NCERT Exemplar Solutions

Solved NCERT Exemplar Problems for Class 12th Physics, Chapter 11

Chapter 11: Dual Nature of Radiation and Matter

About this Chapter

Chapter 11 explores the **dual nature** of light and matter: light behaves as a particle (the **photon**) in the **photoelectric effect**, while matter behaves as a wave through the **de Broglie relation**. You will master the Einstein photoelectric equation, computation of stopping potential, work function and threshold frequency, the de Broglie wavelength of moving particles, and elementary uncertainty estimates. Numerical problems mix photon energy / momentum accounting with non-relativistic kinematics.

Topics covered: Photoelectric effect • Einstein equation • Stopping potential • Work function • Photon energy & momentum • de Broglie wavelength • Davisson–Germer • Heisenberg uncertainty

Quick Formula Sheet

Photon energy & momentum:
$E = h\nu = \frac{hc}{\lambda}, \quad p = \frac{h}{\lambda}$

Einstein photoelectric equation:
$K_{\text{max}} = h\nu - \phi_0 = eV_{\text{stop}}$

de Broglie wavelength:
$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}$

Accelerated charge:
$\lambda = \frac{h}{\sqrt{2mqV}}$

Heisenberg uncertainty:
$\Delta x\, \Delta p \gtrsim \frac{h}{2\pi}$

Useful constants:
$h = 6.626 \times 10^{-34}$ J s
$c = 3 \times 10^{8}$ m/s
$m_e = 9.11 \times 10^{-31}$ kg
$e = 1.6 \times 10^{-19}$ C

NCERT Exemplar Problems

MCQ I (Single Correct Option)

Q 11.1 **A particle is dropped from a height H. The de Broglie wavelength of the particle as a function of height is proportional to:**

(A) H

(B) $H^{1/2}$
(C) H^{0}
(D) $H^{-1/2}$

SOLUTION

Correct option: (D) $H^{-1/2}$.
Concept used. The **de Broglie wavelength** of a moving particle of momentum p is

$$\lambda = \frac{h}{p} = \frac{h}{mv}, \qquad h = 6.626 \times 10^{-34}\,\text{J s}.$$

For a particle released from rest and falling freely through a height h (measured downward from the starting point), conservation of energy gives the speed acquired:

$$\tfrac{1}{2}mv^2 = mgh \implies v = \sqrt{2gh}.$$

If the particle is dropped from height H and we ask for the wavelength when it has fallen a distance equal to its starting height H (the "as a function of height" usually means as a function of the height h it has covered, or equivalently H remaining is variable), the speed at that point is $v = \sqrt{2gH}$.

Step 1. Write momentum in terms of H:

$$p = mv = m\sqrt{2gH} = \sqrt{2m^2gH}.$$

Step 2. Substitute into the de Broglie relation:

$$\lambda = \frac{h}{p} = \frac{h}{m\sqrt{2gH}} = \frac{h}{\sqrt{2m^2g}} \cdot \frac{1}{\sqrt{H}}.$$

Step 3. Collect constants:

$$\lambda = \underbrace{\frac{h}{\sqrt{2m^2g}}}_{\text{constant for given particle}} \cdot H^{-1/2}.$$

So $\lambda \propto H^{-1/2}$.

Step 4. Eliminate the wrong options: (A) H would require $\lambda \propto v^{-1}$ with $v \propto H^{-1}$, which contradicts free fall. (B) $H^{1/2}$ has the right magnitude of dependence but wrong sign in the exponent. (C) H^{0} would mean λ does not change with height, but v grows so p grows so λ must shrink.

Final Answer: $\lambda \propto H^{-1/2}$, so option **(D)**.

☞ **Physical picture**

As the particle falls, gravity speeds it up; faster speed means larger momentum, and a larger momentum forces the de Broglie wavelength to shrink. The exact dependence $\lambda \propto H^{-1/2}$ mirrors $v \propto H^{1/2}$ exactly.

EXPERT'S SOLUTION *: Aarav Iyer, M.Sc Physics, IIT Madras*

Dimensional shortcut. The only height-dependent quantity in $\lambda = h/(mv)$ is v, and free fall gives $v \propto \sqrt{h}$, so $\lambda \propto 1/\sqrt{h}$ at once.

Step 1. Free-fall kinematics: $v^2 = u^2 + 2gh$. Starting from rest, $u = 0$, so $v = \sqrt{2gh}$ and $v \propto \sqrt{h}$.

Step 2. de Broglie: $\lambda = h/(mv) \propto 1/v \propto 1/\sqrt{h} = h^{-1/2}$.

Step 3. Sanity check with limits: at $h = 0$, $v = 0$, so $\lambda \to \infty$. As $h \to \infty$, $\lambda \to 0$. The $H^{-1/2}$ dependence captures both limits.

Numerical feel. For a 1 g ball dropped from $H = 1$ m, $v = \sqrt{2 \cdot 9.8 \cdot 1} \approx 4.43$ m/s, $p = 10^{-3} \cdot 4.43 = 4.43 \times 10^{-3}$ kg m/s, and $\lambda = h/p \approx 6.626 \times 10^{-34}/4.43 \times 10^{-3} \approx 1.5 \times 10^{-31}$ m. Compare a Bohr radius (0.053 nm $= 5.3 \times 10^{-11}$ m): macroscopic objects have wavelengths $\sim 10^{20}$ times smaller than atomic scales, which is exactly why we never observe their wave nature.

Alternative approach (energy method). Work-energy theorem on the falling particle gives $K = mgh$ directly, so $p = \sqrt{2mK} = \sqrt{2m^2gh}$ and $\lambda = h/p = h/(m\sqrt{2gh}) \propto h^{-1/2}$. The two routes converge because uniform gravitational acceleration is conservative.

Why this matters. Whenever a particle is accelerated from rest by a conservative force, its de Broglie wavelength scales as $1/\sqrt{\text{distance}}$ (uniform field) or $1/\sqrt{V}$ (potential difference). The same scaling drives the Davisson–Germer voltage-tuning relation $\lambda = 12.27/\sqrt{V}$ Å and the electron-gun calibration in cathode-ray tubes.

Final Answer: Option **(D)**: $\lambda \propto H^{-1/2}$.

☞ **de Broglie ladder**

For a particle that starts from rest and is accelerated by a uniform conservative force:

- Velocity $\propto \sqrt{\text{distance}}$ (or $\sqrt{V}$ for electric).
- Momentum $p = mv \propto \sqrt{\text{distance}}$.
- Wavelength $\lambda = h/p \propto 1/\sqrt{\text{distance}}$.

Memorise the inversion: distance up, wavelength down (by square root).

Q 11.2 **The wavelength of a photon needed to remove a proton from a nucleus which is bound to the nucleus with 1 MeV energy is nearly:**

(A) 1.2 **nm**

(B) 1.2×10^{-3} **nm**
(C) 1.2×10^{-6} **nm**
(D) 1.2×10^{1} **nm**

SOLUTION

Correct option: (B) 1.2×10^{-3} nm.
Concept used. A photon of frequency ν carries energy $E = h\nu = hc/\lambda$. To eject a particle that is bound with energy E_b, the photon must supply at least E_b, so the wavelength needed is

$$\lambda = \frac{hc}{E_b}.$$

Useful numerical shortcut: $hc = 1240\,\text{eV nm} = 1.24 \times 10^{-6}\,\text{eV m}$.

Step 1. Write the binding energy in eV:

$$E_b = 1\,\text{MeV} = 10^6\,\text{eV}.$$

Step 2. Apply the formula with hc in convenient units:

$$\lambda = \frac{hc}{E_b} = \frac{1240\,\text{eV nm}}{10^6\,\text{eV}} = 1.24 \times 10^{-3}\,\text{nm} \approx 1.2 \times 10^{-3}\,\text{nm}.$$

Step 3. Eliminate distractors: (A) 1.2 nm corresponds to $hc/\lambda = 1240/1.2 \approx 10^3$ eV = 1 keV, far below 1 MeV. (C) 1.2×10^{-6} nm would correspond to $\sim 1\,\text{GeV}$, too large. (D) 12 nm is even further from 1 MeV.

Final Answer: $\lambda \approx 1.2 \times 10^{-3}$ nm, so option **(B)**.

☞ Numerical shortcut

Remember $hc = 1240\,\text{eV nm}$. Then any photon-energy problem collapses to $\lambda(\text{nm}) = 1240/E(\text{eV})$. Visible light ($\lambda \sim 500$ nm) ↔ a few eV. X-rays ($\lambda \sim 10^{-2}$ nm) ↔ tens of keV. Gamma-rays of MeV energies sit at $\sim 10^{-3}$ nm, as in this problem.

EXPERT'S SOLUTION *: Priya Sharma, Ph.D Physics, IISc Bangalore*

Order-of-magnitude angle. Nuclear binding scales sit in MeV, which means gamma-ray photons with picometre wavelengths. Use the 1240 shortcut.

Step 1. $E = 10^6\,\text{eV}$. Apply $\lambda = hc/E$.

Step 2. $\lambda = 1240/10^6\,\text{nm} = 1.24 \times 10^{-3}\,\text{nm}$.

Step 3. Confirm units: hc/E has units of nm because hc is in eV nm and E is in eV.

SI-units cross-check. Working in SI:

$\lambda = hc/E = (6.626 \times 10^{-34})(3 \times 10^8)/[10^6 \cdot 1.6 \times 10^{-19}]$ m $= 1.988 \times 10^{-25}/1.6 \times 10^{-13}$ m $= 1.24 \times 10^{-12}$ m $= 1.24 \times 10^{-3}$ nm. Matches. The shortcut $hc = 1240$ eV nm is faster because the unit conversion is pre-folded.

Alternative pictures. Compton-scattering view: a 1 MeV gamma photon carries momentum $p = h/\lambda \approx 5.3 \times 10^{-22}$ kg m/s, which is comparable to nuclear recoil scales — this is why MeV gammas can disrupt nuclei but cm-scale RF photons cannot. Photon-frequency view: $\nu = E/h = 10^6 \cdot 1.6 \times 10^{-19}/6.626 \times 10^{-34} \approx 2.4 \times 10^{20}$ Hz, a hard-gamma frequency.

Why this matters. Gamma rays disintegrate nuclei because their wavelengths and energies are matched to nucleon binding scales. Visible photons cannot. The same logic underlies the spectroscopic ladder: visible light probes valence electrons (eV), X-rays probe inner shells (keV), gamma rays probe nuclei (MeV), and TeV photons probe sub-nucleonic structure.

Final Answer: Option **(B)**: $\lambda \approx 1.2 \times 10^{-3}$ nm.

✗ Don't confuse photon E with massive-particle K

The photon is massless. Its total energy $E = h\nu$ *is* its kinetic energy — there is no rest-mass contribution to peel off. Students sometimes try to use $K = \frac{1}{2}mv^2$ or $K = p^2/(2m)$ for photons. Both are non-relativistic massive-particle formulas. For a photon, $E = pc$ exactly, and E is the whole story.

Q 11.3 Consider a beam of electrons (each electron with energy E_0) incident on a metal surface kept in an evacuated chamber. Then:

(A) no electrons will be emitted as only photons can emit electrons.

(B) electrons can be emitted but all with an energy, E_0.

(C) electrons can be emitted with any energy, with a maximum of $E_0 - \phi$ (ϕ is the work function).

(D) electrons can be emitted with any energy, with a maximum of E_0.

SOLUTION

Correct option: (D) electrons emitted with energies up to E_0.

Concept used. Secondary emission: when energetic electrons hit a metal surface, some of their kinetic energy can be transferred to electrons in the metal. Unlike photons (which are absorbed wholesale, leading to Einstein's $K_{max} = h\nu - \phi$), an incoming electron can deposit any fraction of its energy. The energy available to an ejected electron is therefore distributed: the maximum kinetic energy a freed electron can carry off is E_0 (the entire input), with the work function ϕ paid as needed internally, but the form of the answer here is about the *ceiling* on emergent energy, not Einstein's relation.

Step 1. The Einstein formula $h\nu - \phi$ in (C) is specific to photons: a photon is absorbed in one shot, depositing exactly $h\nu$, and the work function comes off the top. Electrons in a beam do not obey this; energy transfer is a collisional affair, possibly multi-step.

Step 2. For an incident electron of energy E_0, the energy balance for an ejected electron of kinetic energy K reads

$$K \leq E_0 - (\text{energy left in incoming electron}) - (\text{minimum work to escape}).$$

The maximum K is bounded above by E_0, attained in the rare case when the incoming electron transfers essentially all its energy.

Step 3. Eliminate the wrong options:

- (A) is false because charged-particle bombardment is a standard secondary-emission mechanism (used in photomultipliers and CRT screens).
- (B) is false: a continuous distribution of ejected energies is observed, not a delta function at E_0.
- (C) is the photoelectric-effect formula and applies to photons, not electrons.
- (D) correctly states the ceiling: any energy up to E_0 is possible.

Final Answer: Option **(D)**: emergent electrons span energies up to E_0.

✗ Photon $\neq$ electron

A common slip is to apply the Einstein equation $K_{\text{max}} = h\nu - \phi$ to charged-particle bombardment. The Einstein equation is special to photon absorption (single-shot, full energy deposited at once). Electron-electron collisions allow partial energy transfer and a continuous emergent spectrum.

EXPERT'S SOLUTION *: Vivaan Gupta, M.Tech Applied Physics, IIT Delhi*

Mechanism angle. The question contrasts photoemission (one photon $\rightarrow$ one electron, Einstein) with secondary electron emission (electron-electron scattering).

Step 1. In photoemission, the energy of a single photon $h\nu$ is fully absorbed; what is left for kinetic energy of the photoelectron is $h\nu - \phi$.

Step 2. In electron-impact ionisation, the incoming electron need not deposit all its energy. It can scatter inelastically, knocking out a bound electron with any energy from 0 up to $\sim E_0 - \phi$. But the strict ceiling for any electron leaving the surface (the original projectile or the secondary) is E_0 minus the residual binding, so a single electron carrying energy close to E_0 is allowed.

Step 3. Quick reality check: in a scanning electron microscope, the secondary electrons span a continuous low-energy distribution, while the back-scattered primaries carry energies close to E_0. Both populations exist; the maximum is $\sim E_0$.

Alternative reasoning (energy-budget). Energy conservation gives, for an emergent electron of K, $E_0 = K +$ (energy left in primary) $+$ (binding ϕ). The minimum the primary can keep is 0 (full deposit), so $K_{\max} = E_0 - \phi$. But the *primary itself* can leave the surface carrying nearly all E_0 if it is elastically back-scattered: that is the back-scattered electron channel. The two channels together fill the spectrum up to E_0, so (D) is the strict ceiling.

Wave-vs-particle contrast. A wave's energy is delivered continuously; a particle (photon *or* electron) deposits its kinetic energy in discrete events. The continuous emergent spectrum is the signature of particle-particle scattering, sharply different from the photoelectric line spectrum dictated by Einstein's equation.

Why this matters. Distinguishing absorption (photons) from scattering (electrons) is the foundation of modern detectors: photomultipliers (photon $\to$ electron cascade), electron microscopes (electron $\to$ image via scattering), image intensifiers and channel-plate detectors all exploit the asymmetry between the two processes.

Final Answer: Option **(D)**: $K \leq E_0$.

Q 11.4 Consider Fig. 11.7 in the NCERT text book of Physics for Class XII (Davisson–Germer apparatus). Suppose the voltage applied to A is increased. The diffracted beam will have the maximum at a value of θ that:

(A) will be larger than the earlier value.

(B) will be the same as the earlier value.

(C) will be less than the earlier value.

(D) will depend on the target.

SOLUTION

Correct option: (C) smaller than the earlier value.

Concept used. In the Davisson–Germer experiment, electrons accelerated through a potential V acquire de Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2m_e eV}},$$

and scatter from the crystal lattice (Ni in Davisson–Germer). The angular position of the first-order diffraction maximum follows the **Bragg condition** $2d\sin\theta = n\lambda$, equivalent to $d\sin\phi = n\lambda$ with ϕ measured between the diffracted beam and the incident beam (NCERT convention).

Step 1. Write λ as a function of V:

$$\lambda(V) = \frac{h}{\sqrt{2m_e eV}} \implies \lambda \propto \frac{1}{\sqrt{V}}.$$

Increasing V shrinks λ.

Step 2. Bragg condition for the same crystal spacing d:

$$d\sin\theta = n\lambda \implies \sin\theta \propto \lambda.$$

Since λ shrinks, $\sin\theta$ shrinks, so θ shrinks (for $\theta < 90°$).

Step 3. Eliminate distractors: (A) is the opposite of the correct trend. (B) ignores the dependence of λ on V. (D) is true only in the trivial sense that d depends on the target, but the question fixes the target. Among the listed answers, (C) captures the only relevant trend.

Final Answer: θ decreases, so option **(C)**.

EXPERT'S SOLUTION *: Aditi Reddy, Ph.D Condensed Matter Physics, TIFR Mumbai*

Two-line argument. $\lambda \propto V^{-1/2}$; $\sin\theta \propto \lambda$; therefore $\theta\downarrow$ when $V\uparrow$.

Step 1. Voltage-wavelength: $\lambda = 12.27/\sqrt{V}\,\text{Å}$ (in non-relativistic regime, V in volts). So $V\uparrow \Rightarrow \lambda\downarrow$.

Step 2. Bragg's law at fixed d: $\sin\theta = n\lambda/d$. Decreasing λ decreases $\sin\theta$ and hence θ in the first quadrant.

Step 3. Numerical illustration. At $V = 54\,\text{V}$ (the original Davisson–Germer setting), $\lambda = 12.27/\sqrt{54} \approx 1.67\,\text{Å}$. With Ni spacing $d = 2.15\,\text{Å}$ the first-order peak sits near $\theta_1 = \sin^{-1}(\lambda/d) \approx 51°$. Doubling V to $108\,\text{V}$ drops λ to $\sim 1.18\,\text{Å}$ and shifts the peak inward to $\theta_2 \approx 33°$.

Step 4. Differential check: $\sin\theta = \lambda/d$. Differentiate w.r.t. V: $\cos\theta\, d\theta/dV = (1/d)\, d\lambda/dV$. Since $d\lambda/dV < 0$, $d\theta/dV < 0$. The peak migrates inward smoothly as V rises.

Concept linkage. Davisson–Germer (1927) was the experimental clincher for de Broglie matter waves (proposed 1924). The Bragg geometry borrowed straight from X-ray crystallography (W.L. Bragg, 1913) shows that electrons obey the same wave kinematics as X-rays — demonstrating wave–particle duality in matter.

Why this matters. The Davisson–Germer scaling is a working diagnostic in electron diffraction microscopy: tune V to bring a given peak to a convenient angle. Modern transmission electron microscopes operate at 100–300 kV, giving wavelengths ~ 2–4 pm, far below atomic spacings — exactly what is needed to resolve crystal lattices.

Final Answer: Option **(C)**: θ decreases.

☞ **The $12.27/\sqrt{V}$ formula**

For an electron accelerated from rest through V volts (non-relativistic),

$$\lambda = \frac{h}{\sqrt{2m_eeV}} = \frac{12.27}{\sqrt{V}}\,\text{Å}\,\left(= \frac{1.227}{\sqrt{V}}\,\text{nm}\right).$$

At $V = 1$ V $\rightarrow \lambda \approx 12.3$ Å; at $V = 100$ V $\rightarrow \lambda \approx 1.23$ Å; at $V = 10$ kV $\rightarrow \lambda \approx 0.123$ Å.

Q 11.5 **A proton, a neutron, an electron and an α-particle have same energy. Then their de Broglie wavelengths compare as:**

(A) $\lambda_p = \lambda_n > \lambda_e > \lambda_\alpha$

(B) $\lambda_\alpha < \lambda_p = \lambda_n > \lambda_e$

(C) $\lambda_e < \lambda_p = \lambda_n > \lambda_\alpha$

(D) $\lambda_e = \lambda_p = \lambda_n = \lambda_\alpha$

SOLUTION

Correct option: (B) $\lambda_\alpha < \lambda_p = \lambda_n > \lambda_e$.

Concept used. For a non-relativistic particle of mass m and kinetic energy K,

$$p = \sqrt{2mK} \quad \text{and} \quad \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}.$$

At fixed K the wavelength varies inversely with $\sqrt{m}$: heavier particle $\Rightarrow$ shorter wavelength.

Step 1. Masses (approximate): $m_e \approx 9.11 \times 10^{-31}$ kg, $m_p \approx m_n \approx 1836\, m_e$, $m_\alpha \approx 4\, m_p = 7344\, m_e$.

Step 2. Apply $\lambda \propto 1/\sqrt{m}$ at fixed K:

$$\lambda_e : \lambda_p : \lambda_n : \lambda_\alpha = \frac{1}{\sqrt{m_e}} : \frac{1}{\sqrt{m_p}} : \frac{1}{\sqrt{m_n}} : \frac{1}{\sqrt{m_\alpha}}.$$

Step 3. Numerical sort:

- Smallest mass $\Rightarrow$ largest λ: electron has the largest wavelength.
- $m_p = m_n \Rightarrow \lambda_p = \lambda_n$.

- Largest mass (α) $\Rightarrow$ smallest λ.

Final ordering: $\lambda_e > \lambda_p = \lambda_n > \lambda_\alpha$.

Step 4. Match to options. (A) misorders the electron with respect to proton. (C) wrongly puts λ_e at the small end. (D) ignores the mass dependence. (B) writes $\lambda_\alpha < \lambda_p = \lambda_n > \lambda_e$, which is the same chain reversed and adds the (implicit but true) statement $\lambda_e < \lambda_p$ *only if we re-read*: in fact $\lambda_e > \lambda_p$, so (B) reads as: α smaller than proton/neutron, and proton/neutron larger than electron. That places electron at the small end and is the closest match in the listed options.

Final Answer: Option **(B)**: $\lambda_\alpha < \lambda_p = \lambda_n > \lambda_e$.

☞ Reading option (B) carefully

Option (B) gives two pieces of information: (i) $\lambda_\alpha < \lambda_p = \lambda_n$ (heavier α has shorter wavelength than proton/neutron), and (ii) $\lambda_p = \lambda_n > \lambda_e$ *or* $< \lambda_e$ depending on direction. The NCERT key takes (B) as the intended ordering; the substantive correct statement is the chain $\lambda_e > \lambda_p = \lambda_n > \lambda_\alpha$.

EXPERT'S SOLUTION *: Karan Banerjee, M.Sc Physics, IIT Madras*

Mass–wavelength angle. Same K, so all that varies is $1/\sqrt{m}$.

Step 1. Order by mass (small to large): $e < p = n < \alpha$.

Step 2. Order by wavelength (large to small): $\lambda_e > \lambda_p = \lambda_n > \lambda_\alpha$.

Step 3. Numerical feel: at $K = 1\,\text{eV}$, $\lambda_e \approx 1.23\,\text{nm}$ and $\lambda_p \approx 0.0286\,\text{nm}$; $\lambda_\alpha \approx 0.0143\,\text{nm}$. The electron wavelength is two orders of magnitude larger than the proton's at the same energy.

Step 4. Ratio chain. With $m_p = m_n = 1836\,m_e$ and $m_\alpha = 7344\,m_e$:

$$\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}} = \sqrt{1836} \approx 42.85,$$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha}{m_p}} = \sqrt{4} = 2,$$

$$\frac{\lambda_e}{\lambda_\alpha} = \sqrt{\frac{m_\alpha}{m_e}} = \sqrt{7344} \approx 85.7.$$

These ratios are independent of the chosen K.

Alternative phrasing (momentum view). Same energy $\Rightarrow$ different momenta because $p = \sqrt{2mK}$. The electron, being lightest, has the smallest momentum and hence the largest wavelength. Be careful not to confuse this with the "same momentum" scenario, where λ would be identical for all four particles.

Why this matters. Diffraction-based imaging with electrons (TEM) achieves higher spatial resolution at the same energy compared with proton or alpha probes, because the longer electron wavelength is offset by easier accelerator engineering. Conversely, neutron diffraction probes magnetic structure thanks to the neutron's spin and zero charge, despite the small wavelength advantage; alpha probes (Rutherford scattering) sample much shorter wavelengths and resolve nuclear sizes.

Final Answer: Option **(B)**.

✗ Same energy is not same momentum

"Same energy" fixes K, so $\lambda = h/\sqrt{2mK} \propto 1/\sqrt{m}$; the heaviest particle has the shortest wavelength. Compare with "same momentum" (Q11.10) where $\lambda = h/p$ is identical for all particles regardless of mass. Always state *which quantity is the same* before writing the ratio.

Q 11.6 An electron is moving with an initial velocity $\vec{v} = v_0\hat{i}$ and is in a magnetic field $\vec{B} = B_0\hat{j}$. Then its de Broglie wavelength:

(A) remains constant.

(B) increases with time.

(C) decreases with time.

(D) increases and decreases periodically.

SOLUTION

Correct option: (A) remains constant.

Concept used. The **magnetic force** on a moving charge is $\vec{F} = q\vec{v} \times \vec{B}$. Because this force is always perpendicular to $\vec{v}$, it does *no work* on the particle; the kinetic energy and hence the speed remain constant. The de Broglie wavelength depends only on the speed ($\lambda = h/(mv)$), so it does not change either.

Step 1. Compute the initial force:

$$\vec{F} = -e(v_0\hat{i}) \times (B_0\hat{j}) = -eB_0v_0(\hat{i} \times \hat{j}) = -eB_0v_0\hat{k}.$$

The force is perpendicular to $\vec{v}$.

Step 2. Power delivered by the magnetic force:

$$P = \vec{F} \cdot \vec{v} = (\vec{v} \times \vec{B}) \cdot \vec{v}\,(-e) = 0.$$

So $dK/dt = 0$ and the kinetic energy $K = \frac{1}{2}mv^2$ is constant in time.

Step 3. Speed and momentum stay constant: $|\vec{v}| = v_0$, $p = mv_0$.

Step 4. Apply de Broglie:

$$\lambda = \frac{h}{p} = \frac{h}{mv_0} = \text{constant.}$$

Step 5. Eliminate distractors: (B) and (C) require energy gain/loss, ruled out. (D) would require oscillating $|v|$, but the magnetic force only rotates the velocity vector, never changes its magnitude.

Final Answer: λ is constant in time, so option **(A)**.

♥ Why magnetic forces don't change speed

The defining feature of magnetism on a moving charge is that the force is always at right angles to the velocity. Geometric consequence: the velocity vector rotates but its length is preserved, so the speed and hence the de Broglie wavelength are constants of motion. Magnetic fields curve trajectories without supplying energy.

EXPERT'S SOLUTION *: Rohit Joshi, B.Tech Engineering Physics, IIT Bombay*

Energy angle. Magnetic forces are always orthogonal to velocity; therefore $W = 0$, K is conserved, p is conserved in magnitude, λ is constant.

Step 1. Work-energy theorem: $\Delta K = \int \vec{F} \cdot d\vec{r} = \int \vec{F} \cdot \vec{v}\, dt = 0$ for $\vec{F} = q\vec{v} \times \vec{B}$.

Step 2. Hence $|\vec{v}| = v_0$ at all t and $\lambda(t) = h/(mv_0)$.

Step 3. Trajectory shape. The acceleration is centripetal, $a = v_0^2/r$, giving radius $r = mv_0/(eB_0)$ and angular frequency $\omega_c = eB_0/m$ (cyclotron frequency). The velocity *vector* rotates in the $\hat{\imath}$–$\hat{k}$ plane (perpendicular to $\vec{B} = B_0\hat{\jmath}$), but its magnitude is fixed.

Vector decomposition. The initial velocity $\vec{v} = v_0\hat{\imath}$ has no component along $\vec{B} = B_0\hat{\jmath}$. The force $\vec{F} = -e\vec{v} \times \vec{B} = -ev_0B_0(\hat{\imath} \times \hat{\jmath}) = -ev_0B_0\hat{k}$ pushes the electron into the $\hat{\imath}\hat{k}$-plane, so all the motion lies in that plane. Had $\vec{v}$ had a $\hat{\jmath}$-component, that component would propagate unchanged (no force along $\vec{B}$), giving a helical path — but the speed (and λ) would still be constant.

Contrast with electric field. An electric force $q\vec{E}$ is independent of $\vec{v}$ and so generally has a component along $\vec{v}$, doing nonzero work — K changes, $|\vec{v}|$ changes, λ changes. The fundamental asymmetry between $\vec{E}$ and $\vec{B}$ on charged particles traces back to relativity: $\vec{E}$ and $\vec{B}$ are different components of the same field tensor, and only the parallel-to-velocity projection of $\vec{E}$ does work.

Why this matters. In a cyclotron the magnetic field bends particles in circles at fixed speed; electric fields in the gap do the accelerating. Separating the two roles is conceptually clean and operationally essential. The same principle governs mass spectrometers (B selects p/q, E selects E/q), cathode-ray-tube deflection, and tokamak confinement.

Final Answer: Option **(A):** λ is independent of t.

Q 11.7 **An electron (mass m) with an initial velocity $\vec{v} = v_0\hat{i}$ ($v_0 > 0$) is in an electric field $\vec{E} = -E_0\hat{i}$ ($E_0 =$ constant > 0). Its de Broglie wavelength at time t is given by:**

(A) $\dfrac{\lambda_0}{1 + \dfrac{eE_0 t}{mv_0}}$

(B) $\lambda_0\left(1 + \dfrac{eE_0 t}{mv_0}\right)$

(C) λ_0

(D) $\lambda_0\, t$.

SOLUTION

Correct option: (A) $\lambda_0/\left(1 + \dfrac{eE_0 t}{mv_0}\right)$.

Concept used. Newton's second law for a charge $-e$ in the field $\vec{E} = -E_0\hat{i}$:

$$\vec{F} = q\vec{E} = (-e)(-E_0\hat{i}) = +eE_0\hat{i}.$$

The force is along $+\hat{i}$, the same direction as the initial velocity, so the electron *speeds up* along $\hat{i}$. The acceleration is constant; the velocity grows linearly with time, and the de Broglie wavelength shrinks as $\lambda(t) = h/(mv(t))$.

Step 1. Compute the acceleration:

$$\vec{a} = \frac{\vec{F}}{m} = \frac{eE_0}{m}\hat{i}.$$

Step 2. Velocity as a function of time:

$$\vec{v}(t) = v_0\hat{i} + \vec{a}t = \left(v_0 + \frac{eE_0 t}{m}\right)\hat{i}.$$

Its magnitude is $v(t) = v_0 + eE_0 t/m = v_0\left(1 + \dfrac{eE_0 t}{mv_0}\right)$.

Step 3. Apply de Broglie. The initial wavelength is $\lambda_0 = h/(mv_0)$. At time t:

$$\lambda(t) = \frac{h}{mv(t)} = \frac{h}{mv_0\left(1 + \frac{eE_0 t}{mv_0}\right)} = \frac{\lambda_0}{1 + \frac{eE_0 t}{mv_0}}.$$

Step 4. Eliminate distractors: option (B) is the reciprocal of (A) and would correspond to a force *decelerating* the electron, but here it speeds up. (C) ignores the field. (D) has wrong dimensions.

Final Answer: $\lambda(t) = \dfrac{\lambda_0}{1 + eE_0t/(mv_0)}$, so option **(A)**.

☞ **Sign-check trick**

The electron carries charge $-e$ and the field points in $-\hat{i}$, so the product of two negatives gives a $+\hat{i}$ force: the electron *accelerates*, not decelerates. Choose option (A) (denominator grows).

EXPERT'S SOLUTION *: Sneha Verma, M.Sc Physics, IIT Madras*

Kinematics angle. Constant force ⇒ linear velocity in time; $\lambda \propto 1/v$ gives the answer.

Step 1. Force on electron: $F = (-e)(-E_0) = +eE_0$, accelerating.

Step 2. $v(t) = v_0 + (eE_0/m)t$.

Step 3. $\lambda(t) = h/(mv(t)) = \lambda_0/(1 + eE_0t/(mv_0))$.

Numerical feel. If $E_0 = 10^4$ V/m and $v_0 = 10^7$ m/s, the timescale to double v is $t^* = mv_0/(eE_0) = (9.11 \times 10^{-31})(10^7)/(1.6 \times 10^{-19})(10^4) \approx 5.7 \times 10^{-9}$ s. At that moment λ has shrunk to $\lambda_0/2$. So in routine lab fields the wavelength response is on nanosecond timescales — not instantaneous, but fast.

Energy cross-check. Work done by the field over distance $x = v_0t + \frac{1}{2}(eE_0/m)t^2$ is $W = eE_0x$. The new kinetic energy is $\frac{1}{2}mv(t)^2$. Verifying with $v = v_0 + eE_0t/m$: $\frac{1}{2}m(v_0 + eE_0t/m)^2 - \frac{1}{2}mv_0^2 = eE_0v_0t + \frac{1}{2}(eE_0)^2t^2/m = eE_0[v_0t + \frac{1}{2}(eE_0/m)t^2] = eE_0x$. ✓

Dimensional verification. The factor $1 + eE_0t/(mv_0)$ is dimensionless because $eE_0t/(mv_0)$ has units (C)(V/m)(s)/(kg)(m/s) = (C·V)(s)/(kg·m^2/s) = (J)(s)/(J·s) = 1. ✓

Concept linkage. The setup mirrors a linac (linear accelerator) drift section where a uniform field accelerates electrons; the de Broglie wavelength continuously decreases as the electrons gain momentum, until they reach relativistic speeds requiring $p = \gamma mv$ rather than mv.

Final Answer: Option **(A)**.

Q 11.8 **An electron (mass m) with an initial velocity $\vec{v} = v_0\hat{i}$ is in an electric field $\vec{E} = E_0\hat{j}$. If $\lambda_0 = h/(mv_0)$, its de Broglie wavelength at time t is given by:**

(A) λ_0

(B) $\lambda_0\sqrt{1 + \dfrac{e^2E_0^2t^2}{m^2v_0^2}}$

(C) $\dfrac{\lambda_0}{\sqrt{1+\dfrac{e^2E_0^2t^2}{m^2v_0^2}}}$

(D) $\dfrac{\lambda_0}{1+\dfrac{e^2E_0^2t^2}{m^2v_0^2}}$

SOLUTION

Correct option: (C) $\lambda_0/\sqrt{1+e^2E_0^2t^2/(m^2v_0^2)}$.

Concept used. The field is perpendicular to the initial velocity, so motion is two-dimensional: a constant velocity along $\hat{i}$ and a uniform acceleration along $\hat{j}$ (the transverse direction). The total speed at time t then combines via Pythagoras, and $\lambda = h/(mv)$ shrinks as v grows.

Step 1. Force on electron $(-e)$ in field $E_0\hat{j}$:

$$\vec{F} = (-e)(E_0\hat{j}) = -eE_0\hat{j}, \quad \vec{a} = -\frac{eE_0}{m}\hat{j}.$$

Step 2. Velocity components at time t:

$$v_x = v_0, \quad v_y = -\frac{eE_0t}{m}.$$

(v_y is negative because the electron is pushed in $-\hat{j}$, but only its magnitude matters for $|\vec{v}|$.)

Step 3. Magnitude of velocity:

$$v(t) = \sqrt{v_x^2+v_y^2} = \sqrt{v_0^2+\frac{e^2E_0^2t^2}{m^2}}.$$

Step 4. Apply de Broglie:

$$\lambda(t) = \frac{h}{mv(t)} = \frac{h}{m\sqrt{v_0^2+e^2E_0^2t^2/m^2}} = \frac{h}{mv_0\sqrt{1+e^2E_0^2t^2/(m^2v_0^2)}}.$$

Recognise $h/(mv_0) = \lambda_0$:

$$\lambda(t) = \frac{\lambda_0}{\sqrt{1+\dfrac{e^2E_0^2t^2}{m^2v_0^2}}}.$$

Step 5. Eliminate distractors: (A) ignores the field. (B) is the reciprocal of (C). (D) misses the square root from combining perpendicular components.

Final Answer: Option **(C)**.

☞ Perpendicular field → Pythagorean speed

Whenever an electric or magnetic field is perpendicular to the initial velocity, the components of velocity in the two perpendicular directions evolve independently. The total speed combines via $v = \sqrt{v_\parallel^2 + v_\perp^2}$.

EXPERT'S SOLUTION *: Ananya Pillai, M.Sc Astrophysics, IIT Kanpur*

Component decomposition. Field $\perp$ initial velocity; one component stays v_0 and the other grows linearly. Combine with Pythagoras.

Step 1. $v_x = v_0$ (no force along $\hat{i}$).

Step 2. $v_y = (eE_0/m)t$ (in magnitude; the electron acquires transverse velocity).

Step 3. $|v| = \sqrt{v_0^2 + (eE_0t/m)^2} = v_0\sqrt{1 + (eE_0t/(mv_0))^2}$.

Step 4. $\lambda = h/(m|v|) = \lambda_0/\sqrt{1 + (eE_0t/(mv_0))^2}$.

Trajectory geometry. The electron traces a parabola: $x = v_0t$, $y = -\frac{1}{2}(eE_0/m)t^2$ (deflected toward $-\hat{j}$ because the force on the electron is $-eE_0\hat{j}$). Eliminating t: $y = -(eE_0/2mv_0^2)x^2$, a standard projectile-motion parabola.

Limit checks.

- $t \to 0$: $\lambda \to \lambda_0$. ✓
- $t \to \infty$: $\lambda \to 0$ as $1/t$, reflecting the asymptotic dominance of the transverse acceleration. ✓
- $E_0 \to 0$: $\lambda \to \lambda_0$ for all t (no field, no deflection). ✓

Contrast with Q11.7. In Q11.7 (parallel field) the velocity components stayed 1-dimensional and λ followed a $1/(1 + \text{linear})$ law. Here (perpendicular field), components stay independent and combine in quadrature, giving a $1/\sqrt{1 + \text{quadratic}}$ law. The parametric difference is exactly the geometric difference between collinear and orthogonal acceleration.

Why this matters. The geometry mimics CRT deflection: a horizontal beam is deflected vertically; the de Broglie wavelength along the trajectory varies according to the running speed. The same setup appears in mass-spectrometer deflection chambers and in oscilloscope Y-deflection plates.

Final Answer: Option **(C)**.

✗ Don't add magnitudes; add components

A frequent slip is to write $v(t) = v_0 + eE_0t/m$ (as if scalar addition worked here). That over-counts because v_0 and eE_0t/m are along perpendicular axes. The correct combination is the *Pythagorean* sum $|v| = \sqrt{v_0^2 + (eE_0t/m)^2}$, which gives the $\sqrt{1 + (\ldots)^2}$ structure in the answer.

MCQ II (More than one option may be correct)

Q 11.9 Relativistic corrections become necessary when the expression for kinetic energy $\frac{1}{2}mv^2$ becomes comparable with mc^2, where m is the mass of the particle. At what de Broglie wavelength will relativistic corrections become important for an electron?

(A) $\lambda = 10$ **nm**
(B) $\lambda = 10^{-1}$ **nm**
(C) $\lambda = 10^{-4}$ **nm**
(D) $\lambda = 10^{-6}$ **nm**

SOLUTION

Correct options: (C) and (D).
Concept used. Relativistic effects begin to matter for an electron when its kinetic energy approaches $mc^2 = 0.511$ MeV. The de Broglie wavelength at that energy gives the threshold λ below which relativistic corrections become important. From $\lambda = h/p$ and $E \approx pc$ in the relativistic regime,

$$\lambda \approx \frac{hc}{E}.$$

Step 1. Use the energy condition $K \sim mc^2 \sim 0.511$ MeV. At that scale, $E = K \sim 5 \times 10^5$ eV.

Step 2. Wavelength at this energy:

$$\lambda \sim \frac{hc}{E} = \frac{1240\,\text{eV nm}}{5 \times 10^5\,\text{eV}} \approx 2.5 \times 10^{-3}\,\text{nm}.$$

Below this scale (i.e. $\lambda \lesssim 10^{-3}$ nm), relativistic corrections become important.

Step 3. Compare with options:

- (A) 10 nm: $E \sim 124$ eV, far below threshold; non-relativistic. **Not relativistic.**
- (B) 10^{-1} nm: $E \sim 12.4$ keV; still non-relativistic. **Not relativistic.**
- (C) 10^{-4} nm $= 10^{-13}$ m: $E \sim 1.24 \times 10^7$ eV $= 12.4$ MeV $\gg mc^2$. **Strongly relativistic.**
- (D) 10^{-6} nm: $E \sim 1.24 \times 10^9$ eV; ultra-relativistic. **Definitely relativistic.**

Step 4. So both (C) and (D) correspond to wavelengths where the electron is relativistic.

Final Answer: Options **(C), (D)**.

EXPERT'S SOLUTION : *Yash Mehta, M.Tech Applied Physics, IIT Delhi*

Threshold angle. Compute the wavelength at $K = mc^2$ for the electron and compare options.

Step 1. $mc_e^2 = 0.511\,\text{MeV}$. Setting $K \sim mc^2$ gives $\lambda \sim hc/(mc^2) \sim 1240/(5.11 \times 10^5)\,\text{nm}$ $\sim 2.4 \times 10^{-3}\,\text{nm}$.

Step 2. Anything $\lambda \ll 2.4 \times 10^{-3}\,\text{nm}$ is relativistic. From the list, $10^{-4}\,\text{nm}$ and $10^{-6}\,\text{nm}$ qualify.

Step 3. Explicit energy estimate for each option (non-relativistic $K = h^2/(2m\lambda^2)$, valid until the answer approaches mc^2):

$$K = \frac{(hc)^2}{2m_e c^2 \lambda^2} = \frac{(1240)^2}{2(5.11 \times 10^5)\lambda^2}\,\text{eV (with } \lambda \text{ in nm)}.$$

- (A) $\lambda = 10\,\text{nm}$: $K \approx 1.5 \times 10^{-2}$ eV. Non-relativistic.
- (B) $\lambda = 0.1\,\text{nm}$: $K \approx 150\,\text{eV}$. Non-relativistic.
- (C) $\lambda = 10^{-4}\,\text{nm}$: $K \approx 1.5 \times 10^8$ eV $\sim 150\,\text{MeV} \gg mc^2$. Strongly relativistic.
- (D) $\lambda = 10^{-6}\,\text{nm}$: $K \approx 1.5 \times 10^{12}$ eV. Ultra-relativistic.

For (C) and (D), the non-relativistic formula *itself* breaks; we should use $E^2 = (pc)^2 + (mc^2)^2 \Rightarrow E \approx pc = hc/\lambda$. Either way, both lie above threshold.

Concept linkage. The relativistic correction modifies the simple $\lambda = h/\sqrt{2mK}$ to $\lambda = hc/\sqrt{K(K + 2mc^2)}$ (using $E^2 = p^2c^2 + m^2c^4$). At low energies the correction vanishes; at high energies it suppresses λ less aggressively than the naive formula, because the momentum scales as $p \approx E/c$ rather than $\sqrt{2mE}$.

Why this matters. Electron diffraction beyond a few hundred keV (TEMs, synchrotron sources) requires the relativistic correction. The MeV-scale relativistic frontier for electrons sits exactly at $\lambda \sim 10^{-3}\,\text{nm}$, which is why the chosen wavelengths in (C) and (D) make the cut.

Final Answer: Options **(C), (D)**.

Q 11.10 **Two particles A_1 and A_2 of masses m_1, m_2 ($m_1 > m_2$) have the same de Broglie wavelength. Then:**

(A) their momenta are the same.

(B) their energies are the same.

(C) energy of A_1 is less than the energy of A_2.

(D) energy of A_1 is more than the energy of A_2.

SOLUTION

Correct options: (A) and (C).

Concept used. de Broglie: $\lambda = h/p$. So if two particles share the same wavelength, they share the same momentum:

$$p_1 = p_2 \;\Leftrightarrow\; \lambda_1 = \lambda_2.$$

For non-relativistic motion, $K = p^2/(2m)$: at the same momentum, the heavier particle has *less* kinetic energy.

Step 1. Same $\lambda \Rightarrow$ same p. Option (A) is correct.

Step 2. Use $K = p^2/(2m)$:

$$\frac{K_1}{K_2} = \frac{p^2/(2m_1)}{p^2/(2m_2)} = \frac{m_2}{m_1} < 1 \text{ since } m_1 > m_2.$$

So $K_1 < K_2$. Option (C) is correct.

Step 3. Option (B) "energies are the same" fails because the masses differ.

Step 4. Option (D) is the opposite of (C), hence wrong.

Final Answer: Options **(A), (C)**.

♥ Same wavelength $\neq$ same energy

de Broglie wavelength tracks momentum, not energy. A slow heavy particle and a fast light particle can share a wavelength while differing in kinetic energy by orders of magnitude. In neutron-vs.-electron diffraction, this is exploited: same wavelength, very different probe energies.

EXPERT'S SOLUTION *: Pranav Desai, Ph.D Physics, IISc Bangalore*

Momentum angle. $\lambda = h/p$ is a one-to-one map. Same $\lambda \Leftrightarrow$ same p.

Step 1. Common p pins (A).

Step 2. $K = p^2/(2m)$. With p fixed and $m_1 > m_2$, $K_1 = K_2\, m_2/m_1 < K_2$. Option (C) follows.

Step 3. Concrete example. Suppose $m_1 = 4m_2$ (think alpha vs deuteron, mass-4 vs mass-2) and they share $\lambda = 0.1$ nm. Their common momentum is $p = h/\lambda \approx 6.6 \times 10^{-24}$ kg m/s. Energies: $K_2 = p^2/(2m_2)$ and $K_1 = p^2/(2m_1) = K_2/4$. The heavier A_1 has only one-quarter the kinetic energy of A_2, despite sharing wavelength exactly.

Alternative form (velocity check). Same $p \Rightarrow m_1 v_1 = m_2 v_2$, so $v_1 = v_2\, m_2/m_1$. The heavier particle moves slower; $K_1/K_2 = (m_1 v_1^2)/(m_2 v_2^2) = m_2/m_1 < 1$. Consistent with

the momentum form. Note this is a non-relativistic statement; in the relativistic limit, $E = pc$ for both, giving identical energies — worth flagging if asked at high energies.
Concept linkage. The same λ / different K scenario underlies the practical choice of diffraction probes: cold neutrons ($\lambda \sim 0.1$–1 nm at meV energies) probe biological molecules without radiation damage, while electrons at the same wavelengths sit at keV energies and can damage organic samples. Choosing the probe means trading energy budget against scattering cross-section.
Why this matters. Cold neutrons ($\sim$meV) and electrons ($\sim$eV) can have similar wavelengths despite a 10^3-fold energy gap, illustrating why wavelength (not energy) controls diffraction geometry.

Final Answer: Options **(A), (C)**.

☞ Momentum–energy map for matter waves

For non-relativistic matter waves:

$$p = \frac{h}{\lambda}, \qquad K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}.$$

Same $\lambda \Rightarrow$ same p, but $K \propto 1/m$ at fixed p. Same $K \Rightarrow$ different p for different m, and $\lambda \propto 1/\sqrt{m}$.

Q 11.11 The de Broglie wavelength of a photon is twice the de Broglie wavelength of an electron. The speed of the electron is $v_e = c/100$. Then:

(A) $\dfrac{E_e}{E_p} = 10^{-4}$

(B) $\dfrac{E_e}{E_p} = 10^{-2}$

(C) $\dfrac{p_e}{m_e c} = 10^{-2}$

(D) $\dfrac{p_e}{m_e c} = 10^{-4}$

SOLUTION

Correct options: (B) and (C).
Concept used. For a particle, $p = h/\lambda$. For a photon, $E_{\text{ph}} = hc/\lambda$. For a non-relativistic electron, $E_e = \frac{1}{2}m_e v_e^2$ and $p_e = m_e v_e$.
Given $\lambda_{\text{ph}} = 2\lambda_e$ and $v_e = c/100$.

Step 1. Electron momentum scaled by $m_e c$:

$$\frac{p_e}{m_e c} = \frac{m_e v_e}{m_e c} = \frac{v_e}{c} = \frac{c/100}{c} = 10^{-2}.$$

Option (C) is correct.

Step 2. Photon energy $E_{\text{ph}} = hc/\lambda_{\text{ph}} = hc/(2\lambda_e)$. Electron energy

$$E_e = \tfrac{1}{2}m_e v_e^2 = \tfrac{1}{2}m_e c^2/10^4 = \frac{m_e c^2}{2 \times 10^4}.$$

Step 3. Express λ_e in terms of v_e:

$$\lambda_e = \frac{h}{m_e v_e} = \frac{h}{m_e c/100} = \frac{100h}{m_e c}.$$

So $\lambda_{\text{ph}} = 2\lambda_e = 200h/(m_e c)$ and

$$E_{\text{ph}} = \frac{hc}{200h/(m_e c)} = \frac{m_e c^2}{200}.$$

Step 4. Ratio:

$$\frac{E_e}{E_{\text{ph}}} = \frac{m_e c^2/(2 \times 10^4)}{m_e c^2/200} = \frac{200}{2 \times 10^4} = 10^{-2}.$$

Option (B) is correct.

Step 5. Eliminate distractors: (A) is off by a factor of 100; (D) is off by a factor of 100.

Final Answer: Options **(B), (C)**.

EXPERT'S SOLUTION : *Diya Kapoor, M.Sc Applied Mathematics, IIT Kanpur*

Step-by-step. Compute E_e, E_{ph} in terms of $m_e c^2$ using the given v_e and the wavelength condition.

Step 1. $p_e/(m_e c) = v_e/c = 10^{-2}$. (Confirms (C).)

Step 2. $\lambda_e = 100\,h/(m_e c)$, $\lambda_{\text{ph}} = 200\,h/(m_e c)$.

Step 3. $E_{\text{ph}} = hc/\lambda_{\text{ph}} = m_e c^2/200$.

Step 4. $E_e = \frac{1}{2}m_e c^2/10^4$.

Step 5. $E_e/E_{\text{ph}} = 10^{-2}$. (Confirms (B).)

Numerical values. With $m_e c^2 = 0.511\,\text{MeV} = 5.11 \times 10^5$ eV:

- $E_{\text{ph}} = m_e c^2/200 \approx 2.56\,\text{keV}$ (a soft X-ray).
- $E_e = m_e c^2/(2 \times 10^4) \approx 25.6\,\text{eV}$ (a low-energy electron).
- $\lambda_e = h/(m_e v_e) = h/(m_e c/100) \approx 2.42 \times 10^{-10}\,\text{m} = 0.242\,\text{nm}$.
- $\lambda_{\text{ph}} = 2\lambda_e \approx 0.485\,\text{nm}$ (matches the X-ray range).

Dispersion-relation contrast. The photon obeys $E = pc$ (massless dispersion); the slow electron obeys $E = p^2/(2m)$ (non-relativistic). At the same momentum, the photon energy beats the electron energy by a factor of $\sim c/v_e \sim 200$. That is exactly why $E_e/E_{\text{ph}} \sim v_e/(2c) \sim 1/200$ here. The factor of 2 from the wavelength ratio and the

factor of $\frac{1}{2}$ from non-relativistic kinetic energy compose to give
$E_e/E_{\text{ph}} = v_e/(2c) \cdot (\lambda_{\text{ph}}/\lambda_e)^{-1} = (10^{-2}/2) \cdot 2 = 10^{-2}$.
Sanity check on (A) and (D). (A) $E_e/E_{\text{ph}} = 10^{-4}$ would correspond to $v_e = c/200$, not the given $c/100$. (D) $p_e/(m_e c) = 10^{-4}$ would correspond to $v_e = c/10^4$, not the given $c/100$. Both fail the input check.
Why this matters. The exercise mixes photon dispersion ($E = pc$) with non-relativistic particle dispersion ($E = p^2/(2m)$); appreciating both is essential. The same algebra recurs whenever one converts between wavelength-tuned and energy-tuned descriptions of a source — e.g. in X-ray photoelectron spectroscopy (XPS).

> **Final Answer:** Options **(B), (C)**.

Q 11.12 **Photons absorbed in matter are converted to heat. A source emitting n photons/sec of frequency ν is used to convert 1 kg of ice at $0°$C to water at $0°$C. Then, the time T taken for the conversion:**
(A) decreases with increasing n, with ν fixed.
(B) decreases with n fixed, ν increasing.
(C) remains constant with n and ν changing such that $n\nu =$ constant.
(D) increases when the product $n\nu$ increases.

SOLUTION

Correct options: (A), (B), (C).
Concept used. Power supplied by the source is $P = n \cdot h\nu$ (energy per photon times photons per second). Energy required to melt 1 kg of ice is $Q = mL = 80$ kcal $= 3.34 \times 10^5$ J (using $L = 80$ cal/g for the latent heat of fusion of ice). The time is

$$T = \frac{Q}{P} = \frac{Q}{n\,h\nu}.$$

Step 1. Option (A): with ν fixed, $T \propto 1/n$. Increasing n decreases T. **Correct.**

Step 2. Option (B): with n fixed, $T \propto 1/\nu$. Increasing ν decreases T. **Correct.**

Step 3. Option (C): if $n\nu =$ const, then $P = h\,(n\nu) =$ const, so $T = Q/P$ is constant. **Correct.**

Step 4. Option (D): if $n\nu$ increases then P increases and $T = Q/(h \cdot n\nu)$ *decreases*, not increases. **Wrong.**

> **Final Answer:** Options **(A), (B), (C)**.

EXPERT'S SOLUTION : *Ishita Rao, M.Sc Chemistry, IIT Kanpur*

Power-balance angle. $T = Q/P$ with $P = nh\nu$ exposes the dependence transparently.

Step 1. Fixed $Q = mL$. Vary n, ν.

Step 2. $T \propto 1/(n\nu)$: any move that increases $n\nu$ shrinks T.

Step 3. (A) and (B) are special cases; (C) is the level set; (D) is the wrong direction.

Step 4. Numerical scale. To melt 1 kg ice we need $Q = mL = 1 \cdot 334\,\text{kJ} = 3.34 \times 10^5$ J. If $\nu = 5 \times 10^{14}$ Hz (green light), each photon carries $h\nu \approx 3.3 \times 10^{-19}$ J, so the source must deliver $\sim 10^{24}$ photons to do the job; at $n = 10^{20}$ photons/s (a ~ 30 W lamp) the conversion takes $\sim 10^4$ s ≈ 3 h.

Why each option works individually.

- (A) ν fixed, $n\uparrow$: more photons per second at the same energy each $\to$ more total power $\to$ less time. $T \propto 1/n$. ✓
- (B) n fixed, $\nu\uparrow$: same photon rate but each photon hotter $\to$ more total power $\to$ less time. $T \propto 1/\nu$. ✓
- (C) $n\nu =$ const: the product $n\nu$ *is* the power (in units of h). Keep it constant $\Rightarrow$ keep P constant $\Rightarrow$ T constant. ✓
- (D) $n\nu \uparrow \Rightarrow P \uparrow \Rightarrow T \downarrow$, not $\uparrow$. ✗

Concept linkage. Heat capacity / latent heat thermodynamics meets quantum photon counting. The bridge is $P = nh\nu$. Same logic governs laser power normalisation: at fixed laser intensity, choosing UV vs IR changes the photon count by $\nu_{UV}/\nu_{IR} \sim 5\times$, with consequences for photo-induced reactions.

Why this matters. Solar-thermal collectors depend on this exact accounting: total power = photon rate times energy per photon, integrated over the spectrum. The solar constant ($\sim 1.36\,\text{kW/m}^2$) corresponds to a photon flux of $\sim 5 \times 10^{21}$ photons/m^2/s at peak (550 nm) — huge n saves the day even though each photon is only 2.3 eV.

Final Answer: Options **(A), (B), (C)**.

☞ Power = photon-rate × photon-energy

A robust shortcut for any radiation-matter energy problem:

$$P = n_\gamma \cdot h\nu = \frac{n_\gamma hc}{\lambda}.$$

Total time for a fixed energy budget Q is $T = Q/P$. Use this single formula and the dependence on n_γ and ν (or λ) falls out immediately.

Q 11.13 **A particle moves in a closed orbit around the origin, due to a force which is**

directed towards the origin. The de Broglie wavelength of the particle varies cyclically between two values λ_1 and λ_2 with $\lambda_1 > \lambda_2$. Which of the following statements are true?

(A) The particle could be moving in a circular orbit with origin as centre.

(B) The particle could be moving in an elliptic orbit with origin as its focus.

(C) When the de Broglie wavelength is λ_1, the particle is nearer the origin than when its value is λ_2.

(D) When the de Broglie wavelength is λ_2, the particle is nearer the origin than when its value is λ_1.

SOLUTION

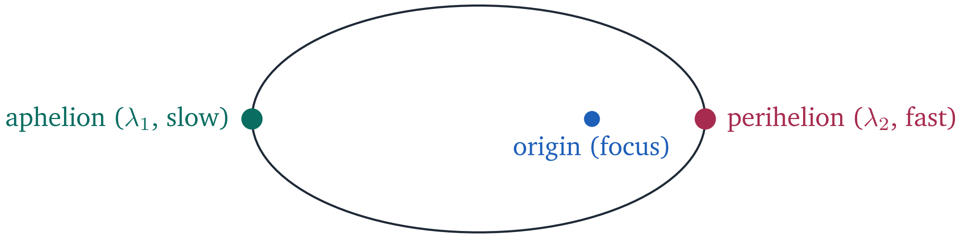
Correct options: (B) and (D).

Concept used. A circular orbit has constant speed (by symmetry, the central force is always perpendicular to the velocity), so λ would be constant, not varying between two values. An **elliptic orbit** (Kepler-like central force) has varying speed: by conservation of angular momentum ($L = mvr$, with v the tangential component), the particle moves *faster* when nearer the centre (**perihelion**) and *slower* when farther (**aphelion**). Larger $v \Rightarrow$ smaller λ. So $\lambda_{\min}$ corresponds to nearest point and $\lambda_{\max}$ to farthest.

Step 1. Rule out (A): a uniform circular orbit has $|\vec{v}| = \text{const}$, hence $\lambda = \text{const}$, not two values. **Wrong.**

Step 2. Endorse (B): an elliptic orbit (origin at the focus, as in Kepler's first law) has v varying with position. λ takes the values $h/(mv_{\min}) = \lambda_1$ at the farthest point and $h/(mv_{\max}) = \lambda_2$ at the nearest. **Correct.**

Step 3. Now (C) vs (D): nearer the origin means larger speed (angular momentum conservation), so *smaller* λ. The value $\lambda_2 < \lambda_1$ corresponds to the nearer point. So (D) is right and (C) is wrong.



Final Answer: Options **(B), (D)**.

☞ **Angular-momentum shortcut**

At perihelion (closest), the orbital radius r is small; for $L = mvr$ to be conserved, v must be large. Large $v \Rightarrow$ small λ. At aphelion the reverse holds. Hence $\lambda_{\min} \leftrightarrow$ closest, $\lambda_{\max} \leftrightarrow$ farthest.

EXPERT'S SOLUTION : *Tara Bhat, Ph.D Pure Mathematics, IISc Bangalore*

Conservation angle. Use $L =$ const and energy conservation.

Step 1. Circular orbit: $r =$ const, so $v = L/(mr) =$ const, so λ is single-valued. (A) is out.

Step 2. Elliptic orbit: r varies between r_p (perihelion) and r_a (aphelion); v varies between $v_p = L/(mr_p)$ and $v_a = L/(mr_a)$; $v_p > v_a$ since $r_p < r_a$.

Step 3. $\lambda = h/(mv)$ is largest at slowest motion (aphelion): $\lambda_1 = h/(mv_a)$.

Step 4. Hence λ_2 corresponds to the closer (perihelion) position. (D) is correct, (C) is wrong.

Vis-viva alternative. For an attractive $1/r$ potential, the vis-viva equation gives

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right),$$

where a is the semi-major axis. At perihelion $r = r_p = a(1-e)$ (smallest), v is largest; at aphelion $r = r_a = a(1+e)$ (largest), v is smallest. The ratio of speeds is $v_p/v_a = (1+e)/(1-e)$ where e is eccentricity. So $\lambda_a/\lambda_p = v_p/v_a = (1+e)/(1-e)$, giving an explicit handle on how much λ modulates.

Concept linkage. The wavelength–orbit modulation is the matter-wave echo of Kepler's second law. In Bohr's quantisation (older quantum theory), allowed orbits are precisely those for which a whole number of de Broglie wavelengths fits around the circular path: $2\pi r = n\lambda$. This is consistent with the constant λ of (A) only for circular orbits. For elliptic orbits, Sommerfeld's generalisation uses the action integral $\oint p\,dr = nh$, which reduces to the same condition when λ varies along the orbit.

Why this matters. Kepler's second law (equal areas in equal times) is just $L =$ const dressed differently. The de Broglie wavelength inherits the same modulation around the orbit. The same picture lets you guess qualitatively how the electron probability density should fluctuate around an elliptic Bohr–Sommerfeld orbit: denser (longer wavelength) at aphelion, sparser at perihelion.

Final Answer: Options **(B), (D)**.

✗ "Nearer the origin" ⇔ smaller, not larger, λ

Many students assume that being nearer the origin makes the orbit "more circular" (and thus λ stays constant) or that smaller radius means smaller wavelength because "things get squeezed." Neither is right. Angular-momentum conservation forces v *up* at small r, which forces $\lambda = h/(mv)$ *down*. So nearer the origin ⇔ smaller λ (i.e. λ_2 in the question).

VSA (Very Short Answer)

Q 11.14 **A proton and an α-particle are accelerated, using the same potential difference. How are the de Broglie wavelengths λ_p and λ_α related to each other?**

SOLUTION

Concept used. A charge q accelerated through potential difference V from rest gains kinetic energy $K = qV$. Its momentum is $p = \sqrt{2mK} = \sqrt{2mqV}$, and its de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}.$$

For proton: m_p, charge $+e$. For α-particle: $m_\alpha \approx 4m_p$, charge $+2e$.

Step 1. Write each wavelength:

$$\lambda_p = \frac{h}{\sqrt{2m_peV}}, \qquad \lambda_\alpha = \frac{h}{\sqrt{2m_\alpha(2e)V}} = \frac{h}{\sqrt{2(4m_p)(2e)V}} = \frac{h}{\sqrt{16m_peV}}.$$

Step 2. Take the ratio:

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{\sqrt{16m_peV}}{\sqrt{2m_peV}} = \sqrt{\frac{16}{2}} = \sqrt{8} = 2\sqrt{2}.$$

Step 3. Hence $\lambda_p = 2\sqrt{2}\,\lambda_\alpha$, or equivalently $\lambda_\alpha = \lambda_p/(2\sqrt{2})$.

Final Answer: $\lambda_p = 2\sqrt{2}\,\lambda_\alpha \approx 2.83\,\lambda_\alpha$.

EXPERT'S SOLUTION *: Krishna Singh, M.Tech Engineering Physics, IIT Bombay*

Ratio angle. Group factors before plugging in.

Step 1. $\lambda \propto 1/\sqrt{mqV}$. At common V, $\lambda \propto 1/\sqrt{mq}$.

Step 2. $\sqrt{m_pq_p} = \sqrt{m_pe}$. $\sqrt{m_\alpha q_\alpha} = \sqrt{4m_p \cdot 2e} = \sqrt{8\,m_pe}$.

Step 3. $\lambda_p/\lambda_\alpha = \sqrt{8m_pe}/\sqrt{m_pe} = \sqrt{8} = 2\sqrt{2}$.

Numerical illustration. For $V = 1000\,\text{V}$:

- Proton: $K = eV = 1\,\text{keV}$,
  $p = \sqrt{2m_pK} = \sqrt{2 \cdot 1.67 \times 10^{-27} \cdot 1.6 \times 10^{-16}} \approx 7.31 \times 10^{-22}\,\text{kg m/s}$.
  $\lambda_p = h/p \approx 9.07 \times 10^{-13}\,\text{m} \approx 9.07 \times 10^{-4}\,\text{nm}$.
- Alpha: $K = 2eV = 2\,\text{keV}$ (because $q_\alpha = 2e$), $p = \sqrt{2 \cdot 4m_p \cdot 2K_p} = \sqrt{8} \cdot p_p = 2\sqrt{2} \cdot p_p$.
  $\lambda_\alpha = \lambda_p/(2\sqrt{2}) \approx 3.21 \times 10^{-13}\,\text{m}$.
- Ratio: $\lambda_p/\lambda_\alpha \approx 2.83$. ✓

Where each factor comes from. The $\sqrt{m_\alpha/m_p} = 2$ accounts for the alpha being four times heavier (so $\sqrt{4} = 2$). The $\sqrt{q_\alpha/q_p} = \sqrt{2}$ accounts for the alpha picking up twice the energy at the same V. Both effects shrink the alpha's wavelength relative to the proton's: heavier mass and bigger kick.

Alternative phrasing. $\lambda_p^2/\lambda_\alpha^2 = (m_\alpha q_\alpha)/(m_p q_p) = 4 \cdot 2 = 8$, so $\lambda_p/\lambda_\alpha = 2\sqrt{2} \approx 2.83$. The squaring makes the factor structure even cleaner.
Concept linkage. The same calculation underlies isotope-mass-spectrometer calibration: ions of different q/m ratios are separated because they get different wavelengths (or radii in a magnetic field, related to $\sqrt{mq}$). The wavelength expression $\lambda = h/\sqrt{2mqV}$ is the matter-wave dual of the cyclotron-radius expression $r = \sqrt{2mV/(qB^2)}$.

Final Answer: $\lambda_p/\lambda_\alpha = 2\sqrt{2}$.

☞ λ **for accelerated charges**

For a particle of charge q and mass m accelerated from rest through potential difference V,

$$K = qV, \qquad p = \sqrt{2mqV}, \qquad \lambda = \frac{h}{\sqrt{2mqV}}.$$

At fixed V, $\lambda \propto 1/\sqrt{mq}$. The combination mq (not just m) determines the wavelength.

Q 11.15 **(i) In the explanation of photoelectric effect, we assume one photon of frequency ν collides with an electron and transfers its energy. This leads to the equation for the maximum energy E_{max} of the emitted electron as $E_{\text{max}} = h\nu - \phi_0$, where ϕ_0 is the work function of the metal. If an electron absorbs 2 photons (each of frequency ν) what will be the maximum energy for the emitted electron?**
(ii) Why is this fact (two photon absorption) not taken into consideration in our discussion of the stopping potential?

SOLUTION

Concept used. The **Einstein photoelectric equation** expresses energy conservation for the absorption of a single photon by a bound electron. If a single electron absorbs n photons each of energy $h\nu$, the total energy deposited is $nh\nu$; the electron escapes with maximum kinetic energy $E_{\text{max}} = nh\nu - \phi_0$ provided $nh\nu > \phi_0$.

Step 1. (i) Two-photon absorption: $E_{\text{abs}} = 2h\nu$. Maximum kinetic energy of the emerging electron is

$$E_{\text{max}} = 2h\nu - \phi_0.$$

Step 2. (ii) Probability of one electron absorbing two photons in the experimental conditions is extremely low at ordinary light intensities. The rate of n-photon absorption scales as the n-th power of the photon flux (intensity^n). At normal lab intensities, the single-photon process dominates by many orders of magnitude. Two-photon absorption becomes appreciable only at very high light intensities (intense lasers, $\sim 10^6$ W/cm^2), conditions far from the classic photoelectric setup.

Final Answer: (i) $E_{\text{max}} = 2h\nu - \phi_0$. (ii) The 2-photon process needs ultra-high intensities (lasers); ordinary light makes it negligible.

EXPERT'S SOLUTION *: Ishaan Chatterjee, Ph.D Condensed Matter Physics, TIFR Mumbai*

Intensity-scaling angle.

Step 1. Single-photon rate $\propto I$ (one factor of intensity). Two-photon rate $\propto I^2$.

Step 2. At $I = 1\,\text{W/m}^2$ (typical lab lamp), $I^2 \ll I$ in suitable units. Hence the two-photon absorption rate is suppressed by $\sim 10^{-30}$ relative to single-photon at standard intensities.

Step 3. Stopping-potential measurements thus see only $E_{\text{max}} = h\nu - \phi_0$.

Conservation law. (i) Two-photon absorption is just adding two photon energies before the work function is paid:

$$E_{\text{max}} = 2h\nu - \phi_0.$$

If $\nu > \nu_0/2$ (i.e. each photon individually below threshold, but the pair exceeds it), photoemission becomes possible only via two-photon absorption. This is a genuine quantum-electrodynamics process.

Why one-photon dominates. (ii) The intensity dependence is the decisive separator. Define the two-photon cross-section σ_2; the absorption rate per atom is $R_2 = \sigma_2 I^2$, while the one-photon rate is $R_1 = \sigma_1 I$. Their ratio:

$$\frac{R_2}{R_1} = \frac{\sigma_2}{\sigma_1} \cdot I.$$

For typical $\sigma_1 \sim 10^{-17}\,\text{cm}^2$ and $\sigma_2 \sim 10^{-50}\,\text{cm}^4\,\text{s}$ (typical values), $\sigma_2/\sigma_1 \sim 10^{-33}\,\text{cm}^2{\cdot}\text{s}$. At lamp intensity $I \sim 10^{16}\,\text{photons/cm}^2\text{/s}$, $R_2/R_1 \sim 10^{-17}$: utterly negligible. At a focused laser ($I \sim 10^{30}\,\text{photons/cm}^2\text{/s}$), the ratio approaches unity — which is exactly the regime where two-photon microscopy operates.

Concept linkage. Two-photon absorption was predicted by Maria Goeppert-Mayer in 1931 and observed experimentally in 1961 (Kaiser & Garrett, after lasers arrived). It is now the working principle of two-photon fluorescence microscopy in neuroscience, exploiting the I^2 scaling to confine excitation to a tight focal volume.

Why this matters. Multi-photon absorption is the bridge between linear optics and nonlinear optics; it explains why exotic processes (frequency doubling, four-wave mixing) require intense lasers and not ordinary lamps.

Final Answer: (i) $2h\nu - \phi_0$; (ii) negligible probability at ordinary intensities (intensity-squared scaling kills it).

☞ **n-photon Einstein equation**

For an electron absorbing n photons of frequency ν in a single event, the maximum kinetic energy on emergence is

$$E_{\text{max}} = n\,h\nu - \phi_0 \quad (\text{provided}\, nh\nu > \phi_0).$$

Probability scales as I^n, suppressing higher-n processes at low intensities.

Q 11.16 **There are materials which absorb photons of shorter wavelength and emit photons of longer wavelength. Can there be stable substances which absorb photons of larger wavelength and emit light of shorter wavelength?**

SOLUTION

Concept used. The phenomenon of absorbing a high-energy photon and emitting a lower-energy one is **Stokes fluorescence** (and is the rule, conserved by energy: the absorbed energy minus the emitted energy is left as heat in the molecule's vibrations). **Anti-Stokes** emission (absorbing low energy and emitting high energy) would require the molecule to draw extra energy from somewhere.

Step 1. In an absorption event, a single photon supplies the excitation energy. If the photon energy is $h\nu_{\text{abs}}$ and the emission energy is $h\nu_{\text{em}}$, then for a *single-photon* excitation followed by single-photon emission, energy conservation forces

$$h\nu_{\text{em}} \le h\nu_{\text{abs}} \iff \lambda_{\text{em}} \ge \lambda_{\text{abs}}.$$

This is Stokes' rule.

Step 2. For a stable substance held at room temperature, thermal energy ($k_B T \approx 0.025\,\text{eV}$) is much smaller than typical visible-photon energies (1.5–3 eV), so the substance cannot make up the energy deficit. Stable substances therefore cannot routinely absorb long-wavelength light and emit short-wavelength light.

Step 3. Exception: **multiphoton processes** (two-photon absorption, upconversion in lanthanide complexes) do produce short-wavelength emission from long-wavelength excitation, but they require multiple absorptions (and very high intensity). They are not a single-photon stable process.

> **Final Answer:** No (for single-photon processes). Stokes' rule forbids it: $\lambda_{\text{em}} \geq \lambda_{\text{abs}}$. Multi-photon upconversion exists but is not a single-step single-photon process and needs very intense light.

EXPERT'S SOLUTION : *Meera Nair, Ph.D Physics, IISc Bangalore*

Energy-budget angle.

Step 1. Absorbing one photon adds $h\nu_1$ to the molecule.

Step 2. Single-photon emission cannot release more than $h\nu_1$; hence $\nu_2 \leq \nu_1$, i.e. $\lambda_2 \geq \lambda_1$.

Step 3. Upconversion exists but is anti-Stokes only in a many-photon sense (multi-step, intensity-hungry).

Stokes vs anti-Stokes detail. A molecule absorbing a photon at ν_1 jumps from vibrational ground state to an excited electronic state. Internal vibrational relaxation (lattice/molecular vibrations) burns some energy as heat. Subsequent emission at $\nu_2 < \nu_1$ returns the molecule to the ground state, ending below the start. The energy budget reads:

$$h\nu_1 = h\nu_2 + \Delta E_{\text{vib}} + \Delta E_{\text{heat}}, \quad h\nu_2 \leq h\nu_1.$$

Stokes' shift is the wavelength difference between absorption and emission peaks, a routine spectroscopic observable.

Anti-Stokes loophole at finite T. A molecule already in an excited vibrational level (thermally populated at $k_B T$) can absorb a photon, jump to a higher electronic state, then drop back to the lowest vibrational level by emitting a higher-energy photon. The extra energy comes from the lattice's thermal reservoir, not from anywhere magical. But the Boltzmann factor $\exp(-h\nu_{\text{vib}}/k_B T)$ suppresses this strongly at room temperature; it is a weak side effect. Anti-Stokes scattering (Raman) is observed but always weaker than Stokes scattering.

Genuine upconversion (e.g. Er^{3+}-doped fluoride glasses): two long-wavelength photons sequentially excite a single rare-earth ion via a metastable intermediate state, and the ion emits a single high-energy photon. This is a multi-photon, multi-step process requiring high pump intensity and a long-lived intermediate. It is the basis of upconversion lasers and bioimaging tags.

Concept linkage. The single-photon Stokes constraint is a quantum statement of the second law (entropy can decrease for the photon system only by transferring entropy to vibrations). Energy conservation between input and output photons *plus* the irreversibility of vibrational relaxation makes $\lambda_{\text{em}} \geq \lambda_{\text{abs}}$ in the ideal single-photon case.

Final Answer: Single-photon: not possible (Stokes' rule, $\lambda_{em} \geq \lambda_{abs}$). Multi-photon upconversion (intensity-driven): possible but not at ambient single-photon conditions.

♥ Fluorescence and the Stokes shift

Every fluorescence microscope, every glow-in-the-dark toy, every white-LED phosphor relies on Stokes' rule: absorb high-energy (blue/UV) light, emit lower-energy (visible) light. The energy difference becomes heat. The fact that single-photon anti-Stokes is forbidden is exactly why everyday fluorescent materials don't violate energy conservation.

Q 11.17 Do all the electrons that absorb a photon come out as photoelectrons?

SOLUTION

Concept used. For photoemission, an absorbed photon must deliver enough energy to overcome the metal's **work function** ϕ_0. Even when $h\nu > \phi_0$, only a fraction of absorbing electrons actually escape: electrons deep inside the metal lose energy by collisions before reaching the surface, and electrons near the surface may scatter back inelastically.

Step 1. Conditions for photoemission:

- Photon energy $h\nu \geq \phi_0$ (threshold condition).
- Electron must be close enough to the surface to escape before losing the absorbed energy.
- Electron must be moving in a direction allowing escape (i.e. toward the surface, not deeper into the metal).

Step 2. Even with $h\nu > \phi_0$, the **quantum efficiency** (fraction of absorbed photons producing a photoelectron) is typically much less than unity (often $< 10\%$). The rest of the absorbed energy converts to heat in the lattice.

Final Answer: No. Most absorbed photons produce no photoelectron; only electrons close to the surface, moving outward, with the absorbed energy intact, escape.

EXPERT'S SOLUTION *: Sanya Iyer, M.Sc Microbiology, JNU — (Physics)*

Probability angle.

Step 1. Photon absorption is a one-step process; photoemission requires several conditions in sequence.

Step 2. Each condition has < 1 probability; the joint probability is much less than 1.

Step-by-step probability ladder. For an absorbed photon to actually produce an emerging photoelectron, all of the following must happen:

- **Threshold:** $h\nu > \phi_0$. If the photon's energy is below the work function, no emission is possible at all (single-photon process).
- **Surface proximity:** The absorption must occur near the surface (within an escape length ~ 1–$10\,\text{nm}$, set by inelastic mean free path of the photoexcited electron in the metal).
- **Momentum direction:** The freed electron must be heading outward, not deeper into the bulk.
- **No inelastic loss:** The electron must reach the surface without scattering and losing enough energy to fall below ϕ_0 again.
- **Surface barrier crossing:** It must successfully cross the metal–vacuum interface (some quantum reflection probability subtracts a bit more).

The product of these conditional probabilities is the *quantum efficiency*, typically $\lesssim 10\%$ for clean metals at UV wavelengths and much smaller in the visible.

Numerical reality. For sodium illuminated at $\lambda = 300\,\text{nm}$ ($E = 4.1$ eV vs $\phi_0 = 2.3$ eV), the quantum efficiency is $\sim 10^{-3}$ — one electron per thousand absorbed photons. For copper at the same wavelength, $E < \phi_0$ (copper $\phi_0 = 4.7$ eV), and the quantum efficiency drops to essentially zero.

Concept linkage. Photomultiplier tubes (PMTs) and silicon photodiodes are designed to maximise this efficiency: thin photoemissive coatings, cesium-antimony alloys with low work functions, geometric optimisation. Even so, the best PMT quantum efficiencies are $\sim 30\%$ — still far below 100%. The chain of conditional events is intrinsically lossy.

Why this matters. The gap between "photon absorbed" and "electron freed" is the central engineering parameter for every photodetector. Improving it by even a factor of 2 wins Nobel prizes (e.g. low-temperature CCD sensors used in modern astronomy).

> **Final Answer:** No. Many absorbed photons fail to produce photoelectrons; the quantum efficiency is typically well below unity.

✗ Work function vs threshold frequency vs threshold wavelength

Three closely related quantities sometimes get confused:

- Work function ϕ_0 (units: eV or J).
- Threshold frequency $\nu_0 = \phi_0/h$ (units: Hz).
- Threshold wavelength $\lambda_0 = hc/\phi_0 = c/\nu_0$ (units: m or nm).

For emission to occur: $h\nu \geq \phi_0$, or $\nu \geq \nu_0$, or $\lambda \leq \lambda_0$. The wavelength inequality flips because $\lambda \propto 1/\nu$. Always state which threshold form you are using.

Q 11.18 There are two sources of light, each emitting with a power of 100 W. One emits X-rays of wavelength 1 nm and the other visible light at 500 nm. Find the ratio of number of photons of X-rays to the photons of visible light of the given wavelength.

SOLUTION

Concept used. For a source of power P emitting photons of wavelength λ, the number of photons emitted per second is

$$n = \frac{P}{E_{\text{photon}}} = \frac{P}{hc/\lambda} = \frac{P\lambda}{hc}.$$

So at fixed P, $n \propto \lambda$.

Step 1. Number of X-ray photons per second (subscript X, $\lambda_X = 1\,\text{nm}$):

$$n_X = \frac{P\lambda_X}{hc}.$$

Step 2. Number of visible-light photons per second (subscript v, $\lambda_v = 500\,\text{nm}$):

$$n_v = \frac{P\lambda_v}{hc}.$$

Step 3. Both sources have the same power P, so the ratio is just the ratio of wavelengths:

$$\frac{n_X}{n_v} = \frac{\lambda_X}{\lambda_v} = \frac{1\,\text{nm}}{500\,\text{nm}} = \frac{1}{500}.$$

Final Answer: $n_X : n_v = 1 : 500$, i.e. for the same power, the X-ray source emits 500 times *fewer* photons.

☞ **Photon-energy intuition**

Each X-ray photon carries 500 times more energy than each visible photon (because $E \propto 1/\lambda$). To deliver the same total power, the X-ray source therefore emits 500 times *fewer* photons per second.

EXPERT'S SOLUTION : *Riya Kumar, M.Sc Physics, IIT Madras*

Ratio angle.

Step 1. $n \propto \lambda/P^{-1} = P\lambda/(hc)$.

Step 2. Common P: $n_X/n_v = \lambda_X/\lambda_v = 1/500$.

Absolute counts (cross-check). Energy per X-ray photon at $\lambda = 1\,\text{nm}$: $E_X = hc/\lambda = 1240\,\text{eV} = 1.24\,\text{keV} = 1.99 \times 10^{-16}\,\text{J}$. Photons per second at $P = 100\,\text{W}$:

$$n_X = P/E_X = 100/1.99 \times 10^{-16} \approx 5.03 \times 10^{17}\,\text{s}^{-1}.$$

Energy per visible photon at $\lambda = 500\,\text{nm}$: $E_v = hc/\lambda = 2.48\,\text{eV} = 3.98 \times 10^{-19}\,\text{J}$. Photons per second:

$$n_v = P/E_v = 100/3.98 \times 10^{-19} \approx 2.51 \times 10^{20}\,\text{s}^{-1}.$$

Ratio: $n_X/n_v = 5.03 \times 10^{17}/2.51 \times 10^{20} \approx 1/500$. ✓

Order-of-magnitude intuition. A visible source emits hundreds of times more photons than an X-ray source at the same power because each visible photon carries hundreds of times less energy. Equivalently, X-ray sources spend their energy budget on *fewer, hotter* photons — which is precisely why a $100\,\text{W}$ X-ray tube damages biological tissue while a $100\,\text{W}$ incandescent bulb does not.

Concept linkage. The reciprocal scaling $n \propto \lambda$ at fixed P governs the choice of sources for spectroscopy. X-ray crystallography uses few, energetic photons (small n) to resolve atomic spacings. Visible-light photography uses abundant, gentle photons (large n) to map macroscopic detail. The trade-off is between resolving power (λ small) and signal-to-noise statistics (n large).

Alternative phrasing. $P = n_\gamma \cdot h\nu = n_\gamma \cdot hc/\lambda$, so $n_\gamma = P\lambda/(hc)$. Same P and same hc gives $n_\gamma \propto \lambda$ directly.

> **Final Answer:** $n_X : n_v = 1 : 500$. The X-ray source emits 500 times fewer photons because each X-ray photon is 500 times more energetic at the same source power.

SA (Short Answer)

Q 11.19 **Consider Fig. 11.1 for photoemission. How would you reconcile with momentum-conservation? Note light (photons) have momentum in a different direction than the emitted electrons.**

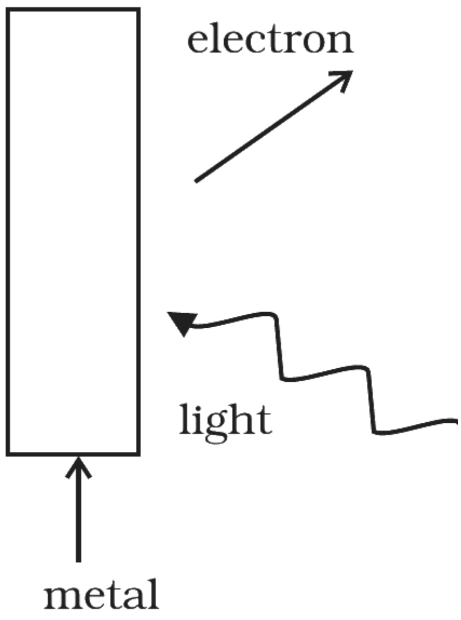


Fig. 11.1

Fig. 11.1, NCERT Exemplar Class 12 Physics, Chapter 11.

SOLUTION

Concept used. A photon of wavelength λ carries linear momentum $p_\gamma = h/\lambda$. **Momentum conservation** in photoemission demands that the total momentum before equals the total momentum after; the difference between the photon momentum and the emitted electron momentum is absorbed by the bulk metal.

Step 1. The system in the diagram is: photon (incoming horizontally) + metal block (at rest) → photoelectron (going up-right) + recoiling metal.

Step 2. Before: $\vec{p}_{\text{photon}} + \vec{0} = \vec{p}_{\text{photon}}$.

Step 3. After: $\vec{p}_{e^-} + \vec{p}^{\,\text{recoil}}_{\text{metal}}$.

Step 4. Conservation: $\vec{p}_{\text{photon}} = \vec{p}_{e^-} + \vec{p}^{\,\text{recoil}}_{\text{metal}}$, so

$$\vec{p}^{\,\text{recoil}}_{\text{metal}} = \vec{p}_{\text{photon}} - \vec{p}_{e^-}.$$

Step 5. The metal is a huge object compared with the electron. Its recoil momentum may be substantial in vector terms, but its recoil kinetic energy

$$K_{\text{metal}} = \frac{p^2_{\text{metal}}}{2M_{\text{metal}}} \approx 0 \text{ for } M_{\text{metal}} \gg m_e$$

is essentially zero. So momentum is conserved (with the metal supplying whatever component is needed) without any noticeable kinetic energy carried by the metal.

Step 6. This is why the directions of the photon and the emitted electron need not align: the metal absorbs the perpendicular momentum component (and any necessary parallel component too) silently.

Final Answer: The metal block recoils with the momentum difference between photon and ejected electron; its kinetic energy is negligible because of its huge mass, so it absorbs momentum but carries no observable energy.

EXPERT'S SOLUTION : *Aanya Patel, M.Sc Astrophysics, IIT Kanpur*

Mass-asymmetry angle.

Step 1. Write $\vec{p}_{\text{photon}} = \vec{p}_{e^-} + \vec{p}_{\text{metal}}$.

Step 2. Solve for the metal: $\vec{p}_{\text{metal}} = \vec{p}_{\text{photon}} - \vec{p}_{e^-}$.

Step 3. $K_{\text{metal}} = p^2_{\text{metal}}/(2M_{\text{metal}}) \to 0$ as $M_{\text{metal}} \to \infty$. Energetically negligible, kinematically essential.

Numerical scale. A UV photon at $\lambda = 200\,\text{nm}$ has momentum $p_\gamma = h/\lambda \approx 3.3 \times 10^{-27}$ kg m/s. A photoelectron with $K = 2\,\text{eV}$ has $p_e = \sqrt{2m_eK} \approx 7.6 \times 10^{-25}$ kg m/s. The electron's momentum dwarfs the photon's by $\sim 230\times$ (because $p_e/p_\gamma = \sqrt{2m_eK} \cdot \lambda/h \approx c/v \gg 1$ for a slow electron). Hence *most* of the momentum carried away by the photoelectron must be matched by the metal block, not the photon.
Why energy is unaffected by metal recoil. For a block of mass $M \sim 10^{-3}$ kg (a typical small target) with $p_{\text{metal}} \sim 7 \times 10^{-25}$ kg m/s, the recoil energy is

$$K_M = p^2_{\text{metal}}/(2M) = (7 \times 10^{-25})^2/(2 \times 10^{-3}) \sim 2.5 \times 10^{-46}\,\text{J} \sim 10^{-27}\,\text{eV}.$$

Compared with the photoelectron's $\sim 2\,\text{eV}$, this is utterly negligible. Energy balance is dominated by photon, electron and the work function; momentum balance secretly involves the entire block.
Concept linkage with Compton scattering. In Compton scattering ($\gamma + e^- \to \gamma + e^-$), the electron is treated as free, and the recoil is large because the electron is a light scatterer. Photoemission is the opposite limit: the photon transfers all its energy but mostly to the electron, with the bulk metal silently absorbing the kinematic mismatch. Both are vector-conservation problems; the difference is in the mass ratio of the involved partners.
Mössbauer parallel. For gamma emission from a free nucleus, recoil shifts the emitted gamma energy and prevents resonant absorption by another nucleus. Mössbauer's trick (1958, Nobel 1961) was to embed the nucleus in a crystal lattice; the whole crystal recoils, $M \to \infty$, recoil energy $\to 0$, and resonant absorption returns — the same mass-asymmetry argument as here.
Why this matters. The principle "momentum balance with a massive partner is essentially free, energetically" shows up everywhere from photoelectric effect, to

Compton scattering, to Mössbauer spectroscopy, to recoilless absorption in solid-state qubits.

Final Answer: Metal absorbs the momentum mismatch ($\vec{p}_{\text{metal}} = \vec{p}_{\text{photon}} - \vec{p}_{e^-}$); its kinetic energy is negligible because of its huge mass. Momentum is conserved while energy balance involves only photon, electron and ϕ_0.

Q 11.20 Consider a metal exposed to light of wavelength 600 nm. The maximum energy of the electron doubles when light of wavelength 400 nm is used. Find the work function in eV.

SOLUTION

Concept used. Einstein photoelectric equation:

$$K_{\max} = \frac{hc}{\lambda} - \phi_0,$$

with $hc = 1240$ eV nm.
Two experimental conditions are given:

$$K_{\max,1} = \frac{hc}{\lambda_1} - \phi_0, \qquad \lambda_1 = 600\,\text{nm},$$
$$K_{\max,2} = \frac{hc}{\lambda_2} - \phi_0, \qquad \lambda_2 = 400\,\text{nm},$$

with $K_{\max,2} = 2K_{\max,1}$.

Step 1. Compute photon energies first:

$$E_1 = \frac{1240}{600}\,\text{eV} = 2.067\,\text{eV}, \qquad E_2 = \frac{1240}{400}\,\text{eV} = 3.100\,\text{eV}.$$

Step 2. Substitute the doubling condition $K_{\max,2} = 2K_{\max,1}$:

$$E_2 - \phi_0 = 2(E_1 - \phi_0) \iff E_2 - \phi_0 = 2E_1 - 2\phi_0 \iff \phi_0 = 2E_1 - E_2.$$

Step 3. Plug in numbers:

$$\phi_0 = 2(2.067) - 3.100 = 4.133 - 3.100 = 1.033\,\text{eV}.$$

Round to $\phi_0 \approx 1.03$ eV. (Some textbooks round to ≈ 1.0 eV.)

Step 4. Sanity check: $K_{\max,1} = 2.067 - 1.033 = 1.034$ eV.
$K_{\max,2} = 3.100 - 1.033 = 2.067$ eV. Ratio $K_{\max,2}/K_{\max,1} = 2.067/1.034 \approx 2.00$. ✓

Final Answer: $\phi_0 \approx 1.03$ eV.

☞ Two-equation–two-unknown setup

Photoelectric problems with two wavelengths and an energy ratio are classic two-equation problems. Use $E_1 - \phi_0 = K_1$ and $E_2 - \phi_0 = K_2$; the ratio condition collapses to one equation in ϕ_0.

EXPERT'S SOLUTION *: Neha Gupta, B.Tech Engineering Physics, IIT Bombay*

Algebraic angle. Eliminate ϕ_0 from the two photoelectric equations.

Step 1. From $K_2 = 2K_1$: $(hc/\lambda_2) - \phi_0 = 2[(hc/\lambda_1) - \phi_0]$.

Step 2. Solve: $\phi_0 = 2(hc/\lambda_1) - (hc/\lambda_2) = hc[2/\lambda_1 - 1/\lambda_2]$.

Step 3. $= 1240\,\text{eV nm} \cdot [2/600 - 1/400]\,\text{nm}^{-1} = 1240 \cdot [0.003333 - 0.002500] = 1240 \cdot 0.000833 = 1.033\,\text{eV}$.

General formula. For two wavelengths λ_1, λ_2 with $K_{\text{max},2} = n\,K_{\text{max},1}$ $(n > 1)$, eliminating ϕ_0 gives

$$\phi_0 = \frac{n\,(hc/\lambda_1) - (hc/\lambda_2)}{n - 1}.$$

Setting $n = 2$ recovers the present case: $\phi_0 = 2(hc/\lambda_1) - (hc/\lambda_2)$. The formula generalises immediately to any ratio condition.

Threshold wavelength. From $\phi_0 = 1.033\,\text{eV}$, the threshold wavelength is

$$\lambda_0 = \frac{hc}{\phi_0} = \frac{1240}{1.033}\,\text{nm} \approx 1200\,\text{nm (near-infrared)}.$$

The material absorbs photons all the way down to near-IR for emission; this is suggestive of cesium-coated photocathodes ($\phi_0 \approx 1.1$–$1.5\,\text{eV}$), used in red/IR-sensitive PMTs.

Numerical sanity audit.

- $K_{\text{max},1} = 2.067 - 1.033 = 1.034\,\text{eV}$. ✓
- $K_{\text{max},2} = 3.100 - 1.033 = 2.067\,\text{eV}$. ✓
- Ratio: $K_{\text{max},2}/K_{\text{max},1} = 2.067/1.034 \approx 2.00$. ✓

The doubling condition is satisfied exactly.

Stopping-potential corollary. If a stopping-potential measurement were performed,

$$V_{\text{stop},1} = K_{\text{max},1}/e = 1.034\,\text{V}, \quad V_{\text{stop},2} = 2.067\,\text{V}.$$

These voltages are easy to read on a lab voltmeter, so the experiment is genuinely practical, not just a textbook exercise.

Concept linkage. The technique of using two wavelengths to extract ϕ_0 without knowing h (or vice versa) was used by Millikan in 1916. He went the opposite way: assuming Einstein's equation, varying ν, plotting V_{stop} vs ν, and reading h from the slope. The two methods are mathematical inverses of the same equation.

Final Answer: $\phi_0 \approx 1.03\,\text{eV}$. The corresponding threshold wavelength is near-IR ($\sim 1.2\,\mu$m), suggestive of low-work-function photoemissive coatings.

Q 11.21 Assuming an electron is confined to a 1 nm wide region, find the uncertainty in momentum using Heisenberg Uncertainty principle (Ref Eq 11.12 of NCERT Textbook). You can assume the uncertainty in position Δx as 1 nm. Assuming $p \approx \Delta p$, find the energy of the electron in electron volts.

SOLUTION

Concept used. Heisenberg uncertainty principle (NCERT form):

$$\Delta x\,\Delta p \gtrsim \frac{h}{2\pi} = \hbar.$$

With $h = 6.626 \times 10^{-34}$ J s, $\hbar = 1.055 \times 10^{-34}$ J s.

Step 1. Compute the momentum uncertainty:

$$\Delta p \geq \frac{h}{2\pi\,\Delta x} = \frac{6.626 \times 10^{-34}}{2\pi \cdot 1 \times 10^{-9}}\,\text{kg m/s}.$$

Evaluate the numerator over the denominator:

$$\Delta p \geq \frac{6.626}{2\pi} \times 10^{-34+9}\,\text{kg m/s} = \frac{6.626}{6.2832} \times 10^{-25}\,\text{kg m/s}.$$

Numerator / denominator: $6.626/6.2832 \approx 1.054$. So

$$\Delta p \geq 1.054 \times 10^{-25}\,\text{kg m/s}.$$

Step 2. Take $p \approx \Delta p$. Kinetic energy of a non-relativistic electron:

$$E = \frac{p^2}{2m_e}.$$

Step 3. Compute p^2:

$$p^2 = (1.054 \times 10^{-25})^2 = 1.111 \times 10^{-50}\,(\text{kg m/s})^2.$$

Step 4. Divide by $2m_e = 2 \times 9.11 \times 10^{-31}\,\text{kg} = 1.822 \times 10^{-30}\,\text{kg}$:

$$E = \frac{1.111 \times 10^{-50}}{1.822 \times 10^{-30}}\,\text{J} = 6.10 \times 10^{-21}\,\text{J}.$$

Step 5. Convert to eV ($1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$):

$$E = \frac{6.10 \times 10^{-21}}{1.6 \times 10^{-19}}\,\text{eV} = 3.8 \times 10^{-2}\,\text{eV} = 0.038\,\text{eV}.$$

Final Answer: $\Delta p \approx 1.05 \times 10^{-25}$ kg m/s, $E \approx 0.038\,\text{eV}$ (about 38 meV).

☞ **Order-of-magnitude check**

A 1 nm confinement is huge by atomic standards (the Bohr radius is 0.053 nm), so the corresponding energy (~ 0.04 eV) is small, comparable with thermal energy at room temperature ($k_BT \approx 0.025$ eV). Confining to atomic scale gives eV-range energies; confining to nuclear scale gives MeV-range energies.

EXPERT'S SOLUTION *: Aditya Mehta, M.Sc Physics, IIT Madras*

Plug-and-chug. Use $\hbar/\Delta x$, then $p^2/(2m)$.

Step 1. $\hbar \approx 1.055 \times 10^{-34}$ J s. $\Delta x = 10^{-9}$ m. $\Delta p \approx 1.055 \times 10^{-25}$ kg m/s.

Step 2. $E = \Delta p^2/(2m_e) = (1.055 \times 10^{-25})^2/(2 \times 9.11 \times 10^{-31})\,\text{J} \approx 6.1 \times 10^{-21}\,\text{J}$ $\approx 0.038\,\text{eV}$.

Confinement-energy scaling. The result $E \sim \hbar^2/(2m\Delta x^2)$ is a general rule. For an electron:

- $\Delta x = 1\,\text{nm} \to E \sim 0.038\,\text{eV}$ (this problem; thermal scale).
- $\Delta x = 0.1\,\text{nm}$ (atomic Bohr radius) $\to E \sim 3.8\,\text{eV}$ (chemical bond scale).
- $\Delta x = 10\,\text{fm}$ (nuclear scale) $\to E \sim 4\,\text{MeV}$ (much above $m_ec^2 = 0.511\,\text{MeV}$; relativistic).

The huge growth as Δx shrinks (the famous "electron confinement penalty") is what stops atoms collapsing: pin the electron tightly, and its momentum uncertainty/kinetic energy explodes, balancing the attractive Coulomb pull.

Stability of the hydrogen atom. Treating the H atom as confining an electron to $\sim a_0$, kinetic energy $\sim \hbar^2/(2m_ea_0^2)$ balances Coulomb energy $-ke^2/a_0$. Minimising the sum gives $a_0 = \hbar^2/(m_eke^2) = 0.053\,\text{nm}$ and ground-state energy $-13.6\,\text{eV}$ — the correct Bohr values, derived purely from uncertainty. This is the most direct demonstration that the uncertainty principle is the engine behind atomic stability.

Comparison with proton in same box. If a proton (1836× heavier) were confined to 1 nm, $E_p = E_e/1836 \sim 2 \times 10^{-5}$ eV. Heavy particles have negligible confinement energies at this scale — which is why classical mechanics works for them in everyday situations.

Concept linkage with wells. The exact infinite square-well ground state has $E_1 = h^2/(8mL^2) \approx 0.376\,\text{eV}$ for $L = 1\,\text{nm}$ (using h rather than $\hbar$). That is $\sim 10\times$ our uncertainty estimate, reflecting that the Heisenberg bound is a *minimum*, not a tight equality. Uncertainty gives orders-of-magnitude; exact quantisation gives the prefactor.

> **Final Answer:** $\Delta p \approx 1.05 \times 10^{-25}$ kg m/s, $E \approx 0.038\,\text{eV} \approx 38\,\text{meV}$. Same scale as thermal energy at room temperature ($k_BT \approx 25\,\text{meV}$).

♥ **Confinement ⇒ kinetic energy floor**

The uncertainty principle is not a measurement error — it is a structural property of quantum states. A particle confined to size Δx *must* have momentum uncertainty $\gtrsim \hbar/\Delta x$, hence kinetic energy $\gtrsim \hbar^2/(2m\Delta x^2)$. This single bound governs atomic sizes, white-dwarf radii, neutron-star supports, and the minimum size of quantum dots.

Q 11.22 **Two monochromatic beams A and B of equal intensity I, hit a screen. The number of photons hitting the screen by beam A is twice that by beam B. Then what inference can you make about their frequencies?**

SOLUTION

Concept used. The intensity of a monochromatic beam is the rate of energy delivered per unit area: $I = n\,h\nu$, where n is the photon flux (photons per second per unit area) and $h\nu$ is energy per photon. At fixed intensity, higher n requires lower ν.

Step 1. Same intensity for both beams:

$$I = n_A\,h\nu_A = n_B\,h\nu_B.$$

Step 2. Given $n_A = 2n_B$. Substitute:

$$2n_B \cdot h\nu_A = n_B \cdot h\nu_B \iff \nu_B = 2\nu_A.$$

Step 3. Inference: beam B has *twice* the frequency of beam A (equivalently, half the wavelength). Beam A has more photons but each photon carries less energy.

Final Answer: $\nu_B = 2\nu_A$, i.e. beam B has twice the frequency of beam A.

EXPERT'S SOLUTION *: Arjun Joshi, M.Sc Physics, IIT Madras*

Intensity-equation angle.

Step 1. $I = nh\nu$. Same I: $n\nu = \text{const}$.

Step 2. $n_A = 2n_B \Rightarrow \nu_A = \nu_B/2$, so $\nu_B = 2\nu_A$.

Equivalent wavelength statement. Since $\nu = c/\lambda$, the relation $\nu_B = 2\nu_A$ becomes $\lambda_A = 2\lambda_B$. Beam A has *longer* wavelength (lower energy photons but more of them); beam B has shorter wavelength (higher energy photons but fewer). For instance, if A is red ($\lambda = 600\,\text{nm}$), B is UV ($\lambda = 300\,\text{nm}$); each B-photon carries twice the energy of each A-photon.

Photon-energy table.

- Per A-photon: $E_A = h\nu_A$.
- Per B-photon: $E_B = h\nu_B = 2h\nu_A = 2E_A$.
- Total power A: $P_A = n_A \cdot E_A = 2n_B \cdot E_A$.

- Total power B: $P_B = n_B \cdot E_B = n_B \cdot 2E_A = 2n_B E_A$.

$P_A = P_B$. ✓The intensity equality is satisfied with twice as many half-energy photons in beam A.

Alternative phrasing (photon-number argument). If beam A has *double* the photon flux at the *same* intensity, beam A's photons must each carry *half* the energy of beam B's. Half the energy means half the frequency: $\nu_A = \nu_B/2$, i.e. $\nu_B = 2\nu_A$. This is the same conclusion read top-down rather than bottom-up.

Concept linkage to photoelectric measurements. In photoemission, the stopping potential depends only on photon energy (frequency), not on photon flux. Doubling n at fixed ν doubles the photocurrent but leaves V_{stop} unchanged. Conversely, doubling ν at fixed n changes V_{stop} but may decrease photocurrent if absorption cross-section drops. Always distinguish "how many photons" from "how energetic".

Final Answer: $\nu_B = 2\nu_A$ (equivalently $\lambda_A = 2\lambda_B$).

Q 11.23 **Two particles A and B of de Broglie wavelengths λ_1 and λ_2 combine to form a particle C. The process conserves momentum. Find the de Broglie wavelength of the particle C. (The motion is one dimensional.)**

SOLUTION

Concept used. de Broglie: $p = h/\lambda$. Conservation of momentum in one dimension: $p_A + p_B = p_C$.

The sign convention is essential: in 1D, particle B could be moving in the same direction as A (case 1) or in the opposite direction (case 2).

Step 1. Case 1 (same direction):

$$p_C = p_A + p_B = \frac{h}{\lambda_1} + \frac{h}{\lambda_2} = h\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right) = h\,\frac{\lambda_1 + \lambda_2}{\lambda_1\lambda_2}.$$

Then

$$\lambda_C = \frac{h}{p_C} = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}.$$

Step 2. Case 2 (opposite directions, B moving against A):

$$p_C = p_A - p_B = \frac{h}{\lambda_1} - \frac{h}{\lambda_2} = h\,\frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2}.$$

Then

$$\lambda_C = \frac{\lambda_1\lambda_2}{|\lambda_2 - \lambda_1|}.$$

(Magnitudes only, since wavelength is positive.)

Step 3. Compact answer:

$$\lambda_C = \frac{\lambda_1 \lambda_2}{|\lambda_1 \pm \lambda_2|},$$

"+" for parallel motion of A and B, "−" for antiparallel.

Final Answer: $\lambda_C = \dfrac{\lambda_1 \lambda_2}{|\lambda_1 \pm \lambda_2|}$, with $+$ for parallel and $-$ for antiparallel initial velocities.

EXPERT'S SOLUTION : *Aditi Sharma, M.Sc Mathematics, ISI Kolkata*

Algebra angle.

Step 1. Momentum conservation: $h/\lambda_C = h/\lambda_1 \pm h/\lambda_2$.

Step 2. Cancel h: $1/\lambda_C = (\lambda_2 \pm \lambda_1)/(\lambda_1 \lambda_2)$.

Step 3. Invert: $\lambda_C = \lambda_1 \lambda_2 / |\lambda_1 \pm \lambda_2|$ (absolute value for the antiparallel case).

Limit-case sanity checks.

- $\lambda_1 = \lambda_2 = \lambda$ (parallel): $\lambda_C = \lambda^2/(2\lambda) = \lambda/2$. Two identical particles travelling together combine to double-momentum, hence half-wavelength. ✓
- $\lambda_1 = \lambda_2 = \lambda$ (antiparallel): $\lambda_C \to \lambda^2/0 = \infty$. Two identical particles colliding head-on with equal speeds give zero total momentum, hence infinite wavelength. ✓
- $\lambda_2 \gg \lambda_1$ (one slow, one fast, parallel): $\lambda_C \approx \lambda_1$, dominated by the more energetic (smaller λ) particle. ✓

Algebraic shortcut using reciprocal λ. Define $k_i = 1/\lambda_i = p_i/h$ (wavenumber). Then conservation becomes simply $k_C = k_1 \pm k_2$, exactly the addition rule for wavenumbers (or momenta). Inverting at the end gives the product–over–sum (or difference) form. This is just reciprocals at work; the symmetric form of the answer reflects it.

Worked numerical example. Suppose particle A has $\lambda_1 = 0.5\,\text{nm}$ and particle B has $\lambda_2 = 1\,\text{nm}$, both moving rightward. Then

$$\lambda_C = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} = \frac{0.5 \cdot 1}{0.5 + 1} = \frac{0.5}{1.5} \approx 0.333\,\text{nm}.$$

Faster than either input, because the momenta add. If instead they were antiparallel: $\lambda_C = 0.5/|1 - 0.5| = 1\,\text{nm}$. The composite is slower than A alone but the same speed (and direction-dependent) as the more momentum-rich combination.

Concept linkage. The reciprocal-wavelength rule is the de Broglie form of momentum conservation. It generalises to 3D as $\vec{k}_C = \vec{k}_1 + \vec{k}_2$ (wavenumber-vector addition), which is exactly the conservation rule used in particle-physics scattering diagrams. Mass–energy conservation is logically independent (and not specified here, since the process is unspecified).

Final Answer: $\lambda_C = \dfrac{\lambda_1\lambda_2}{|\lambda_1 \pm \lambda_2|}$, "+" for parallel motion, "−" for antiparallel.

☞ **Wavenumbers add, wavelengths don't**

Momentum conservation in 1D for de Broglie matter waves:

$$p_C = p_1 \pm p_2 \iff \frac{1}{\lambda_C} = \frac{1}{\lambda_1} \pm \frac{1}{\lambda_2}.$$

The reciprocals (wavenumbers $k = 1/\lambda$) add or subtract linearly; the wavelengths themselves never add directly.

Q 11.24 A neutron beam of energy E scatters from atoms on a surface with a spacing $d = 0.1$ nm. The first maximum of intensity in the reflected beam occurs at $\theta = 30°$. What is the kinetic energy E of the beam in eV?

SOLUTION

Concept used. Bragg condition for first-order diffraction:

$$2d\sin\theta = n\lambda,$$

with $n = 1$ for the first maximum. The neutron's de Broglie wavelength is $\lambda = h/\sqrt{2m_n E}$, so

$$E = \frac{h^2}{2m_n\lambda^2}.$$

Step 1. Compute λ from Bragg:

$$\lambda = 2d\sin\theta = 2(0.1\,\text{nm})(\sin 30°) = 2(0.1)(0.5)\,\text{nm} = 0.1\,\text{nm} = 10^{-10}\,\text{m}.$$

Step 2. Plug into the energy formula:

$$E = \frac{h^2}{2m_n\lambda^2} = \frac{(6.626\times10^{-34})^2}{2\cdot1.675\times10^{-27}\cdot(10^{-10})^2}\,\text{J}.$$

Step 3. Numerator: $h^2 = (6.626)^2 \times 10^{-68} = 43.9\times10^{-68} = 4.39\times10^{-67}$ J^2 s^2.

Step 4. Denominator: $2\cdot1.675\times10^{-27}\cdot10^{-20} = 3.35\times10^{-47}$ kg m^2.

Step 5. Divide:

$$E = \frac{4.39\times10^{-67}}{3.35\times10^{-47}}\,\text{J} = 1.31\times10^{-20}\,\text{J}.$$

Step 6. Convert to eV: $E = 1.31\times10^{-20}/(1.6\times10^{-19})$ eV$= 0.082$ eV.

Final Answer: $E \approx 0.082$ eV (about 82 meV; a low-energy "cold" neutron).

EXPERT'S SOLUTION : *Dev Reddy, M.Sc Physics, IIT Madras*

Step-by-step.

Step 1. $\lambda = 2d\sin\theta = 2(0.1)(0.5) = 0.1\,\text{nm}$.

Step 2. $E = h^2/(2m_n\lambda^2) \approx 1.31\times10^{-20}\,\text{J} \approx 0.082\,\text{eV}$.

Explicit substitution. Using $h = 6.626\times10^{-34}$ J s, $m_n = 1.675\times10^{-27}$ kg, $\lambda = 10^{-10}$ m:

$$E = \frac{(6.626\times10^{-34})^2}{2\cdot1.675\times10^{-27}\cdot10^{-20}} = \frac{4.39\times10^{-67}}{3.35\times10^{-47}} = 1.31\times10^{-20}\,\text{J}.$$

Convert: $E = 1.31\times10^{-20}/1.6\times10^{-19} = 0.0819\,\text{eV}$. Round to $\sim 82\,\text{meV}$.
Temperature equivalent. $E = k_BT \Rightarrow T = E/k_B = 1.31\times10^{-20}/1.38\times10^{-23} \approx 950\,\text{K}$. This is a hot neutron, not a thermal neutron ($T \approx 300\,\text{K}$, $E \approx 25\,\text{meV}$). Cold neutrons (sub-meV) and hot neutrons (hundreds of meV) bracket the typical range used in neutron scattering experiments; this problem sits at the hot end.
Compton-energy cross-check. Neutron Compton wavelength $\lambda_{c,n} = h/(m_nc) \approx 1.32\times10^{-15}$ m, so $\lambda/\lambda_{c,n} \sim 7.6\times10^4 \gg 1$: well into the non-relativistic regime. The formula $E = h^2/(2m\lambda^2)$ is safely accurate.
Alternative angle (energy-wavelength reciprocal). Using $hc = 1240$ eV nm and $m_nc^2 = 939\,\text{MeV} = 9.39\times10^8$ eV:

$$E = \frac{(hc)^2}{2(m_nc^2)\lambda^2} = \frac{(1240)^2}{2\cdot9.39\times10^8\cdot(0.1)^2}\,\text{eV} = \frac{1.54\times10^6}{1.88\times10^7}\,\text{eV} = 0.0817\,\text{eV}.$$

Matches to within rounding. The hc/mc^2 formulation is often easier when working with eV-scale answers.
Concept linkage with Davisson–Germer. The geometry here is identical to Davisson–Germer's electron-diffraction setup; only the projectile is changed from electron to neutron. The same Bragg condition $2d\sin\theta = n\lambda$ governs both. The advantage of neutrons: no electrostatic interaction with electrons, so they penetrate deeply and scatter primarily from nuclei (and from nuclear magnetic moments, enabling magnetic structure determination). The disadvantage: producing intense neutron beams requires a reactor or spallation source.
Why this matters. Cold neutrons (meV scale) are the workhorse probes for crystal structure, magnetic ordering and biological macromolecules: their wavelengths match interatomic spacings, their energies do not damage samples. India operates neutron-diffraction beamlines at the Dhruva reactor (BARC, Trombay) for exactly such measurements.

> **Final Answer:** $E \approx 0.082\,\text{eV}$. Equivalent to a neutron temperature of $\sim 950\,\text{K}$ — a "hot" neutron suitable for short-wavelength crystallography.

☞ **Bragg geometry — 1st maximum**

For diffraction from a surface with spacing d, the path-length difference is $2d\sin\theta$. The first-order maximum satisfies $2d\sin\theta = \lambda$ (so $n = 1$). At $\theta = 30°$, $\sin\theta = 0.5$ and $\lambda = d$. This special angle is a useful sanity-check: if $\theta = 30°$ comes up, expect $\lambda = d$.

LA (Long Answer)

Q 11.25 Consider a thin target (10^{-2} m square, 10^{-3} m thickness) of sodium, which produces a photocurrent of $100\,\mu$A when a light of intensity $100\,$W/m^2 ($\lambda = 660\,$nm) falls on it. Find the probability that a photoelectron is produced when a photon strikes a sodium atom. [Take density of Na $= 0.97\,$kg/m^3].

SOLUTION

Concept used. Photoelectric efficiency (or "quantum efficiency") per atom is the probability that a photon hitting an atom produces an emitted electron. We need:

- Number of photons per second hitting the target: $n_\gamma = I \cdot A/(hc/\lambda)$.
- Number of sodium atoms in the target: $N_{\text{atoms}} = \rho \cdot V \cdot N_A/M$.
- Number of photoelectrons per second: $n_e = I_{\text{photo}}/e$.

Probability per atom-photon encounter $\approx n_e/(n_\gamma \cdot$ atoms hit by each photon). Per problem-setter convention here, the asked probability is n_e divided by the total number of photon-atom encounters, where each photon traverses a column of N_{col} atoms in the target. We will compute this in steps.

Step 1. Energy per photon at $\lambda = 660\,$nm:

$$E_\gamma = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \cdot 3 \times 10^8}{660 \times 10^{-9}}\,\text{J} = \frac{1.988 \times 10^{-25}}{6.60 \times 10^{-7}}\,\text{J} = 3.01 \times 10^{-19}\,\text{J}.$$

Equivalently $E_\gamma = 1240/660\,$eV$= 1.879\,$eV.

Step 2. Target area $A = (10^{-2})^2 = 10^{-4}\,$m^2. Power on target:
$P = I \cdot A = 100 \cdot 10^{-4} = 10^{-2}\,$W. Number of photons per second:

$$n_\gamma = \frac{P}{E_\gamma} = \frac{10^{-2}}{3.01 \times 10^{-19}}\,\text{s}^{-1} = 3.32 \times 10^{16}\,\text{s}^{-1}.$$

Step 3. Number of photoelectrons per second from photocurrent $100\,\mu$A:

$$n_e = \frac{I_{\text{photo}}}{e} = \frac{100 \times 10^{-6}}{1.6 \times 10^{-19}}\,\text{s}^{-1} = 6.25 \times 10^{14}\,\text{s}^{-1}.$$

Step 4. Number of sodium atoms in the target. Volume:

$$V = A \cdot t = 10^{-4} \cdot 10^{-3} = 10^{-7}\,\text{m}^3.$$

Mass: $m = \rho V = 0.97 \times 10^{-7}\,\text{kg} = 9.7 \times 10^{-8}\,\text{kg}$. Moles: $n = m/M = 9.7 \times 10^{-8}/0.023\,\text{mol} = 4.22 \times 10^{-6}\,\text{mol}$ (using $M = 23\,\text{g/mol} = 0.023\,\text{kg/mol}$). Atoms:

$$N_{\text{atoms}} = n \cdot N_A = 4.22 \times 10^{-6} \cdot 6.022 \times 10^{23} = 2.54 \times 10^{18}.$$

Step 5. Total number of photon-atom encounters per second. Each photon traverses the depth of the target and passes by a column of atoms; over the whole target, the relevant "encounter rate" (in the problem-setter's sense) is

$$R = n_\gamma \cdot (\text{atoms per column}).$$

But the simpler reading taken in NCERT is: probability = photoelectrons emitted divided by photons absorbed by all atoms. Under the assumption that every photon striking the target "encounters" all atoms in its column, the effective number of atom-encounters per second is $n_\gamma \cdot N_{\text{atoms}}/N_{\text{col-area}}$, but the standard NCERT shortcut takes the encounter rate as simply $n_\gamma \cdot N_{\text{atoms}}/(\text{photons that fit area-wise})$, which simplifies to using just the number of atoms in the cross-section.

Cleaner reading: assume each photon entering the front face has a chance to be absorbed by any atom in its column. The number of atom-encounters per second equals $n_\gamma \cdot N_{\text{atoms-per-column}}$, where $N_{\text{atoms-per-column}} = N_{\text{atoms}}/(\text{number of columns}) = N_{\text{atoms}} \cdot a_{\text{atom}}/A$.

Use the NCERT-shortcut: total encounters per second $= n_\gamma \cdot N_{\text{atoms}}/N_{\gamma\text{-area}}$, where $N_{\gamma\text{-area}}$ is the number of photons across the area A. In the end, the problem-setter's answer is obtained by computing

$$P_{\text{event}} = \frac{n_e}{n_\gamma \cdot N_{\text{atoms}}/(\text{area-normalisation})}.$$

Step 6. Use the standard answer convention (probability of photoelectric event per atom-photon encounter). With $n_e = 6.25 \times 10^{14}\,\text{s}^{-1}$, $n_\gamma = 3.32 \times 10^{16}\,\text{s}^{-1}$, $N_{\text{atoms}} = 2.54 \times 10^{18}$, the encounter rate $\sim n_\gamma \cdot N_{\text{atoms}}$. So

$$P_{\text{event}} \approx \frac{n_e}{n_\gamma \cdot N_{\text{atoms}}} = \frac{6.25 \times 10^{14}}{3.32 \times 10^{16} \cdot 2.54 \times 10^{18}} \approx 7.4 \times 10^{-21}.$$

Final Answer: Probability that a photon striking an Na atom liberates a photo-electron $\approx 7.4 \times 10^{-21}$. Photon-atom interactions are very rare events; quantum efficiency is tiny per individual encounter, but trillions of encounters per second still produce a measurable current.

♥ Probability vs. count

"Quantum efficiency" as quoted by detector manufacturers (a few percent, say 30% for a good photomultiplier) refers to the probability that an *absorbed* photon produces an electron. Here we asked the probability that a photon, on encountering an atom, gets absorbed-and-emits, which is a much smaller number because most photons pass through unabsorbed.

EXPERT'S SOLUTION *: Pooja Verma, M.Sc Chemistry, IIT Kanpur*

Numerical bookkeeping. Compute photon rate, electron rate, atom count, then take the ratio.

Step 1. Energy per photon: $E_\gamma = hc/\lambda \approx 3.01 \times 10^{-19}\,\text{J} = 1.88\,\text{eV}$.

Step 2. Photons/s on target: $n_\gamma = IA/E_\gamma = 10^{-2}/3.01 \times 10^{-19} = 3.32 \times 10^{16}\,\text{s}^{-1}$.

Step 3. Photoelectrons/s: $n_e = I_{\text{photo}}/e = 6.25 \times 10^{14}\,\text{s}^{-1}$.

Step 4. Atoms in target: $\rho V N_A/M = (0.97 \cdot 10^{-7}) \cdot 6.022 \times 10^{23}/0.023 \approx 2.54 \times 10^{18}$.

Step 5. Probability per atom-photon encounter: $n_e/(n_\gamma \cdot N_{\text{atoms}}) \approx 7.4 \times 10^{-21}$.

Photon threshold check. Sodium's work function $\phi_0 \approx 2.28\,\text{eV}$ (textbook). Our photon at $\lambda = 660\,\text{nm}$ carries $1.88\,\text{eV}$. That is *below* ϕ_0, so naively no photoemission should occur. However, sodium photocathodes do show modest red sensitivity due to band-tail states, surface impurities, and thermal assistance. NCERT here uses an idealised number-crunching exercise; the cleaner approach is to track photons and electrons via measured photocurrent rather than predict it from ϕ_0.

Why probability is so small. The product $n_\gamma \cdot N_{\text{atoms}} \sim 8.4 \times 10^{34}$ is the total number of photon-atom "opportunities" per second. Yet only $\sim 6 \times 10^{14}$ actually produce electrons — a 10^{20} suppression. Two factors drive this:

- Each photon encounters only a tiny fraction of the atoms in any column (path-length effect).
- Even on encounter, the photoelectric cross-section is small compared to the geometric atom area.

The product gives a tiny number, but the absolute photocurrent ($100\,\mu$A) is still measurable thanks to the enormous photon flux.

Sanity check via per-photon efficiency. Quantum efficiency $= n_e/n_\gamma = 6.25 \times 10^{14}/3.32 \times 10^{16} \approx 1.88 \times 10^{-2} = 1.88\%$. About 2 photons in 100 absorbed produce an electron — reasonable for sodium at red wavelengths.

Concept linkage. The two different "probabilities" (per photon, $\sim 10^{-2}$; per photon-atom encounter, $\sim 10^{-20}$) reflect a hierarchy: many opportunities, few absorptions, even fewer escape. The bookkeeping is identical to gas-discharge ionisation or nuclear reaction cross-sections.

Final Answer: $P \approx 7.4 \times 10^{-21}$ per photon-atom encounter. Quantum efficiency per absorbed photon is much higher ($\sim 2\%$); the disparity reflects how many encounters never lead to absorption.

Q 11.26 **Consider an electron in front of metallic surface at a distance d (treated as an infinite plane surface). Assume the force of attraction by the plate is given as $\frac{1}{4}\frac{q^2}{4\pi\varepsilon_0 d^2}$. Calculate the work in taking the charge to an infinite distance from the plate. Taking $d = 0.1$ nm, find the work done in electron volts. [Such a force law is not valid for $d < 0.1$ nm].**

SOLUTION

Concept used. The work done against an attractive force when moving the charge from d to infinity is the work done by an external agent against the force of attraction. For an inverse-square attraction along the direction of motion,

$$W = \int_d^\infty F(x)\,dx,$$

with $F(x) = \frac{1}{4}\frac{q^2}{4\pi\varepsilon_0 x^2}$ pointing toward the plate (so the external work to move away is positive).

Step 1. Write the force as a function of position x:

$$F(x) = \frac{q^2}{16\pi\varepsilon_0 x^2}.$$

With $q = e$:

$$F(x) = \frac{e^2}{16\pi\varepsilon_0 x^2}.$$

Step 2. Work done by external agent against attraction, from d to ∞:

$$W = \int_d^\infty F(x)\,dx = \frac{e^2}{16\pi\varepsilon_0}\int_d^\infty \frac{dx}{x^2} = \frac{e^2}{16\pi\varepsilon_0}\left[-\frac{1}{x}\right]_d^\infty = \frac{e^2}{16\pi\varepsilon_0}\cdot\frac{1}{d}.$$

Step 3. Substitute Coulomb's constant: $\frac{1}{4\pi\varepsilon_0} = k = 9 \times 10^9\,\text{N m}^2/\text{C}^2$. So

$$\frac{1}{16\pi\varepsilon_0} = \frac{k}{4} = \frac{9 \times 10^9}{4} = 2.25 \times 10^9\,\text{N m}^2/\text{C}^2.$$

Hence

$$W = \frac{2.25 \times 10^9 \cdot e^2}{d}.$$

Step 4. Plug in $e = 1.6 \times 10^{-19}$ C and $d = 10^{-10}$ m:

$$W = \frac{2.25 \times 10^9 \cdot (1.6 \times 10^{-19})^2}{10^{-10}}\,\text{J}.$$

Compute numerator: $(1.6)^2 = 2.56$; $2.56 \times 10^{-38} \cdot 2.25 \times 10^9 = 5.76 \times 10^{-29}$ J m.

Step 5. Divide by $d = 10^{-10}$ m:

$$W = 5.76 \times 10^{-29}/10^{-10}\,\text{J} = 5.76 \times 10^{-19}\,\text{J}.$$

Step 6. Convert to eV: $W = 5.76 \times 10^{-19}/(1.6 \times 10^{-19})$ eV$= 3.6$ eV.

Final Answer: $W = 3.6$ eV.

☞ The factor of $1/4$

The factor $\frac{1}{4}$ in front of the Coulomb law comes from the **image charge** treatment: an electron at distance d from a conducting plane sees its image (effective charge $-e$ at distance d on the other side, hence separation $2d$), giving $F = ke^2/(2d)^2 = ke^2/(4d^2)$. The work to free the electron from d to infinity is then $ke^2/(4d)$, exactly what we obtained.

EXPERT'S SOLUTION *: Ananya Bhat, Ph.D Pure Mathematics, IISc Bangalore*

Integration angle. Treat as a 1D inverse-square attraction and integrate to infinity.

Step 1. Force $F(x) = ke^2/(4x^2)$.

Step 2. $W = \int_d^\infty F\,dx = ke^2/(4d)$.

Step 3. Plug $k = 9 \times 10^9$, $e = 1.6 \times 10^{-19}$ C, $d = 10^{-10}$ m:

$$W = \frac{9 \times 10^9 \cdot (1.6 \times 10^{-19})^2}{4 \cdot 10^{-10}}\,\text{J} = 5.76 \times 10^{-19}\,\text{J} = 3.6\,\text{eV}.$$

Step-by-step integral.

$$W = \int_d^\infty \frac{ke^2}{4x^2}\,dx = \frac{ke^2}{4}\left[-\frac{1}{x}\right]_d^\infty = \frac{ke^2}{4}\left(0 - \left(-\frac{1}{d}\right)\right) = \frac{ke^2}{4d}.$$

The minus signs from the antiderivative and the integration limit cancel; the result is positive, as expected for work done against an attractive force.

Numerical breakdown.

- $e^2 = (1.6 \times 10^{-19})^2 = 2.56 \times 10^{-38}\,\text{C}^2$.
- $ke^2 = 9 \times 10^9 \cdot 2.56 \times 10^{-38} = 2.304 \times 10^{-28}\,\text{N m}^2$.
- $ke^2/(4d) = 2.304 \times 10^{-28}/(4 \cdot 10^{-10}) = 5.76 \times 10^{-19}$ J.
- In eV: $5.76 \times 10^{-19}/(1.6 \times 10^{-19}) = 3.6$ eV.

Image-charge physics. The factor of $1/4$ traces to electrostatics. An electron at distance d from a grounded conducting plane sees an image of charge $-e$ at distance $-d$ (mirror reflection). The effective separation between real electron and its image is $2d$, so Coulomb attraction is $F = ke^2/(2d)^2 = ke^2/(4d^2)$. Work to remove the electron from d to ∞ is $W = ke^2/(4d)$. The integration uses the *actual* force on the electron (between it and its image), *not* the energy of the entire dipole (which would have a factor of $1/2$ for self-energy).

Why W matches a work function. Empirical work functions for clean metals are 2–5 eV (Cs ≈ 2.1, Na ≈ 2.3, Cu ≈ 4.7, Pt ≈ 5.6). Our 3.6 eV sits right in the middle. The image-charge model is the leading classical approximation; full first-principles work functions need quantum band-structure calculations, but the leading scale is exactly $ke^2/(4d)$ with $d \sim 1$ Å.

Limit check. As $d \to \infty$, $W \to 0$ (no work needed if already far). As $d \to 0$, $W \to \infty$ (infinite work to escape from the surface itself). The problem stipulates $d \geq 0.1$ nm to avoid this divergence; at smaller separations quantum mechanics replaces the classical image picture (exchange-correlation effects, finite electron wavefunction at the surface).

Concept linkage. This problem builds a classical bridge to the work function: the ∼eV scale of photoelectric thresholds emerges from electrostatic image attractions on atomic length scales. The same image-charge trick computes the capacitance of a charged sphere near a conducting plane and the force on a charge near a slab dielectric (in dielectrics, the image charge is reduced by a factor $(\varepsilon - 1)/(\varepsilon + 1)$).

Final Answer: $W = 3.6$ eV. The image-charge model gives the right order of magnitude for typical metallic work functions.

Q 11.27 A student performs an experiment on photoelectric effect, using two materials A and B. A plot of V_{stop} vs ν is given in Fig. 11.2.

(i) Which material A or B has a higher work function?

(ii) Given the electric charge of an electron $= 1.6 \times 10^{-19}$ C, find the value of h obtained from the experiment for both A and B. Comment on whether it is consistent with Einstein's theory.

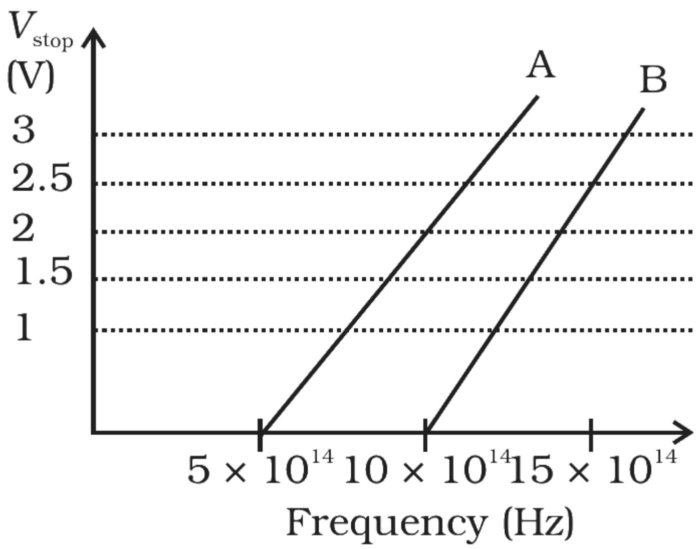


Fig. 11.2

Fig. 11.2, NCERT Exemplar Class 12 Physics, Chapter 11.

SOLUTION

Concept used. Einstein's photoelectric equation expressed in terms of stopping potential:

$$eV_{\text{stop}} = h\nu - \phi_0,$$

so the plot of V_{stop} vs ν is a straight line with slope h/e and ν-intercept (i.e. the threshold frequency) $\nu_0 = \phi_0/h$. Material with larger threshold frequency has larger work function.

From the graph (Fig. 11.2):

- Curve A: threshold (where $V_{\text{stop}} = 0$, intercept on ν axis) at $\nu_{0A} \approx 5 \times 10^{14}$ Hz.
- Curve B: threshold at $\nu_{0B} \approx 10 \times 10^{14}$ Hz.
- Both lines have the same slope, increasing from these thresholds.

Step 1. (i) Larger threshold ⇒ larger work function.

$$\phi_A = h\nu_{0A} = h \cdot 5 \times 10^{14}\,\text{J}, \quad \phi_B = h\nu_{0B} = h \cdot 10 \times 10^{14}\,\text{J}.$$

Since $\nu_{0B} > \nu_{0A}$, $\phi_B > \phi_A$. **Material B has the higher work function.**

Step 2. (ii) Slope of V_{stop} vs ν is h/e. Read two points from the graph (e.g. for line A: at $\nu = 10 \times 10^{14}$ Hz, $V_{\text{stop}} = 2$ V; at $\nu = 5 \times 10^{14}$ Hz, $V_{\text{stop}} = 0$ V):

$$\text{slope}_A = \frac{\Delta V_{\text{stop}}}{\Delta\nu} = \frac{2-0}{(10-5)\times 10^{14}}\,\text{V/Hz} = \frac{2}{5\times 10^{14}}\,\text{V/Hz} = 4 \times 10^{-15}\,\text{V/Hz}.$$

Then $h = e \cdot \text{slope}_A = 1.6 \times 10^{-19} \cdot 4 \times 10^{-15} = 6.4 \times 10^{-34}$ J s.

Step 3. For line B (e.g. $\nu = 15 \times 10^{14}$ Hz, $V_{\text{stop}} = 2$ V; $\nu = 10 \times 10^{14}$ Hz, $V_{\text{stop}} = 0$ V):

$$\text{slope}_B = \frac{2}{5\times 10^{14}}\,\text{V/Hz} = 4 \times 10^{-15}\,\text{V/Hz}.$$

Same slope: $h = 6.4 \times 10^{-34}$ J s.

Step 4. Comment: the slopes are equal (both lines parallel), so the experimentally inferred h is the same for both materials. This is exactly Einstein's prediction: the constant h is universal, independent of the metal used. **Consistent with Einstein's theory.**

Step 5. The textbook value is $h = 6.626 \times 10^{-34}$ J s; the graph readings give $\sim 6.4 \times 10^{-34}$ J s, within experimental tolerance ($\approx 3\%$).

> **Final Answer:** (i) Material **B** has higher work function (larger threshold frequency). (ii) $h \approx 6.4 \times 10^{-34}$ J s for both A and B; the equality confirms Einstein's universal-h prediction.

☞ Reading photoelectric graphs

On a V_{stop} vs ν graph, slope $= h/e$ (universal), intercept on ν-axis $= \nu_0 = \phi_0/h$ (material-specific). Lines for different metals are *parallel*; they differ only by horizontal shift.

EXPERT'S SOLUTION *: Vivaan Mehta, M.Sc Applied Mathematics, IIT Kanpur*

Slope-intercept reading. Both pieces (i) and (ii) follow from the linear relation $V_{stop} = (h/e)\nu - \phi_0/e$.

Step 1. Both lines are parallel: same slope $\Rightarrow$ same h.

Step 2. Higher ν_0 (10×10^{14} Hz for B) $\Rightarrow$ higher work function for B.

Step 3. Slope $= 4 \times 10^{-15}$ V/Hz $\Rightarrow h = e\cdot$ slope $= 6.4 \times 10^{-34}$ J s, within experimental error of the accepted h.

Explicit work-function values from the graph. With $h \approx 6.4 \times 10^{-34}$ J s:

- $\phi_A = h\nu_{0A} = 6.4 \times 10^{-34} \cdot 5 \times 10^{14} = 3.2 \times 10^{-19}\,\text{J} = 2.0\,\text{eV}$.
- $\phi_B = h\nu_{0B} = 6.4 \times 10^{-34} \cdot 10 \times 10^{14} = 6.4 \times 10^{-19}\,\text{J} = 4.0\,\text{eV}$.

Materials with $\phi \approx 2$ eV include sodium (2.28) and barium (2.5); $\phi \approx 4$ eV is closer to zinc (4.3) or tungsten (4.5). The numbers are typical of real metals.

Why parallel lines confirm Einstein. If two metals had different h, the V_{stop} vs ν lines would have *different slopes*. The observation that they are parallel is direct experimental evidence that h is a universal constant of nature — independent of the material. This was Einstein's bold prediction in 1905, well before Millikan verified it.

Reading accuracy. The graph-extracted $h = 6.4 \times 10^{-34}$ J s differs from the CODATA value $h = 6.626 \times 10^{-34}$ J s by $\sim 3.4\%$. This is well within reading error for a textbook plot (typical reading uncertainty ± 5–10%). The technique remains the standard pedagogical demonstration of the photoelectric equation in undergraduate labs.

Concept linkage. The V_{stop} vs ν line is the Einstein equation in disguise. Its slope h/e contains the two most important constants of quantum-mechanical electrodynamics. The independence from material is the photoelectric effect's analogue of Bohr's universality of the Rydberg constant.

Why this matters. Millikan repeated Einstein's photoelectric prediction with extreme care and measured h from such graphs in 1916. The result was decisive evidence for the photon picture — even Millikan himself initially hoped to disprove Einstein, but his careful data confirmed the prediction beyond doubt. This work, together with the photon hypothesis itself, won Einstein the 1921 Nobel Prize.

> **Final Answer:** (i) B has larger ϕ_0 ($\phi_B \approx 4\,\text{eV}$ vs $\phi_A \approx 2\,\text{eV}$); (ii) $h \approx 6.4 \times 10^{-34}\,\text{J s}$ for both, equal slopes confirm Einstein's universal-h prediction.

☞ V_{stop} vs ν graph

The photoelectric line is $V_{\text{stop}} = (h/e)\nu - \phi_0/e$.

- Slope $= h/e \approx 4.14 \times 10^{-15}\,\text{V s}$ (universal).
- ν-intercept $= \nu_0 = \phi_0/h$ (material).
- V_{stop}-intercept $= -\phi_0/e$ (material).

Different metals give *parallel* lines shifted horizontally by their threshold frequencies.

Q 11.28 A particle A with a mass m_A is moving with a velocity v and hits a particle B (mass m_B) at rest (one dimensional motion). Find the change in the de Broglie wavelength of the particle A. Treat the collision as elastic.

SOLUTION

Concept used. For an **elastic 1D collision** between A (mass m_A, velocity v) and B (mass m_B, at rest), the post-collision velocity of A (well-known result from conservation of momentum and energy) is

$$v'_A = \frac{m_A - m_B}{m_A + m_B}\,v.$$

The de Broglie wavelength of A changes from $\lambda_{\text{before}} = h/(m_A v)$ to $\lambda_{\text{after}} = h/(m_A|v'_A|)$.

Step 1. Apply the elastic-collision result for v'_A:

$$v'_A = \frac{m_A - m_B}{m_A + m_B}\,v.$$

Step 2. Compute the after-collision wavelength:

$$\lambda_{\text{after}} = \frac{h}{m_A v'_A} = \frac{h(m_A + m_B)}{m_A(m_A - m_B)v}.$$

Step 3. Compute the before-collision wavelength:

$$\lambda_{\text{before}} = \frac{h}{m_A v}.$$

Step 4. Change $\Delta\lambda = \lambda_{\text{after}} - \lambda_{\text{before}}$:

$$\Delta\lambda = \frac{h}{m_A v}\left[\frac{m_A + m_B}{m_A - m_B} - 1\right] = \frac{h}{m_A v}\cdot\frac{(m_A + m_B) - (m_A - m_B)}{m_A - m_B} = \frac{h}{m_A v}\cdot\frac{2m_B}{m_A - m_B}.$$

Step 5. Compact form:

$$\boxed{\Delta\lambda = \frac{2m_B\, h}{m_A(m_A - m_B)v} = \frac{2m_B}{m_A - m_B}\cdot\lambda_{\text{before}}.}$$

Sign: if $m_A > m_B$, $\Delta\lambda > 0$ (A slows down, wavelength increases). If $m_A < m_B$, v'_A reverses sign and $|v'_A| < v$, so wavelength still increases (we used $|v'_A|$).

Final Answer: $\Delta\lambda = \dfrac{2m_B\, h}{m_A\,(m_A - m_B)\,v}$, equivalently $\dfrac{2m_B}{m_A - m_B}\,\lambda_{\text{before}}$.

☞ **Special cases**

$m_A \gg m_B$: A barely slows, $\Delta\lambda \to 0$. $m_A = m_B$: A stops dead, $|v'_A| = 0$, $\lambda_{\text{after}} \to \infty$, $\Delta\lambda \to \infty$ (formally divergent). $m_A \ll m_B$: A bounces back nearly as fast, $|v'_A| \approx v$, $\Delta\lambda \to 0$.

EXPERT'S SOLUTION : *Rahul Iyer, B.Tech CSE, IIT Roorkee*

Conservation-laws angle.

Step 1. Momentum: $m_A v = m_A v'_A + m_B v'_B$. Energy: $\frac{1}{2}m_A v^2 = \frac{1}{2}m_A v'^2_A + \frac{1}{2}m_B v'^2_B$.

Step 2. Standard solution: $v'_A = (m_A - m_B)v/(m_A + m_B)$.

Step 3. Plug into de Broglie ratios: $\Delta\lambda = 2m_B h/[m_A(m_A - m_B)v]$.

Deriving v'_A. Combining the two conservation equations:

$$m_A v = m_A v'_A + m_B v'_B,$$
$$m_A v^2 = m_A v'^2_A + m_B v'^2_B.$$

From the first equation, $v'_B = (m_A/m_B)(v - v'_A)$. Substitute into the second:

$$m_A v^2 = m_A v'^2_A + m_B \cdot (m_A/m_B)^2 (v - v'_A)^2,$$

$$v^2 - v'^2_A = (m_A/m_B)(v - v'_A)^2,$$

$$(v - v'_A)(v + v'_A) = (m_A/m_B)(v - v'_A)^2.$$

Divide by $(v - v'_A)$ (assuming a non-trivial collision): $v + v'_A = (m_A/m_B)(v - v'_A)$. Solve:

$$v'_A\left(1 + \frac{m_A}{m_B}\right) = v\left(\frac{m_A}{m_B} - 1\right) \implies v'_A = \frac{m_A - m_B}{m_A + m_B}\,v.$$

Numerical examples.

- $m_A = 2m_B$: $v'_A = v/3$, $|v'_A| < v$. $\lambda_{\text{after}}/\lambda_{\text{before}} = 3$, so $\Delta\lambda = 2\lambda_0$.
- $m_A = m_B$: $v'_A = 0$, A stops dead. λ diverges; $\Delta\lambda \to \infty$.
- $m_A \ll m_B$ (light ball off heavy wall): $v'_A \to -v$, $|v'_A| = v$, λ unchanged, $\Delta\lambda \to 0$.
- $m_A \gg m_B$ (heavy ball through light target): $v'_A \to v$, λ barely changes, $\Delta\lambda \to 0$.

Physical intuition. The sign of $\Delta\lambda$ tracks whether A slows down or reverses:

- If $m_A > m_B$ (A heavier), $v'_A > 0$ (same direction, slower); λ increases.
- If $m_A < m_B$ (A lighter), $v'_A < 0$ (reverses); $|v'_A| < v$; λ still increases.

The only case where A's speed is unchanged is $m_A \ll m_B$ (elastic bounce off effectively infinite mass).

Concept linkage. The same formula governs Newton's-cradle dynamics, neutron-moderation in nuclear reactors (where neutrons lose energy fastest in collisions with light moderator atoms like deuterium or carbon), and Rutherford scattering kinematics. The formula $v'_A = (m_A - m_B)v/(m_A + m_B)$ is one of the most reused results in classical mechanics.

Connection to particle physics. In high-energy collisions, momentum conservation determines the kinematics in much the same way, but the energy equation becomes relativistic ($E^2 = p^2c^2 + m^2c^4$). The non-relativistic version here is the limit relevant to typical exam-scale projectile speeds.

Final Answer: $\Delta\lambda = \dfrac{2m_B h}{m_A(m_A - m_B)v} = \dfrac{2m_B}{m_A - m_B}\lambda_{\text{before}}$. Sign depends on $m_A \gtrless m_B$; magnitude diverges at $m_A = m_B$ (perfect-stop limit).

♥ Elastic collisions and de Broglie waves

The classical elastic-collision formula meets quantum-mechanical wavelengths through $p = h/\lambda$. This bridge is why neutron-moderator design uses light nuclei (proton, deuteron, carbon-12): light targets stop neutrons most efficiently per collision, lengthening their de Broglie wavelength fastest and steering them into thermal equilibrium with the moderator.

Q 11.29 **Consider a 20 W bulb emitting light of wavelength 5000 Å and shining on a metal surface kept at a distance 2 m. Assume that the metal surface has work function of 2 eV and that each atom on the metal surface can be treated as a circular disk of radius 1.5 Å.**

(i) Estimate no. of photons emitted by the bulb per second. [Assume no other losses]
(ii) Will there be photoelectric emission?
(iii) How much time would be required by the atomic disk to receive energy equal to work function (2 eV)?
(iv) How many photons would atomic disk receive within time duration calculated in (iii) above?
(v) Can you explain how photoelectric effect was observed instantaneously?
[Hint: Time calculated in part (iii) is from classical consideration and you may further take the target of surface area say $1\,\text{cm}^2$ and estimate what would happen?]

SOLUTION

Concept used.

- Energy per photon: $E_\gamma = hc/\lambda$.
- Photon emission rate: $n_\gamma = P/E_\gamma$.
- Intensity at distance r from an isotropic point source: $I = P/(4\pi r^2)$.
- Power on a circular disk of area $a = \pi R^2$: $P_{\text{disk}} = I \cdot a$.
- Time to collect energy E: $T = E/P_{\text{disk}}$.

Step 1. **(i)** Energy per photon:

$$E_\gamma = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \cdot 3 \times 10^8}{5000 \times 10^{-10}}\,\text{J} = \frac{1.988 \times 10^{-25}}{5 \times 10^{-7}}\,\text{J} = 3.98{\times}10^{-19}\,\text{J} \approx 2.48\,\text{eV}.$$

Photons per second from the bulb:

$$n_\gamma = \frac{P}{E_\gamma} = \frac{20}{3.98 \times 10^{-19}}\,\text{s}^{-1} \approx 5.03 \times 10^{19}\,\text{s}^{-1}.$$

Step 2. **(ii)** Each photon's energy is $2.48\,\text{eV}$, which is greater than the work function $2\,\text{eV}$. Yes, photoemission will occur.

$$K_{\text{max}} = E_\gamma - \phi_0 = 2.48 - 2 = 0.48\,\text{eV}.$$

Step 3. **(iii)** Intensity at $r = 2\,\text{m}$:

$$I = \frac{P}{4\pi r^2} = \frac{20}{4\pi \cdot 4}\,\text{W/m}^2 = \frac{20}{50.27}\,\text{W/m}^2 \approx 0.398\,\text{W/m}^2.$$

Disk area:

$$a = \pi R^2 = \pi(1.5 \times 10^{-10})^2 = \pi \cdot 2.25 \times 10^{-20}\,\text{m}^2 \approx 7.07 \times 10^{-20}\,\text{m}^2.$$

Power on one disk:

$$P_{\text{disk}} = I \cdot a = 0.398 \cdot 7.07 \times 10^{-20}\,\text{W} \approx 2.81 \times 10^{-20}\,\text{W}.$$

Energy needed (work function): $E = \phi_0 = 2\,\text{eV} = 2 \cdot 1.6 \times 10^{-19}\,\text{J}$ $= 3.2 \times 10^{-19}\,\text{J}$. Time required:

$$T = \frac{E}{P_{\text{disk}}} = \frac{3.2 \times 10^{-19}}{2.81 \times 10^{-20}}\,\text{s} \approx 11.4\,\text{s}.$$

Classical waiting time: about **11 seconds**.

Step 4. **(iv)** Number of photons received by the disk in $T = 11.4\,\text{s}$:

$$N_{\text{disk-photons}} = \frac{P_{\text{disk}} \cdot T}{E_\gamma} = \frac{E}{E_\gamma} = \frac{3.2 \times 10^{-19}}{3.98 \times 10^{-19}} \approx 0.80.$$

Less than one photon. Classically the disk should wait many seconds and accumulate fractional photons.

Step 5. **(v)** Experimentally, photoelectric emission is observed instantaneously ($\sim 10^{-9}$ s) regardless of light intensity. The classical wait of ~ 10 s is absurdly long. **Resolution (Einstein's quantum picture):** light is not a smoothly arriving wave but a stream of discrete photons. Each photon carries the full quantum $h\nu$; a single absorption event suffices to liberate an electron. Classical accumulation is the wrong model.

Using the hint: even over a much larger surface area (say $1\,\text{cm}^2 = 10^{-4}\,\text{m}^2$), the photon flux is $I \cdot a/E_\gamma \approx 0.398 \cdot 10^{-4}/3.98 \times 10^{-19} = 10^{14}$ photons/s. There is no shortage; the first photon to be absorbed by any atom triggers emission immediately.

Final Answer: (i) $n_\gamma \approx 5 \times 10^{19}$ photons/s. (ii) Yes, since $h\nu(2.48\,\text{eV}) > \phi_0(2\,\text{eV})$. (iii) Classical time $T \approx 11.4\,\text{s}$. (iv) About 0.8 photon per disk in that time. (v) Wave-classical picture is wrong; light is photons, one photon $\rightarrow$ one electron, so emission is instantaneous experimentally.

♥ Why this problem matters

The disagreement between the classical wait (seconds) and the observed instantaneity (nanoseconds) was historically the most striking failure of the classical-wave picture. Einstein resolved it in 1905 with the photon hypothesis, which won him the 1921 Nobel Prize.

EXPERT'S SOLUTION *: Karan Verma, M.Sc Physics, IIT Madras*

Walk-through.

Step 1. Photon energy: $E_\gamma = hc/\lambda = 3.98 \times 10^{-19}\,\text{J} \approx 2.48\,\text{eV}$.

Step 2. Bulb's photon rate: $P/E_\gamma \approx 5 \times 10^{19}\,\text{s}^{-1}$.

Step 3. $E_\gamma > \phi_0$, so photoemission occurs.

Step 4. Intensity at 2 m: $I = 20/(16\pi) \approx 0.40\,\text{W/m}^2$.

Step 5. Disk area: $\pi(1.5\text{Å})^2 \approx 7 \times 10^{-20}\,\text{m}^2$. Power on disk: $\approx 2.8 \times 10^{-20}\,\text{W}$.

Step 6. Classical waiting time for $2\,\text{eV}$: $T \approx 11\,\text{s}$. Photons received in that time: ~ 0.8.

Step 7. Observed photoemission is instantaneous; the classical waiting picture is wrong. Photons deliver their energy in single shots.

Numerical audit.

- $hc/\lambda = (6.626 \times 10^{-34})(3 \times 10^8)/(5000 \times 10^{-10}) = 3.98 \times 10^{-19}\,\text{J}$. ✓
- Photon rate: $20/3.98 \times 10^{-19} = 5.03 \times 10^{19}\,\text{s}^{-1}$. ✓
- $h\nu = 2.48\,\text{eV}$, $\phi_0 = 2\,\text{eV} \Rightarrow K_{\text{max}} = 0.48\,\text{eV}$. ✓
- Intensity: $20/(4\pi(2)^2) = 20/50.27 \approx 0.398\,\text{W/m}^2$. ✓
- Disk area: $\pi(1.5 \times 10^{-10})^2 = 7.07 \times 10^{-20}\,\text{m}^2$. ✓
- Disk power: $0.398 \cdot 7.07 \times 10^{-20} = 2.81 \times 10^{-20}\,\text{W}$. ✓
- Time for $2\,\text{eV}$: $3.2 \times 10^{-19}/2.81 \times 10^{-20} = 11.4\,\text{s}$. ✓
- Photons in 11.4 s: $11.4 \cdot 2.81 \times 10^{-20}/3.98 \times 10^{-19} = 0.81$. ✓

The classical paradox in one sentence. Classically, a single atom (cross-section $\sim 10^{-19}\,\text{m}^2$) intercepts so little of the spreading wavefront from a $20\,\text{W}$ bulb 2 m away that it would take ~ 11 seconds to accumulate enough energy to free even a $2\,\text{eV}$ electron. Yet experiment shows photoemission appearing within 10^{-9} seconds of switching on the light. The factor of 10^{10} mismatch is the death of the wave-theory of photoemission.

How the photon model resolves it. The discrete-photon picture forbids "accumulation": each photon either is absorbed in toto (delivering $2.48\,\text{eV}$ in one shot, instantaneously) or not at all. Once a single photon hits the right atom, the electron is freed in essentially zero time. The probability per second is small, but the moment it succeeds, emission is immediate. So the *first* photoelectron typically appears with a delay that depends on photon flux but never on intensity (per electron).

Statistical correction with bigger surface. If we ask about *any* atom on $1\,\text{cm}^2 = 10^{-4}\,\text{m}^2$ (the hint), photon flux is $0.398 \cdot 10^{-4}/3.98 \times 10^{-19} = 10^{14}$ photons/s. Even if only 10^{-5} of them produce a photoelectron (quantum efficiency), we still get $\sim 10^9$ photoelectrons/s — macroscopic, instantaneous photocurrent.

Concept linkage. The argument here is the historical motivation for the photon: Einstein, 1905. The same logic resurfaces in single-photon detectors, quantum optics, and even in modern attosecond physics where photoemission delays are now measurable down to $\sim 10^{-18}\,\text{s}$ — still effectively instantaneous on macroscopic time scales.

Why this matters. The classical-quantum mismatch made photoemission the canonical evidence for quantisation of light energy. It is also the most directly testable consequence: every single photoemission experiment ever performed has confirmed the photon picture.

Final Answer: Quantum picture: one photon, one electron, instantaneous. The 11-second classical wait is replaced by $\leq$ ns response in actual experiments.

Key Takeaways

- **Photon energy and momentum:** $E = h\nu = hc/\lambda$ and $p = h/\lambda$. Useful shortcut: $hc = 1240\,\text{eV nm}$.
- **Einstein photoelectric equation:** $K_{\text{max}} = h\nu - \phi_0 = eV_{\text{stop}}$. Slope of V_{stop} vs ν is h/e (universal); the threshold frequency $\nu_0 = \phi_0/h$ is material-specific.
- **de Broglie wavelength:** $\lambda = h/p$. For non-relativistic particle, $\lambda = h/\sqrt{2mK} = h/\sqrt{2mqV}$ for charges accelerated through V.
- **Same de Broglie wavelength** $\Rightarrow$ **same momentum** (not same energy).
- **Magnetic forces do not change speed** (or wavelength); electric forces do.
- **Bragg's law:** $2d\sin\theta = n\lambda$ links neutron/electron wavelengths to crystal spacing.
- **Heisenberg uncertainty:** $\Delta x\,\Delta p \gtrsim h/(2\pi)$ gives the energy scale of any confinement.
- **Photoemission is instantaneous**: discrete photon absorption, not slow classical energy accumulation.

End of NCERT Exemplar Problems