Chapter 11

Dual Nature of Radiation & Matter

Light behaves both as a wave (interference, diffraction) and as a stream of particles called ~~quanta~~ photons ($E = h\nu$).

Big question :

If light has particle nature, can matter (electron, proton...) have wave nature ?
de Broglie's bold answer : YES.

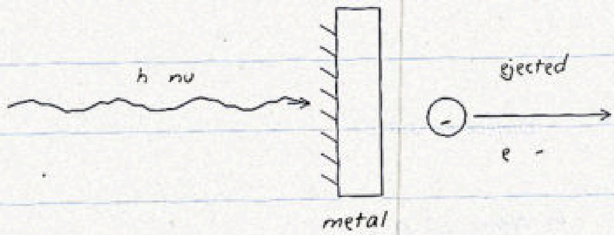


Fig. photon strikes metal -> electron is ejected

Topics ahead

1. Electron emission ; work function
2. Photoelectric effect ; Einstein eqn.
3. Matter waves ; Davisson - Germer.

Electron Emission

Metals contain free electrons . But at room temperature they cannot escape the surface . A potential barrier holds them in.

Work function W

Minimum energy needed by a free electron to just escape the metal surface .

$$\boxed{W = h \nu_0}$$

<- threshold
<- frequency

Unit : joule (J) or electron - volt (eV)

$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Typical W values

Caesium (Cs) : 2.14 eV

Potassium : 2.30 eV

Sodium : 2.75 eV

Zinc : 3.60 eV

Copper : 4.65 eV

Platinum : 5.65 eV

Smaller W -> easier to eject electrons ; Cs , K , Na are used in photo - cells.

(low W matches visible light frequencies.)

Types of Electron Emission

Energy to overcome W can be supplied in four physical ways :

(1) Thermionic emission

Heat the metal . Free electrons gain thermal energy and escape .

Used in : CRT cathode , vacuum ~~diode~~ values.

(2) Photoelectric emission

Light of suitable frequency falls on metal . Photons transfer energy to electrons .

Used in : photocells , solar cells.

(3) Field (cold) emission

A very strong external electric field (10 8 V/m) pulls electrons out of the surface (no heating needed).

(4) Secondary emission

Fast incoming particles knock out further electrons from the metal surface .

Used in : photomultiplier tubes.

All four overcome the same barrier = W.

Photoelectric Effect

Emission of electrons from a metal surface when light of suitable frequency falls on it.

Historical background

Hertz (1887) : UV light on Zn knob made sparks jump more easily across a gap.

Hallwachs & Lenard (1886 - 1902) studied the effect systematically :

* Negative Zn plate lost charge in UV light.
* Positive Zn plate did NOT lose charge.
* So UV ejected negative particles (electrons).

Key features (observed)

1. Below a threshold frequency nu0 , no emission - whatever the intensity.
2. Above nu0 , emission is instantaneous (< 10 -9 s) .
3. Photocurrent is proportional to intensity.
4. Max KE of photoelectrons depends only on nu , NOT on intensity.

Classical wave theory cannot explain these.

Experimental Setup

Evacuated glass tube with a quartz window
(UV passes through quartz, not ordinary glass).
Two electrodes : emitter C , collector A.

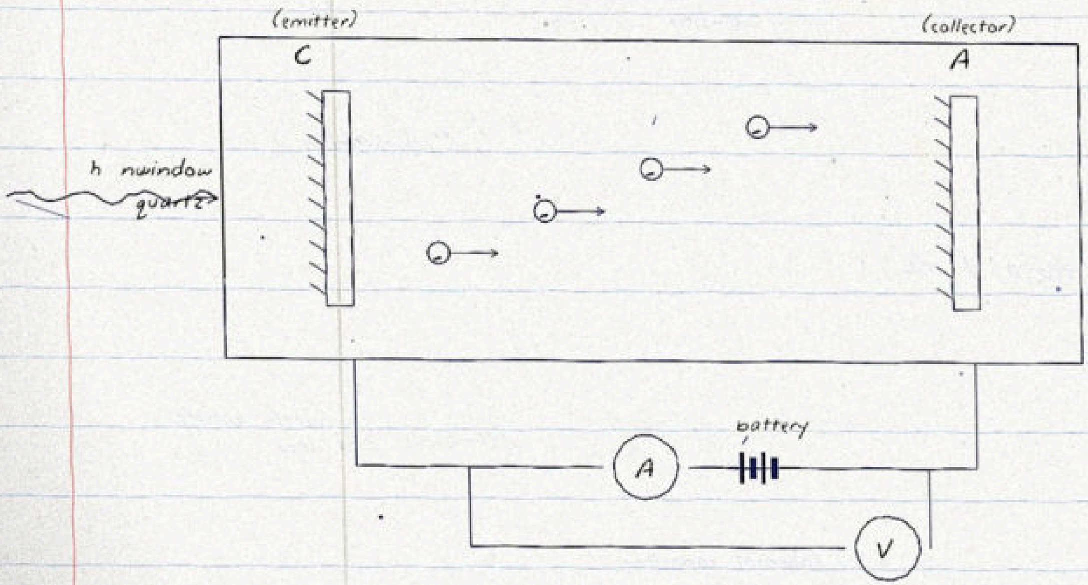


Fig. photoelectric experiment apparatus

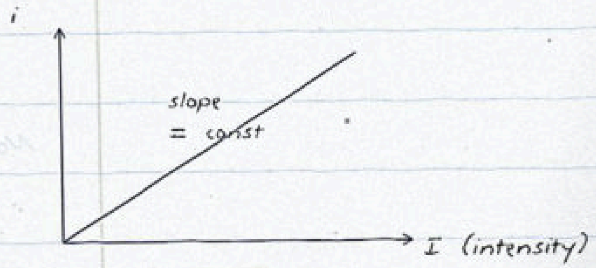
Ammeter A reads photocurrent i .
Voltmeter V reads p.d. between C and A.
Polarity can be reversed by a commutator.

Effect of Intensity on Current

Keep frequency nu and collector p.d. fixed.
Vary intensity I of incident light.

Observation :
Photocurrent i is directly proportional to the intensity of incident light.

$\boxed{i \text{ proportional to } I}$ <- linear



Why ? (photon view)

Higher intensity = more photons per second.
More photons -> more electrons ejected per s.
But each photon still carries energy E = h nu.

Hence intensity controls NUMBER, not energy of emitted photoelectrons.

Effect of Potential : V - I Curve

Keep nu and I (intensity) fixed. Vary the potential of collector A w.r.t. emitter C.

Positive V : i rises, reaches saturation.
Negative V : i falls ; at V = -VO i = 0.
VO is called the stopping potential.

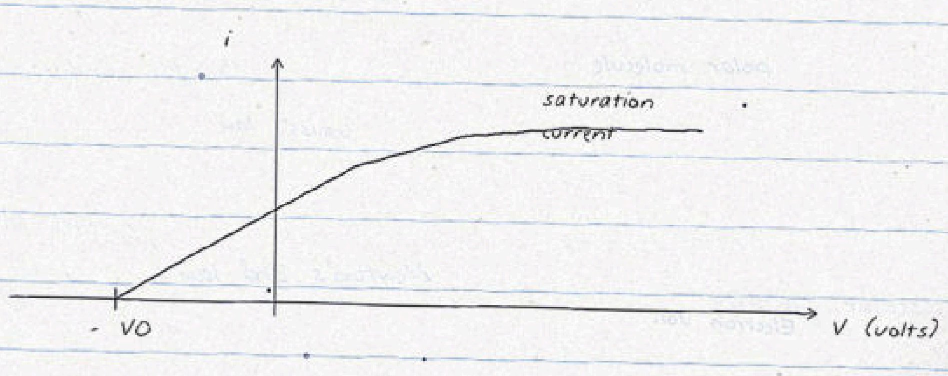


Fig. V - I characteristic of photocell

Saturation i -> all emitted electrons reach A.
At V = -VO even the fastest electron is stopped just before reaching A.

Stopping Potential VO

Definition : the negative collector voltage that just stops the fastest photoelectron .

Energy balance :

$$\boxed{e V_0 = (1/2) m v^2_{max} = KE_{max}}$$

<- definition
<- of VO

Three key results

① VO is independent of intensity.

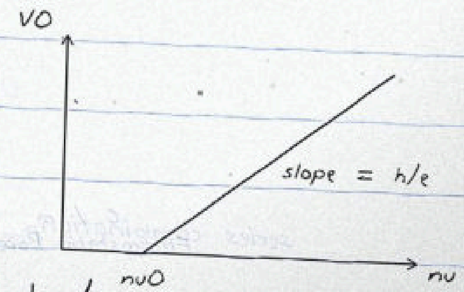
More photons , but same KE per electron.

② VO increases linearly with nu .

$$V_0 = (h/e) \nu - W/e$$

③ VO is zero at nu = nu0 .

Below nu0 no emission at all .



Slope of VO vs nu line = h / e .

Failure of Wave Theory

Classical wave theory predicts that energy of a wave is shared by the whole wavefront and depends only on intensity (amplitude 2).

Contradiction (1)

Wave view : enough intensity should always eject electrons, regardless of frequency.

Actual obs. : no emission below ~~intensity~~ nu0.

Contradiction (2)

Wave view : weak light should take time to build up energy ; emission delayed by seconds.

Actual obs. : emission is instantaneous (10 -9 s).

Contradiction (3)

Wave view : KE of electron should grow with intensity.

Actual obs. : KE_max depends only on nu .

Conclusion

Light energy comes in discrete packets , each interacting with ONE electron at a time .

Einstein (1905) : introduce photons .

Einstein's Photoelectric Equation

Light of frequency nu is made of photons. each of energy $E = h \nu$. One photon transfers ALL its energy to a single electron.

Energy balance on the surface :

$$h \nu = W + (1/2) m v^2_{max}$$

$$\boxed{h \nu = W + KE_{max}}$$

<- Einstein
<- 1905

Substituting KE_max = e VO :

$$\boxed{h \nu = W + e V_0}$$

Special case : nu = nuO

Then KE_max = 0 , so :

$W = h \nu_0$

i.e. $\nu_0 = W / h$

Below nuO , KE_max would be negative - impossible , so no emission occurs .

This is the meaning of threshold frequency.

Stopping-Potential Formula

From $h\,nu = W + e\,VO$:

$$\boxed{V_0 = (h/e)\cdot \nu - W/e}$$

<- linear in nu

- Compare with $y = mx + c$:

  slope $m = h/e$

  intercept $c = -W/e$

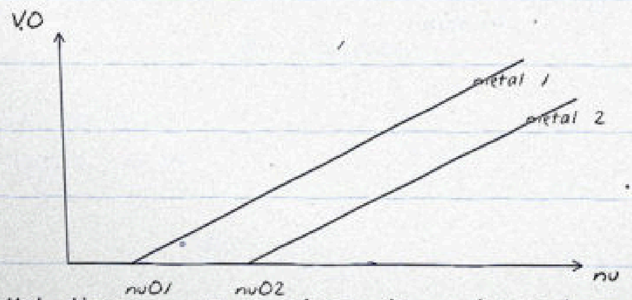
  x - intercept = $nu0 = W/h$

- **Millikan's Verification (1916)**

Plotted VO vs nu for several metals.

Lines were parallel, slope $= h/e$.

Gave $h = 6.57 \times 10^{-34}$ J s (good match).



Parallel lines -> slope is universal $= h/e$.

Photon - the Particle of Light

Photon = quantum of EM radiation.
Each photon carries definite energy and momentum, travels at speed of light c.

Energy of a photon

$$\boxed{E = h\nu = hc/\lambda}$$

<- frequency
<- or wavelength

Momentum of a photon

Photon is massless but carries momentum :

$$\boxed{p = E/c = h\nu/c = h/\lambda}$$

Properties of photon

* Rest mass = 0 (only relativistic mass).
* Electric charge = 0, unaffected by E, B.
* Energy of photon does not change with intensity - only the number of photons.
* Photons follow ~~Newton~~ Bose - Einstein statistics.
* Interaction : all - or - nothing with one e-.

Photon has wave character (nu, lambda) too.

Worked Example 1

Q. Light of wavelength 488 nm hits a Cs plate (W = 2.14 eV). Find KE_max of the photoelectron and stopping potential V0.

Sol :

Photon energy E = h c / lambda

$$E = (6.63 \times 10^{-34})(3 \times 10^{8}) / (488 \times 10^{-9})$$

$$= 4.08 \times 10^{-19}\ J$$

$$= 4.08 / 1.6\ eV = 2.55\ eV$$

$$KE_{max} = h\nu - W = E - W$$

$$= 2.55 - 2.14 = 0.41\ eV$$

$$\boxed{KE_{max} = 0.41\ eV}$$

Stopping potential V0 = KE_max / e

$$V0 = 0.41\ eV / e = 0.41\ V.$$

$$\boxed{V_0 = 0.41\ V}$$

Worked Example 2

Q. Two photoelectric measurements on a metal :

nu1 = 6.0 x 10 14 Hz , VO_1 = 0.42 V

nu2 = 7.5 x 10 14 Hz , VO_2 = 1.04 V

Find h, then W and nu0.

Sol :

h nu = W + e VO for both readings .

Subtract : h (nu2 - nu1) = e (VO_2 - VO_1)

h = e (VO_2 - VO_1) / (nu2 - nu1)

= (1.6 x 10 -19)(1.04 - 0.42)

/ (7.5 - 6.0) x 10 14

= (1.6 x 0.62) / 1.5 x 10 -33

approx 6.6 x 10 -34 J s . (= h)

Now W = h nu1 - e VO_1

= (6.6 x 10 -34)(6.0 x 10 14)

- (1.6 x 10 -19)(0.42)

= 3.96 x 10 -19 - 0.67 x 10 -19

approx 3.29 x 10 -19 J = 2.06 eV.

$\boxed{W = 2.06 \text{ eV}, \quad \nu_0 = W/h = 5.0 \times 10}$ 14 Hz

Visible light (red - violet) can eject electrons.

Wave Nature of Matter

Photons (mass = 0) have momentum p = h / lambda.
L. de Broglie (1924) : why not extend this to all material particles ?

de Broglie hypothesis

A moving particle of momentum p has a wave associated with it, of wavelength :

$$\boxed{\lambda = h / p = h / (m v)}$$ <- matter <- wave

Implication : lambda is small for heavy objects (large p), large for light particles.

From energy

If KE = (1/2) m v 2 , then p = sqrt(2 m KE).

$$\boxed{\lambda = h / \mathrm{sqrt}(2\ m\ KE)}$$

- For a charged particle accelerated through potential V : KE = q V .

$$\boxed{\lambda = h / \mathrm{sqrt}(2\ m\ q\ V)}$$

de Broglie Wave Picture

Matter wave is a probability wave, not a mechanical wave. Amplitude squared -> probability of finding the particle.

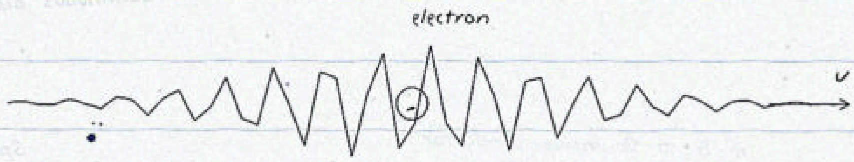


Fig. matter wave packet associated with a moving e-

Macro vs Micro

* Heavy objects (m large) -> lambda tiny, wave behaviour undetectable.
* Light particles (e-, p, n) -> lambda atomic spacing -> diffraction observed.

lambda for charged particles depends on charge and mass, not on the medium.

All particles have wave nature (de Broglie).

Electron Wavelength Formula

An electron is accelerated from rest through a potential difference V.

Energy gained = (1/2) m v 2 = e V

v = sqrt(2 e V / m)

p = m v = sqrt(2 m e V)

$\boxed{\lambda = h \,/\, \mathrm{sqrt}(2\ m\ e\ V)}$ <-exact

Putting numbers (h, m, e in SI) :

lambda = (6.63 x 10 -34) /

sqrt(2 x 9.1 x 10 -31 x 1.6 x 10 -19 V)

= 1.227 x 10 -9 / sqrt(V) m

= 12.27 / sqrt(V) angstrom

$\boxed{\lambda\ (\text{in A}) = 12.27 \,/\, \mathrm{sqrt}(V)}$ <-useful <-shortcut

Sample values

V = 100 V -> lambda = 1.23 A

V = 1000 V -> lambda = 0.39 A

V = 10 kV -> lambda = 0.12 A

(comparable to atomic spacings 1 A).

Worked Example 3

Q. Find de Broglie wavelength of :

(a) electron with $v = 5.4 \times 10^{6}$ m/s

(b) cricket ball $m = 0.15$ kg , $v = 20$ m/s

(c) proton $v = 1.0 \times 10^{5}$ m/s

Sol :

(a) $m_e = 9.11 \times 10^{-31}$ kg

$\lambda = h / m v$

$= (6.63 \times 10^{-34}) / (9.11 \times 10^{-31} \times 5.4 \times 10^{6})$

$\approx 1.35 \times 10^{-10}$ m $= 1.35$ Å

(b) $m v = 0.15 \times 20 = 3.0$ kg m/s

$\lambda = 6.63 \times 10^{-34} / 3.0$

$\approx 2.2 \times 10^{-34}$ m (TINY!)

(c) $m_p = 1.67 \times 10^{-27}$ kg

$\lambda = h / m_p v$

$= 6.63 \times 10^{-34} / (1.67 \times 10^{-27} \times 10^{5}$

$\approx 3.97 \times 10^{-12}$ m $= 0.04$ Å

Lesson : for the ball , $\lambda \ll$ atomic size $\rightarrow$ wave nature undetectable .

For electrons , wave nature is dominant.

Davisson - Germer Experiment

1927 - first direct experimental proof of electron waves, by Davisson and Germer.

Idea

If electrons have wave nature, a beam should show diffraction maxima off a crystal lattice, like X - rays do.

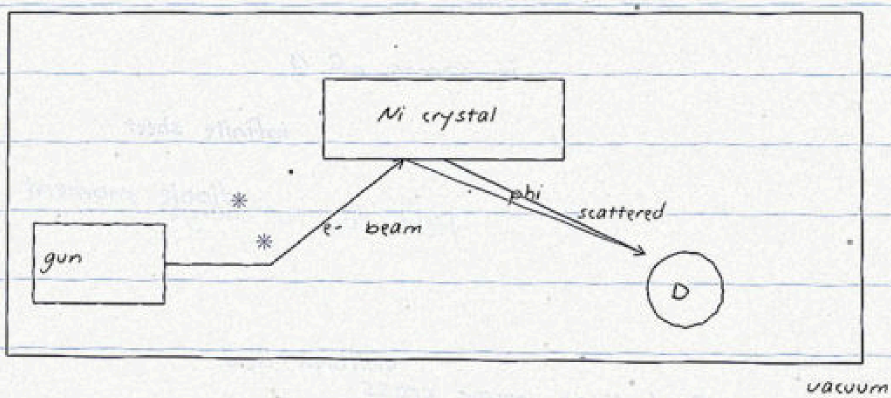


Fig. Davisson - Germer setup (top view)

Procedure

Vary V (accel.) and angle phi ∴ Measure intensity of scattered electrons at D.

Davisson - Germer Result

Sharp peak in scattered intensity at :

$V = 54 \, V$, phi = 50 degrees.

de Broglie prediction

At $V = 54 \, V$,

lambda = 12.27 / sqrt(54) A

approx 1.67 A.

Bragg prediction

For Ni crystal , d = 0.91 A , phi = 50 .

lambda = 2 d sin(phi / 2)

= 2 (0.91) sin 25

approx 1.65 A (matches?)

$\boxed{\text{lambda (de Broglie) = lambda (Bragg)}}$ <- wave <- nature ?

Significance

* Confirmed de Broglie hypothesis for e-.
* Foundation of ~~matter~~ wave mechanics .
* Led to development of the electron microscope (G. P. Thomson, 1928).
* Nobel Prize to Davisson & Thomson (1937).

Heisenberg's Uncertainty

Consequence of wave nature : one cannot fix both the position AND the momentum of a particle with arbitrary precision .

λ x . delta x . delta p >= h / (4 pi)

(Often written delta x . delta p >= hbar / 2)

Physical picture

A perfectly localized particle (delta x -> 0) needs a wave packet of infinitely many wavelengths -> momentum totally uncertain.

A pure plane wave (one lambda) has fixed p but is spread over all space (delta x -> infty).

Numerical feel

Electron in an atom : delta x 10 -10 m.

delta p >= h / (4 pi delta x)

approx 5 x 10 -25 kg m/s

delta v >= delta p / m approx 6 x 10 5 m/s.

Huge - classical orbits are not meaningful.

Photon Energy - Practical Cases

Compute E = h c / lambda for typical light.
(h c approx 1240 eV nm - handy shortcut)

$$\boxed{E\ (eV) = 1240\ /\ \lambda\ (\text{in nm})}$$ <- useful !

Worked numbers

Red light : lambda = 700 nm
E = 1240 / 700 approx 1.77 eV

Green : lambda = 550 nm
E = 1240 / 550 approx 2.25 eV

Violet : lambda = 400 nm
E = 1240 / 400 approx 3.10 eV

X - ray : lambda = 0.1 nm
E = 1240 / 0.1 = 12 400 eV = 12.4 keV

Notice : visible light just clears W for Cs, K, Na ; but cannot eject from Cu, Pt .

X - rays easily eject - even from heavy metals with KE in keV range.

Photo - cell Applications

Device based on photoelectric effect :

* Evacuated glass tube .
* Concave Cs - coated cathode catches light.
* Anode collects photoelectrons.
* Series with battery + microammeter.

Uses

* Automatic doors , burglar alarms.
* Sound track readers in old cinema film.
* Light meters in cameras.
* Counters in solar / lunar probes.
* Photo - diodes / photo - transistors.

Electron microscope

Uses matter waves of accelerated electrons.
Since lambda_e << lambda_visible , resolution is FAR better than an optical microscope.

At V = 60 kV , lambda approx 0.05 A .
Resolution few angstroms -> individual atoms can be imaged.

Also : electron diffraction for crystal study.

Chapter Summary - Formulas

Photoelectric effect

$E = h \nu = h c / \lambda$ (photon energy)

$p = h / \lambda$ (photon momentum)

$h \nu = W + KE_{max}$ (Einstein)

$e V_0 = KE_{max}$ (stopping pot.)

$W = h \nu_0$ (threshold)

$V_0 = (h/e) \nu - W/e$ (linear graph)

Matter waves

$\lambda = h / p = h / m v$

$\lambda = h / \text{sqrt}(2 \, m \, KE)$

$\lambda = h / \text{sqrt}(2 \, m \, q \, V)$

$\lambda \, (A) = 12.27 / \text{sqrt}(V)$ for e-

Constants to remember

$h = 6.626 \times 10^{-34}$ J s

$e = 1.602 \times 10^{-19}$ C

$m_e = 9.11 \times 10^{-31}$ kg

$c = 3 \times 10^{8}$ m / s

$h c = 1240$ eV. nm (shortcut)

1 eV $= 1.6 \times 10^{-19}$ J

Uncertainty : delta x . delta p >= h / (4 pi)

End of Chapter 11 - Dual Nature.