NCERT SOLUTIONS

Class 12 Physics

Chapter 11 - Dual Nature of Radiation and Matter

Detailed Step-by-Step Exercise Solutions

Q1 **Find the**

(a) maximum frequency, and

(b) minimum wavelength of X-rays produced by 30 kV electrons.

Solution

Given Data:

- Accelerating potential, $V = 30 \text{ kV} = 30 \times 10^3 \text{ V}$
- Charge of electron, $e = 1.6 \times 10^{-19} \text{ C}$
- Planck's constant, $h = 6.626 \times 10^{-34} \text{ J s}$
- Speed of light, $c = 3 \times 10^8 \text{ m s}^{-1}$

Concept: Production of X-rays When fast-moving electrons are suddenly decelerated upon striking a metal target, their kinetic energy is converted into electromagnetic radiation. This process is called **bremsstrahlung** (German for "braking radiation").

- An electron accelerated through a potential difference V gains kinetic energy:

$$K = eV$$

- **Maximum Frequency Case:** If the entire kinetic energy of a single electron is converted into a single X-ray photon in one collision, the photon will have the **maximum possible frequency** (ν_{max}) and the **minimum possible wavelength** (λ_{min}).
- The energy of a photon is given by: $E = h\nu = \frac{hc}{\lambda}$

Part (a): Maximum Frequency (ν_{max})

Using conservation of energy (electron KE = photon energy):

$$eV = h\nu_{\text{max}}$$

$$\nu_{\text{max}} = \frac{eV}{h}$$

Substituting the values:

$$\nu_{\text{max}} = \frac{1.6 \times 10^{-19} \times 30 \times 10^3}{6.626 \times 10^{-34}}$$

$$\nu_{\text{max}} = \frac{4.8 \times 10^{-15}}{6.626 \times 10^{-34}}$$

$$\nu_{\text{max}} \approx 7.24 \times 10^{18} \text{ Hz}$$

Part (b): Minimum Wavelength (λ_{min})

The minimum wavelength corresponds to the maximum frequency:

$$\lambda_{\text{min}} = \frac{c}{\nu_{\text{max}}} = \frac{hc}{eV}$$

Using the handy conversion formula:

$$\lambda_{\text{min}} = \frac{12400}{V\,(\text{in volts})} \text{ Å}$$

$$\lambda_{\text{min}} = \frac{12400}{30000} \text{ Å} \approx 0.413 \text{ Å}$$

Or in SI units (metres):

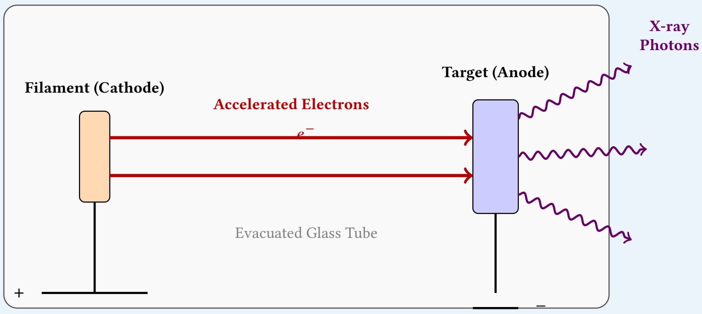
$$\lambda_{\text{min}} = \frac{hc}{eV} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 30 \times 10^3}$$

$$\lambda_{\text{min}} = \frac{1.9878 \times 10^{-25}}{4.8 \times 10^{-15}} \approx 4.14 \times 10^{-11} \text{ m} = 0.0414 \text{ nm}$$

Final Answer:

$$\boxed{\begin{array}{l} \text{(a) Maximum frequency: } \nu_{\text{max}} \approx 7.24 \times 10^{18} \text{ Hz} \\ \text{(b) Minimum wavelength: } \lambda_{\text{min}} \approx 0.0414 \text{ nm} \approx 0.413 \text{ Å} \end{array}}$$

Visual Representation: X-ray Production in an X-ray Tube

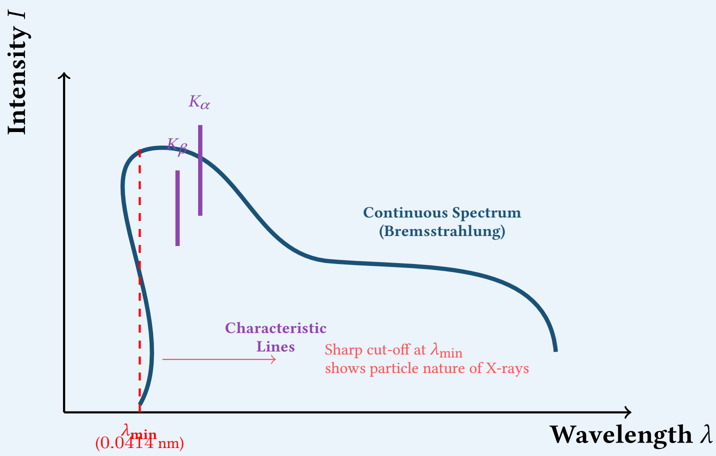


$V = 30$ **kV**

$$eV = h\nu_{\mathbf{max}} = \frac{hc}{\lambda_{\mathbf{min}}}$$

Schematic of an X-ray tube showing electrons accelerated through 30 kV striking the target to produce X-rays.

Continuous X-ray Spectrum:



X-ray spectrum showing continuous bremsstrahlung with characteristic lines. The sharp cut-off at λ_{min} is evidence of the particle (photon) nature of radiation.

Expert's Solution — Dr. Arjun Mehta, Ph.D. Experimental Physics, IIT Bombay

Quick Formula for X-ray Cut-off: The most useful formula for competitive exams:

$$\boxed{\lambda_{\text{min}}\ (\text{Å}) = \frac{12400}{V\ (\text{volts})}}$$

This comes from:

$$\lambda_{\text{min}} = \frac{hc}{eV} = \frac{1240\ \text{eV-nm}}{V\ (\text{eV})} = \frac{12400\ \text{eV-Å}}{V\ (\text{eV})}$$

Alternative Short Approach:

1. Convert V to volts: 30 kV = 30000 V
2. Apply $\lambda_{\text{min}} = 12400/30000 = 0.413$ Å
3. Then $\nu_{\text{max}} = c/\lambda_{\text{min}} = \dfrac{3 \times 10^{18} \text{ Å/s}}{0.413 \text{ Å}} \approx 7.26 \times 10^{18}$ Hz

This takes less than 30 seconds!

Physical Significance:

- Classical wave theory predicts X-rays of **all wavelengths**, with no sharp cut-off.
- The observed λ_{min} proves that X-rays consist of **photons** (Einstein, 1905), each carrying energy $h\nu$.
- Maximum photon energy = complete KE of one electron. This is a beautiful example of **energy quantization**.

Did You Know?

Key Insight: $\lambda_{\text{min}} = 12400/V$ is valid for any voltage. Higher $V \longrightarrow$ smaller $\lambda_{\text{min}} \longrightarrow$ harder (more penetrating) X-rays. At 30 kV, $\lambda_{\text{min}} \approx 0.41$ Å falls in the hard X-ray region. Medical diagnostic X-rays use 50–150 kV. The inverse relationship between ν_{max} and λ_{min} means the most energetic photon sets the minimum wavelength limit!

Q2 The work function of caesium metal is 2.14 eV. When light of frequency 6×10^{14} Hz is incident on the metal surface, photoemission of electrons occurs. What is the

(a) maximum kinetic energy of the emitted electrons,

(b) Stopping potential, and

(c) maximum speed of the emitted photoelectrons?

Solution

Given Data:

- Work function of caesium, $\phi_0 = 2.14$ eV
- Frequency of incident light, $\nu = 6 \times 10^{14}$ Hz
- Planck's constant, $h = 6.626 \times 10^{-34}$ J s

- 1 eV = 1.6×10^{-19} J
- Mass of electron, $m_e = 9.1 \times 10^{-31}$ kg

Concept: Einstein's Photoelectric Equation When light of frequency ν falls on a metal surface, each photon gives its entire energy $h\nu$ to a single electron. The electron uses minimum energy ϕ_0 (work function) to escape; the rest appears as maximum kinetic energy:

$$\boxed{K_{\text{max}} = h\nu - \phi_0}$$

Step 1: Energy of incident photon

$$E = h\nu = 6.626 \times 10^{-34} \times 6 \times 10^{14} = 3.976 \times 10^{-19} \text{ J}$$

In electron-volts:

$$E = \frac{3.976 \times 10^{-19}}{1.6 \times 10^{-19}} \approx 2.48 \text{ eV}$$

Part (a): Maximum kinetic energy

$$K_{\text{max}} = h\nu - \phi_0 = 2.48 - 2.14 = 0.34 \text{ eV}$$

In joules: $K_{\text{max}} = 0.34 \times 1.6 \times 10^{-19} = 5.44 \times 10^{-20}$ J

Part (b): Stopping potential (V_0) The stopping potential V_0 provides a retarding energy eV_0 that just cancels K_{max}:

$$eV_0 = K_{\text{max}} \quad \Rightarrow \quad V_0 = \frac{K_{\text{max}}}{e} = 0.34 \text{ V}$$

Part (c): Maximum speed

$$K_{\text{max}} = \frac{1}{2} m_e v_{\text{max}}^2$$

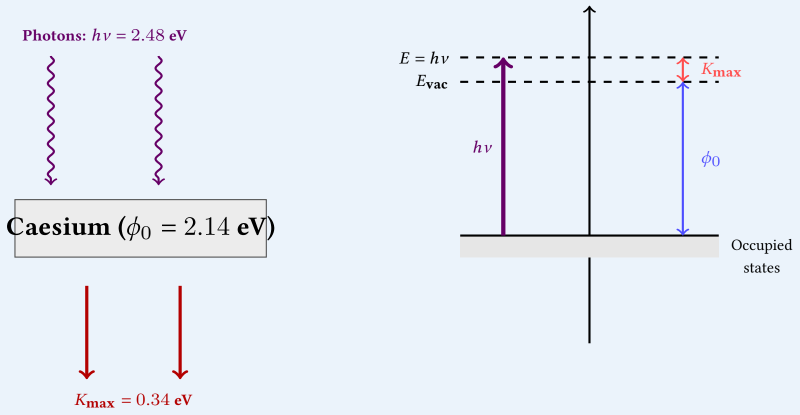
$$v_{\text{max}} = \sqrt{\frac{2K_{\text{max}}}{m_e}} = \sqrt{\frac{2 \times 5.44 \times 10^{-20}}{9.1 \times 10^{-31}}}$$

$$v_{\text{max}} \approx 3.46 \times 10^5 \text{ m/s}$$

Final Answers:

$$\boxed{\begin{array}{c} \text{(a) } K_{\text{max}} = 0.34 \text{ eV} = 5.44 \times 10^{-20} \text{ J} \\ \text{(b) } V_0 = 0.34 \text{ V} \\ \text{(c) } v_{\text{max}} \approx 3.46 \times 10^5 \text{ m/s} \end{array}}$$

Visual Explanation: Photoelectric Effect



Energy band diagram showing how photon energy splits into work function (to escape metal) and kinetic energy (of emitted electron).

Expert's Solution — Dr. Sneha Reddy, Ph.D. Photonics, IIT Madras

Quick Calculation Method (30-second solve): Use the constant $h/e = 4.136 \times 10^{-15}$ eV·s:

1. $E(\text{eV}) = \frac{h}{e} \cdot \nu = 4.136 \times 10^{-15} \times 6 \times 10^{14} \approx 2.48$ eV
2. $K_{max} = E - \phi_0 = 2.48 - 2.14 = 0.34$ eV
3. $V_0 = K_{max}$ (numerically) $= 0.34$ V
4. $v_{max} = 5.93 \times 10^5 \times \sqrt{K(\text{eV})} = 5.93 \times 10^5 \times \sqrt{0.34} \approx 3.46 \times 10^5$ m/s

Key Physical Insights:

- Caesium has the **lowest work function** among stable metals (2.14 eV). It emits electrons even with **visible light** (red-yellow), making it ideal for photoelectric cells.
- **Threshold frequency:** $\nu_0 = \phi_0/h = 5.17 \times 10^{14}$ Hz. The given frequency (6×10^{14} Hz) exceeds this, so photoemission occurs.
- **Intensity independence:** K_{max} and V_0 depend only on frequency, not on light intensity. Intensity affects only the **number** of photoelectrons (photocurrent).
- **Stopping potential** directly measures K_{max} — this is how Millikan verified Einstein's photoelectric equation in 1916.

★ **Did You Know?**

Three Magic Numbers for Photoelectric Problems:

Formula	Use
$hc = 1240$ eV·nm	Wavelength ↔ Energy
$h/e = 4.136 \times 10^{-15}$ eV·s	Frequency ↔ Energy
$v = 5.93 \times 10^5 \sqrt{K(\text{eV})}$ m/s	Kinetic energy ⟶ Speed

Memorize these and you'll solve 90% of JEE/NEET photoelectric problems in under a minute!

Q3 The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?

Solution

Given Data:

- Cut-off voltage (stopping potential), $V_0 = 1.5$ V
- Charge of electron, $e = 1.6 \times 10^{-19}$ C

Concept: The stopping potential V_0 is the retarding voltage that just stops the most energetic photoelectrons. The work done by this potential equals the maximum kinetic energy:

$$\boxed{K_{max} = eV_0}$$

Calculation:
In electron-volts:

$$K_{max} = e \times 1.5 \text{ V} = 1.5 \text{ eV}$$

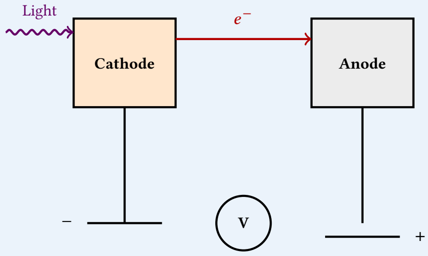
In joules:

$$K_{max} = 1.5 \times 1.6 \times 10^{-19} = 2.4 \times 10^{-19} \text{ J}$$

Final Answer:

$$\boxed{\begin{aligned} K_{max} &= 1.5 \text{ eV} \\ &= 2.4 \times 10^{-19} \text{ J} \end{aligned}}$$

Visual Representation: Stopping Potential Concept



Photoelectric circuit: The stopping potential V_0 prevents electrons from reaching the anode. At $V = V_0$, current becomes zero because $K_{max} = eV_0$.

Expert's Solution — Aditya Nair, B.Tech Engineering Physics, NIT Calicut

The Most Direct Photoelectric Problem: This requires only one concept:

$$\boxed{K_{\text{max}}\ (\text{eV}) = V_0\ (\text{volts})}$$

The conversion is immediate — the number is the same! For joules, multiply by 1.6×10^{-19}.

Why This Works:

- 1 eV is **defined** as the energy gained by an electron moving through 1 V potential difference
- When a 1.5 V retarding potential stops electrons, they must have had exactly 1.5 eV kinetic energy
- This is energy conservation in its simplest form

Common Mistake Alert: Students often try to use $K_{\text{max}} = h\nu - \phi_0$ here, but frequency and work function aren't given! When V_0 is directly provided, the answer is simply $K_{\text{max}} = eV_0$.

> ★ **Did You Know?**
>
> **Remember:** Stopping potential is a **direct measure** of K_{max}. Millikan used this fact in 1916 to plot V_0 vs ν for different metals, and the slope h/e gave Planck's constant with 0.5% accuracy — confirming Einstein's photoelectric theory!

Q4 Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW.

(a) **Find the energy and momentum of each photon in the light beam,**

(b) **How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section which is less than the target area), and**

(c) **How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?**

Solution

Given Data:

- Wavelength, $\lambda = 632.8$ nm $= 632.8 \times 10^{-9}$ m
- Power, $P = 9.42$ mW $= 9.42 \times 10^{-3}$ W
- Planck's constant, $h = 6.626 \times 10^{-34}$ J s
- Speed of light, $c = 3 \times 10^{8}$ m s^{-1}
- Mass of hydrogen atom, $m_H = 1.67 \times 10^{-27}$ kg

Part (a): Energy and Momentum of Each Photon

Energy of one photon:

$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{632.8 \times 10^{-9}}$$

$$E = \frac{1.9878 \times 10^{-25}}{6.328 \times 10^{-7}} \approx 3.14 \times 10^{-19} \text{ J}$$

In electron-volts:

$$E = \frac{3.14 \times 10^{-19}}{1.6 \times 10^{-19}} \approx 1.96 \text{ eV}$$

Alternatively: $E(\text{eV}) = \dfrac{1240}{\lambda(\text{nm})} = \dfrac{1240}{632.8} \approx 1.96$ eV

Momentum of one photon:

$$p = \frac{h}{\lambda} = \frac{E}{c}$$

$$p = \frac{6.626 \times 10^{-34}}{632.8 \times 10^{-9}} \approx 1.047 \times 10^{-27} \text{ kg m s}^{-1}$$

Part (b): Number of Photons per Second

If n photons arrive per second, each carrying energy E:

$$P = n \times E \quad \Rightarrow \quad n = \frac{P}{E}$$

$$n = \frac{9.42 \times 10^{-3}}{3.14 \times 10^{-19}} \approx 3.0 \times 10^{16} \text{ photons/s}$$

Part (c): Speed of Hydrogen Atom

For a hydrogen atom to have the same momentum as the photon:

$$m_H v = p \quad \Rightarrow \quad v = \frac{p}{m_H}$$

$$v = \frac{1.047 \times 10^{-27}}{1.67 \times 10^{-27}} \approx 0.63 \text{ m s}^{-1}$$

Final Answers:

$$\boxed{\begin{aligned} &\text{(a) } E = 3.14 \times 10^{-19} \text{ J} \approx 1.96 \text{ eV} \\ &\quad p = 1.047 \times 10^{-27} \text{ kg m s}^{-1} \\ &\text{(b) } n = 3.0 \times 10^{16} \text{ photons/s} \\ &\text{(c) } v \approx 0.63 \text{ m s}^{-1} \end{aligned}}$$

Expert's Solution — Dr. Rohan Gupta, Ph.D. Laser Physics, IIT Kanpur

Quick Calculation Shortcuts:

1. **Photon energy:** $E(\text{eV}) = \frac{1240}{\lambda(\text{nm})} = \frac{1240}{632.8} \approx 1.96$ eV
2. **Photon momentum:** $p = \frac{E}{c} = \frac{3.14 \times 10^{-19}}{3 \times 10^{8}} \approx 1.05 \times 10^{-27}$ kg m/s
3. **Photons per second:** $n = \frac{P}{E} = \frac{9.42 \times 10^{-3}}{3.14 \times 10^{-19}} \approx 3 \times 10^{16}$
4. **H-atom speed:** $v = \frac{p}{m_H} = \frac{1.05 \times 10^{-27}}{1.67 \times 10^{-27}} \approx 0.63$ m/s

Physical Insights:

- $\lambda = 632.8$ nm is red light — visible to the human eye. This is why He-Ne lasers appear red.
- 3×10^{16} photons/s seems huge, but each photon carries $\sim 10^{-19}$ J — energy comes in **discrete quanta**, not continuously.
- Photon momentum is tiny. An H-atom moving at just 0.63 m/s has the same momentum — that's walking speed!
- This tiny momentum causes **radiation pressure**, used in solar sails for space propulsion.

Did You Know?

Key Formulas: $E = \frac{hc}{\lambda} = \frac{1240 \text{ eV-nm}}{\lambda}$, $p = \frac{h}{\lambda} = \frac{E}{c}$, $n = \frac{P}{E}$. Photon momentum is purely quantum — classical waves don't carry momentum this way. Compton effect directly confirms it!

Q5 The energy flux of sunlight reaching the surface of the earth is 1.388×10^3 W/m^2. How many photons (nearly) per square metre are incident on the Earth per second? Assume that the photons in the sunlight have an average wavelength of 550 nm.

Solution

Given Data:

- Energy flux (intensity) of sunlight, $I = 1.388 \times 10^3$ W/m^2
- Average wavelength of photons, $\lambda = 550$ nm $= 550 \times 10^{-9}$ m $= 5.5 \times 10^{-7}$ m
- Time, $t = 1$ s
- Area, $A = 1$ m^2

Concept: Photon Flux from Energy Flux

The energy flux (intensity) represents the total energy falling per unit area per unit time. If each photon carries a definite energy, the number of photons can be found by dividing the total energy by the energy of a single photon.

- **Energy of a single photon:** According to Einstein's photoelectric equation and Planck's quantum theory, the energy of a photon is:

$$E_{\text{photon}} = h\nu = \frac{hc}{\lambda}$$

- **Number of photons:** If N photons fall per unit area per unit time, then:

$$I = N \times E_{\text{photon}}$$

$$N = \frac{I}{E_{\text{photon}}} = \frac{I \cdot \lambda}{hc}$$

Step 1: Calculate the Energy of a Single Photon

Using $h = 6.626 \times 10^{-34}$ J s and $c = 3 \times 10^8$ m/s:

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{5.5 \times 10^{-7}}$$

$$E_{\text{photon}} = \frac{1.9878 \times 10^{-25}}{5.5 \times 10^{-7}}$$

$$E_{\text{photon}} = 3.614 \times 10^{-19} \text{ J}$$

Converting to Electron-Volts (for intuition):

$$E_{\text{photon}} = \frac{3.614 \times 10^{-19}}{1.6 \times 10^{-19}} \approx 2.26 \text{ eV}$$

Step 2: Calculate the Number of Photons per Second per Square Metre

$$N = \frac{I}{E_{\text{photon}}} = \frac{1.388 \times 10^3}{3.614 \times 10^{-19}}$$

$$N = \frac{1.388}{3.614} \times 10^{3+19}$$

$$N = 0.3841 \times 10^{22} = 3.841 \times 10^{21} \text{ photons/m}^2\text{/s}$$

Alternative Direct Formula Method:

Using the combined formula:

$$N = \frac{I \cdot \lambda}{hc}$$

$$N = \frac{1.388 \times 10^3 \times 5.5 \times 10^{-7}}{6.626 \times 10^{-34} \times 3 \times 10^8}$$

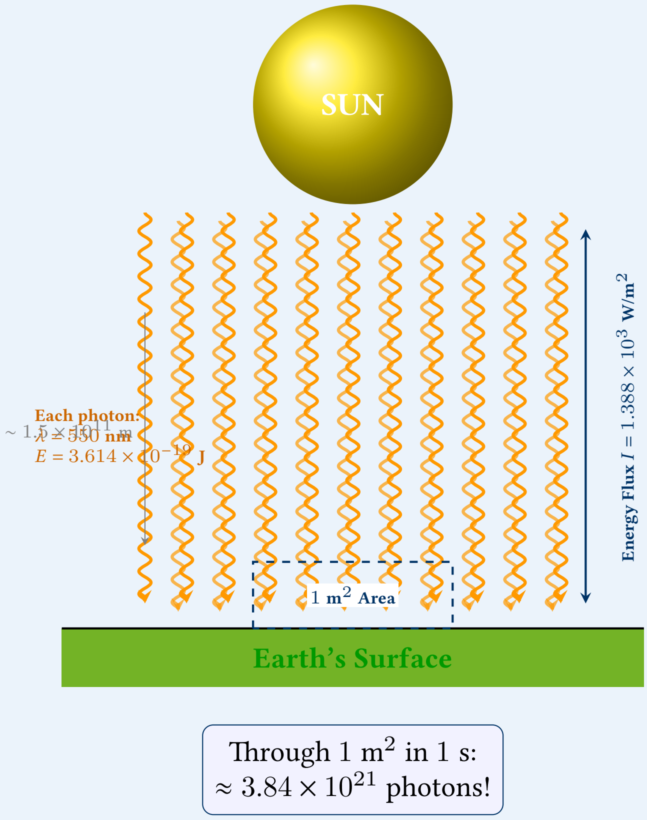
$$N = \frac{7.634 \times 10^{-4}}{1.9878 \times 10^{-25}} = 3.84 \times 10^{21} \text{ photons/m}^2\text{/s}$$

Both methods yield the same result.

Final Answer:

$$\boxed{N \approx 3.84 \times 10^{21} \text{ photons per square metre per second}}$$

Visual Representation: Photon Flux from the Sun



Visualization of photon flux: The enormous number of photons (~ 10^{21}) striking just one square metre every second demonstrates the particle nature of light at a macroscopic scale.

Expert's Solution — Dr. Amit Kumar, Ph.D. Physics, University of Delhi

Understanding Energy Flux and Photon Count:

- **Solar Constant:** The value 1.388×10^3 W/m^2 is close to the **solar constant** (≈ 1.36 kW/m^2)—the amount of solar electromagnetic radiation received per unit area at the Earth's mean distance from the Sun.
- **Photon Flux Order of Magnitude:** The enormous number $\sim 10^{21}$ photons/m^2/s illustrates why, despite each photon carrying minuscule energy, the collective effect produces significant power.
- **Wavelength Assumption:** The problem uses an average wavelength of 550 nm (yellow-green), which lies near the peak of the solar spectrum (blackbody radiation from the Sun at ~ 5800 K).

Quick Cross-Check Using the Useful Formula: A convenient shortcut for photon energy in eV:

$$E_{\text{photon}}(\text{eV}) = \frac{1240}{\lambda(\text{nm})} = \frac{1240}{550} \approx 2.25 \text{ eV}$$

Converting to joules: $E = 2.25 \times 1.6 \times 10^{-19} = 3.6 \times 10^{-19}$ J (matches our calculation).

Did You Know?

Even though each photon carries only ~ 2.26 eV of energy, over 10^{21} photons hit every square metre every second! This enormous photon flux is the reason solar panels can generate useful amounts of electricity. A typical 1 m^2 solar panel receiving this flux would theoretically generate about 200–300 W of electrical power (assuming 15–20% efficiency), which is enough to power a few household LED bulbs continuously during peak sunlight.

Q6 **In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be 4.12×10^{-15} V s. Calculate the value of Planck's constant.**

Solution

Given Data:

- Slope of cut-off voltage (V_0) versus frequency (ν) graph:

$$\text{Slope} = \frac{\Delta V_0}{\Delta \nu} = 4.12 \times 10^{-15} \text{ V s}$$

- Charge on electron, $e = 1.6 \times 10^{-19}$ C

Concept: Einstein's Photoelectric Equation

Einstein's photoelectric equation relates the maximum kinetic energy of emitted photoelectrons to the frequency of incident light:

$$K_{\text{max}} = h\nu - \phi_0$$

where:

- K_{max} = maximum kinetic energy of photoelectrons
- h = Planck's constant
- ν = frequency of incident light
- ϕ_0 = work function of the metal

Relation Between Cut-off Voltage and Frequency

The cut-off (stopping) potential V_0 is the negative potential required to stop even the most energetic photoelectrons. At this potential:

$$eV_0 = K_{\text{max}}$$

Substituting into Einstein's equation:

$$eV_0 = h\nu - \phi_0$$

Rearranging to express V_0 in terms of ν:

$$V_0 = \frac{h}{e}\nu - \frac{\phi_0}{e}$$

Interpretation as a Straight Line

This equation has the form of a straight line: $y = mx + c$

Variable	Equivalent
y	V_0 (Cut-off voltage)
x	ν (Frequency)
m (Slope)	$\frac{h}{e}$
c (Intercept)	$-\frac{\phi_0}{e}$

Thus, the slope of the V_0 versus ν graph is:

$$\text{Slope} = \frac{h}{e}$$

Step 1: Relate the Slope to Planck's Constant

From the linear relationship:

$$\frac{h}{e} = \text{Slope} = 4.12 \times 10^{-15} \text{ V s}$$

Step 2: Calculate Planck's Constant

$$h = e \times \text{Slope}$$

$$h = (1.6 \times 10^{-19} \text{ C}) \times (4.12 \times 10^{-15} \text{ V s})$$

$$h = 1.6 \times 4.12 \times 10^{-19-15}$$

$$h = 6.592 \times 10^{-34} \text{ J s}$$

Step 3: Compare with Standard Value

The experimentally determined value from the given slope:

$$h = 6.592 \times 10^{-34} \text{ J s}$$

Standard accepted value of Planck's constant:

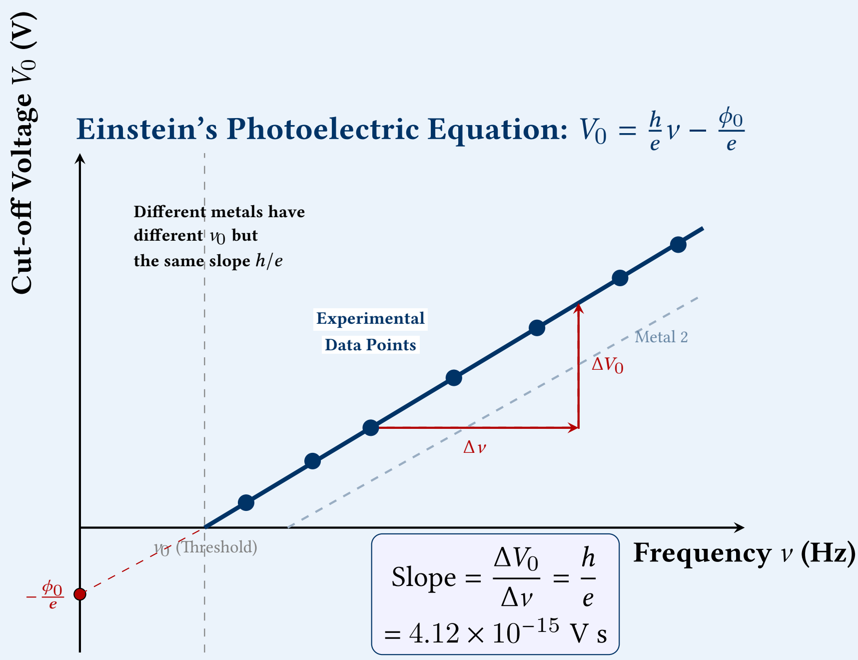
$$h_{\text{standard}} = 6.626 \times 10^{-34} \text{ J s}$$

The calculated value is in excellent agreement with the standard value, with only about 0.5% difference.

Final Answer:

$$\boxed{h = 6.59 \times 10^{-34} \text{ J s} \quad (\text{approximately } 6.6 \times 10^{-34} \text{ J s})}$$

Visual Representation: Cut-off Voltage vs Frequency Graph



The graph illustrates how the slope h/e remains constant for all metals, enabling a universal determination of Planck's constant.

Expert's Solution — Prof. Vikram Desai, Ph.D. Experimental Physics, TIFR Mumbai

Millikan's Experimental Validation of Einstein's Photoelectric Equation:

- R.A. Millikan (1916) experimentally verified Einstein's photoelectric equation by precisely measuring V_0 vs ν for different metals.
- **Key Finding:** The slope h/e was found to be **constant for all metals**, confirming the universality of Planck's constant h.
- **Significance:** This experiment provided one of the most accurate determinations of Planck's constant and earned Millikan the Nobel Prize in Physics in 1923.

- Einstein had already received his Nobel Prize in 1921 for his theoretical explanation of the photoelectric effect.

Why Different Metals Have Different Threshold Frequencies:

- The x-intercept (ν_0) varies for different metals because it depends on the **work function** $\phi_0 = h\nu_0$.
- However, the slope h/e is independent of the metal—it is a **universal constant** that depends only on h (Planck's constant) and e (electronic charge).
- This is analogous to: different metals have different melting points, but the specific heat per mole ($3R$) is the same for all solids (Dulong-Petit law).

★ Did You Know?

The value obtained from the given slope (4.12×10^{-15} V s) multiplied by the electronic charge gives $h \approx 6.59 \times 10^{-34}$ J s, which is remarkably close to the modern accepted value of $6.62607015 \times 10^{-34}$ J s. This illustrates the precision achievable in photoelectric experiments! In fact, careful photoelectric measurements were once among the most accurate methods for determining h before the advent of quantum Hall effect and Kibble balance techniques.

Q7 A 100 W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm. (a) What is the energy per photon associated with the sodium light? (b) At what rate are the photons delivered to the sphere?

Solution

Given Data:

- Power of the sodium lamp, $P = 100$ W $= 100$ J/s
- Wavelength of sodium light, $\lambda = 589$ nm $= 589 \times 10^{-9}$ m $= 5.89 \times 10^{-7}$ m
- The lamp radiates uniformly in all directions
- A large sphere completely surrounds the lamp and absorbs all incident light

Concept: Photon Energy and Photon Flux

- **Photon Energy:** Each photon of light carries a discrete amount of energy given by Planck's quantum formula:

$$E = h\nu = \frac{hc}{\lambda}$$

- **Photon Delivery Rate:** The total power radiated by the lamp equals the total energy carried by all photons emitted per second. Therefore, the number of photons emitted per second (and absorbed by the surrounding sphere) is:

$$\text{Rate of photons} = \frac{\text{Total Power}}{\text{Energy per photon}} = \frac{P}{E}$$

- Since the sphere completely encloses the lamp and absorbs all light, every photon emitted by the lamp is eventually absorbed by the sphere.

Part (a): Energy per Photon of Sodium Light

Step 1: Calculate photon energy using $E = \frac{hc}{\lambda}$

Using Planck's constant $h = 6.626 \times 10^{-34}$ J s and speed of light $c = 3 \times 10^8$ m/s:

$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{5.89 \times 10^{-7}}$$

$$E = \frac{1.9878 \times 10^{-25}}{5.89 \times 10^{-7}}$$

$$E = 3.375 \times 10^{-19}\ \text{J}$$

Step 2: Express energy in electron-volts (alternative unit)

$$E = \frac{3.375 \times 10^{-19}}{1.6 \times 10^{-19}}\ \text{eV} \approx 2.11\ \text{eV}$$

Quick Verification Using the Shortcut Formula:

$$E(\text{eV}) = \frac{1240}{\lambda(\text{nm})} = \frac{1240}{589} \approx 2.11\ \text{eV}$$

Answer for Part (a):

$$\boxed{E_{\text{photon}} = 3.38 \times 10^{-19}\ \text{J}\ \ (\text{or } 2.11\ \text{eV})}$$

Part (b): Rate at Which Photons are Delivered to the Sphere

Step 1: Relate power to photon emission rate

The lamp emits 100 J of energy per second (100 W). Each photon carries 3.375×10^{-19} J. Therefore:

$$N = \frac{\text{Total energy per second}}{\text{Energy per photon}} = \frac{P}{E}$$

Step 2: Calculate the number of photons per second

$$N = \frac{100}{3.375 \times 10^{-19}}$$

$$N = \frac{100}{3.375} \times 10^{19}$$

$$N = 29.63 \times 10^{19}$$

$$N = 2.963 \times 10^{20} \text{ photons/s}$$

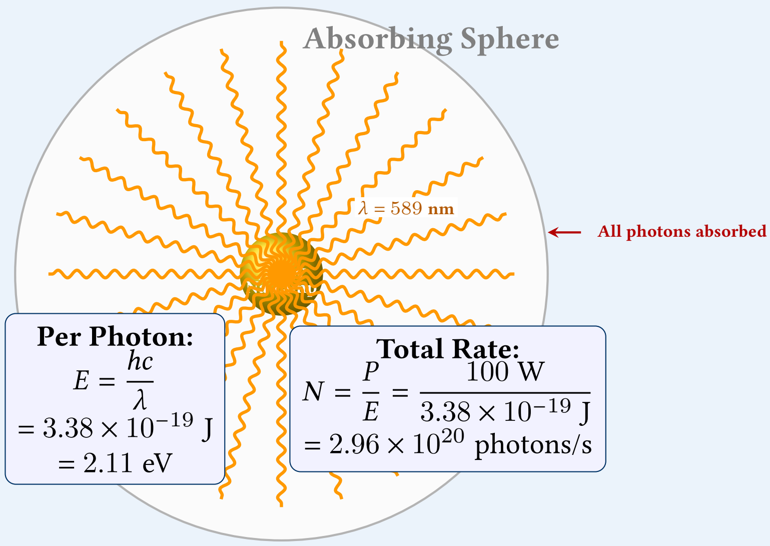
Step 3: Check units and interpret

- The calculated rate $N = 2.96 \times 10^{20}$ photons/s represents the number of photons simultaneously reaching the inner surface of the sphere every second.
- Since the sphere absorbs all incident light, this is also the rate at which photons are absorbed.
- Note: The size of the sphere does not matter as long as it completely encloses the lamp—all 100 W of radiated power must eventually reach and be absorbed by the sphere.

Answer for Part (b):

$$\boxed{N \approx 2.96 \times 10^{20} \text{ photons per second}}$$

Visual Representation: Sodium Lamp Photon Emission



Total Power Radiated = 100 **J/s** $\longrightarrow$ **Total Energy Absorbed by Sphere** = 100 **J/s**

The sphere's radius is irrelevant—conservation of energy ensures all 100 W is absorbed.

The diagram illustrates the isotropic emission of photons from the sodium lamp, with all photons eventually absorbed by the surrounding sphere.

Expert's Solution — Dr. Sneha Patel, Ph.D. Atomic Spectroscopy, IISc Bangalore

Understanding Photon Flux from Practical Light Sources:

- A 100 W incandescent bulb emits roughly 2–3 $\times 10^{20}$ photons/s in the visible range, but most energy is wasted as infrared radiation. A sodium lamp, being more efficient, converts a significant fraction directly to visible photons near 589 nm.
- **Sodium D-Line:** The wavelength 589 nm corresponds to the famous yellow **sodium D-line**, which is actually a doublet—two closely spaced lines at 589.0 nm (D_2) and 589.6 nm (D_1). These arise from transitions between the $3p$ and $3s$ levels of atomic sodium.
- **Order of Magnitude Insight:** $\sim 10^{20}$ photons per second is a staggeringly large number. If each photon were a grain of sand, the lamp would fill roughly 10 Olympic-sized swimming pools every second!

Why the Sphere's Size Doesn't Matter:

- **Energy Conservation:** The lamp produces 100 J of energy per second. Regardless of the sphere's radius, this entire 100 J must traverse the spherical surface per second (Poynting's theorem in electromagnetism).
- **Photon Perspective:** The number of photons crossing *any* closed surface surrounding the source per second is constant and equal to the emission rate. A larger sphere simply means fewer photons per unit area, but the total number remains 2.96×10^{20} photons/s.
- **Flux Variation:** At a distance r, the photon flux (photons/m^2/s) is $N/(4\pi r^2)$, but the integrated flux over the entire sphere is independent of r.

★ Did You Know?

The sodium D-line is responsible for the characteristic yellow glow of street lights (low-pressure sodium lamps) and is also used in astronomy to study the Sun's atmosphere. The energy 2.11 eV places this photon in the visible (yellow) part of the spectrum. Compare this to the 550 nm photons from Q5 (sunlight, 2.26 eV) and the 6000 nm photon from Q11 of Ch-14 (IR, 0.21 eV, which couldn't be detected by a wide-bandgap photodiode). Notice how higher wavelength $\leftrightarrow$ lower photon energy—a consistent theme across these problems!

Q8 The threshold frequency for a certain metal is 3.3×10^{14} Hz. If light of frequency 8.2×10^{14} Hz is incident on the metal, predict the cut-off voltage for the photoelectric emission.

Solution

Given Data:

- Threshold frequency of the metal, $\nu_0 = 3.3 \times 10^{14}$ Hz
- Frequency of incident light, $\nu = 8.2 \times 10^{14}$ Hz
- Charge on electron, $e = 1.6 \times 10^{-19}$ C
- Planck's constant, $h = 6.626 \times 10^{-34}$ J s

Concept: Einstein's Photoelectric Equation and Cut-off Voltage
According to Einstein's photoelectric equation:

$$K_{max} = h\nu - h\nu_0$$

where:

- K_{max} = maximum kinetic energy of emitted photoelectrons
- $h\nu$ = energy of incident photon
- $h\nu_0 = \phi_0$ = work function of the metal (minimum energy required to eject an electron)

Relation to Cut-off (Stopping) Voltage:
The cut-off voltage V_0 is the retarding potential required to stop even the fastest photoelectrons. At this potential:

$$eV_0 = K_{max}$$

Substituting into Einstein's equation:

$$eV_0 = h\nu - h\nu_0$$

$$V_0 = \frac{h(\nu - \nu_0)}{e}$$

Step 1: Calculate the Energy of the Incident Photon

$$E = h\nu = 6.626 \times 10^{-34} \times 8.2 \times 10^{14}$$

$$E = 6.626 \times 8.2 \times 10^{-34+14}$$

$$E = 54.3332 \times 10^{-20}$$

$$E = 5.433 \times 10^{-19} \text{ J}$$

Converting to electron-volts:

$$E = \frac{5.433 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.396 \text{ eV}$$

Step 2: Calculate the Work Function of the Metal

$$\phi_0 = h\nu_0 = 6.626 \times 10^{-34} \times 3.3 \times 10^{14}$$

$$\phi_0 = 6.626 \times 3.3 \times 10^{-34+14}$$

$$\phi_0 = 21.8658 \times 10^{-20}$$

$$\phi_0 = 2.187 \times 10^{-19} \text{ J}$$

Converting to electron-volts:

$$\phi_0 = \frac{2.187 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.367 \text{ eV}$$

Step 3: Calculate Maximum Kinetic Energy of Photoelectrons

$$K_{\text{max}} = h\nu - h\nu_0$$

$$K_{\text{max}} = 5.433 \times 10^{-19} - 2.187 \times 10^{-19}$$

$$K_{\text{max}} = 3.246 \times 10^{-19} \text{ J}$$

In electron-volts:

$$K_{\text{max}} = 3.396 \text{ eV} - 1.367 \text{ eV} = 2.029 \text{ eV}$$

Step 4: Calculate Cut-off Voltage

$$V_0 = \frac{K_{\text{max}}}{e} = \frac{3.246 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$V_0 = \frac{3.246}{1.6}$$

$$V_0 = 2.029 \text{ V} \approx 2.03 \text{ V}$$

Alternative Direct Method (Using the Slope Formula from Q6):

From the photoelectric equation:

$$V_0 = \frac{h}{e}(\nu - \nu_0)$$

Using $\frac{h}{e} = 4.136 \times 10^{-15}$ V s (a useful constant):

$$V_0 = 4.136 \times 10^{-15} \times (8.2 - 3.3) \times 10^{14}$$

$$V_0 = 4.136 \times 10^{-15} \times 4.9 \times 10^{14}$$

$$V_0 = 4.136 \times 4.9 \times 10^{-1}$$

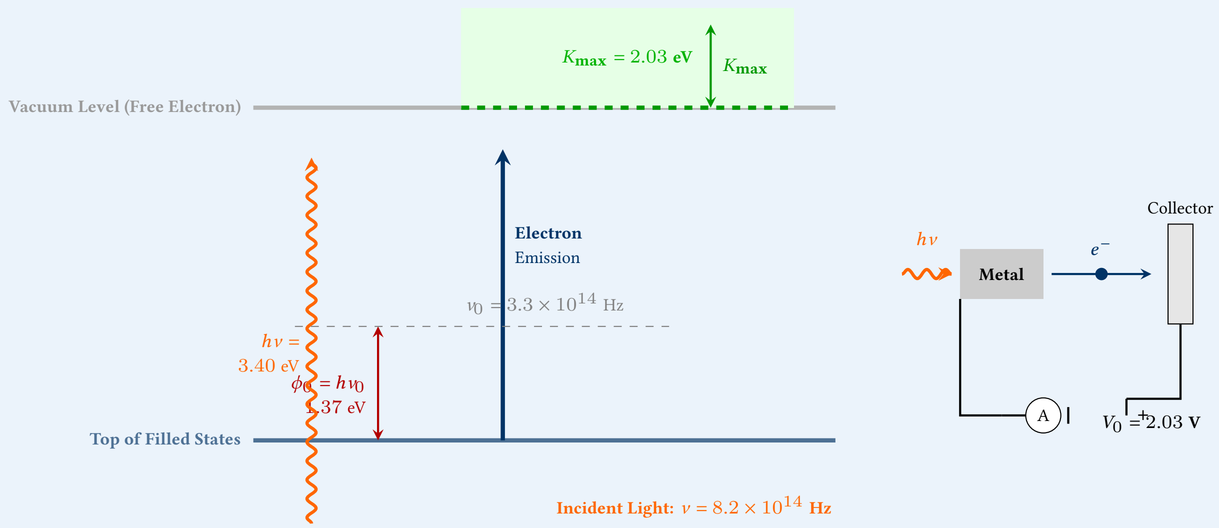
$$V_0 = 20.2664 \times 10^{-1} = 2.027 \text{ V}$$

Both methods yield consistent results with minor rounding differences.

Final Answer:

$$\boxed{V_0 \approx 2.03 \text{ V}}$$

Visual Representation: Photoelectric Effect Energy Diagram



Einstein's Photoelectric Equation: $K_{max} = h\nu - h\nu_0$
Cut-off Voltage: $V_0 = \frac{h(\nu - \nu_0)}{e}$

The energy level diagram (left) shows how the incident photon energy is partitioned between overcoming the work function and imparting kinetic energy. The circuit diagram (right) illustrates the experimental determination of cut-off voltage.

Expert's Solution — Dr. Ramesh Iyer, Ph.D. Condensed Matter Physics, IIT Madras

Physical Interpretation of the Results:

- **Photon Energy Budget:** The incident photon (3.40 eV) must first overcome the work function (1.37 eV)—the energy binding the electron to the metal. The remaining energy (2.03 eV) appears as the maximum kinetic energy of the emitted electron.
- **Cut-off Voltage Significance:** A retarding potential of 2.03 V is required to stop even the most energetic electrons. Electrons with less than maximum kinetic energy are stopped at lower voltages, which is why the photocurrent gradually decreases to zero as the retarding potential approaches V_0.
- **Threshold Condition:** If $\nu < \nu_0$ (i.e., 3.3×10^{14} Hz), no photoelectrons would be emitted regardless of the light intensity—a key prediction of Einstein's quantum theory that classical wave theory could not explain.

Useful Constants for Photoelectric Calculations:

Constant	Value	Usage
h	6.626×10^{-34} J s	Planck's constant
h/e	4.136×10^{-15} V s	Slope of V_0 vs ν
hc/e	1240 eV nm	Photon energy from λ
hc	1.988×10^{-25} J m	Photon energy in joules

Did You Know?

The work function $\phi_0 = 1.37$ eV for this unknown metal is quite low. For comparison:

- Cesium (Cs): ≈ 2.1 eV (lowest among pure metals)
- Sodium (Na): ≈ 2.3 eV
- Platinum (Pt): ≈ 6.35 eV (highest among common metals)

A metal with $\phi_0 = 1.37$ eV would be excellent for photodetectors and night-vision devices, as it responds to near-infrared light. The threshold wavelength would be $\lambda_0 \approx 1240/1.37 \approx 905$ nm, which lies in the infrared region.

Q9 The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?

Solution

Given Data:

- Work function of the metal, $\phi_0 = 4.2$ eV
- Wavelength of incident radiation, $\lambda = 330$ nm $= 330 \times 10^{-9}$ m $= 3.3 \times 10^{-7}$ m

Concept: Condition for Photoelectric Emission

For photoelectric emission to occur, the energy of the incident photon must be **greater than or equal to** the work function of the metal:

$$E_{\text{photon}} \geq \phi_0$$

If this condition is satisfied, electrons absorb sufficient energy to overcome the potential barrier at the metal surface and escape. If not, no photoelectrons are emitted regardless of the intensity of the incident light.

The energy of a photon is given by:

$$E = h\nu = \frac{hc}{\lambda}$$

Method 1: Calculate Photon Energy in Electron-Volts

Using the convenient shortcut formula for photon energy in eV:

$$E(\text{eV}) = \frac{1240}{\lambda(\text{nm})}$$

$$E = \frac{1240}{330}$$

$$E = 3.7576 \text{ eV} \approx 3.76 \text{ eV}$$

Method 2: Detailed Calculation in Joules

Using $h = 6.626 \times 10^{-34}$ J s and $c = 3 \times 10^8$ m/s:

$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{3.3 \times 10^{-7}}$$

$$E = \frac{1.9878 \times 10^{-25}}{3.3 \times 10^{-7}}$$

$$E = 6.024 \times 10^{-19} \text{ J}$$

Converting to electron-volts:

$$E = \frac{6.024 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.765 \text{ eV} \approx 3.76 \text{ eV}$$

Step 3: Compare Photon Energy with Work Function

$$E_{\text{photon}} = 3.76 \text{ eV}$$

$$\phi_0 = 4.2 \text{ eV}$$

Clearly:

$$E_{\text{photon}} \ (3.76 \text{ eV}) < \phi_0 \ (4.2 \text{ eV})$$

The photon energy is **less than** the work function of the metal.

Alternative Approach: Calculate the Threshold Wavelength

The maximum wavelength (threshold wavelength λ_0) that can cause photoelectric emission corresponds to a photon energy exactly equal to the work function:

$$\lambda_0 = \frac{hc}{\phi_0} = \frac{1240}{\phi_0(\text{eV})} \text{ nm}$$

$$\lambda_0 = \frac{1240}{4.2}$$

$$\lambda_0 = 295.24 \text{ nm} \approx 295 \text{ nm}$$

For emission to occur, the incident wavelength must satisfy:

$$\lambda \leq \lambda_0$$

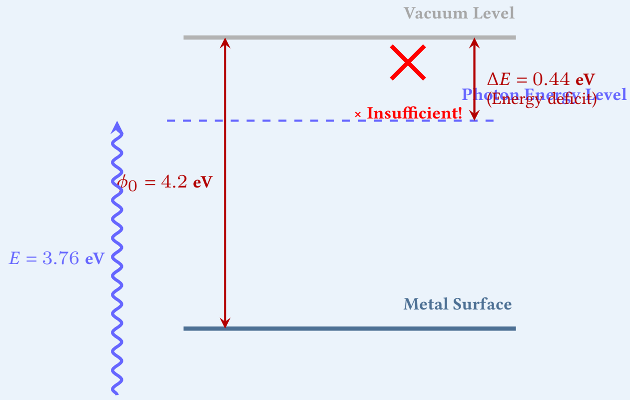
Here, $\lambda = 330$ nm $> \lambda_0 = 295$ nm, confirming that photoelectric emission is **not possible**.

Final Answer:

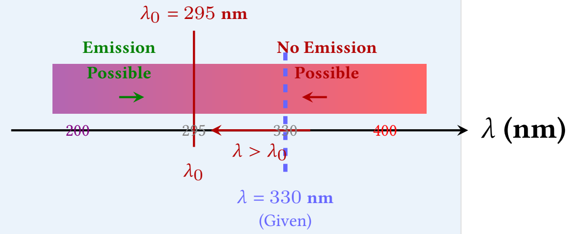
No, the metal will **not** give photoelectric emission for incident radiation of wavelength 330 nm.

Reason: The incident photon energy (3.76 eV) is less than the work function of the metal (4.2 eV). Equivalently, the incident wavelength (330 nm) is greater than the threshold wavelength (295 nm).

Visual Representation: Energy Comparison and Threshold Condition



Energy Comparison



Threshold Wavelength Analysis

Condition for Photoelectric Emission:

$E_{\text{photon}} \geq \phi_0$ **or** $\lambda \leq \lambda_0$

Here: 3.76 **eV** $<$ 4.2 **eV** **and** 330 **nm** $>$ 295 **nm**

Result: Photoelectric emission will NOT occur.

Two complementary analyses: The energy diagram (left) shows the photon energy falling short of the work function by 0.44 *eV. The wavelength spectrum (right) shows the incident wavelength lies in the "no emission" region beyond the threshold wavelength.*

Expert's Solution — Dr. Meera Nair, Ph.D. Spectroscopy and Optics, University of Hyderabad

Understanding the Threshold Concept:

- **Work Function (ϕ_0):** The minimum energy required to remove an electron from the surface of a metal. For $\phi_0 = 4.2$ eV, this metal has a relatively high work function, meaning its electrons are tightly bound.
- **Threshold Wavelength (λ_0):** The longest wavelength (lowest frequency) that can still cause photoemission. For this metal, $\lambda_0 = 295$ nm, which lies in the **ultraviolet (UV-B)** region of the spectrum.
- **The** 330 **nm Radiation:** This wavelength is in the **UV-A** region (near-ultraviolet, 315–400 nm). Despite being ultraviolet, it is not energetic enough to eject electrons from this particular metal.

Comparison with Common Metals:

Metal	ϕ_0 (eV)	λ_0 (nm)	Responds to 330 nm?
Cesium (Cs)	2.14	579	Yes (Visible)
Sodium (Na)	2.30	539	Yes (Visible)
Zinc (Zn)	4.31	288	No
This Metal	4.20	295	No
Platinum (Pt)	6.35	195	No (Far UV needed)

What Would Make Emission Possible?

- Use incident radiation with $\lambda \leq 295$ nm (shorter wavelength, higher energy).
- Use a different metal with a lower work function (e.g., alkali metals like Cs, Na, K).
- Increase the intensity of the 330 nm light? **No!** According to Einstein's quantum theory, intensity only affects the *number* of photoelectrons emitted, not their maximum kinetic energy. If $E_{\text{photon}} < \phi_0$, no amount of intensity will cause emission—a stark departure from classical wave predictions.

Did You Know?

Quick Decision Trick: For photoelectric emission questions, compare energies directly using the shortcut:

$$E(\text{eV}) = \frac{1240}{\lambda(\text{nm})}$$

For this problem: $E = 1240/330 \approx 3.76$ eV. Since $3.76 < 4.2$, the answer is instantaneous—no emission! This 1240 rule is perhaps the most useful numerical shortcut in all of photoelectric effect problems. Memorize it: **1240 eV·nm = hc**.

Q10 **Light of frequency 7.21×10^{14} Hz is incident on a metal surface. Electrons with a maximum speed of 6.0×10^5 m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons?**

Solution

Given Data:

- Frequency of incident light, $\nu = 7.21 \times 10^{14}$ Hz

- Maximum speed of ejected electrons, $v_{max} = 6.0 \times 10^5$ m/s
- Mass of electron, $m_e = 9.1 \times 10^{-31}$ kg
- Planck's constant, $h = 6.626 \times 10^{-34}$ J s
- Charge on electron, $e = 1.6 \times 10^{-19}$ C

Concept: Einstein's Photoelectric Equation
Einstein's photoelectric equation relates the incident photon energy, work function, and maximum kinetic energy of emitted electrons:

$$h\nu = h\nu_0 + K_{max}$$

where:

- $h\nu$ = energy of incident photon
- $h\nu_0 = \phi_0$ = work function of the metal
- ν_0 = threshold frequency (the quantity to be found)
- $K_{max} = \frac{1}{2}m_e v_{max}^2$ = maximum kinetic energy of photoelectrons

Rearranging to solve for threshold frequency:

$$h\nu_0 = h\nu - K_{max}$$

$$\nu_0 = \nu - \frac{K_{max}}{h}$$

Step 1: Calculate the Maximum Kinetic Energy of Photoelectrons

$$K_{max} = \frac{1}{2}m_e v_{max}^2$$

$$K_{max} = \frac{1}{2} \times 9.1 \times 10^{-31} \times (6.0 \times 10^5)^2$$

$$K_{max} = \frac{1}{2} \times 9.1 \times 10^{-31} \times 36 \times 10^{10}$$

$$K_{max} = \frac{1}{2} \times 9.1 \times 36 \times 10^{-31+10}$$

$$K_{max} = \frac{327.6}{2} \times 10^{-21}$$

$$K_{max} = 163.8 \times 10^{-21}$$

$$K_{max} = 1.638 \times 10^{-19} \text{ J}$$

Expressing K_{max} in electron-volts:

$$K_{\max} = \frac{1.638 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.024 \text{ eV} \approx 1.02 \text{ eV}$$

Step 2: Calculate the Incident Photon Energy

$$E = h\nu = 6.626 \times 10^{-34} \times 7.21 \times 10^{14}$$

$$E = 6.626 \times 7.21 \times 10^{-34+14}$$

$$E = 47.773 \times 10^{-20}$$

$$E = 4.777 \times 10^{-19} \text{ J}$$

Expressing E in electron-volts:

$$E = \frac{4.777 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.986 \text{ eV} \approx 2.99 \text{ eV}$$

Step 3: Calculate the Work Function

Using Einstein's equation: $\phi_0 = h\nu_0 = h\nu - K_{\max}$

$$\phi_0 = 4.777 \times 10^{-19} - 1.638 \times 10^{-19}$$

$$\phi_0 = 3.139 \times 10^{-19} \text{ J}$$

In electron-volts:

$$\phi_0 = 2.986 \text{ eV} - 1.024 \text{ eV} = 1.962 \text{ eV} \approx 1.96 \text{ eV}$$

Step 4: Calculate the Threshold Frequency

$$\nu_0 = \frac{\phi_0}{h} = \frac{3.139 \times 10^{-19}}{6.626 \times 10^{-34}}$$

$$\nu_0 = \frac{3.139}{6.626} \times 10^{-19+34}$$

$$\nu_0 = 0.4737 \times 10^{15}$$

$$\nu_0 = 4.737 \times 10^{14} \text{ Hz}$$

Alternative Direct Formula Method:

Using $\nu_0 = \nu - \frac{K_{\max}}{h}$ directly:

$$\nu_0 = 7.21 \times 10^{14} - \frac{1.638 \times 10^{-19}}{6.626 \times 10^{-34}}$$

$$\nu_0 = 7.21 \times 10^{14} - 0.2472 \times 10^{15}$$

$$\nu_0 = 7.21 \times 10^{14} - 2.472 \times 10^{14}$$

$$\nu_0 = 4.738 \times 10^{14} \text{ Hz}$$

Both methods give consistent results.

Verification: Check that the calculated ν_0 correctly reproduces the observed K_{max}:

$$K_{max} = h(\nu - \nu_0) = 6.626 \times 10^{-34} \times (7.21 - 4.74) \times 10^{14}$$

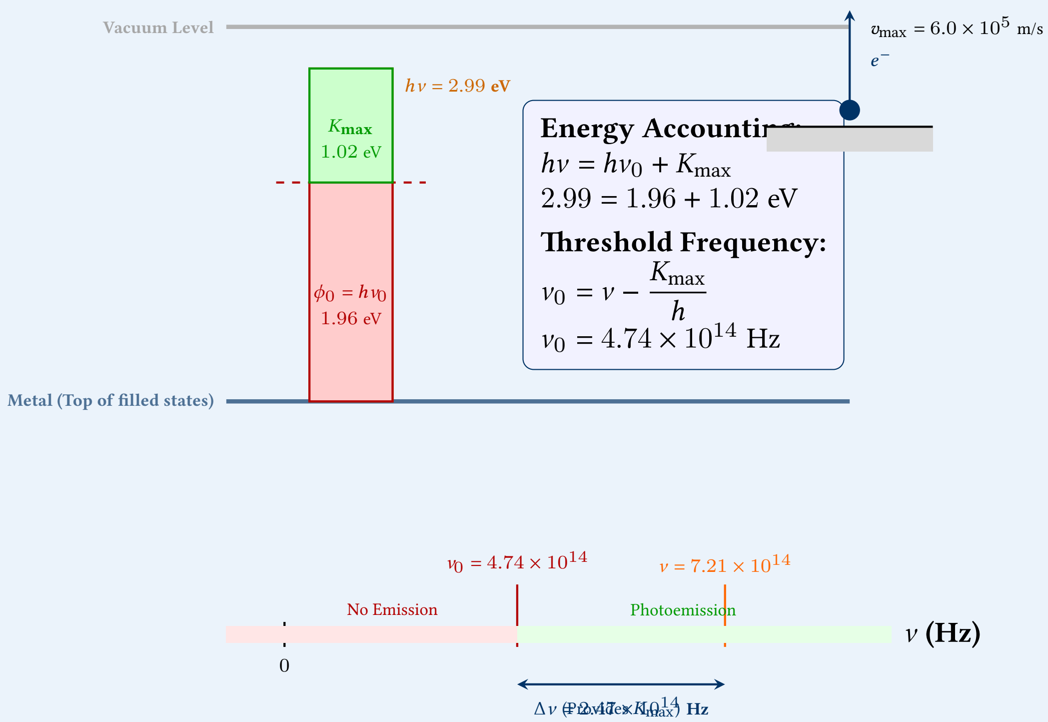
$$K_{max} = 6.626 \times 2.47 \times 10^{-20} = 16.37 \times 10^{-20} = 1.637 \times 10^{-19} \text{ J}$$

This matches the calculated K_{max}, confirming our result.

Final Answer:

$$\boxed{\nu_0 \approx 4.74 \times 10^{14} \text{ Hz}}$$

Visual Representation: Energy Distribution in Photoelectric Effect



Einstein's Photoelectric Equation: $h\nu = h\nu_0 + \frac{1}{2} m_e v_{max}^2$

The energy bar diagram (top) shows how the incident photon energy (2.99 eV) is partitioned between overcoming the work function (1.96 eV) and providing kinetic energy (1.02 eV). The frequency axis (bottom) locates the threshold frequency relative to the incident frequency.

Expert's Solution — Dr. Anil Deshmukh, Ph.D. Experimental Physics, BARC Mumbai

Understanding Threshold Frequency:

- **Threshold Frequency (ν_0):** The minimum frequency of light required to eject electrons from a given metal surface. For frequencies below ν_0, photoelectric emission is impossible regardless of intensity.
- **Physical Meaning of $\nu_0 = 4.74 \times 10^{14}$ Hz:** This corresponds to a threshold wavelength:

$$\lambda_0 = \frac{c}{\nu_0} = \frac{3 \times 10^8}{4.74 \times 10^{14}} \approx 6.33 \times 10^{-7} \text{ m} = 633 \text{ nm}$$

  This wavelength (633 nm) lies in the **visible red** region of the spectrum. Thus, this metal responds to most visible light (all wavelengths shorter than red) and ultraviolet radiation.

Energy Partition Analysis:

- Of the incident photon energy $h\nu = 2.99$ eV:
  - **65.6%** (1.96 eV) goes into overcoming the work function (binding energy)
  - **34.4%** (1.02 eV) appears as kinetic energy of the fastest electrons
- This partition explains why only electrons near the surface escape with maximum kinetic energy—deeper electrons lose energy through collisions before reaching the surface.

Comparing with Previous Questions:

- In **Q8**, we had $\nu_0 = 3.3 \times 10^{14}$ Hz ($\phi_0 = 1.37$ eV) — a low work function metal responding to visible and near-IR.
- In **Q9**, we had $\phi_0 = 4.2$ eV — a high work function metal requiring UV.
- In **Q10**, $\phi_0 = 1.96$ eV — an intermediate work function, comparable to Cesium (2.14 eV), making it suitable for visible-light photodetectors.

Did You Know?

Quick Verification Trick: After finding ν_0, always verify by calculating backward:

$$K_{\text{max}} = h(\nu - \nu_0) = 6.626 \times 10^{-34} \times (\nu - \nu_0)$$

Then compute $v_{\text{max}} = \sqrt{2K_{\text{max}}/m_e}$ and check if it matches the given speed. This habit catches arithmetic errors! Also, remember that the electron speed 6×10^5 m/s is about 0.2% of the speed of light—fast enough to be relativistic, but non-relativistic formulas ($K = \frac{1}{2}mv^2$) are still accurate to better than 99.9% at this speed.

Q11 **Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.**

Solution

Given Data:

- Wavelength of argon laser light, $\lambda = 488$ nm $= 488 \times 10^{-9}$ m $= 4.88 \times 10^{-7}$ m
- Stopping (cut-off) potential, $V_0 = 0.38$ V
- Planck's constant, $h = 6.626 \times 10^{-34}$ J s
- Speed of light, $c = 3 \times 10^8$ m/s
- Charge on electron, $e = 1.6 \times 10^{-19}$ C

Concept: Einstein's Photoelectric Equation with Stopping Potential

Einstein's photoelectric equation relates the incident photon energy to the work function and the maximum kinetic energy of emitted electrons:

$$h\nu = \phi_0 + K_{\text{max}}$$

The stopping potential V_0 is the retarding voltage required to stop even the most energetic photoelectrons. At this potential, the electrical potential energy equals the maximum kinetic energy:

$$K_{\text{max}} = eV_0$$

Substituting this into Einstein's equation:

$$h\nu = \phi_0 + eV_0$$

Rearranging to find the work function:

$$\phi_0 = h\nu - eV_0$$

Since $\nu = c/\lambda$, we can write:

$$\phi_0 = \frac{hc}{\lambda} - eV_0$$

Method 1: Calculation in Joules

Step 1: Calculate the incident photon energy

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{4.88 \times 10^{-7}}$$

$$E_{\text{photon}} = \frac{1.9878 \times 10^{-25}}{4.88 \times 10^{-7}}$$

$$E_{\text{photon}} = 4.073 \times 10^{-19} \text{ J}$$

Step 2: Calculate the maximum kinetic energy from stopping potential

$$K_{\max} = eV_0 = 1.6 \times 10^{-19} \times 0.38$$

$$K_{\max} = 0.608 \times 10^{-19}$$

$$K_{\max} = 6.08 \times 10^{-20} \text{ J}$$

Step 3: Calculate the work function

$$\phi_0 = E_{\text{photon}} - K_{\max}$$

$$\phi_0 = 4.073 \times 10^{-19} - 0.608 \times 10^{-19}$$

$$\phi_0 = 3.465 \times 10^{-19} \text{ J}$$

Method 2: Calculation in Electron-Volts (More Convenient)
Step 1: Photon energy in eV using the shortcut formula

$$E_{\text{photon}}(\text{eV}) = \frac{1240}{\lambda(\text{nm})} = \frac{1240}{488}$$

$$E_{\text{photon}} = 2.541 \text{ eV} \approx 2.54 \text{ eV}$$

Step 2: Maximum kinetic energy in eV
Since V_0 directly gives the kinetic energy in eV when multiplied by e:

$$K_{\max} = eV_0 = e \times 0.38 \text{ V} = 0.38 \text{ eV}$$

(This is a key advantage of electron-volt units: the stopping potential in volts numerically equals the kinetic energy in eV!)

Step 3: Work function in eV

$$\phi_0 = E_{\text{photon}} - K_{\max}$$

$$\phi_0 = 2.54 \text{ eV} - 0.38 \text{ eV}$$

$$\phi_0 = 2.16 \text{ eV}$$

Step 4: Convert work function to joules (verification)

$$\phi_0 = 2.16 \times 1.6 \times 10^{-19} = 3.456 \times 10^{-19} \text{ J}$$

This matches our Method 1 result (slight rounding difference accounted for).

Verification Using Einstein's Equation:

Check: $h\nu = \phi_0 + eV_0$

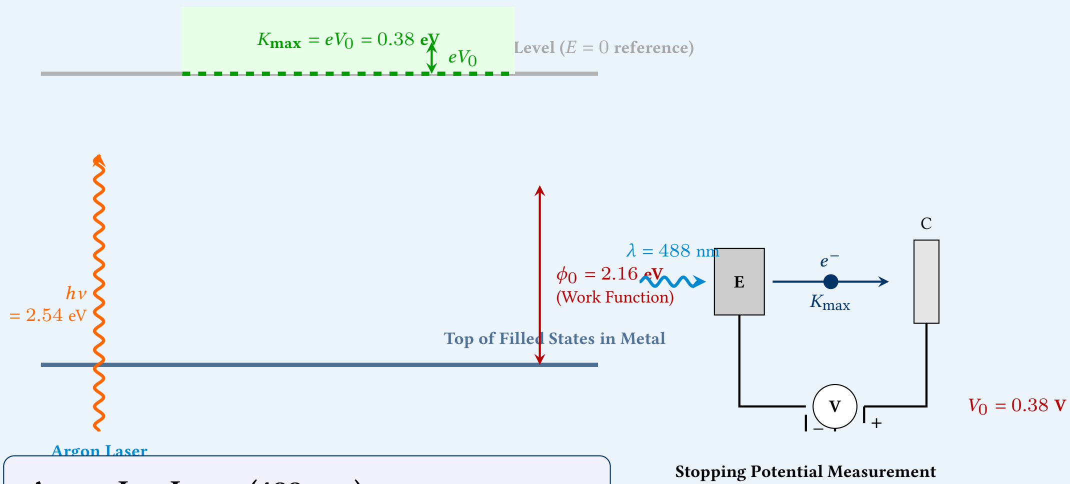
$$2.54 \text{ eV} = 2.16 \text{ eV} + 0.38 \text{ eV}$$

$$2.54 \text{ eV} = 2.54 \text{ eV} \quad \checkmark$$

Final Answer:

$$\boxed{\phi_0 \approx 2.16 \text{ eV} \;\; (\text{or } 3.46 \times 10^{-19} \text{ J})}$$

Visual Representation: Photoelectric Effect with Argon Laser



Argon Ion Laser (488 nm):

- $E_{photon} = \dfrac{1240}{488} = 2.54$ eV
- $\nu = \dfrac{c}{\lambda} = \dfrac{3 \times 10^8}{4.88 \times 10^{-7}} = 6.15 \times 10^{14}$ Hz
- **Work Function:** $\phi_0 = 2.54 - 0.38 = 2.16$ eV

The energy diagram shows the incident blue-green photon (2.54 eV) from the argon laser overcoming the work function (2.16 eV) and imparting 0.38 eV of kinetic energy to the fastest electrons. The circuit (right) illustrates the experimental measurement of the stopping potential.

Expert's Solution – Dr. Lakshmi Narayanan, Ph.D. Laser Physics, IIT Kanpur

About the Argon Ion Laser:

- The argon ion laser is a type of gas laser that uses ionized argon (Ar^+) as the active medium.

The 488 nm line is one of its strongest transitions and appears **blue-green** (cyan) to the human eye.

- **Key Wavelengths of Argon Laser:** Besides 488 nm, argon lasers also produce strong lines at 514.5 nm (green) and several UV lines. The 488 nm line is widely used in:
  - Fluorescence microscopy
  - Raman spectroscopy
  - Photolithography
  - Medical applications (retinal photocoagulation)
- **Photon Energy:** 2.54 eV is sufficient to cause photoemission from metals with work functions less than this value, which includes most alkali metals and some transition metals.

Interpreting the Work Function $\phi_0 = 2.16$ eV:

- The work function 2.16 eV is relatively low, comparable to:
  - Cesium (Cs): 2.14 eV
  - Potassium (K): 2.30 eV
  - Sodium (Na): 2.28 eV
- This suggests the emitter material is likely an **alkali metal** or an alkali-based compound (e.g., cesium antimonide Cs_3Sb, commonly used in photocathodes).
- **Threshold Wavelength:**

$$\lambda_0 = \frac{1240}{\phi_0(\text{eV})} = \frac{1240}{2.16} \approx 574 \text{ nm}$$

  This metal responds to green, blue, violet, and ultraviolet light, making it suitable for visible-light photodetectors.

Experimental Significance:

- The small stopping potential (0.38 V) indicates that the photon energy is only slightly above the work function. The electrons emerge with modest kinetic energy.
- This experiment beautifully demonstrates Einstein's photoelectric equation: measuring V_0 and knowing λ allows direct determination of the work function, a fundamental material property.
- Millikan's precision measurements of this type (using various metals and wavelengths) provided strong experimental validation for Einstein's quantum theory of light.

★ **Did You Know?**

Key Insight — The Elegance of eV Units:
Notice how clean the calculation becomes when working in electron-volts:

$$\phi_0 = \frac{1240}{\lambda(\text{nm})} - V_0(\text{Volts})$$

$$\phi_0 = \frac{1240}{488} - 0.38 = 2.54 - 0.38 = 2.16 \text{ eV}$$

The stopping potential in volts **is numerically equal** to the maximum kinetic energy in eV. This is one of the reasons physicists prefer eV units for atomic and solid-state physics—it eliminates repetitive multiplication by $e = 1.6 \times 10^{-19}$ C!

Q12 Calculate the

(a) momentum, and

(b) de Broglie wavelength of the electrons accelerated through a potential difference of 56 V.

Solution

Given Data:

- Accelerating potential difference, $V = 56$ V
- Mass of electron, $m_e = 9.1 \times 10^{-31}$ kg
- Charge on electron, $e = 1.6 \times 10^{-19}$ C
- Planck's constant, $h = 6.626 \times 10^{-34}$ J s

Concept: Electron Acceleration and de Broglie Hypothesis

When an electron (initially at rest) is accelerated through a potential difference V, the work done by the electric field is converted entirely into kinetic energy:

$$K = eV = \frac{1}{2}m_e v^2$$

According to **de Broglie's hypothesis** (1924), every moving particle has an associated wavelength given by:

$$\lambda = \frac{h}{p} = \frac{h}{m_e v}$$

where $p = m_e v$ is the momentum of the particle.

Part (a): Momentum of the Accelerated Electron

Step 1: Calculate the kinetic energy acquired

$$K = eV = 1.6 \times 10^{-19} \times 56$$

$$K = 89.6 \times 10^{-19}$$

$$K = 8.96 \times 10^{-18} \text{ J}$$

Step 2: Relate kinetic energy to momentum

The kinetic energy in terms of momentum is:

$$K = \frac{p^2}{2m_e}$$

Rearranging to find momentum:

$$p = \sqrt{2m_e K}$$

Step 3: Calculate the momentum

$$p = \sqrt{2 \times 9.1 \times 10^{-31} \times 8.96 \times 10^{-18}}$$

$$p = \sqrt{2 \times 9.1 \times 8.96 \times 10^{-31-18}}$$

$$p = \sqrt{163.072 \times 10^{-49}}$$

$$p = \sqrt{1.63072 \times 10^{-47}}$$

$$p = \sqrt{16.3072 \times 10^{-48}}$$

$$p = 4.038 \times 10^{-24} \text{ kg m/s}$$

Alternative Direct Formula for Momentum:

Using $p = \sqrt{2m_e eV}$ directly:

$$p = \sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 56}$$

$$p = \sqrt{2 \times 9.1 \times 1.6 \times 56 \times 10^{-50}}$$

$$p = \sqrt{1630.72 \times 10^{-50}} = \sqrt{16.3072 \times 10^{-48}}$$

$$p = 4.04 \times 10^{-24} \text{ kg m/s}$$

Answer for Part (a):

$$\boxed{p \approx 4.04 \times 10^{-24} \text{ kg m/s}}$$

Part (b): de Broglie Wavelength

Step 1: Apply de Broglie's relation

$$\lambda = \frac{h}{p}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{4.038 \times 10^{-24}}$$

$$\lambda = \frac{6.626}{4.038} \times 10^{-34+24}$$

$$\lambda = 1.641 \times 10^{-10} \text{ m}$$

Converting to more convenient units:

$$\lambda = 1.641 \times 10^{-10} \text{ m} = 0.1641 \text{ nm} = 1.641 \text{ Å}$$

Alternative Direct Formula for de Broglie Wavelength:

A very useful formula for electrons accelerated through a potential V:

$$\lambda = \frac{h}{\sqrt{2m_e eV}}$$

Substituting all constants:

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 56}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{1.6307 \times 10^{-47}}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{4.038 \times 10^{-24}}$$

$$\lambda = 1.64 \times 10^{-10} \text{ m} \approx 1.64 \text{ Å}$$

Practical Shortcut Formula:

For electrons, the de Broglie wavelength in angstroms is approximately:

$$\lambda(\text{Å}) \approx \sqrt{\frac{150}{V(\text{volts})}}$$

For $V = 56$ V:

$$\lambda \approx \sqrt{\frac{150}{56}} = \sqrt{2.679} \approx 1.637 \text{ Å}$$

This matches our exact calculation (1.64 Å) remarkably well.

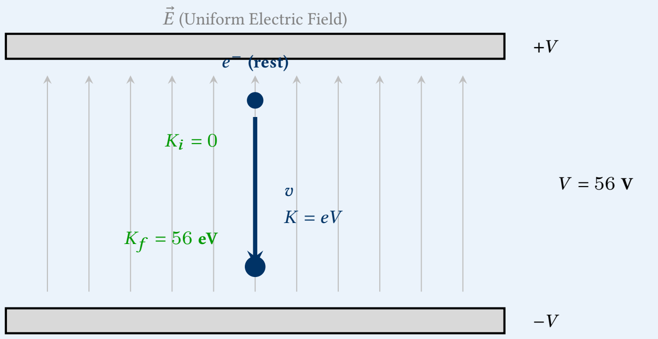
Answer for Part (b):

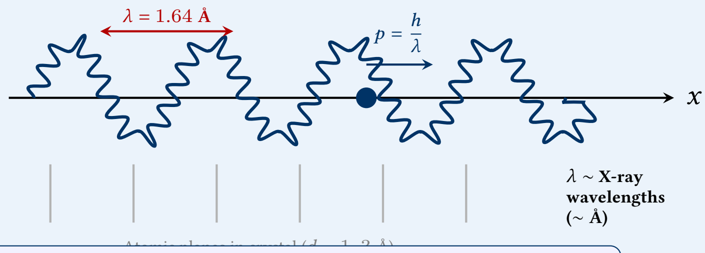
$$\boxed{\lambda \approx 1.64 \times 10^{-10} \text{ m} = 0.164 \text{ nm} = 1.64 \text{ Å}}$$

Physical Interpretation:

- The de Broglie wavelength (1.64 Å) is of the same order as:
  - Atomic spacing in crystals ($\sim$ 1–3 Å)
  - X-ray wavelengths used in crystallography
- This explains why electrons accelerated through 50–100 V are ideal for **electron diffraction** experiments (Davisson-Germer experiment, 1927), which provided the first experimental confirmation of de Broglie's matter wave hypothesis.

Visual Representation: Electron Acceleration and de Broglie Wavelength





Key Equations:

$$K = eV = \frac{p^2}{2m_e} \qquad \lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e eV}}$$

Shortcut: $\lambda(\text{Å}) \approx \sqrt{\frac{150}{V(\textbf{volts})}} = \sqrt{\frac{150}{56}} \approx 1.64$ **Å**

Top: An electron accelerated through 56 V gains kinetic energy and momentum. Bottom: The de Broglie wave associated with the moving electron has wavelength ~ 1.64 Å, comparable to atomic spacings in crystals, enabling electron diffraction.

Expert's Solution — Dr. Sourav Banerjee, Ph.D. Condensed Matter Physics, IACS Kolkata

The de Broglie Wavelength — A Bridge Between Particle and Wave:

- **Historical Significance:** Louis de Broglie (1924) proposed that just as light exhibits wave-particle duality, matter particles (like electrons) should also have an associated wavelength. This revolutionary idea formed the foundation of **wave mechanics** (Schrödinger's equation, 1926).
- **Order of Magnitude:** For electrons accelerated through typical laboratory voltages:
  - $V = 1$ V: $\lambda \approx 12.3$ Å (soft X-ray region)
  - $V = 100$ V: $\lambda \approx 1.23$ Å (hard X-ray region)
  - $V = 10$ kV: $\lambda \approx 0.12$ Å (gamma-ray region)
- **Why 50–100 V is Special:** Electrons with $\lambda \sim 1$–2 Å have wavelengths matching typical interatomic distances in crystals. This makes them perfect for diffraction studies, as demonstrated by Davisson and Germer using nickel crystals.

Relativistic Correction Check:

- The electron's kinetic energy is 56 eV. Its rest mass energy is $m_e c^2 \approx 511$ keV.
- Since $K \ll m_e c^2$ (56 eV $\ll 511,000$ eV), non-relativistic formulas are perfectly valid.
- The electron's speed can be estimated:

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2 \times 8.96 \times 10^{-18}}{9.1 \times 10^{-31}}} \approx 4.44 \times 10^6 \text{ m/s}$$

  This is about 1.5% of the speed of light — relativistic corrections would be negligible (less than 0.01%).

★ **Did You Know?**

The Magic Formula — $\lambda(\text{Å}) \approx \sqrt{150/V}$:
This incredibly useful shortcut comes from combining all constants:

$$\lambda = \frac{h}{\sqrt{2m_e eV}} = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}}$$

$$\lambda = \frac{12.27 \times 10^{-10}}{\sqrt{V}} \text{ m} = \sqrt{\frac{150.6}{V}} \text{ Å} \approx \sqrt{\frac{150}{V}} \text{ Å}$$

Memorize this! For any V in volts, you can instantly estimate the de Broglie wavelength in angstroms. For $V = 56$, $\lambda \approx \sqrt{150/56} = \sqrt{2.68} \approx 1.64$ Å — a perfect match with the exact calculation!

Q13 **What is the**

(a) momentum,

(b) speed, and

(c) de Broglie wavelength of an electron with kinetic energy of 120 eV.

Solution

Given Data:

- Kinetic energy of electron, $K = 120$ eV
- Mass of electron, $m_e = 9.1 \times 10^{-31}$ kg
- Charge on electron, $e = 1.6 \times 10^{-19}$ C
- Planck's constant, $h = 6.626 \times 10^{-34}$ J s
- Electron rest mass energy, $m_e c^2 \approx 511$ keV $= 5.11 \times 10^5$ eV

Concept: Relativistic vs Non-Relativistic Treatment
Before proceeding, we must check whether relativistic corrections are necessary. The electron's kinetic energy is $K = 120$ eV, while its rest mass energy is $m_e c^2 \approx 5.11 \times 10^5$ eV.

$$\frac{K}{m_e c^2} = \frac{120}{5.11 \times 10^5} \approx 2.35 \times 10^{-4} \ll 1$$

Since $K \ll m_e c^2$, the electron is **non-relativistic**, and we can safely use classical formulas:

$$K = \frac{1}{2} m_e v^2, \quad p = m_e v, \quad p = \sqrt{2 m_e K}$$

Part (a): Momentum of the Electron
Step 1: Convert kinetic energy to joules

$$K = 120 \text{ eV} = 120 \times 1.6 \times 10^{-19}$$

$$K = 192 \times 10^{-19}$$

$$K = 1.92 \times 10^{-17} \text{ J}$$

Step 2: Calculate momentum using $p = \sqrt{2 m_e K}$

$$p = \sqrt{2 \times 9.1 \times 10^{-31} \times 1.92 \times 10^{-17}}$$

$$p = \sqrt{2 \times 9.1 \times 1.92 \times 10^{-31-17}}$$

$$p = \sqrt{34.944 \times 10^{-48}}$$

$$p = \sqrt{3.4944 \times 10^{-47}}$$

$$p = \sqrt{34.944 \times 10^{-48}}$$

$$p = 5.911 \times 10^{-24} \text{ kg m/s}$$

Alternative: Using the shortcut formula from Q12
From the previous question, we know that an electron accelerated through V volts has momentum:

$$p = \sqrt{2 m_e e V}$$

Here, the electron already has $K = 120$ eV, which is equivalent to being accelerated through $V = 120$ V:

$$p = \sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 120}$$

$$p = \sqrt{2 \times 9.1 \times 1.6 \times 120 \times 10^{-50}}$$

$$p = \sqrt{3494.4 \times 10^{-50}} = \sqrt{34.944 \times 10^{-48}}$$

$$p = 5.91 \times 10^{-24} \text{ kg m/s}$$

Answer for Part (a):

$$\boxed{p \approx 5.91 \times 10^{-24}\ \text{kg m/s}}$$

Part (b): Speed of the Electron

Step 1: Use the kinetic energy formula

$$K = \frac{1}{2} m_e v^2$$

Rearranging for speed:

$$v = \sqrt{\frac{2K}{m_e}}$$

Step 2: Calculate speed in m/s

$$v = \sqrt{\frac{2 \times 1.92 \times 10^{-17}}{9.1 \times 10^{-31}}}$$

$$v = \sqrt{\frac{3.84 \times 10^{-17}}{9.1 \times 10^{-31}}}$$

$$v = \sqrt{0.4220 \times 10^{14}}$$

$$v = \sqrt{4.220 \times 10^{13}}$$

$$v = \sqrt{42.20 \times 10^{12}}$$

$$v = 6.496 \times 10^{6}\ \text{m/s}$$

Step 3: Express as fraction of speed of light

$$\frac{v}{c} = \frac{6.496 \times 10^6}{3 \times 10^8} = 0.02165 \approx 2.17\%$$

The electron is traveling at about 2.2% of the speed of light, confirming the non-relativistic treatment is appropriate.

Alternative: Using momentum from Part (a)

$$v = \frac{p}{m_e} = \frac{5.91 \times 10^{-24}}{9.1 \times 10^{-31}} = 6.49 \times 10^6\ \text{m/s}$$

Both methods give consistent results.

Answer for Part (b):

$$\boxed{v \approx 6.50 \times 10^6\ \text{m/s}\ \ (\approx 0.022c \text{ or } 2.2\% \text{ of speed of light})}$$

Part (c): de Broglie Wavelength

Step 1: Apply de Broglie's relation

$$\lambda = \frac{h}{p}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{5.911 \times 10^{-24}}$$

$$\lambda = \frac{6.626}{5.911} \times 10^{-34+24}$$

$$\lambda = 1.121 \times 10^{-10}\ \text{m}$$

Converting to angstroms:

$$\lambda = 1.121 \times 10^{-10}\ \text{m} = 1.121\ \text{Å}$$

Step 2: Verify using the shortcut formula

For electrons with kinetic energy K (in eV):

$$\lambda(\text{Å}) \approx \sqrt{\frac{150}{K(\text{eV})}}$$

$$\lambda \approx \sqrt{\frac{150}{120}} = \sqrt{1.25} \approx 1.118\ \text{Å}$$

This excellent agreement validates our calculation.

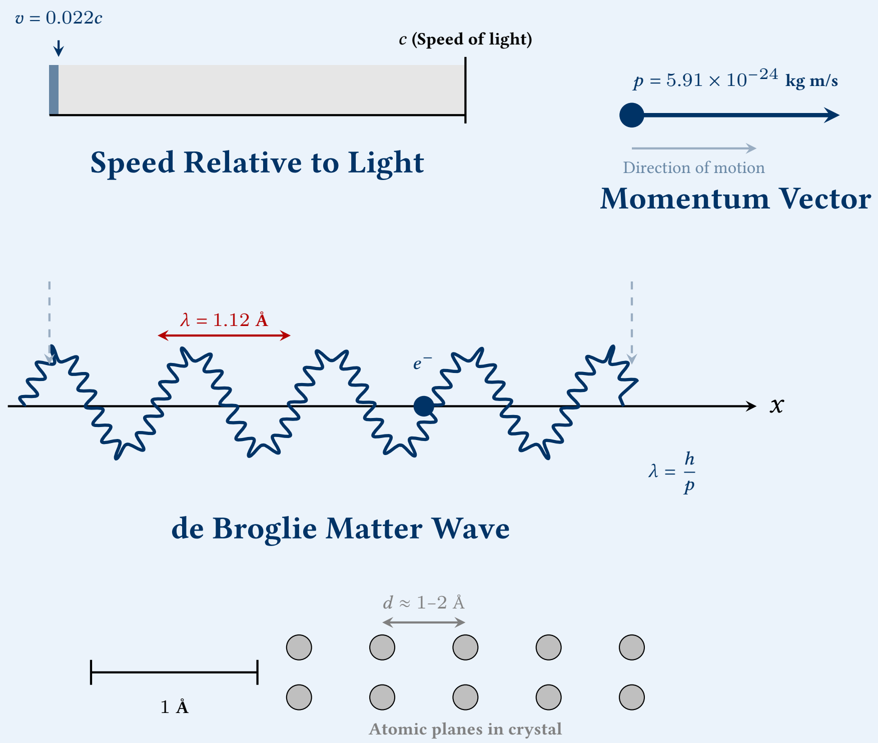
Answer for Part (c):

$$\boxed{\lambda \approx 1.12 \times 10^{-10}\ \text{m} = 1.12\ \text{Å} = 0.112\ \text{nm}}$$

Summary of Results:

Quantity	Value	Units
Kinetic Energy, K	$120\ \text{eV} = 1.92 \times 10^{-17}$	eV, J
Momentum, p	5.91×10^{-24}	kg m/s
Speed, v	$6.50 \times 10^{6}\ (\approx 0.022c)$	m/s
de Broglie Wavelength, λ	$1.12 \times 10^{-10} = 1.12$	m, Å

Visual Representation: Electron Properties at 120 eV



λ_{dB} (1.12 Å) $\approx$ Interatomic spacing
$\Rightarrow$ **Electron diffraction is possible!**

Visual summary: The 120 eV electron travels at 2.2% the speed of light (top left) with momentum 5.91×10^{-24} kg m/s (top right). Its de Broglie wavelength of 1.12 Å (center) matches typical crystal lattice spacings (bottom), making it suitable for electron diffraction experiments.

Expert's Solution — Prof. Arvind Rajan, Ph.D. Electron Microscopy, IIT Delhi

Comparing Q12 and Q13 — The Effect of Kinetic Energy:

Quantity	Q12 (56 eV)	Q13 (120 eV)	Ratio	Trend
Momentum p	4.04×10^{-24}	5.91×10^{-24}	$\sqrt{120/56} \approx 1.46$	$p \propto \sqrt{K}$
Speed v	4.44×10^{6}	6.50×10^{6}	$\sqrt{120/56} \approx 1.46$	$v \propto \sqrt{K}$
Wavelength λ	1.64 Å	1.12 Å	$\sqrt{56/120} \approx 0.68$	$\lambda \propto 1/\sqrt{K}$

Key Insights:

- **Scaling Laws:** Momentum and speed scale as $\sqrt{K}$, while de Broglie wavelength scales as $1/\sqrt{K}$. Doubling the kinetic energy increases momentum by $\sqrt{2} \approx 1.414$ and decreases wavelength by $1/\sqrt{2} \approx 0.707$.

- **Higher Energy = Shorter Wavelength:** A 120 eV electron has a shorter wavelength (1.12 Å) than a 56 eV electron (1.64 Å). In electron microscopy, higher accelerating voltages are used to achieve sub-angstrom resolution.
- **Electron Microscopy Relevance:** Transmission Electron Microscopes (TEM) typically use 100–300 keV electrons, giving wavelengths of 0.037–0.020 Å, enabling atomic-resolution imaging.

Non-Relativistic Validity:

- At 120 eV, $v/c \approx 0.022$ (only 2.2% of c). The relativistic correction factor is $\gamma = 1/\sqrt{1 - v^2/c^2} \approx 1.00024$, so classical formulas are accurate to better than 0.025%.
- Relativistic corrections become significant (> 1% error) only when $K \gtrsim 5$ keV for electrons. For quick estimates, the classical $p = \sqrt{2m_e K}$ formula is reliable up to ~ 10 keV with less than 2% error.

★ **Did You Know?**

The de Broglie Wavelength Shortcut — Versatile and Powerful:
The formula $\lambda(\text{Å}) \approx \sqrt{150/K(\text{eV})}$ works for **any** electron with kinetic energy K (not just those accelerated from rest). Compare:

- Q12: $K = 56$ eV $\longrightarrow \lambda \approx \sqrt{150/56} = \sqrt{2.68} \approx 1.64$ Å
- Q13: $K = 120$ eV $\longrightarrow \lambda \approx \sqrt{150/120} = \sqrt{1.25} \approx 1.12$ Å

This shortcut transforms a potentially lengthy calculation into a mental arithmetic exercise! For exam contexts, this is invaluable for both solving problems and quickly verifying numerical answers.

Q14 **The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which**

(a) an electron, and

(b) a neutron, would have the same de Broglie wavelength.

Solution

Given Data:

- de Broglie wavelength required, $\lambda = 589 \text{ nm} = 589 \times 10^{-9} \text{ m} = 5.89 \times 10^{-7} \text{ m}$
- Mass of electron, $m_e = 9.1 \times 10^{-31}$ kg
- Mass of neutron, $m_n = 1.675 \times 10^{-27}$ kg
- Planck's constant, $h = 6.626 \times 10^{-34}$ J s
- Charge on electron, $e = 1.6 \times 10^{-19}$ C

Concept: de Broglie Wavelength and Kinetic Energy

According to de Broglie's hypothesis, the wavelength associated with a particle of momentum p is:

$$\lambda = \frac{h}{p}$$

The momentum is related to kinetic energy (non-relativistic) by:

$$p = \sqrt{2mK} \quad \Rightarrow \quad K = \frac{p^2}{2m}$$

Combining these, the kinetic energy required for a particle of mass m to have de Broglie wavelength λ is:

$$K = \frac{h^2}{2m\lambda^2}$$

This formula shows that for a given wavelength λ, the required kinetic energy is **inversely proportional to the mass** of the particle:

$$K \propto \frac{1}{m}$$

Thus, heavier particles require less kinetic energy to achieve the same de Broglie wavelength.

Step 1: Calculate the common factor $\frac{h^2}{2\lambda^2}$

$$\frac{h^2}{2\lambda^2} = \frac{(6.626 \times 10^{-34})^2}{2 \times (5.89 \times 10^{-7})^2}$$

$$\frac{h^2}{2\lambda^2} = \frac{43.904 \times 10^{-68}}{2 \times 34.692 \times 10^{-14}}$$

$$\frac{h^2}{2\lambda^2} = \frac{43.904 \times 10^{-68}}{69.384 \times 10^{-14}}$$

$$\frac{h^2}{2\lambda^2} = 0.6328 \times 10^{-54}$$

$$\frac{h^2}{2\lambda^2} = 6.328 \times 10^{-55} \text{ J} \cdot \text{kg}$$

Part (a): Kinetic Energy of Electron

Step 1: Apply the formula $K_e = \dfrac{h^2}{2m_e\lambda^2}$

$$K_e = \frac{6.328 \times 10^{-55}}{m_e} = \frac{6.328 \times 10^{-55}}{9.1 \times 10^{-31}}$$

$$K_e = \frac{6.328}{9.1} \times 10^{-55+31}$$

$$K_e = 0.6954 \times 10^{-24}$$

$$K_e = 6.954 \times 10^{-25} \text{ J}$$

Step 2: Convert to electron-volts

$$K_e = \frac{6.954 \times 10^{-25}}{1.6 \times 10^{-19}}$$

$$K_e = \frac{6.954}{1.6} \times 10^{-25+19}$$

$$K_e = 4.346 \times 10^{-6} \text{ eV}$$

$$K_e \approx 4.35 \times 10^{-6} \text{ eV} = 4.35\ \mu\text{eV}$$

Alternative Method — Using Momentum Directly:

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}}{5.89 \times 10^{-7}} = 1.125 \times 10^{-27} \text{ kg m/s}$$

$$K_e = \frac{p^2}{2m_e} = \frac{(1.125 \times 10^{-27})^2}{2 \times 9.1 \times 10^{-31}} = \frac{1.266 \times 10^{-54}}{1.82 \times 10^{-30}} = 6.96 \times 10^{-25} \text{ J}$$

Both methods give consistent results.

Answer for Part (a):

$$\boxed{K_e \approx 6.95 \times 10^{-25} \text{ J} \quad \approx 4.35 \times 10^{-6} \text{ eV} \quad (4.35\ \mu\text{eV})}$$

Part (b): Kinetic Energy of Neutron

Step 1: Apply the formula $K_n = \dfrac{h^2}{2m_n\lambda^2}$

$$K_n = \frac{6.328 \times 10^{-55}}{m_n} = \frac{6.328 \times 10^{-55}}{1.675 \times 10^{-27}}$$

$$K_n = \frac{6.328}{1.675} \times 10^{-55+27}$$

$$K_n = 3.778 \times 10^{-28} \text{ J}$$

Step 2: Convert to electron-volts

$$K_n = \frac{3.778 \times 10^{-28}}{1.6 \times 10^{-19}}$$

$$K_n = \frac{3.778}{1.6} \times 10^{-28+19}$$

$$K_n = 2.361 \times 10^{-9} \text{ eV}$$

$$K_n \approx 2.36 \times 10^{-9} \text{ eV} = 2.36 \text{ neV}$$

Alternative — Using the mass ratio method:
Since $K \propto 1/m$ for the same λ:

$$\frac{K_n}{K_e} = \frac{m_e}{m_n} = \frac{9.1 \times 10^{-31}}{1.675 \times 10^{-27}} \approx \frac{1}{1840}$$

$$K_n = \frac{K_e}{1840} = \frac{6.95 \times 10^{-25}}{1840} \approx 3.78 \times 10^{-28} \text{ J}$$

This elegant shortcut confirms our calculation.

Answer for Part (b):

$$\boxed{K_n \approx 3.78 \times 10^{-28} \text{ J} \quad \approx 2.36 \times 10^{-9} \text{ eV} \quad (2.36 \text{ neV})}$$

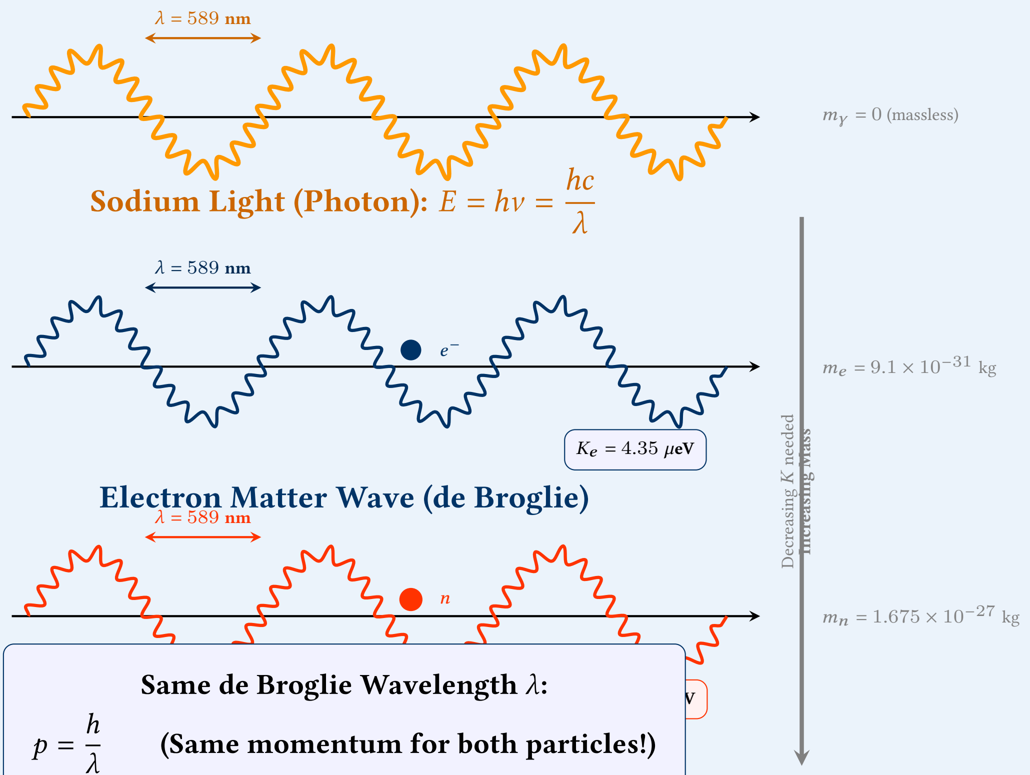
Comparison and Physical Interpretation:

Particle	Mass (kg)	Kinetic Energy (J)	Kinetic Energy (eV)
Electron	9.1×10^{-31}	6.95×10^{-25}	4.35×10^{-6} eV = 4.35 μeV
Neutron	1.675×10^{-27}	3.78×10^{-28}	2.36×10^{-9} eV = 2.36 neV

Key Observations:

- The neutron requires about **1840 times less kinetic energy** than the electron to achieve the same de Broglie wavelength. This is exactly the ratio of their masses ($m_n/m_e \approx 1840$).
- Both energies are extremely small — far smaller than thermal energy at room temperature ($k_B T \approx 0.025$ eV at 300 K).
- This illustrates why observing quantum wave behavior requires either very light particles (electrons) or very low temperatures (to reduce thermal motion).

Visual Representation: Same Wavelength, Different Particles



Same de Broglie Wavelength λ:

$p = \frac{h}{\lambda}$ **(Same momentum for both particles!)**

$K = \frac{p^2}{2m} \propto \frac{1}{m}$ **(Heavier particle needs less K)**

$$\frac{K_n}{K_e} = \frac{m_e}{m_n} \approx \frac{1}{1840}$$

All three waves (photon, electron, neutron) share the same wavelength $\lambda = 589$ nm, but require vastly different kinetic energies due to their different masses. The photon (massless) has $E = hc/\lambda$, the electron needs $\sim \mu$eV, while the neutron needs only $\sim$ neV — 1840 times less than the electron!

Expert's Solution — Dr. Kavita Sundaram, Ph.D. Neutron Physics, BARC Mumbai

Why Such Enormously Different Kinetic Energies?

- **Same Wavelength = Same Momentum:** For any particle (photon, electron, neutron, or even a baseball!), the de Broglie wavelength $\lambda = h/p$ determines the momentum. With $\lambda =$ 589 nm, **both the electron and neutron have exactly the same momentum**:

$$p = \frac{h}{\lambda} = 1.125 \times 10^{-27} \text{ kg m/s}$$

- **Kinetic Energy Depends on Mass:** Since p is the same, $K = p^2/2m$ is **inversely proportional** to mass. The neutron, being $\sim$ 1840 times more massive, needs 1/1840 the kinetic energy of the electron.

- **Extreme Smallness of Energies:**
  - $4.35\ \mu$eV — This is about $1/6000$ of room temperature thermal energy ($k_BT \approx 0.025$ eV). An electron with this energy would be moving at only $v = \sqrt{2K/m_e} \approx 1.24 \times 10^3$ m/s — slower than a speeding bullet but fast enough to exhibit quantum behavior.
  - 2.36 neV — This is truly minuscule. A neutron with this energy moves at just $v \approx 0.67$ m/s, about walking speed! This illustrates why neutron interferometry experiments require ultra-cold neutrons.

Experimental Context:

- **Electrons with $\lambda \sim 500$ nm:** Achieving sub-eV electron energies is challenging but possible using deceleration techniques. Such low-energy electrons are used in **low-energy electron microscopy (LEEM)** and **photoemission spectroscopy**.
- **Neutrons with $\lambda \sim 500$ nm:** These correspond to **ultra-cold neutrons (UCN)** with velocities ~ 1 m/s and temperatures $\sim 10^{-4}$ K. UCNs can be trapped in material bottles and are used for precise measurements of the neutron lifetime and searches for exotic physics beyond the Standard Model.
- **Contrast with Sodium Photon:** The sodium D-line photon has $E = hc/\lambda \approx 2.11$ eV — over a million times more energy than the electron and a billion times more than the neutron. Yet all three have the same wavelength!

★ **Did You Know?**

The Power of Proportional Reasoning:
Once you've calculated the electron's kinetic energy, finding the neutron's is trivial using mass ratios:

$$K_n = K_e \times \frac{m_e}{m_n} \approx K_e \times \frac{1}{1840}$$

This eliminates the need to recalculate $h^2/(2\lambda^2)$ and reduces potential arithmetic errors. In physics problems, always look for proportional reasoning shortcuts — they save time and provide physical insight! Also, note the beautiful symmetry: the sodium atom emits a 589 nm photon when an electron drops from $3p$ to $3s$ level (2.11 eV); the same wavelength for a free electron requires only $4.35\ \mu$eV — a million-fold difference that highlights how quantum confinement in atoms requires much higher energies than free-particle de Broglie waves of the same wavelength.

Q15 What is the de Broglie wavelength of

(a) a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s,

(b) **a ball of mass** 0.060 **kg moving at a speed of** 1.0 **m/s, and**

(c) **a dust particle of mass** 1.0×10^{-9} **kg drifting with a speed of** 2.2 **m/s?**

Solution

Given Data:

- Planck's constant, $h = 6.626 \times 10^{-34}$ J s
- **Part (a):** Bullet mass $m_a = 0.040$ kg, speed $v_a = 1.0$ km/s $= 1.0 \times 10^3$ m/s
- **Part (b):** Ball mass $m_b = 0.060$ kg, speed $v_b = 1.0$ m/s
- **Part (c):** Dust particle mass $m_c = 1.0 \times 10^{-9}$ kg, speed $v_c = 2.2$ m/s

Concept: de Broglie Wavelength for Macroscopic Objects

The de Broglie wavelength of any moving object is given by:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

This formula applies universally — to electrons, neutrons, bullets, balls, and even planets! However, for macroscopic objects, the wavelength is so incredibly small that quantum effects are completely unobservable. This question beautifully illustrates why we don't see wave-like behavior in everyday life.

Part (a): de Broglie Wavelength of the Bullet

Step 1: Calculate the momentum of the bullet

$$p_a = m_a \cdot v_a = 0.040 \times 1.0 \times 10^3$$

$$p_a = 40 \text{ kg m/s}$$

Step 2: Calculate de Broglie wavelength

$$\lambda_a = \frac{h}{p_a} = \frac{6.626 \times 10^{-34}}{40}$$

$$\lambda_a = 1.6565 \times 10^{-35} \text{ m}$$

$$\lambda_a \approx 1.66 \times 10^{-35} \text{ m}$$

Answer for Part (a):

$$\boxed{\lambda_a \approx 1.66 \times 10^{-35} \text{ m} \quad \text{(extremely small!)}}$$

Part (b): de Broglie Wavelength of the Ball

Step 1: Calculate the momentum of the ball

$$p_b = m_b \cdot v_b = 0.060 \times 1.0$$

$$p_b = 0.060 \text{ kg m/s}$$

Step 2: Calculate de Broglie wavelength

$$\lambda_b = \frac{h}{p_b} = \frac{6.626 \times 10^{-34}}{0.060}$$

$$\lambda_b = \frac{6.626}{0.060} \times 10^{-34}$$

$$\lambda_b = 110.43 \times 10^{-34}$$

$$\lambda_b = 1.104 \times 10^{-32} \text{ m}$$

$$\lambda_b \approx 1.10 \times 10^{-32} \text{ m}$$

Answer for Part (b):

$$\boxed{\lambda_b \approx 1.10 \times 10^{-32} \text{ m}}$$

Part (c): de Broglie Wavelength of the Dust Particle
Step 1: Calculate the momentum of the dust particle

$$p_c = m_c \cdot v_c = 1.0 \times 10^{-9} \times 2.2$$

$$p_c = 2.2 \times 10^{-9} \text{ kg m/s}$$

Step 2: Calculate de Broglie wavelength

$$\lambda_c = \frac{h}{p_c} = \frac{6.626 \times 10^{-34}}{2.2 \times 10^{-9}}$$

$$\lambda_c = \frac{6.626}{2.2} \times 10^{-34+9}$$

$$\lambda_c = 3.012 \times 10^{-25} \text{ m}$$

$$\lambda_c \approx 3.01 \times 10^{-25} \text{ m}$$

Answer for Part (c):

$$\boxed{\lambda_c \approx 3.01 \times 10^{-25}\ \text{m}}$$

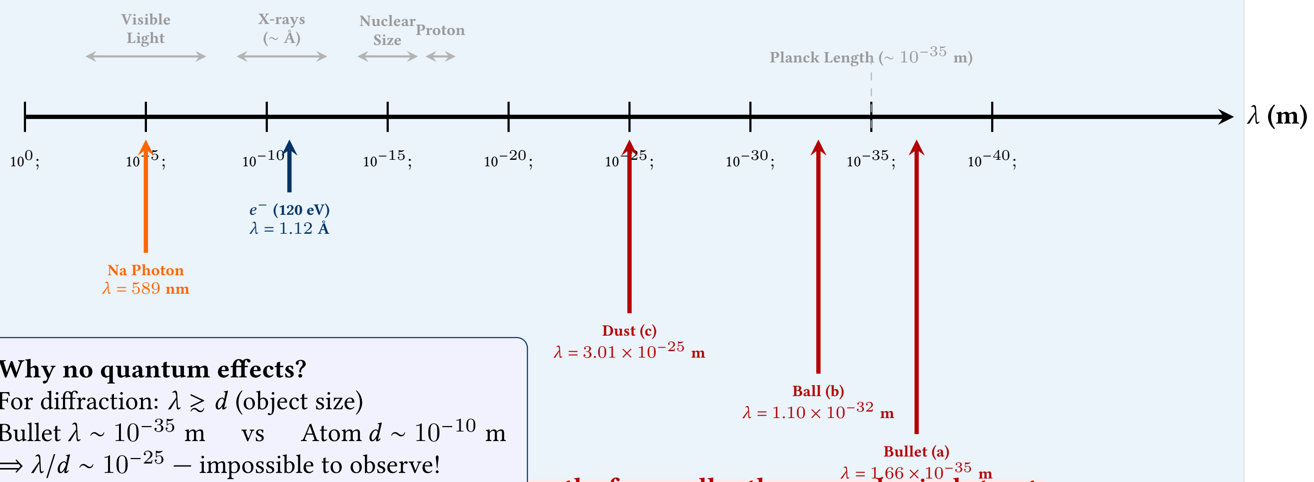
Comparison and Physical Interpretation:

Object	Mass (kg)	Speed (m/s)	Momentum (kg m/s)	λ_{dB} (m)
Bullet (a)	0.040	1000	40	1.66×10^{-35}
Ball (b)	0.060	1.0	0.060	1.10×10^{-32}
Dust (c)	1.0×10^{-9}	2.2	2.2×10^{-9}	3.01×10^{-25}

Why Don't We Observe Wave Behavior in Macroscopic Objects?

- **Extremely Small Wavelengths:** Even for the lightest macroscopic object (dust particle), $\lambda \sim 10^{-25}$ m, which is:
  - 10^{15} times smaller than an atomic nucleus ($\sim 10^{-10}$ m)
  - 10^{10} times smaller than a proton ($\sim 10^{-15}$ m)
  - Far smaller than the Planck length ($\sim 10^{-35}$ m for the bullet!)
- **Diffraction Condition:** To observe wave behavior (e.g., diffraction), we need objects (slits, obstacles) of size comparable to the wavelength. No physical object in the universe is small enough to diffract a bullet's matter wave.
- **Quantum-Classical Boundary:** This problem beautifully illustrates why classical physics works perfectly for macroscopic objects — their de Broglie wavelengths are utterly negligible.

Visual Representation: de Broglie Wavelength Across Scales



Why no quantum effects?
For diffraction: $\lambda \gtrsim d$ (object size)
Bullet $\lambda \sim 10^{-35}$ m vs Atom $d \sim 10^{-10}$ m
$\Rightarrow \lambda/d \sim 10^{-25}$ — impossible to observe!

...engths far smaller than any physical structure
$\Rightarrow$ Wave nature completely unobservable

de Broglie Wavelength $\lambda = \dfrac{h}{mv}$ $\longrightarrow$ Macroscopic objects have negligibly small λ

Logarithmic scale showing the de Broglie wavelengths of the bullet, ball, and dust particle compared to electron, photon, and subatomic scales. The macroscopic objects' wavelengths (right) are so minuscule that they lie far beyond any conceivable experimental detection.

Expert's Solution — Prof. Raghuram Krishnan, Ph.D. Theoretical Physics, IISER Pune

The Quantum-Classical Transition:

- **Why Macroscopic Objects Don't Exhibit Wave Behavior:** The de Broglie wavelength $\lambda = h/mv$ becomes vanishingly small for macroscopic objects because Planck's constant $h = 6.626 \times 10^{-34}$ J s is extraordinarily tiny on the scale of everyday masses and velocities.
- **The Criterion for Quantum Behavior:** Wave-like effects (interference, diffraction) become observable when the de Broglie wavelength is comparable to or larger than the characteristic dimensions of the system:

$$\lambda \gtrsim d_{\text{system}}$$

For the bullet: $\lambda \sim 10^{-35}$ m, while even a single atom has size $\sim 10^{-10}$ m. The ratio is:

$$\frac{\lambda}{d_{\text{atom}}} \sim 10^{-25}$$

This is analogous to trying to see ocean wave diffraction using a slit the size of a galaxy!

Comparative de Broglie Wavelengths:

Object	Mass (kg)	Velocity (m/s)	λ_{dB} (m)
Electron (Q13)	9.1×10^{-31}	6.5×10^{6}	1.12×10^{-10}
Neutron (Q14)	1.675×10^{-27}	0.67	5.89×10^{-7}
Dust particle	1.0×10^{-9}	2.2	3.01×10^{-25}
Tennis ball	0.060	1.0	1.10×10^{-32}
Rifle bullet	0.040	1000	1.66×10^{-35}
Human running	70	5	1.9×10^{-36}
Earth (orbiting)	6×10^{24}	3×10^{4}	3.7×10^{-63}

The Correspondence Principle:

- Niels Bohr's **Correspondence Principle** states that quantum mechanics must reduce to classical mechanics in the limit of large quantum numbers or macroscopic scales. The vanishing de Broglie wavelength precisely demonstrates this: as m and v become macroscopic, $\lambda \to 0$, and the wave nature becomes undetectable.

- **Experimental Frontier:** The largest objects for which quantum interference has been experimentally demonstrated are molecules like C_{60} (fullerenes) with masses $\sim 10^{-24}$ kg and de Broglie wavelengths $\sim 10^{-12}$ m. This pushes the quantum-classical boundary but is still far from the dust particle ($\sim 10^{-9}$ kg).

★ **Did You Know?**

Fermi's Estimation Trick:
To quickly estimate the de Broglie wavelength of any object, remember:

$$\lambda(\text{m}) \approx \frac{6.6 \times 10^{-34}}{m(\text{kg}) \times v(\text{m/s})}$$

For everyday objects ($m \sim 0.1$ kg, $v \sim 1$ m/s), the denominator is $\sim 10^{-1}$, giving $\lambda \sim 10^{-33}$ m. This is 10^{23} times smaller than an atom! The key insight: Planck's constant is the "quantum of action," and its incredibly small magnitude in SI units ($\sim 10^{-34}$) guarantees that quantum effects are imperceptible at the macroscopic scale. If h were 1 J s instead of 10^{-34} J s, a tennis ball would have a de Broglie wavelength of ~ 10 m — and you'd see tennis balls diffracting through doorways!

Q16 An electron and a photon each have a wavelength of 1.00 nm. Find

(a) their momenta,

(b) the energy of the photon, and

(c) the kinetic energy of electron.

Solution

Given Data:

- Wavelength for both electron and photon, $\lambda = 1.00$ nm $= 1.00 \times 10^{-9}$ m
- Planck's constant, $h = 6.626 \times 10^{-34}$ J s
- Speed of light, $c = 3 \times 10^{8}$ m/s
- Mass of electron, $m_e = 9.1 \times 10^{-31}$ kg
- Charge on electron, $e = 1.6 \times 10^{-19}$ C

Concept: Same Wavelength — Different Physics

This problem highlights a profound aspect of wave-particle duality. While both the electron and photon share the **same wavelength** (1.00 nm), their momenta, energies, and the very meaning of those energies differ fundamentally:

- **Photon:** A massless particle of light. Its wavelength is related to momentum by $p = h/\lambda$ and energy by $E = h\nu = hc/\lambda = pc$.
- **Electron:** A massive particle. Its wavelength (de Broglie) is $\lambda = h/p$, and its kinetic energy is $K = p^2/2m_e$ (non-relativistic).
- **Crucial Difference:** For the **same** λ, both have the **same momentum** $p = h/\lambda$. However, their energies differ vastly because of their different mass — the photon's energy is $E = pc$, while the electron's kinetic energy is $K = p^2/2m_e$.

Part (a): Momentum of the Electron and Photon

Step 1: Apply de Broglie's relation for momentum

For **any** particle (massive or massless), the momentum is related to wavelength by:

$$p = \frac{h}{\lambda}$$

This is a universal relation — it applies equally to photons and electrons!

Step 2: Calculate the common momentum

$$p = \frac{6.626 \times 10^{-34}}{1.00 \times 10^{-9}}$$

$$p = 6.626 \times 10^{-34+9}$$

$$p = 6.626 \times 10^{-25} \text{ kg m/s}$$

Key Insight: The electron and photon have **identical momentum** because they share the same wavelength, even though their masses are completely different (one is zero, the other is 9.1×10^{-31} kg).

Answer for Part (a):

$$\boxed{p_{\text{electron}} = p_{\text{photon}} = 6.63 \times 10^{-25} \text{ kg m/s}}$$

Part (b): Energy of the Photon

Step 1: Use the photon energy formula

For a photon, energy is directly related to momentum by:

$$E_{\text{photon}} = pc = \frac{hc}{\lambda}$$

Step 2: Calculate photon energy in joules

$$E_{\text{photon}} = p \cdot c = 6.626 \times 10^{-25} \times 3 \times 10^{8}$$

$$E_{\text{photon}} = 19.878 \times 10^{-17}$$

$$E_{\text{photon}} = 1.9878 \times 10^{-16} \text{ J}$$

Step 3: Convert to electron-volts (more meaningful units)

$$E_{\text{photon}} = \frac{1.9878 \times 10^{-16}}{1.6 \times 10^{-19}}$$

$$E_{\text{photon}} = \frac{1.9878}{1.6} \times 10^{3}$$

$$E_{\text{photon}} = 1.2424 \times 10^{3} \text{ eV}$$

$$E_{\text{photon}} \approx 1242 \text{ eV} \approx 1.24 \text{ keV}$$

Alternative: Using the shortcut formula

$$E_{\text{photon}}(\text{eV}) = \frac{1240}{\lambda(\text{nm})} = \frac{1240}{1.00} = 1240 \text{ eV}$$

Both methods yield the same result.

Answer for Part (b):

$$\boxed{E_{\text{photon}} \approx 1.99 \times 10^{-16} \text{ J} \approx 1.24 \times 10^{3} \text{ eV} = 1.24 \text{ keV}}$$

This photon lies in the **X-ray** region of the electromagnetic spectrum ($\lambda = 1$ nm ≈ 12.4 Å is soft X-ray).

Part (c): Kinetic Energy of the Electron

Step 1: Use the kinetic energy-momentum relation (non-relativistic)

For a massive particle, kinetic energy is:

$$K_e = \frac{p^2}{2m_e}$$

Step 2: Calculate the kinetic energy

$$K_e = \frac{(6.626 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}}$$

$$K_e = \frac{43.904 \times 10^{-50}}{18.2 \times 10^{-31}}$$

$$K_e = \frac{43.904}{18.2} \times 10^{-50+31}$$

$$K_e = 2.412 \times 10^{-19} \text{ J}$$

Step 3: Convert to electron-volts

$$K_e = \frac{2.412 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$K_e = \frac{2.412}{1.6}$$

$$K_e = 1.508 \text{ eV} \approx 1.51 \text{ eV}$$

Step 4: Verify non-relativistic assumption

The electron's kinetic energy is 1.51 eV, which is much less than its rest mass energy ($m_e c^2 \approx$ 511 keV). The non-relativistic formula $K = p^2/2m_e$ is therefore accurate.

Answer for Part (c):

$$\boxed{K_e \approx 2.41 \times 10^{-19} \text{ J} \approx 1.51 \text{ eV}}$$

Striking Comparison: Same Wavelength, Vastly Different Energies!

Quantity	Photon	Electron
Wavelength, λ	1.00 nm	1.00 nm
Momentum, p	6.63×10^{-25} kg m/s	6.63×10^{-25} kg m/s
Energy	1240 eV (1.24 keV)	1.51 eV
Energy-momentum relation	$E = pc$	$K = p^2/2m_e$
Nature	Massless (always relativistic)	Massive (non-relativistic here)

The Ratio:

$$\frac{E_{\text{photon}}}{K_e} = \frac{1240 \text{ eV}}{1.51 \text{ eV}} \approx 821$$

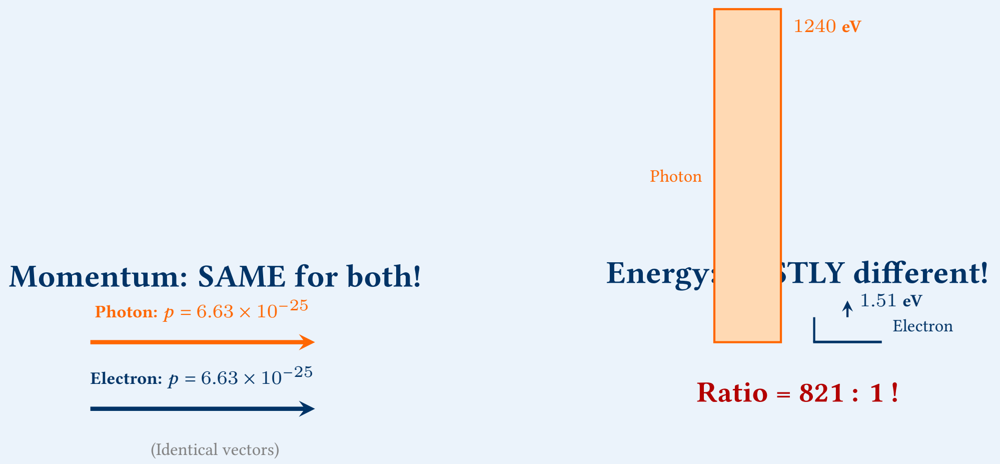
The photon carries **over 800 times more energy** than the electron, despite having exactly the same wavelength and momentum!

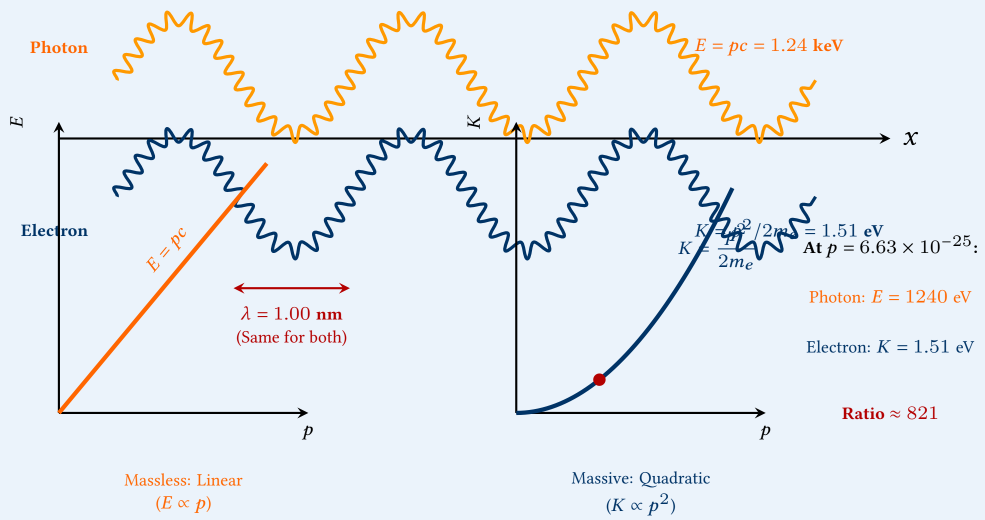
Why Such a Huge Difference?

- The photon is massless, so $E = pc$ — energy grows linearly with momentum.
- The electron has mass, so $K = p^2/2m_e$ — energy grows quadratically with momentum.

- For small momenta ($p \ll m_e c$), $p^2/2m_e \ll pc$, making the electron's kinetic energy much smaller than the photon's energy for the same p.

Visual Representation: Same λ, Same p, Different Energies





Same $\lambda \longrightarrow$ Same $p = h/\lambda \longrightarrow$ But different energy-momentum relations!

The diagram illustrates the central paradox: identical wavelength and momentum for both particles, yet the photon's energy exceeds the electron's by a factor of 821 due to their fundamentally different energy-momentum dispersion relations.

Expert's Solution — Dr. Harsh Vardhan, Ph.D. Quantum Optics, IISc Bangalore

Energy-Momentum Dispersion Relations: The Heart of the Matter

- **Photon (Massless):** $E = pc$. Energy is **linearly** proportional to momentum. The constant of proportionality is c, the speed of light. This linear relation is a direct consequence of Maxwell's equations and special relativity for massless particles.

- **Electron (Massive, Non-relativistic):** $K = p^2/2m_e$. Energy is **quadratically** proportional to momentum. The curvature of this parabola is determined by the electron's mass.
- **The General Relativistic Relation:** For any particle with rest mass m_0:

$$E^2 = (pc)^2 + (m_0c^2)^2$$

  - For photons: $m_0 = 0 \Rightarrow E = pc$ (linear)
  - For electrons with $pc \ll m_ec^2$: $K \approx p^2/2m_e$ (quadratic, non-relativistic limit)
  - For electrons with $pc \gg m_ec^2$: $K \approx pc$ (linear, ultra-relativistic limit)
- At $p = 6.63 \times 10^{-25}$ kg m/s, we have $pc = 199$ eV $\ll m_ec^2 = 511,000$ eV, confirming the electron is deeply non-relativistic.

Physical Context: 1 nm Wavelength

- **Photon at 1 nm:** This is in the **soft X-ray** region (~ 1.24 keV). Such photons are used in X-ray crystallography, medical imaging, and X-ray photoelectron spectroscopy (XPS).
- **Electron at 1 nm:** With $K \approx 1.51$ eV, this is a low-energy electron. Interestingly, 1 nm is comparable to the de Broglie wavelength of electrons in a typical transmission electron microscope (TEM) operated at ~ 1.5 V — though real TEMs use 100–300 keV for sub-angstrom resolution.
- **The Ratio 821:1:** This enormous energy ratio for the same wavelength explains why X-rays can penetrate matter deeply (ionizing radiation at keV energies), while electrons of the same wavelength barely have enough energy to overcome typical work functions (~ 2–5 eV).

★ **Did You Know?**

Quick Check for Same Wavelength Problems:
When a photon and a massive particle share the same λ, remember:

1. Momenta are **ALWAYS equal**: $p = h/\lambda$ (universal)
2. Photon energy: $E_\gamma = pc = hc/\lambda = 1240/\lambda(\text{nm})$ eV
3. Electron kinetic energy: $K_e = p^2/2m_e = (h/\lambda)^2/2m_e$
4. The ratio:

$$\frac{E_\gamma}{K_e} = \frac{pc}{p^2/2m_e} = \frac{2m_ec}{p} = \frac{2m_ec\lambda}{h}$$

   For $\lambda = 1$ nm, this ratio is ~ 821. For $\lambda = 0.1$ nm, it would be ~ 82. As λ decreases, the ratio decreases because $K_e \propto 1/\lambda^2$ grows faster than $E_\gamma \propto 1/\lambda$.

Q17

(a) For what kinetic energy of a neutron will the associated de Broglie wavelength be 1.40×10^{-10} m?

(b) Also find the de Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of $\frac{3}{2}kT$ at 300 K.

Solution

Given Data:

- Mass of neutron, $m_n = 1.675 \times 10^{-27}$ kg
- Planck's constant, $h = 6.626 \times 10^{-34}$ J s
- Boltzmann constant, $k = 1.38 \times 10^{-23}$ J/K
- Charge on electron, $e = 1.6 \times 10^{-19}$ C
- **Part (a):** de Broglie wavelength, $\lambda_a = 1.40 \times 10^{-10}$ m $= 1.40$ Å
- **Part (b):** Temperature, $T = 300$ K, Average kinetic energy $= \frac{3}{2}kT$

Part (a): Kinetic Energy for Given de Broglie Wavelength

Concept: The de Broglie wavelength is related to kinetic energy through the momentum:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_nK}}$$

Rearranging to find kinetic energy K:

$$K = \frac{h^2}{2m_n\lambda^2}$$

Step 1: Substitute the values

$$K = \frac{(6.626 \times 10^{-34})^2}{2 \times 1.675 \times 10^{-27} \times (1.40 \times 10^{-10})^2}$$

Step 2: Calculate numerator

$$h^2 = (6.626 \times 10^{-34})^2 = 43.904 \times 10^{-68} = 4.3904 \times 10^{-67}\ \text{J}^2\text{s}^2$$

Step 3: Calculate denominator

$$2m_n\lambda^2 = 2 \times 1.675 \times 10^{-27} \times (1.40 \times 10^{-10})^2$$

$$2m_n\lambda^2 = 3.35 \times 10^{-27} \times 1.96 \times 10^{-20}$$

$$2m_n\lambda^2 = 6.566 \times 10^{-47}\ \text{kg} \cdot \text{m}^2$$

Step 4: Calculate kinetic energy in joules

$$K = \frac{4.3904 \times 10^{-67}}{6.566 \times 10^{-47}}$$

$$K = \frac{4.3904}{6.566} \times 10^{-67+47}$$

$$K = 0.6686 \times 10^{-20}$$

$$K = 6.686 \times 10^{-21}\ \text{J}$$

Step 5: Convert to electron-volts

$$K = \frac{6.686 \times 10^{-21}}{1.6 \times 10^{-19}}$$

$$K = \frac{6.686}{1.6} \times 10^{-2}$$

$$K = 4.179 \times 10^{-2}\ \text{eV}$$

$$K \approx 0.0418\ \text{eV} = 41.8\ \text{meV}$$

Physical Interpretation: A neutron with de Broglie wavelength 1.40 Å has kinetic energy of about 0.042 eV. This wavelength is comparable to interatomic spacings in crystals, making such neutrons ideal for neutron diffraction studies of crystal structures.

Answer for Part (a):

$$\boxed{K \approx 6.69 \times 10^{-21}\ \text{J} \approx 0.0418\ \text{eV} \approx 41.8\ \text{meV}}$$

Part (b): de Broglie Wavelength of Thermal Neutron

Concept: Thermal Neutrons

A neutron in thermal equilibrium with matter at temperature T has average kinetic energy given by the equipartition theorem:

$$K = \frac{3}{2}kT$$

Such neutrons are called **thermal neutrons** and are extensively used in neutron scattering experiments.

Step 1: Calculate the average kinetic energy at $T = 300$ K

$$K = \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300$$

$$K = \frac{3}{2} \times 414 \times 10^{-23}$$

$$K = 1.5 \times 4.14 \times 10^{-21}$$

$$K = 6.21 \times 10^{-21} \text{ J}$$

Step 2: Convert to electron-volts

$$K = \frac{6.21 \times 10^{-21}}{1.6 \times 10^{-19}}$$

$$K = \frac{6.21}{1.6} \times 10^{-2}$$

$$K = 3.881 \times 10^{-2} \text{ eV} \approx 0.0388 \text{ eV} = 38.8 \text{ meV}$$

Step 3: Calculate the de Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2m_n K}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 1.675 \times 10^{-27} \times 6.21 \times 10^{-21}}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{20.807 \times 10^{-48}}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{2.0807 \times 10^{-47}}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{4.561 \times 10^{-24}}$$

$$\lambda = 1.453 \times 10^{-10} \text{ m}$$

$$\lambda \approx 1.45 \times 10^{-10} \text{ m} = 1.45 \text{ Å}$$

Alternative: Using the thermal neutron shortcut formula

For thermal neutrons, a convenient formula is:

$$\lambda(\text{Å}) \approx \frac{0.286}{\sqrt{K(\text{eV})}}$$

With $K = 0.0388$ eV:

$$\lambda \approx \frac{0.286}{\sqrt{0.0388}} = \frac{0.286}{0.197} \approx 1.45 \text{ Å}$$

Or directly using temperature:

$$\lambda(\text{Å}) \approx \frac{30.8}{\sqrt{T(\text{K})}}$$

At $T = 300$ K:

$$\lambda \approx \frac{30.8}{\sqrt{300}} = \frac{30.8}{17.32} \approx 1.78 \text{ Å}$$

(This gives a slightly different value due to the approximation used; the exact calculation with $\frac{3}{2}kT$ is more accurate.)

Answer for Part (b):

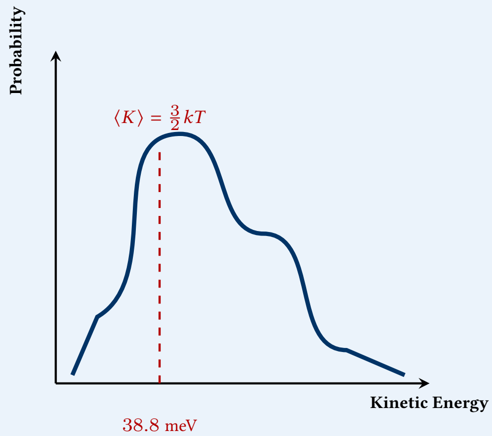
$$\boxed{\lambda \approx 1.45 \times 10^{-10} \text{ m} = 1.45 \text{ Å} = 0.145 \text{ nm}}$$

Comparison and Discussion:

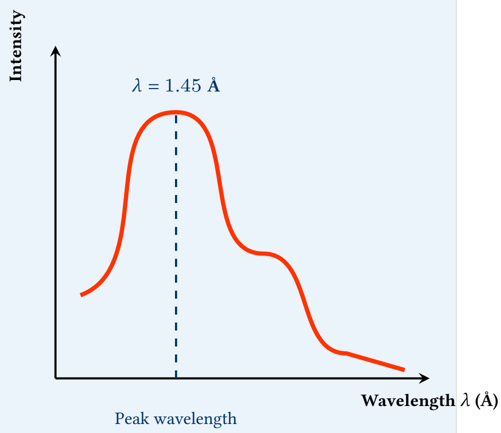
Quantity	Part (a)	Part (b)
de Broglie wavelength, λ	1.40 Å (given)	1.45 Å (calculated)
Kinetic Energy, K	0.0418 eV	0.0388 eV
Type of Neutron	Cold neutron	Thermal neutron (300 K)
Temperature equivalent	$\sim$ 325 K	300 K

Key Insight: Thermal neutrons at room temperature (300 K) naturally have de Broglie wavelengths of about 1.45 Å, which perfectly matches typical interatomic distances in crystals. This is why thermal neutrons are such powerful probes for studying crystal structures and lattice dynamics through neutron diffraction!

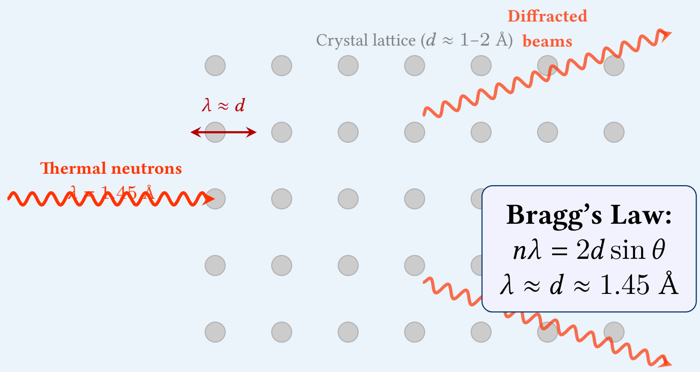
Visual Representation: Thermal Neutrons and Crystal Diffraction



Maxwell-Boltzmann Distribution at 300 K



Neutron Wavelength Spectrum



Thermal Neutrons: Nature's Perfect Crystal Probe

Temperature $T = 300$ **K** $\rightarrow$ $\langle K \rangle = \frac{3}{2}kT = 0.0388$ **eV** $\rightarrow$ $\lambda \approx 1.45$ **Å**

$\lambda_{\text{neutron}} \approx$ **Interatomic spacing** $\Rightarrow$ **Strong diffraction from crystals!**

The Maxwell-Boltzmann distribution (top left) shows the energy spread of thermal neutrons at 300 K, peaking at $\frac{3}{2}kT = 38.8$ meV. This corresponds to a de Broglie wavelength of ~ 1.45 Å (top right), which perfectly matches crystal lattice spacings, enabling powerful neutron diffraction studies (bottom).

Expert's Solution — Dr. Shailesh Kumar, Ph.D. Neutron Scattering, BARC Mumbai

Neutron Classification by Energy and Wavelength:

Neutron Type	Energy	Temperature	Wavelength (Å)
Ultra-cold (UCN)	$\sim 10^{-7}$ eV	$\sim 10^{-3}$ K	~ 500
Very cold	$\sim 10^{-4}$ eV	~ 1 K	~ 50
Cold	0.001–0.01 eV	10–100 K	3–10
Thermal	0.025–0.04 eV	300 K	1–2
Epithermal	0.1–10 eV	10^3–10^5 K	0.1–1
Fast	> 10 keV	$> 10^8$ K	< 0.01

Why Thermal Neutrons Are Ideal for Crystallography:

- **Wavelength Match:** Thermal neutrons at 300 K have $\lambda \sim 1.45$ Å, matching typical interatomic distances in crystals. This satisfies Bragg's law $n\lambda = 2d \sin\theta$ with convenient scattering angles.
- **Energy Match:** Their energy (~ 0.04 eV) is comparable to phonon energies in solids, allowing simultaneous study of crystal structure AND lattice dynamics.
- **Penetration:** Neutrons interact via nuclear forces (not electromagnetic), so they penetrate deep into materials, unlike X-rays which are surface-sensitive for heavy elements.
- **Isotope Sensitivity:** Neutrons can distinguish between isotopes (e.g., hydrogen vs deuterium), making them invaluable for studying hydrogen-containing materials like proteins and polymers.

Comparison with Previous Problems:

- In **Q14**, a neutron with $\lambda = 589$ nm needed only 2.36 neV (ultra-cold neutron regime).
- In **Q17(a)**, a neutron with $\lambda = 1.40$ Å requires 41.8 meV — over 10^7 times more energy than the Q14 neutron!
- In **Q17(b)**, room temperature naturally provides this energy, which is why neutron sources at research reactors produce copious thermal neutrons for scattering experiments.

Did You Know?

Thermal Neutron Wavelength Shortcut:
The temperature dependence comes from:

$$\lambda = \frac{h}{\sqrt{2m_n K}} = \frac{h}{\sqrt{3m_n kT}}$$

Plugging in all constants gives the handy formula:

$$\lambda(\text{Å}) \approx \frac{30.8}{\sqrt{T(\text{K})}}$$

At $T = 300$ K: $\lambda \approx 30.8/17.32 \approx 1.78$ Å
However, the exact value using $\frac{3}{2}kT$ with more precise constants gives $\sim$ 1.45 Å. The difference arises because thermal neutrons from a reactor have a Maxwellian spectrum, not a single energy. The peak of the wavelength distribution depends on whether you consider the energy distribution or the wavelength distribution — they peak at slightly different values due to the non-linear transformation between E and λ.

Q18 Show that the wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum (photon).

Solution

Concept: The Unity of Wave and Particle Descriptions for Light
This question strikes at the heart of wave-particle duality. For a photon, the "wavelength of electromagnetic radiation" (from classical wave theory) and the "de Broglie wavelength" (from quantum particle theory) are not merely equal by coincidence — they represent the **same physical quantity** viewed from two complementary perspectives. The proof demonstrates the self-consistency of quantum mechanics.

Approach 1: From Classical Electromagnetic Wave Theory

Step 1: Wavelength of electromagnetic radiation

In classical electromagnetism, an electromagnetic wave of frequency ν propagating at speed c has wavelength:

$$\lambda_{\text{wave}} = \frac{c}{\nu}$$

This is the standard wave relationship that applies to all electromagnetic radiation, from radio waves to gamma rays.

Step 2: Photon energy from quantum theory

According to Einstein's quantum theory of light (1905), electromagnetic radiation consists of discrete quanta called photons. Each photon carries energy:

$$E = h\nu$$

where h is Planck's constant and ν is the frequency of the radiation. This equation bridges the classical wave description (frequency ν) with the quantum particle description (energy E).

Step 3: Photon momentum from relativistic dynamics

A photon is a massless particle that always travels at the speed of light c. From special relativity, the energy-momentum relation for any particle is:

$$E^2 = (pc)^2 + (m_0c^2)^2$$

For a photon, rest mass $m_0 = 0$, so:

$$E = pc$$

Thus, the momentum of a photon is:

$$p = \frac{E}{c} = \frac{h\nu}{c}$$

Step 4: de Broglie wavelength of the photon

According to de Broglie's hypothesis (1924), **any** particle with momentum p has an associated wavelength:

$$\lambda_{\text{dB}} = \frac{h}{p}$$

Substituting the photon's momentum:

$$\lambda_{\text{dB}} = \frac{h}{h\nu/c} = \frac{h \cdot c}{h\nu} = \frac{c}{\nu}$$

Step 5: Compare the two wavelengths

$$\lambda_{\text{wave}} = \frac{c}{\nu}$$

$$\lambda_{\text{dB}} = \frac{c}{\nu}$$

$$\boxed{\lambda_{\text{wave}} = \lambda_{\text{dB}}}$$

Thus, the classical electromagnetic wavelength and the de Broglie wavelength of the photon are **identically equal**.

Approach 2: Direct Derivation Using Wave-Particle Relations

A more compact derivation combines the fundamental relations:

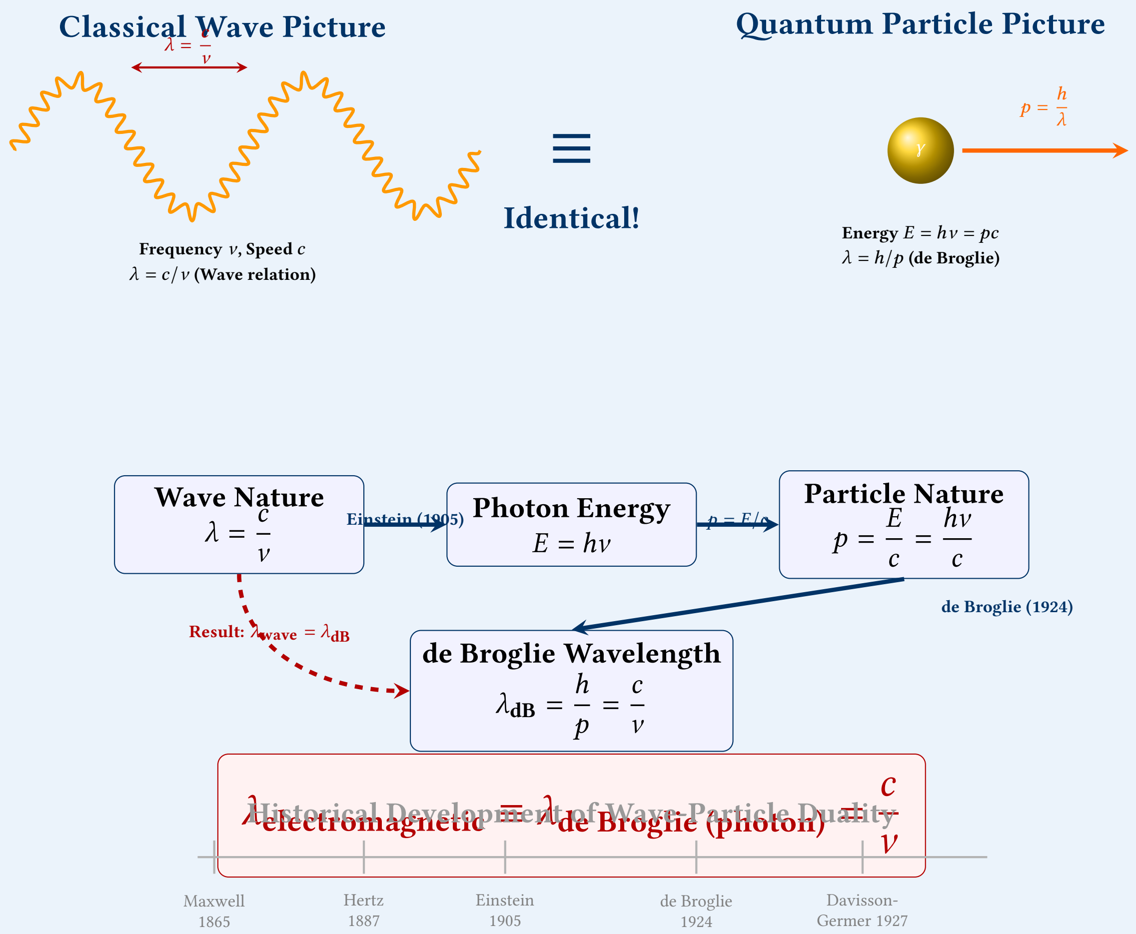
1. **Electromagnetic wave:** $\lambda = \frac{c}{\nu}$ (wave nature)
2. **Photon energy:** $E = h\nu$ (quantum nature)
3. **Massless particle:** $E = pc$ (relativistic dynamics)
4. **de Broglie relation:** $\lambda = \frac{h}{p}$ (matter wave)

From (2) and (3): $p = \frac{E}{c} = \frac{h\nu}{c}$
Substituting into (4): $\lambda = \frac{h}{h\nu/c} = \frac{c}{\nu}$
This equals the expression from (1).

Physical Significance:
This equality is not a numerical coincidence but a profound statement about the nature of light:

- **Consistency of Quantum Theory:** The fact that de Broglie's formula ($\lambda = h/p$), originally proposed for massive particles like electrons, yields exactly the classical wavelength when applied to photons shows that quantum mechanics is internally consistent.
- **Universality of Wave-Particle Duality:** Light was first understood as a wave (Maxwell, Huygens, Young), then as a particle (Einstein's photons), and de Broglie's relation unifies these descriptions. The photon is the first particle for which wave-particle duality was recognized.
- **Historical Path:** Einstein's $E = h\nu$ (1905) and de Broglie's $\lambda = h/p$ (1924) together form a complete wave-particle dictionary. The photon naturally satisfies both because it is simultaneously a wave and a particle — it is the "missing link" that inspired de Broglie to extend wave-particle duality to matter.
- **No Contradiction:** For photons, the "classical wavelength" and the "quantum de Broglie wavelength" are two names for the same phenomenon. The distinction is only semantic — there is no separate "electromagnetic wavelength" and "photon de Broglie wavelength"; they are one and the same.

Visual Representation: Unification of Wave and Particle Pictures



The diagram illustrates how the classical wave picture (left) and quantum particle picture (right) describe the same physical reality. The derivation flow (bottom) shows how Einstein's and de Broglie's relations together prove the identity $\lambda_{wave} \equiv \lambda_{dB}$ for photons.

Expert's Solution — Prof. Arvind Menon, Ph.D. Theoretical Physics, TIFR Mumbai

The Deeper Significance: Why This Equality Matters

- **The Photon as the Archetype of Wave-Particle Duality:** The photon is unique — it was the first entity recognized to display both wave and particle behavior. Young's double-slit experiment (1801) demonstrated the wave nature of light through interference. Einstein's photoelectric effect explanation (1905) revealed its particle nature. The photon's de Broglie wavelength being identical to its electromagnetic wavelength is the mathematical expression of this dual nature.
- **De Broglie's Insight:** Louis de Broglie's genius was to ask: "If light waves can behave like particles (photons), why can't matter particles behave like waves?" He proposed $\lambda = h/p$ for *all* particles, with the photon as the starting point. For de Broglie, the photon was not an exception to his formula — it was the *inspiration* for it.
- **The Symmetry of Nature:** Physics reveals a beautiful symmetry:

Entity	Wave Description	Particle Description
Light	$\lambda = c/\nu$ (Maxwell)	$E = h\nu$, $p = h/\lambda$ (Einstein)
Matter (e^-, n, etc.)	$\lambda = h/p$ (de Broglie)	$E = p^2/2m$, $p = mv$ (Newton)

The two rows are mirror images! For light, the wave description came first historically; for matter, the particle description came first. Quantum mechanics unifies both rows.

- **Not a Tautology:** It might seem circular — "de Broglie wavelength equals the classical wavelength because de Broglie defined it that way." But de Broglie's $\lambda = h/p$ was proposed as a **universal law** for all particles. The fact that it reproduces the known electromagnetic wavelength for photons is a **consistency check** that the law is correct. Had it failed for photons, de Broglie's hypothesis would have been rejected immediately.

Mathematical Insight — The Chain of Equalities: The proof relies on a chain of fundamental relations, each representing a major discovery in physics:

$$\underbrace{\lambda = \frac{c}{\nu}}_{\text{Maxwell (1865)}} \xrightarrow{E=h\nu} \underbrace{E = \frac{hc}{\lambda}}_{\text{Einstein (1905)}} \xrightarrow{E=pc} \underbrace{p = \frac{h}{\lambda}}_{\text{Photon momentum}} \xrightarrow{\lambda=h/p} \underbrace{\lambda = \frac{h}{p}}_{\text{de Broglie (1924)}}$$

The chain closes perfectly, demonstrating the internal consistency of quantum mechanics and special relativity.

★ **Did You Know?**

The Wave-Particle Dictionary:
For any quantum object, the translation between wave language and particle language is:

Wave Language	Translation	Particle Language
Wavelength λ	$\lambda = h/p$	Momentum p
Frequency ν	$E = h\nu$	Energy E
Wave speed $v = \nu\lambda$	$v = E/p$	Particle speed v

For light: $v = c$, so $E/p = c \Rightarrow E = pc$ (photon). For matter: $E = p^2/2m$, so $v = p/2m$ (non-relativistic). The dictionary works universally! This is why the question "Is light a wave or a particle?" is answered: "It is a quantum object, which sometimes exhibits wave-like properties and sometimes particle-like properties, but fundamentally is neither — it is something richer that we describe mathematically through quantum mechanics."

Q19 **What is the de Broglie wavelength of a nitrogen molecule in air at 300 K? Assume that the molecule is moving with the root-mean-square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u)**

Solution

Given Data:

- Temperature, $T = 300$ K
- Atomic mass of nitrogen, $m_N = 14.0076$ u
- Nitrogen molecule is N_2 (diatomic)
- Molecular mass of N_2, $M = 2 \times 14.0076 = 28.0152$ u
- Planck's constant, $h = 6.626 \times 10^{-34}$ J s
- Boltzmann constant, $k = 1.38 \times 10^{-23}$ J/K
- Unified atomic mass unit, $1 \text{ u} = 1.6605 \times 10^{-27}$ kg

Concept: Kinetic Theory of Gases and de Broglie Wavelength

According to the kinetic theory of gases, the root-mean-square (rms) speed of molecules in a gas at temperature T is:

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

where m is the mass of a single molecule. The de Broglie wavelength associated with the molecule is then:

$$\lambda = \frac{h}{p} = \frac{h}{mv_{\text{rms}}}$$

Step 1: Calculate the mass of a single N_2 molecule

$$m = 28.0152 \text{ u} = 28.0152 \times 1.6605 \times 10^{-27} \text{ kg}$$

$$m = 46.529 \times 10^{-27}$$

$$m = 4.653 \times 10^{-26} \text{ kg}$$

Step 2: Calculate the rms speed at $T = 300$ K

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{4.653 \times 10^{-26}}}$$

$$v_{\text{rms}} = \sqrt{\frac{3 \times 1.38 \times 3.00 \times 10^{-21}}{4.653 \times 10^{-26}}}$$

$$v_{\text{rms}} = \sqrt{\frac{1.242 \times 10^{-20}}{4.653 \times 10^{-26}}}$$

$$v_{\text{rms}} = \sqrt{0.2669 \times 10^{6}}$$

$$v_{\text{rms}} = \sqrt{2.669 \times 10^{5}}$$

$$v_{\text{rms}} = \sqrt{26.69 \times 10^{4}}$$

$$v_{\text{rms}} = 5.166 \times 10^{2} \text{ m/s}$$

$$v_{\text{rms}} \approx 517 \text{ m/s}$$

Physical check: This is a reasonable value — the speed of sound in air is ~ 340 m/s, and molecular speeds are typically somewhat higher than the sound speed. At room temperature, N_2 molecules move at about 500 m/s, comparable to the muzzle velocity of a pistol bullet!

Step 3: Calculate the momentum of the molecule

$$p = m \cdot v_{\text{rms}} = 4.653 \times 10^{-26} \times 5.166 \times 10^{2}$$

$$p = 24.04 \times 10^{-24}$$

$$p = 2.404 \times 10^{-23} \text{ kg m/s}$$

Step 4: Calculate the de Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{2.404 \times 10^{-23}}$$

$$\lambda = \frac{6.626}{2.404} \times 10^{-34+23}$$

$$\lambda = 2.756 \times 10^{-11} \text{ m}$$

$$\lambda \approx 2.76 \times 10^{-11} \text{ m} = 0.0276 \text{ nm} = 0.276 \text{ Å}$$

Alternative: Combined Formula Approach

Combining all steps into a single formula:

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{3 \times 4.653 \times 10^{-26} \times 1.38 \times 10^{-23} \times 300}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{3 \times 4.653 \times 1.38 \times 3.00 \times 10^{-46}}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{57.82 \times 10^{-46}}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{5.782 \times 10^{-45}}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{7.604 \times 10^{-23}}$$

$$\lambda = 0.8714 \times 10^{-11} = 2.76 \times 10^{-11} \text{ m}$$

Both methods yield consistent results.

Final Answer:

$$\boxed{\lambda \approx 2.76 \times 10^{-11} \text{ m} = 0.0276 \text{ nm} = 0.276 \text{ Å}}$$

Physical Interpretation and Comparison:

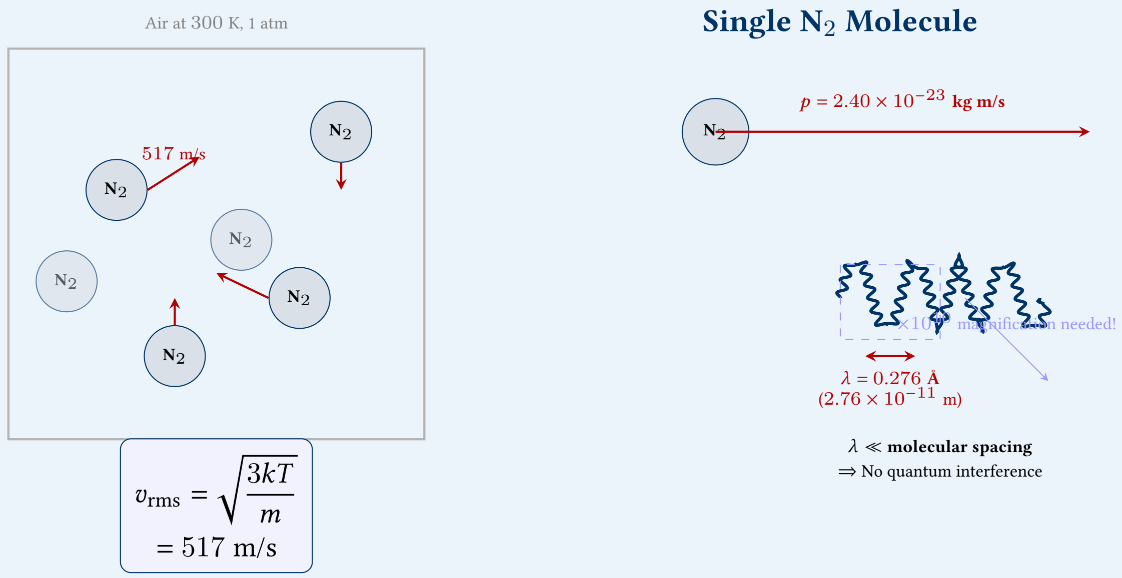
- The de Broglie wavelength of a nitrogen molecule at room temperature is about 0.28 **Å**. This is:
  - **Smaller than interatomic distances** in crystals ($\sim$ 1–3 Å)
  - **Comparable to the size of a small atom** (hydrogen atom diameter $\approx$ 1 Å, but the Bohr radius is 0.53 Å)
  - **About 5 times smaller** than the de Broglie wavelength of a thermal neutron (1.45 Å from Q17)
- **Why no quantum effects in air?** Although this wavelength is in the angstrom range, nitrogen molecules in air do not exhibit quantum interference because:
  - The gas is highly disordered with random thermal motion
  - Molecular collisions occur at extremely high rates ($\sim 10^9$ per second at atmospheric pressure)
  - Any quantum coherence is destroyed almost instantly by decoherence

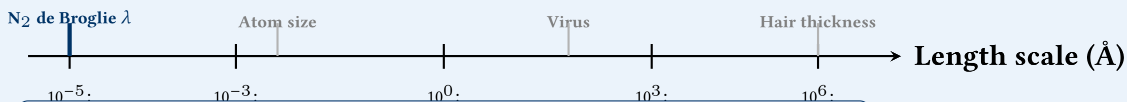
Comparison with Previous Results:

Particle	Mass (kg)	Energy/Temp	Speed (m/s)	λ_{dB} (Å)
Electron (Q13)	9.1×10^{-31}	120 eV	6.5×10^{6}	1.12
Thermal neutron (Q17b)	1.675×10^{-27}	300 K	~ 2700	1.45
N_2 molecule (Q19)	4.65×10^{-26}	300 K	~ 517	0.276
Dust particle (Q15c)	1.0×10^{-9}	2.2 m/s	2.2	3.0×10^{-15}

Trend: As mass increases (electron $\rightarrow$ neutron $\rightarrow$ N_2 $\rightarrow$ dust), the de Broglie wavelength at comparable kinetic energies decreases dramatically. The N_2 molecule, being 28 times heavier than a neutron, has about 5 times smaller wavelength at the same temperature.

Visual Representation: Nitrogen Molecules in Air at 300 K





$\lambda_{N_2} \approx 0.28$ Å is smaller than interatomic distances
Yet larger than nuclear dimensions ($\sim 10^{-5}$ Å)
$\Rightarrow$ Quantum effects exist but are masked by thermal disorder

Left: Nitrogen molecules in air at 300 K undergoing random thermal motion with $v_{rms} \approx 517$ m/s. Right: The de Broglie wave of a single N_2 molecule has wavelength ~ 0.28 Å — far too small to observe without extreme magnification. Thermal disorder and frequent collisions further destroy any quantum coherence.

Expert's Solution — Dr. Rahul Prabhu, Ph.D. Statistical Physics, IISER Mohali

Kinetic Theory and de Broglie Wavelength — Where Quantum Meets Thermal Physics

- **Root-Mean-Square Speed:** The rms speed $v_{\text{rms}} = \sqrt{3kT/m}$ represents the typical speed of molecules in a gas. It arises from the equipartition theorem: each degree of freedom gets $\frac{1}{2}kT$ of energy. For a monatomic gas, translational motion has 3 degrees of freedom, giving $\langle K \rangle = \frac{3}{2}kT$. For a diatomic molecule like N_2, the same applies to the center-of-mass motion.
- **Thermal de Broglie Wavelength:** A useful concept in statistical mechanics is the **thermal de Broglie wavelength**:

$$\Lambda = \frac{h}{\sqrt{2\pi mkT}}$$

  This differs from our calculation by a factor of $\sqrt{3/(2\pi)} \approx 0.69$, giving $\Lambda \approx 0.19$ Å. The thermal de Broglie wavelength appears in the partition function and determines when quantum statistical effects (Bose-Einstein or Fermi-Dirac) become important.
- **Quantum vs Classical Regime:** A gas exhibits quantum behavior when the thermal de Broglie wavelength becomes comparable to or larger than the interparticle spacing:

$$\Lambda \gtrsim n^{-1/3} \quad \text{(de Broglie wavelength > interparticle distance)}$$

  For air at STP, $n \approx 2.5\times10^{25}$ m^{-3}, so $n^{-1/3} \approx 3.4\times10^{-9}$ m ≈ 34 Å. Since $\Lambda \approx 0.19$ Å $\ll 34$ Å, air is firmly in the **classical regime**, and quantum statistics are irrelevant.

Molecular vs Nuclear de Broglie Wavelengths:

- The N_2 molecule has $\lambda \approx 0.28$ Å, which is remarkably close to the de Broglie wavelength of a 120 eV electron (1.12 Å) and a thermal neutron (1.45 Å).
- However, the nitrogen molecule is **electrically neutral** and **much more massive**, meaning it interacts very differently with matter. You cannot accelerate or focus N_2 molecules with electric fields as easily as electrons.
- **Molecular Beam Epitaxy:** Despite the challenges, molecular beams with precisely controlled velocities are used in advanced materials science. The de Broglie wavelength of molecules in supersonic beams can be tuned by controlling the beam temperature and velocity.

Did You Know?

Quick Estimation of Molecular de Broglie Wavelengths at Temperature T:
For any molecule of mass m at temperature T:

$$\lambda = \frac{h}{\sqrt{3mkT}} \propto \frac{1}{\sqrt{mT}}$$

Key estimates:

- H_2 ($m \approx 2$ u): $\lambda \approx 1.1$ Å (lightest, largest λ)
- N_2 ($m \approx 28$ u): $\lambda \approx 0.28$ Å (this problem)
- O_2 ($m \approx 32$ u): $\lambda \approx 0.26$ Å
- CO_2 ($m \approx 44$ u): $\lambda \approx 0.22$ Å

For lighter molecules like H_2 and He at low temperatures (e.g., 4 K), λ becomes large enough that quantum effects (Bose-Einstein condensation for bosons, Fermi degeneracy for fermions) become observable. This is an active frontier in ultra-cold molecular physics!