrical Invariant

The angular fringe width $\theta_\beta = \lambda/d$ is one of the most elegant results in wave optics because of its **independence from the screen distance** D. This makes it a *geometrical invariant* of the experimental setup for a given wavelength and slit separation.

Why Does Immersion in Water Change the Angular Width?

Let's analyze this from first principles. The condition for the n^{th} bright fringe is:

$$d \sin\theta_n = n\lambda$$

For the $(n+1)^{\text{th}}$ bright fringe:

$$d \sin\theta_{n+1} = (n+1)\lambda$$

Subtracting:

$$d(\sin\theta_{n+1} - \sin\theta_n) = \lambda$$

For small angles, $\sin\theta \approx \theta$, so:

$$d(\theta_{n+1} - \theta_n) = \lambda$$

$$\theta_\beta = \theta_{n+1} - \theta_n = \frac{\lambda}{d}$$

When immersed in water, λ decreases to λ/n, while d remains constant. Hence:

$$\theta_\beta^{(\text{water})} = \frac{\lambda}{n \cdot d} = \frac{\theta_\beta^{(\text{air})}}{n}$$

The Key Insight:

The $D = 1$ m given in the problem is **not needed** for finding the angular width. It's only useful if we wanted to calculate the *linear* fringe width $\beta = \theta_\beta \cdot D$. The problem gives this information to test whether students recognize that angular width is independent of screen distance.

What Changes and What Stays Constant?
Let's examine each parameter when the apparatus is immersed in water:

Parameter	In Air	In Water
Wavelength λ	600 nm	450 nm (Decreases)
Slit separation d	d	d (Unchanged)
Screen distance D	1 m	1 m (Unchanged)
Linear fringe width β	$\beta_{air} = \frac{\lambda D}{d}$	$\beta_{water} = \frac{\lambda D}{nd} = \frac{\beta_{air}}{n}$
Angular fringe width θ_β	$0.2°$	$0.15°$ (Decreases)

Calculating Linear Fringe Widths (For Interest):
From $\theta_\beta = \beta/D$:

$$\beta_{air} = \theta_\beta^{(air)} \times D = (3.49 \times 10^{-3}) \times 1 \approx 3.49 \text{ mm}$$

$$\beta_{water} = \frac{\beta_{air}}{n} = \frac{3.49}{4/3} \approx 2.62 \text{ mm}$$

General Principle:

Whenever the entire YDSE apparatus is immersed in a medium of refractive index n:

- **Wavelength** becomes λ/n
- **Fringe width** becomes β/n
- **Angular fringe width** becomes θ_β/n
- The **positions of bright fringes** shift closer to the center
- The **intensity pattern** maintains its $\cos^2$ shape but is compressed

Did You Know?

Quick Formula for Immersion Problems:
When YDSE apparatus is completely immersed in a medium of refractive index n:

$$\boxed{\beta_{\text{medium}} = \frac{\beta_{\text{air}}}{n} \quad ; \quad \theta_{\beta,\text{medium}} = \frac{\theta_{\beta,\text{air}}}{n}}$$

Mental Check:

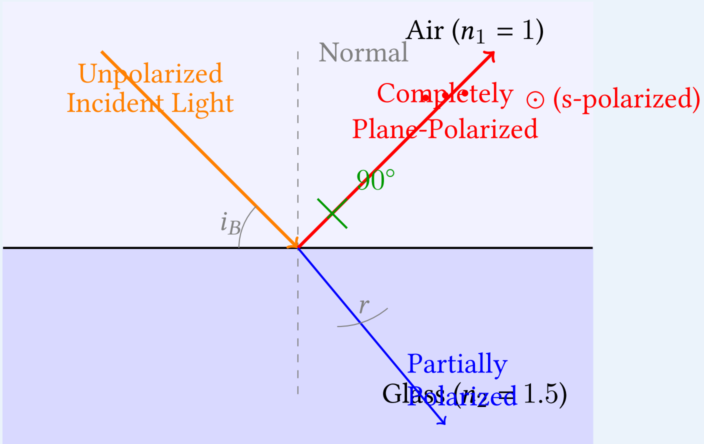
- Water has $n = 4/3 \approx 1.33$
- So everything reduces by a factor of $3/4$
- $0.2° \times 3/4 = 0.15°$ ✓

Distractor Alert: In problems like this, always identify which parameters are relevant. The screen distance D was given but doesn't affect angular width. Don't get confused by extraneous information!

Q8 What is the Brewster angle for air to glass transition? (Refractive index of glass = 1.5.)

Solution

Understanding Brewster's Law
When unpolarized light is incident on a transparent medium, the reflected and refracted rays are partially polarized. At a particular angle of incidence called the **Brewster angle** (or polarizing angle), the reflected light becomes **completely plane-polarized** with its electric field vector perpendicular to the plane of incidence.



Brewster's Law:
At the Brewster angle, the reflected and refracted rays are **perpendicular** to each other. This fundamental condition leads to Brewster's law:

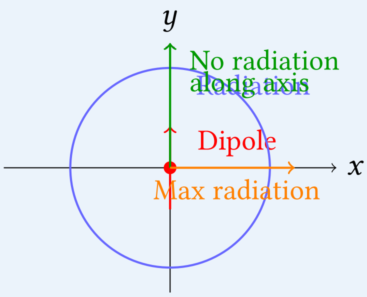
Brewster's Law:

$$\boxed{\tan i_B = \frac{n_2}{n_1} = n_{21}}$$

where:

- i_B = Brewster angle (polarizing angle)
- n_1 = refractive index of the incident medium
- n_2 = refractive index of the refracting medium
- $n_{21} = n_2/n_1$ = relative refractive index

Physical Reason for Polarization:
At the Brewster angle, the electric dipoles in the medium oscillate parallel to the refracted ray's electric field. Since an oscillating dipole does **not** radiate along its axis of oscillation, the component of the electric field parallel to the plane of incidence is not reflected — only the perpendicular component survives in the reflected beam.



Step 1: Identify the Given Parameters

- Incident medium: Air, $n_1 = 1$
- Refracting medium: Glass, $n_2 = 1.5$
- Transition: Air $\longrightarrow$ Glass

Step 2: Apply Brewster's Law
Using the formula $\tan i_B = n_2/n_1$:

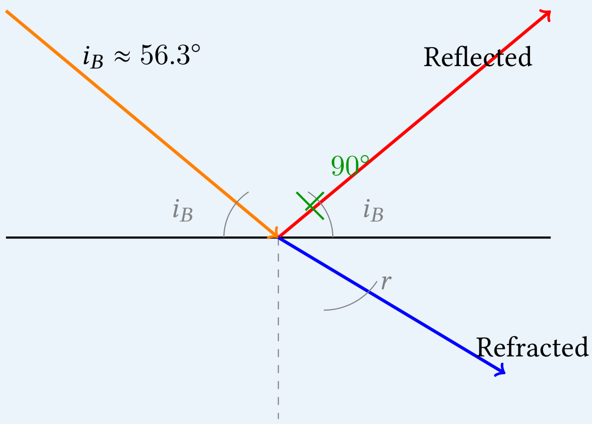
$$\tan i_B = \frac{1.5}{1} = 1.5$$

Step 3: Calculate the Brewster Angle

$$i_B = \tan^{-1}(1.5)$$

Calculating:

$$i_B = \tan^{-1}(1.5) \approx 56.31°$$



$$i_B + r = 90° \quad \text{(Brewster's condition)}$$

Step 4: Verify the 90° Condition
According to Brewster's law, at the polarizing angle:

$$i_B + r = 90°$$

Let's verify using Snell's law:

$$n_1 \sin i_B = n_2 \sin r$$
$$1 \times \sin(56.31°) = 1.5 \times \sin r$$
$$\sin r = \frac{\sin(56.31°)}{1.5} = \frac{0.832}{1.5} \approx 0.5547$$
$$r = \sin^{-1}(0.5547) \approx 33.69°$$

Check: $i_B + r = 56.31° + 33.69° = 90°$ ✓

Verification of Perpendicularity:

$$i_B + r = 56.31° + 33.69° = 90°$$

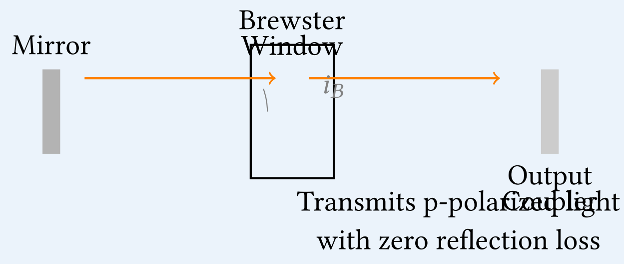
The reflected and refracted rays are indeed perpendicular, confirming that we have found the correct Brewster angle.

Answer:

$$\boxed{i_B = \tan^{-1}(1.5) \approx 56.3°}$$

The Brewster angle for the air-to-glass transition (with $n_{\text{glass}} = 1.5$) is approximately 56.3°.

Practical Application — Brewster Windows:
Brewster's angle is used in laser technology. A glass window placed at the Brewster angle inside a laser cavity transmits p-polarized light with **zero reflection loss**, making it ideal for producing highly polarized laser beams.



Expert's Solution — Meera Nambiar, B.Tech Photonics, IIIT Delhi

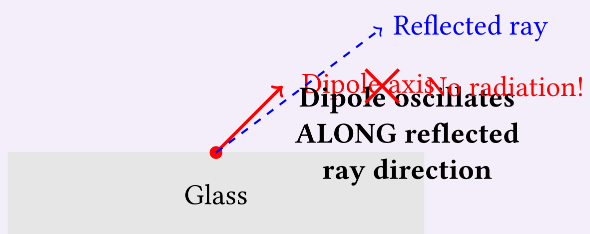
Brewster's Angle: The Polarization Gate

Brewster's angle is arguably the most elegant way to produce completely polarized light without any specialized polarizing materials. Discovered by Sir David Brewster in 1815, it remains a cornerstone of optical physics.

Why Does Complete Polarization Occur at Brewster's Angle?

The microscopic explanation involves the behavior of atomic dipoles in the medium:

1. The incident light's electric field causes electrons in the glass to oscillate.
2. These oscillating dipoles re-radiate electromagnetic waves — this is the physical origin of the reflected beam.
3. An oscillating dipole radiates maximum energy perpendicular to its axis and **zero energy along its axis**.
4. At Brewster's angle, the **refracted ray** is perpendicular to the reflected ray.
5. The component of the electric field **parallel** to the plane of incidence (p-polarized) would cause dipoles to oscillate in a direction **exactly pointing toward** the reflected ray.
6. Since a dipole doesn't radiate along its axis, the p-polarized component is **completely absent** in the reflected beam!



Brewster Angle for Different Transitions:

Transition	n_1	n_2	Brewster Angle
Air → Water	1.0	1.33	$\tan^{-1}(1.33) \approx 53.1°$
Air → Glass	1.0	1.5	$\tan^{-1}(1.5) \approx 56.3°$
Air → Diamond	1.0	2.42	$\tan^{-1}(2.42) \approx 67.5°$
Water → Air	1.33	1.0	$\tan^{-1}(0.752) \approx 36.9°$
Glass → Air	1.5	1.0	$\tan^{-1}(0.667) \approx 33.7°$

Interesting Fact: For the reverse transition (glass to air), the Brewster angle is:

$$i_B^{(\text{glass} \to \text{air})} = \tan^{-1}\left(\frac{1}{1.5}\right) = \tan^{-1}(0.667) \approx 33.7°$$

Note that $56.3° + 33.7° = 90°$, which is not a coincidence — the Brewster angles for the forward and reverse transitions are complementary!

Applications of Brewster's Angle:

Application	How Brewster Angle is Used
Brewster Windows	Zero-loss transmission in lasers
Polarizing Sunglasses	Reduce glare from horizontal surfaces
Photography	Polarizing filters eliminate reflections
Optical Instruments	Anti-reflection at specific angles
Fiber Optics	Control polarization in waveguides

Did You Know?

Key Points to Remember for Exams:

- **Brewster's Law:** $\tan i_B = n_{21} = n_2/n_1$
- **Perpendicularity Condition:** At i_B, reflected ray ⊥ refracted ray, i.e., $i_B + r = 90°$
- **Polarization:** Reflected light is **completely s-polarized** (perpendicular to plane of incidence)
- **Transmitted light:** Partially p-polarized (can be made fully polarized using multiple plates — *pile-of-plates polarizer*)
- **If $n_1 > n_2$:** Brewster angle is less than 45°
- **If $n_2 > n_1$:** Brewster angle is greater than 45°

Mental Shortcut:

$$\text{For air to medium: } i_B \approx 45° + \frac{(n-1)}{0.1} \times 1.1°$$

For $n = 1.5$: $i_B \approx 45° + 5 \times 1.1° \approx 50.5°$ (rough estimate; exact is 56.3°)

Q9 **Light of wavelength 5000 Å falls on a plane reflecting surface. What are the wavelength and frequency of the reflected light? For what angle of incidence is the reflected ray normal to the incident ray?**

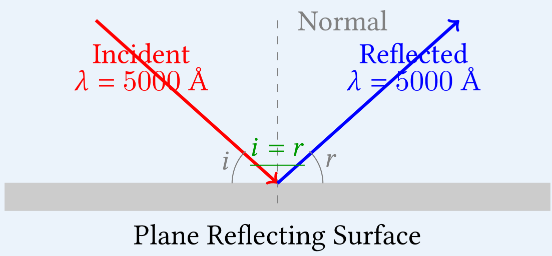
Solution

Understanding the Problem
This problem has two distinct parts:

(i) Finding the wavelength and frequency of reflected light

(ii) Finding the angle of incidence for which the reflected ray is perpendicular to the incident ray

Part 1: Wavelength and Frequency of Reflected Light
When light reflects from a plane surface, it remains in the **same medium**. Unlike refraction, reflection does not change the medium of propagation. Therefore, all fundamental wave characteristics remain unchanged.



Same medium $\Rightarrow$ Same λ, Same ν, Same v

Given Data:

- Wavelength of incident light: $\lambda = 5000$ Å $= 5000 \times 10^{-10}$ m $= 5.0 \times 10^{-7}$ m
- Speed of light in air: $c = 3.0 \times 10^{8}$ m/s

Step 1: Calculate the Frequency
The frequency of light is its fundamental property and does not change during reflection. Using $c = \nu\lambda$:

$$\nu = \frac{c}{\lambda}$$

$$\nu = \frac{3.0 \times 10^{8}\ \text{m/s}}{5.0 \times 10^{-7}\ \text{m}}$$

$$\nu = \frac{3.0}{5.0} \times 10^{8+7} = 0.6 \times 10^{15}\ \text{Hz}$$

$$\nu = 6.0 \times 10^{14}\ \text{Hz}$$

Step 2: Identify Wavelength and Frequency of Reflected Light
Since the reflected ray propagates in the same medium (air):

- **Wavelength:** $\lambda_{\text{reflected}} = 5000\ \text{Å} = 5.0 \times 10^{-7}$ m
- **Frequency:** $\nu_{\text{reflected}} = 6.0 \times 10^{14}$ Hz
- **Speed:** $v_{\text{reflected}} = c = 3.0 \times 10^{8}$ m/s

Key Principle:
Reflection does **not** change
the medium of propagation.
Therefore, **speed**, **wavelength**, and
frequency all remain unchanged.

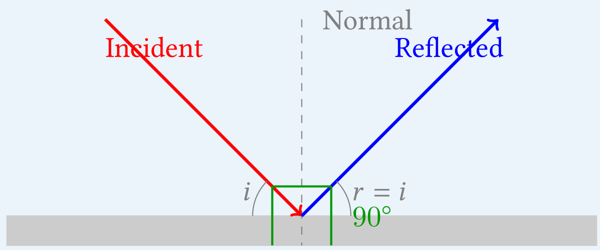
Answer (Part 1):

$$\boxed{\lambda_{\text{reflected}} = 5000\ \text{Å} = 5.0 \times 10^{-7}\ \text{m}}$$

$$\boxed{\nu_{\text{reflected}} = 6.0 \times 10^{14}\ \text{Hz}}$$

The wavelength and frequency of reflected light are identical to those of the incident light.

Part 2: Angle of Incidence for Perpendicular Incident and Reflected Rays
We need to find the angle of incidence i such that the **reflected ray is normal (perpendicular) to the incident ray**.



$$i + r = 90°$$
$$\Rightarrow 2i = 90°$$

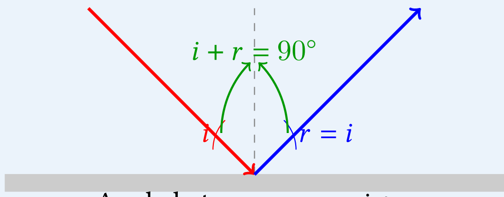
Step 1: Set Up the Geometric Condition
From the law of reflection, the angle of incidence equals the angle of reflection:

$$i = r$$

For the incident and reflected rays to be perpendicular:

$$i + r = 90°$$

This is because the angle between the incident ray and the reflected ray in the diagram is $i + r$ (the sum of the angles they make with the normal on opposite sides).



Angle between rays = $i + r$
(Both measured from normal
on opposite sides)

Step 2: Solve for the Angle
Substituting $r = i$ into $i + r = 90°$:

$$i + i = 90°$$

$$2i = 90°$$

$$i = 45°$$

Step 3: Verify with the Law of Reflection
At $i = 45°$, the reflected angle is also $r = 45°$ (by law of reflection). The total deviation of the reflected ray from the incident ray is:

$$\text{Deviation} = 180° - (i + r) = 180° - 90° = 90°$$

A deviation of $90°$ means the reflected ray is indeed perpendicular to the incident ray. ✓

General Rule:

For a reflected ray to be **perpendicular** to the incident ray:

$$\boxed{i = 45°}$$

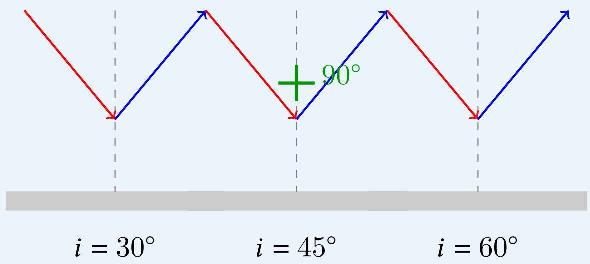
This is independent of the wavelength of light and depends purely on geometry. It follows from the law of reflection ($i = r$) and the perpendicularity condition ($i + r = 90°$).

Answer (Part 2):

$$\boxed{i = 45°}$$

When the angle of incidence is **45°**, the reflected ray is normal (perpendicular) to the incident ray.

Visual Summary:



Only at **45°** are the rays perpendicular to each other

Expert's Solution — Arjun Menon, B.Tech Applied Physics, NIT Surathkal

The Invariance of Frequency Upon Reflection

This problem highlights a fundamental principle that often confuses students: **reflection preserves all wave characteristics**. Unlike refraction, where wavelength and speed change (but frequency remains constant), reflection keeps everything the same because the medium doesn't change.

Why Frequency is the Invariant:

The frequency of light is determined **solely by the source** — it's the rate at which the source oscillates. Whether the light travels through vacuum, air, water, or gets reflected, the source doesn't change its oscillation frequency. Hence:

$$\boxed{\nu_{\text{incident}} = \nu_{\text{reflected}} = \nu_{\text{refracted}} = \nu_{\text{source}}}$$

The Fundamental Invariant:

Process	Medium Change?	Effect on ν, λ, v
Reflection	No	All unchanged
Refraction	Yes	ν unchanged, λ and v change
Scattering	No (usually)	ν unchanged
Absorption/re-emission	—	ν may change

The 45-Degree Reflection: A Geometrical Gem

The condition for perpendicular incident and reflected rays is purely geometric. Let's prove it more formally:

Consider the incident ray making angle i with the normal, and the reflected ray making angle r with the normal on the opposite side. The total angle between the two rays is:

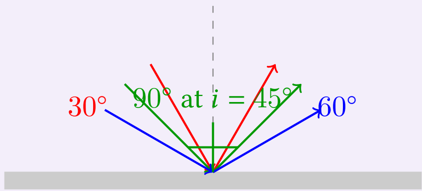
$$\theta = 180^\circ - (i + r) \quad \text{(supplement of the sum)}$$

Actually, let's be more precise. Looking at the geometry:

- The incident ray is at angle i above the surface (measuring from the surface)
- The reflected ray is at angle $r = i$ above the surface on the other side
- The angle between them $= i + r = 2i$

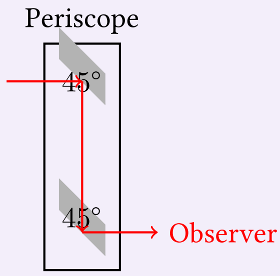
For perpendicular rays:

$$2i = 90^\circ \quad \Rightarrow \quad i = 45^\circ$$



Applications of 45-Degree Reflection:

- **Periscopes:** Use two 45° mirrors/prisms to bend light by 90° at each reflection.
- **Corner reflectors:** Three mutually perpendicular mirrors reflect light back exactly parallel to incident direction.
- **Optical squares:** Used in surveying to establish perpendicular lines.
- **Laser beam steering:** 45° mirrors direct laser beams in optical setups.



Did You Know?

Quick Memory Aids:

- **Reflection → No medium change → ALL properties unchanged**
- **Refraction → Medium changes → Only ν unchanged, λ and v change**
- **Perpendicular rays on reflection:** $i = 45^\circ$ *always* (independent of λ, material, etc.)
- **General formula for angle between incident and reflected rays:**

$$\boxed{\theta_{\text{between rays}} = 180^\circ - 2i}$$

For $\theta = 90^\circ$: $180^\circ - 2i = 90^\circ \Rightarrow i = 45^\circ$

Frequency Calculation Shortcut:

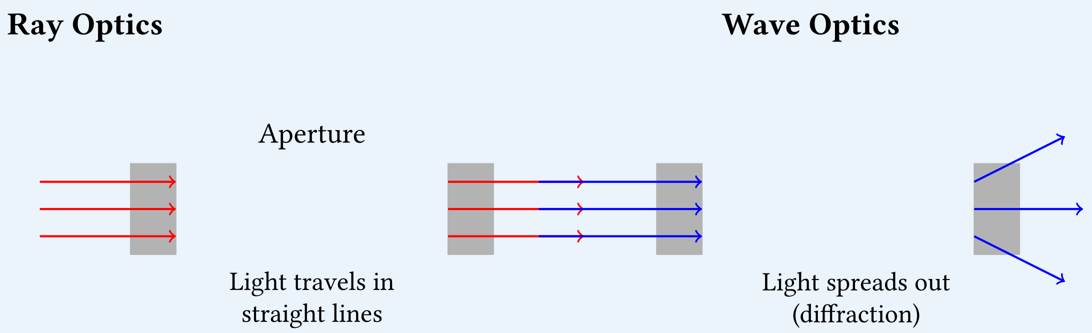
$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{5000 \times 10^{-10}} = \frac{3 \times 10^{18}}{5000} = 6 \times 10^{14} \text{ Hz}$$

Remember: $1 \text{ Å} = 10^{-10}$ m

Q10 **Estimate the distance for which ray optics is a good approximation for an aperture of 4 mm and wavelength 400 nm.**

Solution

Understanding the Validity of Ray Optics
Ray optics (geometrical optics) treats light as traveling in straight lines, ignoring its wave nature. However, when light passes through apertures or encounters obstacles, diffraction effects become significant. The validity of ray optics depends on the relative size of the wavelength compared to the aperture dimensions.



The Fresnel Distance: The Criterion for Ray Optics Validity
The transition region between ray optics and wave optics is characterized by a critical distance called the **Fresnel distance** (Z_F). Ray optics is a good approximation when the observation distance D is much less than Z_F.

Fresnel Distance:

$$\boxed{Z_F = \frac{a^2}{\lambda}}$$

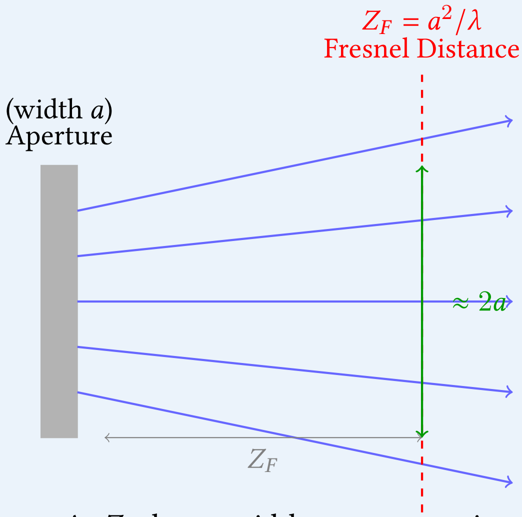
where:

- Z_F = Fresnel distance (characteristic distance for diffraction spreading)
- a = size of the aperture (or obstacle)
- λ = wavelength of light

Criterion for Ray Optics Validity:

$$D \ll Z_F \quad \text{or} \quad D \lesssim \frac{Z_F}{10}$$

Physical Meaning:
The Fresnel distance gives the approximate distance at which the spreading of light due to diffraction becomes comparable to the size of the aperture itself.



At Z_F, beam width $\approx$ aperture size

Given Data:

- Aperture size: $a = 4$ mm $= 4 \times 10^{-3}$ m
- Wavelength: $\lambda = 400$ nm $= 4.0 \times 10^{-7}$ m

Step 1: Calculate the Fresnel Distance

$$Z_F = \frac{a^2}{\lambda}$$

Substitute the values:

$$Z_F = \frac{(4 \times 10^{-3}\ \text{m})^2}{4.0 \times 10^{-7}\ \text{m}}$$
$$Z_F = \frac{16 \times 10^{-6}\ \text{m}^2}{4.0 \times 10^{-7}\ \text{m}}$$
$$Z_F = \frac{16}{4.0} \times \frac{10^{-6}}{10^{-7}}\ \text{m}$$
$$Z_F = 4 \times 10^1\ \text{m}$$
$$Z_F = 40\ \text{m}$$

Calculation:

$$a = 4\ \text{mm} = 4 \times 10^{-3}\ \text{m}$$
$$\lambda = 400\ \text{nm} = 400 \times 10^{-9}\ \text{m} = 4 \times 10^{-7}\ \text{m}$$
$$Z_F = \frac{a^2}{\lambda} = \frac{16 \times 10^{-6}}{4 \times 10^{-7}} = 4 \times 10^1 = 40\ \text{m}$$

Step 2: Interpret the Result
The Fresnel distance $Z_F = 40$ m means:

- For distances **much less than 40 m**, the diffraction spreading is negligible, and **ray optics is a good approximation**.
- For distances **comparable to or greater than 40 m**, diffraction effects become significant, and **wave optics** must be used.

Practically, ray optics holds well for distances up to about $Z_F/10 \approx 4$ m and is reasonably valid up to $Z_F/2 \approx 20$ m.

Step 3: Physical Verification — Angular Spread of Diffraction
The angular spread of the central maximum in diffraction from a single slit of width a is:

$$\theta \approx \frac{\lambda}{a}$$

At a distance D, the linear spread is:

$$\text{Spread} \approx \theta \cdot D = \frac{\lambda D}{a}$$

When this spread becomes comparable to the aperture size a, diffraction dominates:

$$\frac{\lambda D}{a} \approx a \quad \Rightarrow \quad D \approx \frac{a^2}{\lambda} = Z_F$$

At $D = Z_F = 40$ m, the spread is approximately equal to the aperture width (4 mm), confirming our result.

Angular Spread of Central Maximum:

$$\theta = \frac{\lambda}{a} = \frac{4 \times 10^{-7}}{4 \times 10^{-3}} = 10^{-4} \text{ rad}$$

At $D = 40$ m: Spread $= \theta \times D = 10^{-4} \times 40 = 4 \times 10^{-3}$ m $= 4$ mm

Spread equals aperture size $\Rightarrow$ Wave effects dominate beyond this point.

Answer:

$$\boxed{Z_F = \frac{a^2}{\lambda} = \frac{(4 \times 10^{-3})^2}{4 \times 10^{-7}} = 40 \text{ m}}$$

Ray optics is a good approximation for distances **much less than 40 m** (typically $D \lesssim 4$ m for excellent accuracy). Beyond this distance, diffraction effects become increasingly significant, and wave optics must be applied.

General Insight:
The criterion $Z_F = a^2/\lambda$ reveals that:

- **Larger apertures** (a large) $\rightarrow Z_F$ is large $\rightarrow$ Ray optics valid over longer distances
- **Smaller wavelengths** (λ small) $\rightarrow Z_F$ is large $\rightarrow$ Ray optics valid over longer distances
- For **macroscopic objects** ($a \gg \lambda$), Z_F is enormous, explaining why we rarely observe diffraction in daily life

Expert's Solution — Anushka Dwivedi, B.Tech Optics & Optoelectronics, NIT Jalandhar

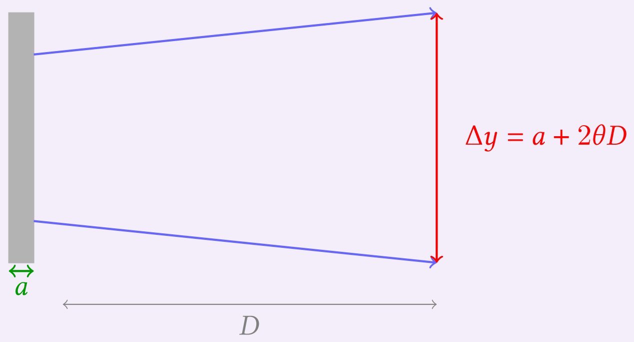
Fresnel Distance: The Boundary Between Ray and Wave Optics
The Fresnel distance is arguably the most practical criterion for determining when we can safely use the simpler ray optics (geometrical optics) instead of the more complex wave optics treatment. Let's explore this concept deeply.
The Physics Behind $Z_F = a^2/\lambda$:
Consider a plane wave incident on an aperture of width a. According to Huygens' principle, each point in the aperture acts as a source of secondary wavelets. The diffraction (angular spreading) angle is:

$$\theta \approx \frac{\lambda}{a}$$

After traveling a distance D, the wave spreads laterally by:

$$\Delta y \approx \theta D = \frac{\lambda D}{a}$$

When $\Delta y \approx 2a$,
wave effects dominate



The **additional spreading** beyond the geometric shadow is $\theta D = \lambda D/a$. When this additional spreading becomes comparable to the aperture size itself:

$$\frac{\lambda D}{a} \approx a \quad \Rightarrow \quad D \approx \frac{a^2}{\lambda} = Z_F$$

Classification of Diffraction Regimes:

Regime	Distance D	Type of Diffraction	Optics to Use
$D \ll Z_F$	$D < Z_F/10$	Negligible diffraction	**Ray Optics**
$D \sim Z_F$	$Z_F/10 < D < 10Z_F$	Fresnel Diffraction	**Wave Optics**
$D \gg Z_F$	$D > 10Z_F$	Fraunhofer Diffraction	**Wave Optics**

Real-World Examples:
Let's calculate Z_F for some everyday situations:

Scenario	a	λ	Z_F
Human eye pupil (daylight)	2 mm	550 nm	~ 7.3 m
Camera lens ($f/8$)	5 mm	550 nm	~ 45 m
Telescope aperture	10 cm	550 nm	~ 18 km
Doorway (sound)	1 m	0.5 m (sound)	~ 2 m
This problem	**4 mm**	**400 nm**	**40 m**

Notice that for sound waves passing through a doorway, Z_F is only about 2 m — that's why we can hear sounds around corners (diffraction) but can't see around corners (ray optics for light)!

Why We Don't See Diffraction in Daily Life:

For typical apertures ($a \sim 1$ mm to 1 cm) and visible light ($\lambda \sim 500$ nm):

$$Z_F = \frac{a^2}{\lambda} \sim \frac{10^{-6}}{5 \times 10^{-7}} = 2 \text{ m} \quad \text{to} \quad \frac{10^{-4}}{5 \times 10^{-7}} = 200 \text{ m}$$

Since we usually observe at distances $\ll Z_F$, ray optics works perfectly for most everyday situations.

Did You Know?

Key Formula for Exams:

$$\boxed{Z_F = \frac{a^2}{\lambda}}$$

Condition for Ray Optics:

$$\boxed{D \ll \frac{a^2}{\lambda}}$$

Important relationships:

- $Z_F \propto a^2$ — doubling aperture quadruples Z_F
- $Z_F \propto 1/\lambda$ — shorter wavelength gives larger Z_F
- Unit check: $\frac{\text{m}^2}{\text{m}} = \text{m}$ ☒

Common mistake: Students often write $Z_F = \lambda/a^2$ (inverted). Remember: the distance should be **large** for large apertures — this helps catch the error.
Numerical Trick: For a in mm and λ in nm:

$$Z_F(\text{in m}) = \frac{a^2(\text{mm}^2)}{\lambda(\text{nm})} \times 10^3$$

Here: $Z_F = \frac{4^2}{400} \times 10^3 = \frac{16}{400} \times 10^3 = 40$ m ☒