NCERT Exemplar Solutions

Solved NCERT Exemplar Problems for Class 12th Physics, Chapter 12

Chapter 12: Atoms

About this Chapter

Chapter 12 builds the quantum picture of the atom. We move from **Rutherford's nuclear model** to **Bohr's quantisation** of angular momentum, derive the radii and energies of hydrogen-like orbits, and explain the line spectra (Lyman, Balmer, Paschen). These Exemplar problems probe the limits of the Bohr model with numerical estimates, hypothetical force laws, reduced-mass corrections, and Auger transitions.

Topics covered: Bohr's model • Hydrogen spectrum • Energy levels • Spectral series • Reduced mass • Limitations of Bohr model

Quick Formula Sheet

Bohr radius:

$$r_n = \frac{n^2 a_0}{Z},\ a_0 = 53\,\text{pm}$$

Energy of nth level:

$$E_n = -\frac{13.6\,Z^2}{n^2}\ \text{eV}$$

Rydberg formula:

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Angular momentum:

$$L = n\hbar = \frac{nh}{2\pi}$$

NCERT Exemplar Problems

MCQ I (Single Correct Option)

Q 12.1 **Taking the Bohr radius as $a_0 = 53\,\text{pm}$, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about:**

(a) $53\,$**pm**

(b) $27\,$**pm**

(c) $18\,$**pm**

(d) $13\,$**pm**

SOLUTION

Correct option: (c) 18 pm.

Concept used. In Bohr's model the radius of the nth permitted orbit in a hydrogen-like atom (one electron, nuclear charge $+Ze$) is

$$r_n = \frac{n^2 a_0}{Z},$$

where $a_0 = 53\,\text{pm}$ is the Bohr radius (the ground-state radius of ordinary hydrogen, $Z = 1$, $n = 1$). For lithium the atomic number is $Z = 3$, so Li^{++} (a lithium atom that has lost two of its three electrons) is a one-electron hydrogen-like ion with $Z = 3$.

Step 1. Identify the parameters for Li^{++} in the ground state: $Z = 3$, $n = 1$.

Step 2. Substitute into the Bohr radius formula:

$$r_1 = \frac{1^2 \times a_0}{3} = \frac{a_0}{3}.$$

Step 3. Plug in $a_0 = 53\,\text{pm}$:

$$r_1 = \frac{53\,\text{pm}}{3} = 17.67\,\text{pm} \approx 18\,\text{pm}.$$

Step 4. Compare with the options: (a) 53 matches hydrogen, not Li^{++}; (b) 27 would correspond to $Z = 2$ (He^+); (d) 13 is too small for $Z = 3$.

Final Answer: Option **(c)**: $r \approx 18\,\text{pm}$.

☞ Why the radius shrinks with Z

A higher nuclear charge pulls the electron closer, so for the same quantum number n a heavier hydrogen-like nucleus has a smaller orbit. The factor is exactly $1/Z$.

EXPERT'S SOLUTION *: Aarav Sharma, M.Sc Physics, IIT Madras*

Quick reading. The Bohr radius formula $r_n = n^2 a_0/Z$ has two knobs: principal quantum number n (squared, in numerator) and atomic number Z (linear, in denominator). For ground state, $n = 1$ is fixed, leaving only Z.

Step 1. Recognise Li^{++} as hydrogen-like: lithium ($Z = 3$) stripped of two electrons leaves one electron orbiting a $+3e$ nucleus.

Step 2. Apply $r_1 = a_0/Z$ with $Z = 3$:

$$r_1 = \frac{53}{3}\,\text{pm} = 17.67\,\text{pm}.$$

Step 3. Round to the precision of the options: $17.67 \approx 18\,\text{pm}$.

Why this matters. The same logic delivers the radius of any hydrogen-like ion (He$^+$, Be^{+++}, ...); just divide a_0 by Z and multiply by n^2.

Final Answer: $r_{\mathrm{Li}^{++}} \approx 18\,\mathrm{pm}$.

Q 12.2 **The binding energy of a H-atom, considering an electron moving around a fixed nucleus (proton), is $B = -\dfrac{me^4}{8n^2\varepsilon_0^2h^2}$ (m = electron mass). If one decides to work in a frame of reference where the electron is at rest, the proton would be moving around it. By similar arguments, the binding energy would be $B = -\dfrac{Me^4}{8n^2\varepsilon_0^2h^2}$ (M = proton mass). This last expression is not correct because:**

(a) n would not be integral.
(b) Bohr-quantisation applies only to electron.
(c) the frame in which the electron is at rest is not inertial.
(d) the motion of the proton would not be in circular orbits, even approximately.

SOLUTION

Correct option: (c) the frame in which the electron is at rest is not inertial.

Concept used. Newton's laws (and the derivation of Bohr's orbits, which uses centripetal force = Coulomb force) only hold in an **inertial frame**: a frame moving at constant velocity relative to the fixed stars, with no proper acceleration. In a hydrogen atom the electron orbits the (nearly fixed) proton; in the electron's rest frame the proton accelerates around the electron, so that frame itself is non-inertial.

Step 1. Recall the Bohr derivation. The radius formula $r_n = n^2a_0/Z$ and energy $E_n \propto m$ arise from balancing $\dfrac{mv^2}{r} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{r^2}$ and quantising $L = mvr = n\hbar$. Both steps assume Newton's second law holds, i.e. the lab frame is inertial.

Step 2. In the proton's frame the electron moves in a closed orbit, so the proton frame is (nearly) inertial; fine.

Step 3. Switch to the electron's rest frame. Here the electron itself is following a circle in the lab frame, so its rest frame is constantly accelerating. Newton's laws need fictitious-force corrections; the naive replacement $m \to M$ in the binding energy ignores those, so the formula fails.

Step 4. Eliminate the other options: (a) n is still integer (the quantisation rule does not depend on which particle is moving); (b) Bohr-quantisation applies to any orbiting particle in an inertial frame; (d) approximate circular orbits do exist relative to the centre of mass.

Final Answer: Option **(c)**: the electron's rest frame is non-inertial, so the simple Bohr derivation cannot be transplanted into it.

EXPERT'S SOLUTION *: Vivaan Iyer, Ph.D Physics, IISc Bangalore*

Strategic angle. The right way to handle the proton's motion is the *reduced-mass* trick: use $\mu = mM/(m+M)$ in place of m, and continue working in an inertial frame (centre-of-mass frame).

Step 1. In the centre-of-mass frame both particles move on circles about the COM. This frame is inertial.

Step 2. The two-body problem reduces to a single particle of mass $\mu = mM/(m+M)$ moving around the COM at distance r.

Step 3. Since $M \gg m$, $\mu \approx m$, recovering the usual Bohr formula. There is no symmetric formula with $m \to M$; that would only be valid in the electron's frame, which is not inertial.

Why this matters. Choosing an inertial frame is a prerequisite for Newtonian mechanics; otherwise pseudo-forces intrude. This same idea reappears in Q 12.25 (deuterium isotope shift via reduced mass).

Alternative method (reduced-mass derivation). Set up the two-body Coulomb problem in the COM frame. Let $\vec{r} = \vec{r}_e - \vec{r}_p$ be the relative coordinate. Newton's laws in the COM frame yield

$$\mu\ddot{\vec{r}} = -\frac{e^2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2},$$

where $\mu = mM/(m+M)$. This is mathematically identical to a single particle of mass μ orbiting a fixed centre. The Bohr energy formula is then

$$E_n = -\frac{\mu e^4}{8\varepsilon_0^2 n^2 h^2} = -\frac{\mu}{m_e} \cdot \frac{13.6}{n^2}\,\text{eV}.$$

For hydrogen $\mu/m_e \approx 1 - m_e/m_p \approx 0.99946$, so the "corrected" binding is ≈ 13.593 eV. The asymmetric naive formula with $m \to M$ would give $E_n \propto M$, i.e. ~ 1800 times deeper; absurd, and traceable to the non-inertial framing.

Order-of-magnitude cross-check. A pseudo-force in the non-inertial frame would have magnitude $m\omega^2 r$ with $\omega = v/a_0 \sim 4 \times 10^{16}\,\text{s}^{-1}$; that is comparable to the real Coulomb force, so dropping it cannot be a small correction.

Final Answer: (c); the electron-rest frame is non-inertial.

✗ Don't swap m for M blindly

The Coulomb force depends on the charges, not on which particle is "chosen" to be at rest. Replacing $m \to M$ in the energy formula secretly assumes the electron's frame is inertial; it isn't. The correct correction is to use the reduced mass μ, not the proton mass.

Q 12.3 The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because:

(a) of the electrons not being subject to a central force.

(b) of the electrons colliding with each other.

(c) of screening effects.

(d) the force between the nucleus and an electron will no longer be given by Coulomb's law.

SOLUTION

Correct option: (a) of the electrons not being subject to a central force.

Concept used. A central force is one that points always toward (or away from) a fixed centre and whose magnitude depends only on the distance from that centre. Bohr's derivation (circular orbit + quantisation) is built entirely on the assumption that the electron experiences exactly such a force, supplied by the nucleus alone.

Step 1. For hydrogen, the electron feels only the Coulomb attraction from the proton, which is purely radial; a central force. So Bohr's model works.

Step 2. In a many-electron atom, each electron feels the nucleus *plus* the repulsions from all the other electrons. Those electron-electron forces do not point toward the nucleus and depend on the angular positions of all the other electrons. The net force is no longer central.

Step 3. Without a central force, angular momentum about the nucleus is not conserved and circular Bohr orbits with fixed radii cease to be exact solutions.

Step 4. Why the other options are weaker: (b) collisions are not the primary reason; (c) screening is a consequence of, not the underlying cause of, the central-force breakdown; (d) the electron-nucleus force is still Coulombic, it is the electron-electron pieces that spoil centrality.

> **Final Answer:** Option **(a)**: in a many-electron atom the net force on each electron is not central.

EXPERT'S SOLUTION *: Arjun Patel, M.Tech Applied Physics, IIT Delhi*

Structural observation. Bohr's success rests on three ingredients: Coulomb force, central direction, single electron. Drop any one and the simple model breaks.

Step 1. Coulomb's law still holds for each pair of charges in a many-electron atom; so option (d) is wrong.

Step 2. Screening (c) is a real effect, but it is the symptom: the outer electron sees a partially shielded nucleus precisely *because* inner electrons exert their own non-central forces.

Step 3. Collisions (b) are negligible in the bound, quasi-stationary electron cloud of an atom.

Step 4. Therefore the root cause is the loss of centrality: option (a).

Why this matters. Real multi-electron atoms need self-consistent-field methods (Hartree-Fock, density functional theory) precisely because the central-force approximation is only a zeroth-order starting point.

Concept linkage. The breakdown of Bohr's central-force hypothesis is exactly why the periodic table needs s, p, d, f subshells (set by the orbital quantum number ℓ). In a true central field, energies depend on n alone; in the screened multi-electron field, levels with different ℓ split apart, giving rise to the energy ordering $4s < 3d$ that explains the filling of transition metals.

Order-of-magnitude check. For lithium ($Z = 3$), Bohr would predict the outermost electron at $E \approx -13.6 \cdot 9/4 = -30.6$ eV; the experimental first ionisation energy is 5.39 eV; six times smaller. The disagreement is enormous, confirming that the simple Bohr extension fails badly the moment electron-electron interaction enters.

> **Final Answer:** (a).

☞ **Central force**

A force $\vec{F}$ is central if (i) it points along the line joining the source to the test particle and (ii) its magnitude depends only on the radial separation r. Central forces conserve orbital angular momentum and admit planar circular orbits; both pillars of the Bohr derivation.

Q 12.4 **For the ground state, the electron in the H-atom has an angular momentum $= \hbar$, according to the simple Bohr model. Angular momentum is a vector and hence there will be infinitely many orbits with the vector pointing in all possible directions. In actuality, this is not true,**

(a) because Bohr model gives incorrect values of angular momentum.

(b) because only one of these would have a minimum energy.

(c) angular momentum must be in the direction of spin of electron.

(d) because electrons go around only in horizontal orbits.

SOLUTION

Correct option: (a) because the Bohr model gives incorrect values of angular momentum.

Concept used. The Bohr model postulates $L = n\hbar$, so the ground-state ($n = 1$) angular momentum is $\hbar$. But quantum mechanics (the more accurate theory) gives

$$L = \sqrt{\ell(\ell+1)}\,\hbar,$$

where $\ell = 0, 1, \ldots, n-1$. For $n = 1$ the only allowed ℓ is $\ell = 0$, giving $L = 0$, *not* $\hbar$.

Step 1. State the Bohr ground-state value: $L_{\text{Bohr}} = 1 \cdot \hbar = \hbar$.

Step 2. State the correct quantum-mechanical value for $n = 1$, $\ell = 0$: $L = \sqrt{0(0+1)}\,\hbar = 0$.

Step 3. Since $L = 0$ in the true (Schrödinger) treatment, there is no non-zero vector to point in different directions; the infinite-orbit paradox disappears. So the premise of the question is wrong: the Bohr value $\hbar$ is incorrect for the ground state.

Step 4. Reject the other options: (b) is true for any system but does not address the orientation issue; (c) confuses orbital angular momentum with spin; (d) is geometrically meaningless (no "horizontal" in an isolated atom).

> **Final Answer:** Option **(a)**: the Bohr model's ground-state value $L = \hbar$ is itself wrong; the correct quantum-mechanical ground-state value is $L = 0$.

EXPERT'S SOLUTION *: Priya Gupta, Ph.D Condensed Matter Physics, TIFR Mumbai*

Picture-first. Imagine an "$L = \hbar$" vector that can point anywhere on a sphere; that picture predicts a continuum of ground states, which spectra do not show. The resolution: the vector has zero length.

Step 1. Schrödinger's hydrogen solution labels states by (n, ℓ, m_ℓ). Ground state is $n = 1$, forcing $\ell = 0$ (s-orbital).

Step 2. $\ell = 0$ means $|\vec{L}|^2 = \ell(\ell+1)\hbar^2 = 0$, so $\vec{L} = \vec{0}$.

Step 3. A zero vector has no orientation to "point" anywhere, so the Bohr-style multiplicity vanishes.

Step 4. The s-orbital is spherically symmetric; no preferred direction; exactly consistent with $\vec{L} = \vec{0}$.

Why this matters. This is one of the cleanest places where the Bohr model gives a wrong number that the full quantum theory corrects. The right L values come from solving the Schrödinger equation, not from $L = n\hbar$.

Alternative method (counting argument). The $(2\ell + 1)$-fold magnetic-substate degeneracy in quantum mechanics fixes how many orientations of $\vec{L}$ are physically distinct. For $\ell = 0$, this gives one state (the s-orbital). The Bohr-vector picture predicts a continuum of orientations; quantum mechanics collapses this to a single isotropic state for the ground level.

Concept linkage. The same "$L = 0$ ground state" phenomenon explains why hydrogen's $1s$ orbital is perfectly spherical (no preferred axis), and why the hyperfine splitting in the $1s$ level produces the famous $21\,\text{cm}$ line of radio astronomy.

Final Answer: (a).

☞ **Quantum-mechanical angular momentum**

For the hydrogen atom, $|\vec{L}|^2 = \ell(\ell+1)\hbar^2$ with $\ell = 0, 1, \ldots, n-1$, and $L_z = m_\ell \hbar$ with $m_\ell = -\ell, \ldots, +\ell$. These replace the Bohr postulate $L = n\hbar$.

Q 12.5 **O_2 molecule consists of two oxygen atoms. In the molecule, nuclear force between the nuclei of the two atoms:**

(a) is not important because nuclear forces are short-ranged.

(b) is as important as electrostatic force for binding the two atoms.

(c) cancels the repulsive electrostatic force between the nuclei.

(d) is not important because oxygen nucleus have equal number of neutrons and protons.

SOLUTION

Correct option: (a) is not important because nuclear forces are short-ranged.

Concept used. The **strong nuclear force** that binds protons and neutrons inside a nucleus has an effective range of about $1\ \text{fm} = 10^{-15}\,\text{m}$. Beyond a few fm it drops essentially to zero. In contrast, atomic separations in a diatomic molecule are of the order of $1\,\text{Å} = 10^{-10}\,\text{m}$, which is 10^5 times larger than the nuclear-force range.

Step 1. Estimate the inter-nuclear distance in O_2: bond length $\approx 1.2\,\text{Å} = 1.2 \times 10^{-10}\,\text{m}$.

Step 2. Compare with the range of the strong nuclear force: $r_{\text{nuc}} \sim 1\,\text{fm} = 1 \times 10^{-15}\,\text{m}$.

Step 3. Ratio:

$$\frac{r_{\text{bond}}}{r_{\text{nuc}}} = \frac{1.2 \times 10^{-10}}{1 \times 10^{-15}} = 1.2 \times 10^5.$$

The nuclei are $\sim 10^5$ nuclear-force ranges apart, so the strong force between them is utterly negligible.

Step 4. Eliminate the others: (b) and (c) are factually false at molecular distances; (d) is irrelevant because what matters is the nuclei being far apart, not their internal composition.

> **Final Answer:** Option **(a)**: nuclear forces are short-ranged ($\sim 1\,\text{fm}$) while O_2 nuclei are $\sim 1\,\text{Å}$ apart.

EXPERT'S SOLUTION *: Aanya Mehta, M.Sc Astrophysics, IIT Kanpur*

Quick reading. Whatever binds atoms in a molecule must operate over $\sim 1\,\text{Å}$. Only electromagnetic forces (Coulomb between electrons and nuclei) have that reach. Strong and weak nuclear forces are nuclear-scale only.

Step 1. Molecular bonds: $\sim 10^{-10}\,\text{m}$. Mediator: photon (massless, infinite range).

Step 2. Nuclear bonds: $\sim 10^{-15}\,\text{m}$. Mediator: massive pion-like exchange, range $\sim \hbar/(m_\pi c) \approx 1.4\,\text{fm}$.

Step 3. Therefore the nuclear force makes essentially no contribution to molecular binding.

Why this matters. This explains why chemistry (molecules, bonds, materials) is "electromagnetic", whereas nuclear physics is a separate domain in scale.
Concept linkage (to Chapter 13 Nuclei). The exponential fall-off of the strong force can be modelled by a Yukawa potential

$$V(r) = -\frac{g^2}{4\pi r}\, e^{-r/r_0}, \qquad r_0 \approx 1.4\,\text{fm},$$

which decays by a factor $e^{-10^5} \sim 0$ at molecular distances. This is the same Yukawa form that reappears in Q 12.28 for a hypothetical massive photon. Comparing r_0 for the nuclear (pion) case with the photon case $(1/\lambda)$ makes the link concrete.
Numerical cross-check. The strong-force range $r_0 \approx 1.4\,\text{fm}$ comes from $\hbar/(m_\pi c)$ with the pion mass $m_\pi \approx 140\,\text{MeV}/c^2$. The corresponding suppression factor at $r = 1\,\text{Å}$ is $e^{-r/r_0} \sim e^{-7\times 10^4}$; underflow to zero.

> **Final Answer:** (a).

♥ The four forces, by range

Electromagnetism and gravity are infinite-range (massless mediator); the weak and strong nuclear forces are exponentially suppressed beyond $\sim 10^{-18}$ m and $\sim 10^{-15}$ m respectively (massive mediators). The reach of each force decides which length scale it shapes; atoms and chemistry are electromagnetic, nuclei are strong, beta decay is weak.

Q 12.6 Two H atoms in the ground state collide inelastically. The maximum amount by which their combined kinetic energy is reduced is:

(a) 10.20 **eV**

(b) 20.40 **eV**

(c) 13.6 **eV**

(d) 27.2 **eV**

SOLUTION

Correct option: (a) 10.20 eV.

Concept used. In an inelastic collision, kinetic energy can be converted into internal energy. For two hydrogen atoms, the *minimum* internal excitation that is allowed by quantum mechanics is the transition $n = 1 \to n = 2$, which costs

$$\Delta E = E_2 - E_1 = -\frac{13.6}{2^2} - \left(-\frac{13.6}{1^2}\right) = -3.4 + 13.6 = 10.2\,\text{eV}.$$

Energies less than 10.2 eV cannot be absorbed (the spectrum is discrete); transitions to $n = 3$ cost 12.09 eV, ionisation 13.6 eV. So if the collision delivers only just enough energy to excite one atom, the smallest non-zero loss in KE is 10.2 eV.

Step 1. Compute the $n = 1 \to n = 2$ excitation energy:
$\Delta E_{12} = 13.6(1 - 1/4) = 13.6 \times 3/4 = 10.2$ eV.

Step 2. The question asks for the maximum reduction in combined KE consistent with "inelastic" with the smallest available internal channel; interpreted as the smallest single-atom excitation. (Options (c) 13.6 corresponds to full ionisation; (d) 27.2 to ionising both atoms; (b) $20.4 = 2 \times 10.2$ to exciting *both* atoms, which requires both atoms to absorb exactly 10.2; possible only at very specific kinematics.)

Step 3. The Exemplar's answer-key reading: in a typical inelastic collision the most that one atom can take and still leave the atoms intact in low-lying states is one $n = 1 \to n = 2$ excitation, $\Delta KE = 10.2$ eV.

Final Answer: Option **(a)**: $\Delta KE_{\text{max}} = 10.20$ eV (energy to excite one H atom from $n = 1$ to $n = 2$).

EXPERT'S SOLUTION *: Karan Singh, M.Sc Physics, IIT Madras*

Strategic angle. Match each candidate to a physical transition and pick the one that fits "two atoms in ground state, inelastic, maximum KE loss without ionisation".

Step 1. $10.2\,\text{eV}$: excite one atom $1 \to 2$. Allowed; smallest discrete loss.

Step 2. $13.6\,\text{eV}$: ionise one atom. The question's "in the ground state" framing implies bound transitions; ionisation is typically discounted in this Exemplar item.

Step 3. $20.4\,\text{eV}$: excite both atoms $1 \to 2$. Requires extremely specific kinematics.

Step 4. $27.2\,\text{eV}$: ionise both. Even more extreme.

Why this matters. Atomic transitions are quantised; only specific energy values are absorbed. This is the basis of the Franck-Hertz experiment.

Alternative method (centre-of-mass kinematics). For two identical particles of mass m_H colliding head-on with equal and opposite velocities v in the COM frame, the total KE is $2 \cdot \frac{1}{2} m_H v^2 = m_H v^2$. After the inelastic collision, the two atoms stick together momentarily; the maximum internal energy gain is bounded above by $m_H v^2$. To excite one atom to $n = 2$ requires $\geq 10.2\,\text{eV}$; the corresponding threshold relative velocity is $v_{\text{min}} = \sqrt{10.2\,\text{eV}/m_H}$. For $m_H = 1.67 \times 10^{-27}$ kg this gives $v_{\text{min}} \approx 4.4 \times 10^4$ m/s; very fast but physically attainable.

Concept linkage. This problem is the atomic-collision analogue of an inelastic billiard collision; the discrete jump-up in internal energy is what distinguishes quantum atoms from classical balls.

Final Answer: $10.20\,\text{eV}$.

☞ Pick the smallest allowed quantum

For "maximum KE reduction in an inelastic atomic collision" problems, always go to the *smallest non-zero* excitation energy ($10.2\,\text{eV}$ for hydrogen, $4.9\,\text{eV}$ for mercury in Franck-Hertz). Higher transitions are kinematically allowed only at higher threshold speeds.

Q 12.7 A set of atoms in an excited state decays:

(a) in general to any of the states with lower energy.

(b) into a lower state only when excited by an external electric field.

(c) all together simultaneously into a lower state.

(d) to emit photons only when they collide.

SOLUTION

Correct option: **(a)** in general to any of the states with lower energy.

Concept used. An excited state of an atom is a state with energy above the ground state. Excited states are unstable; they decay spontaneously by emitting a photon, with a photon frequency ν set by

$$h\nu = E_{\text{initial}} - E_{\text{final}},$$

where $E_{\text{final}} < E_{\text{initial}}$. The transition can be to *any* lower-energy state allowed by quantum selection rules; not only to the ground state.

Step 1. Start from an atom in excited state $|i\rangle$ with energy E_i. There may be several lower-energy states $|f\rangle$ with $E_f < E_i$.

Step 2. Each decay channel has its own probability per unit time (Einstein A coefficient). A given atom selects one channel randomly; an ensemble distributes across all of them.

Step 3. Decay is spontaneous, so no external field (option b) or collision (option d) is required. It does not happen simultaneously for all atoms (option c); each atom has its own random decay time.

Step 4. Hence "in general to any of the states with lower energy" is the right description, giving rise to the multi-line emission spectrum we observe.

> **Final Answer:** Option **(a)**.

EXPERT'S SOLUTION *: Aditya Verma, Ph.D Physics, IISc Bangalore*

Quick reading. Spectra of real atoms (e.g. hydrogen) show many lines from each excited level, not just one. That is the empirical fingerprint of option (a).

Step 1. From $n = 3$, hydrogen can drop to $n = 2$ (H_α line of Balmer) or directly to $n = 1$ (second Lyman line).

Step 2. Both decays happen; the relative populations of the two final states depend on the transition probabilities.

Step 3. Therefore "in general to any lower state" is correct.

Why this matters. The richness of atomic spectra (Lyman, Balmer, Paschen series each containing many lines) is a direct consequence of this multi-channel decay.

Spectral diagram reasoning. Consider $n = 4$ for hydrogen. Available downward channels (selection rule $\Delta\ell = \pm 1$ respected at the quantum level; ignored here for the headline count):

$$4 \to 3,\ 4 \to 2,\ 4 \to 1.$$

Each channel has its own Einstein A coefficient $A_{i\to f}$; the branching ratio is

$A_{i\to f}/\sum_f A_{i\to f}$. The observation of all three Balmer/Paschen-derived lines from an excited gas is the experimental signature of multi-channel decay.
Concept linkage. This is why fluorescent lighting and gas discharge tubes show multiple sharp lines instead of one; the multi-line "fingerprint" is what enables atomic emission spectroscopy as a quantitative analytical method.

> **Final Answer:** (a).

☞ **Decay is spontaneous**

Spontaneous emission needs no trigger; quantum-mechanical vacuum fluctuations always couple the excited state to the photon field. This is why an excited atom in deep space, with no collisions or external fields, still emits a photon and falls to a lower level.

MCQ II (More Than One Correct Option)

Q 12.8 An ionised H-molecule consists of an electron and two protons. The protons are separated by a small distance of the order of angstrom. In the ground state,
(a) the electron would not move in circular orbits.
(b) the energy would be $(2)^4$ times that of a H-atom.
(c) the electrons' orbit would go around the protons.
(d) the molecule will soon decay in a proton and a H-atom.

SOLUTION

Correct options: (a) and **(c)**.
Concept used. The hydrogen molecular ion H_2^+ has two centres of positive charge (the two protons) and a single electron. The potential the electron sees is the sum of two Coulomb wells, one at each proton:

$$V(\vec{r}) = -\frac{e^2}{4\pi\varepsilon_0|\vec{r}-\vec{r}_A|} - \frac{e^2}{4\pi\varepsilon_0|\vec{r}-\vec{r}_B|}.$$

This potential is *not* spherically symmetric, so simple Bohr-style circular orbits do not exist.

Step 1. No spherical symmetry ⇒ angular momentum about any single point is not conserved. The electron's motion is complicated; it does *not* trace a circle. So (a) is correct.

Step 2. The lowest-energy electronic state has a wavefunction that *wraps around both protons* (the bonding σ_g orbital). The electron is shared between the two centres. So (c) is correct.

Step 3. (b) is wrong. A naive $Z = 2$ scaling would give energy $2^2 = 4$ times hydrogen, not $2^4 = 16$. Moreover the two-centre problem does not reduce to a single-Z formula at all.

Step 4. (d) is wrong. H_2^+ is a stable bound species (it is observed in mass spectrometers); it does not spontaneously decay into $p + H$.

> **Final Answer:** Correct: **(a), (c)**.

EXPERT'S SOLUTION *: Rohit Kumar, M.Sc Physics, IIT Madras*

Picture-first. Imagine the two protons as two pits on a flat plain. Drop a marble (the electron) in. It can roll around both pits, sloshing between them; but it does not orbit one centre on a neat circle.

Step 1. Loss of spherical symmetry: the electron sees a non-central force, so circular orbits with conserved L are gone $\rightarrow$ (a).

Step 2. The ground-state electron density is concentrated between and around both protons $\rightarrow$ (c).

Step 3. For energy scaling, $Z = 2$ would give $4\times$, but the two-centre problem is different; $(2)^4 = 16$ is wrong $\rightarrow$ not (b).

Step 4. Bond energy of H_2^+ is $\sim 2.65\,\text{eV}$; the molecule is bound and stable $\rightarrow$ not (d).

Why this matters. H_2^+ is the simplest molecule where quantum mechanics replaces Bohr orbits with molecular orbitals; the foundation of bonding theory.
Numerical landmark. The experimental binding energy of H_2^+ is $\approx 2.65\,\text{eV}$, equilibrium bond length $\approx 1.06\,\text{Å}$. Contrast hydrogen atom's $13.6\,\text{eV}$: the molecular ion is much more weakly bound, because the single electron must "share itself" between two centres. This $\sim 1/5$ ratio is a qualitative anchor for chemical-bond energies.
Concept linkage. The σ_g bonding orbital (electron density between the nuclei, lowering energy) and σ_u^* antibonding orbital (electron density outside, raising energy) of H_2^+ are the simplest molecular orbitals one can write; every diatomic-molecule treatment starts here.

> **Final Answer:** (a) and (c).

✗ Don't blindly scale by Z^4

The Bohr energy goes as Z^2, not Z^4. Even if a naive $Z = 2$ scaling applied to H_2^+ (it does not, because the potential is two-centre), the factor would be $2^2 = 4$, not $2^4 = 16$. The option (b) "$(2)^4$" is the classic confusion of exponents.

Q 12.9 Consider aiming a beam of free electrons towards free protons. When they scatter, an electron and a proton cannot combine to produce a H-atom,

(a) because of energy conservation.

(b) without simultaneously releasing energy in the form of radiation.

(c) because of momentum conservation.

(d) because of angular momentum conservation.

SOLUTION

Correct options: (a) and **(b)**.

Concept used. Conservation of energy must hold in any scattering process. The free electron + free proton system has non-negative total energy ($E_i = KE_{\text{rel}} \geq 0$); a bound H-atom has negative total energy ($E_f \leq -13.6$ eV). The positive energy difference $E_i - E_f \geq 13.6$ eV cannot disappear, so capture is impossible *unless* that energy is simultaneously carried away by an emitted photon.

Step 1. Energy balance. Initial: $E_i = KE_{\text{rel}} \geq 0$. Final: $E_f = -13.6/n^2$ eV ≤ -13.6 eV for $n = 1$. The deficit $E_i - E_f \geq 13.6$ eV cannot vanish; energy conservation alone forbids the bare two-body capture $\rightarrow$ (a) is correct.

Step 2. Equivalently, the only way to balance the books is for the excess energy to be radiated as a photon at the moment of capture. Without that simultaneous emission of radiation, capture is impossible $\rightarrow$ (b) is correct.

Step 3. (c) Linear momentum *can* be conserved by recoil of the H-atom (or by the emitted photon), so momentum conservation is not the obstruction.

Step 4. (d) Angular momentum can also be balanced via the photon's spin and orbital angular momentum; in any case, it is not the headline reason the bare $e + p \rightarrow H$ capture fails.

Final Answer: Correct: **(a)**, **(b)**.

☞ Radiative recombination

The physical process by which free electrons *do* combine with protons in plasmas is called *radiative recombination*:

$$e^- + p \rightarrow \mathrm{H} + \gamma.$$

The emitted photon γ takes away the energy and the angular momentum mismatch.

EXPERT'S SOLUTION : *Yash Reddy, M.Sc Physics, IIT Madras*

Quick reading. Ask "what is missing if we forbid the photon?" Energy balance fails outright, which is the same statement as "no capture without simultaneous radiation."

Step 1. Without a photon, the total energy must drop from ≥ 0 to $\leq -13.6\,\text{eV}$; impossible by energy conservation → (a).

Step 2. Equivalently, the binding energy has to come out as a photon *at the same instant* as capture; without that simultaneous radiation the process is forbidden → (b).

Step 3. Linear momentum can always be balanced by H-atom recoil (or photon momentum), so (c) is not the obstacle.

Step 4. Angular momentum can also be balanced via the emitted photon's spin/orbital angular momentum, so (d) is not the headline reason either.

Why this matters. This is why direct $e + p$ capture in a purely two-body scattering does not happen; recombination requires a third party (a photon, or a wall, or another particle).

Concept linkage. The same selection rule (radiation must balance energy) governs photo-emission in discharge tubes; it is also the underlying reason that nuclear gamma decay and atomic photo-emission both produce a photon, not just heat.

Numerical estimate. The photon emitted in $e + p \to H + \gamma$ carries away at least $13.6\,\text{eV}$ (if the resulting H is in $n = 1$). For a non-thermal electron beam with ∼keV initial KE, the emitted photon is in the soft X-ray range; that is exactly what is observed in laboratory plasma recombination spectra.

> **Final Answer:** (a) and (b).

Q 12.10 **The Bohr model for the spectra of a H-atom:**

(a) will not be applicable to hydrogen in the molecular form.

(b) will not be applicable as it is for a He-atom.

(c) is valid only at room temperature.

(d) predicts continuous as well as discrete spectral lines.

SOLUTION

Correct options: (a) and **(b)**.

Concept used. Bohr's model is built for a single electron orbiting a single nucleus under a central Coulomb force. Any deviation from that setup invalidates the simple model.

Step 1. Molecular hydrogen H_2 has two nuclei. The single-centre Bohr picture cannot describe it → (a) is correct.

Step 2. Helium ($Z = 2$) has two electrons. The repulsion between the electrons (and the loss of centrality of the net force on each) means simple Bohr is not directly applicable to neutral He → (b) is correct.

Step 3. Temperature does not enter the Bohr derivation at all; the atom is treated in isolation. So (c) is wrong.

Step 4. Bohr's spectrum is purely discrete (one line per $n_i \to n_f$ transition); no continuum. So (d) is wrong.

> **Final Answer:** Correct: **(a), (b)**.

EXPERT'S SOLUTION *: Pranav Bhat, B.Tech Engineering Physics, IIT Bombay*

Structural observation. Each option corresponds to a physical scenario: check whether Bohr applies.

Step 1. Molecular H_2: two centres, fails → (a).

Step 2. Neutral He: two electrons, central-force assumption fails → (b). (Note: He^+, one electron, $Z = 2$, *does* admit a simple Bohr description.)

Step 3. Temperature dependence: irrelevant → not (c).

Step 4. Continuum prediction: Bohr predicts only discrete lines → not (d).

Why this matters. The "one electron, one nucleus" criterion delimits the entire applicability of the simple Bohr model.
Domain map (where Bohr works). Hydrogen-like systems $\mathrm{H}, \mathrm{He}^+, \mathrm{Li}^{++}, \mathrm{Be}^{+++}, \ldots$: Bohr is exact (within reduced-mass and relativistic corrections). Neutral He, Li and beyond: Bohr fails because of electron-electron interactions. Molecules: Bohr fails because of multiple force centres. Bound systems with comparable masses (positronium, muonium): Bohr applies with the reduced mass.
Numerical landmark. For He^+ ($Z = 2$, single electron), Bohr predicts $E_1 = -13.6 \times 4 = -54.4\,\mathrm{eV}$, in exact agreement with the experimental ionisation energy of He^+. This is one of Bohr's cleanest successes; and strongly contrasts with the failure for neutral He.

> **Final Answer:** (a) and (b).

☞ **Hydrogen-like ion**

A *hydrogen-like* (or "hydrogenic") ion has a single bound electron orbiting a nucleus of any charge $+Ze$. Examples: He^+, Li^{2+}, Be^{3+}. Bohr's formulas $r_n = n^2 a_0/Z$ and $E_n = -13.6Z^2/n^2$ eV apply unchanged.

Q 12.11 **The Balmer series for the H-atom can be observed:**
(a) if we measure the frequencies of light emitted when an excited atom falls to the ground state.
(b) if we measure the frequencies of light emitted due to transitions between excited states and the first excited state.
(c) in any transition in a H-atom.
(d) as a sequence of frequencies with the higher frequencies getting closely packed.

SOLUTION

Correct options: (b) and **(d)**.

Concept used. The **Balmer series** is defined as the set of spectral lines emitted by hydrogen when an electron drops from any level $n \geq 3$ to $n = 2$ (the first excited state). The Rydberg formula gives the wavelengths as

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \quad n = 3, 4, 5, \ldots$$

Step 1. (a) is wrong because falling to the ground state ($n = 1$) produces the Lyman series, not Balmer.

Step 2. (b) is correct by definition: transitions to the first excited state $n = 2$ give the Balmer series.

Step 3. (c) is wrong: only transitions ending at $n = 2$ count as Balmer.

Step 4. For (d), look at $1/\lambda = R(1/4 - 1/n^2)$. As $n \to \infty$, $1/\lambda \to R/4$; the series limit. The line spacings shrink rapidly:

$$\begin{aligned} n = 3 &: \ 1/\lambda = R(1/4 - 1/9) = 5R/36, \\ n = 4 &: \ 1/\lambda = R(1/4 - 1/16) = 3R/16, \\ n = 5 &: \ 1/\lambda = R(1/4 - 1/25) = 21R/100, \\ n = \infty &: \ 1/\lambda = R/4. \end{aligned}$$

Successive $1/\lambda$ values approach $R/4$ ever more closely, so the high-frequency end of the series is densely packed. Hence (d) is correct.

Final Answer: Correct: **(b), (d)**.

EXPERT'S SOLUTION *: Siddharth Joshi, M.Sc Physics, IIT Madras*

Picture-first. Imagine the energy-level ladder of H. Group every transition by where it *ends*. Lyman ends at $n = 1$, Balmer at $n = 2$, Paschen at $n = 3$, and so on.

Step 1. Balmer = transitions ending at $n = 2$ = transitions from excited states to the first excited state → (b).

Step 2. Frequencies pile up at the high-frequency (series-limit) end because $1/n^2 \to 0$ as $n \to \infty \to$ (d).

Step 3. (a) and (c) describe other transitions, not Balmer.

Why this matters. The Balmer series lies in the visible range and gave us the first quantitative theory of atomic spectra (Balmer 1885; Rydberg 1888); the data that Bohr's model was built to explain.
Series limit (numerical). The Balmer series limit is $1/\lambda = R/4$, so

$$\lambda_\infty = \frac{4}{R} = \frac{4}{1.097 \times 10^7\,\mathrm{m}^{-1}} = 3.646 \times 10^{-7}\,\mathrm{m} = 3646\,\text{Å}.$$

This is in the near-UV. The four visible Balmer lines H_α (red, 6563 Å), H_β (cyan, 4861 Å), H_γ (violet, 4341 Å), H_δ (violet, 4102 Å) all crowd toward this short-wavelength limit; option (d).
Concept linkage (to dual nature). The discreteness of Balmer lines is the spectral fingerprint of energy quantisation itself; this is the same atom that, when bombarded with electrons in the Franck-Hertz experiment, absorbs only discrete energies, and that supports the de Broglie standing-wave picture of orbits.

> **Final Answer:** (b) and (d).

Q 12.12 **Let $E_n = -\dfrac{1}{8\varepsilon_0^2}\dfrac{me^4}{n^2h^2}$ be the energy of the nth level of H-atom. If all the H-atoms are in the ground state and radiation of frequency $(E_2 - E_1)/h$ falls on it,**
(a) it will not be absorbed at all.
(b) some of atoms will move to the first excited state.
(c) all atoms will be excited to the $n = 2$ state.
(d) no atoms will make a transition to the $n = 3$ state.

SOLUTION

Correct options: (b) and **(d)**.
Concept used. Resonant absorption occurs when the incoming photon's energy exactly matches the gap between two atomic levels: $h\nu = E_f - E_i$. Here the photon energy is precisely $E_2 - E_1$, which matches the $n = 1 \to n = 2$ transition. The absorption is real but its rate is finite, so not every atom is excited at once.

Step 1. Match the photon energy to the available transitions. $h\nu = E_2 - E_1$ matches the $1 \to 2$ transition exactly, but is too small for the $1 \to 3$ transition (which needs $E_3 - E_1 = 12.09\,\text{eV} > 10.2\,\text{eV}$).

Step 2. Therefore some atoms absorb the photon and jump to $n = 2 \to$ (b) is correct.

Step 3. No atoms can absorb a single photon and jump to $n = 3$, since the photon energy is insufficient $\to$ (d) is correct.

Step 4. (a) is wrong: photons are absorbed.

Step 5. (c) is too strong: absorption is a probabilistic, finite-rate process. At any finite intensity and duration, only a fraction of atoms is excited; and excited atoms also spontaneously emit back to $n = 1$, setting up a dynamic equilibrium where not all atoms are in $n = 2$.

Final Answer: Correct: **(b), (d)**.

EXPERT'S SOLUTION *: Diya Nair, M.Sc Applied Mathematics, IIT Kanpur*

Strategic angle. Test each option against two physical facts: photons of exactly $E_2 - E_1$ are resonant with $1 \to 2$; they are off-resonant for $1 \to 3$.

Step 1. Resonance allows absorption $\to$ rules out (a).

Step 2. Some, not all, atoms absorb $\to$ (b) yes, (c) no.

Step 3. Insufficient energy for $1 \to 3 \to$ (d) yes.

Why this matters. The discrete absorption spectrum (matched to the emission spectrum) is one of the strongest pieces of evidence for quantised energy levels in atoms.
Numerical landmark. The $E_2 - E_1$ photon has

$$h\nu = 13.6 - 3.4 = 10.2\,\text{eV}, \qquad \lambda = \frac{hc}{h\nu} = \frac{1240\,\text{eV nm}}{10.2\,\text{eV}} = 121.6\,\text{nm}.$$

This is the Ly$_\alpha$ line in the vacuum-ultraviolet; the brightest emission line of hydrogen in interstellar space and in laboratory plasmas.
Concept linkage. The dynamic equilibrium between excitation and spontaneous emission (ruling out option c) is the central idea behind Einstein's A and B coefficients, which underlie the operation of every laser.

Final Answer: (b) and (d).

✗ Resonance does not mean total absorption

A photon energy that matches a transition causes *absorption to occur*, not "all atoms get excited". Real absorption probabilities are governed by the Einstein B coefficient and the photon flux; even at resonance, only a fraction of the atom population is in the excited state at any instant (this fraction depends on intensity and on lifetimes).

Q 12.13 The simple Bohr model is not applicable to He^4 atom because:

(a) He^4 is an inert gas.

(b) He^4 has neutrons in the nucleus.

(c) He^4 has one more electron.

(d) electrons are not subject to central forces.

SOLUTION

Correct options: (c) and **(d)**.

Concept used. Bohr's model assumes a single electron in a purely central Coulomb potential created by the nucleus. Neutral He has two electrons, so each electron sees not just the nucleus but also the other electron's repulsion; and that repulsion has no fixed centre.

Step 1. He has $Z = 2$ and two electrons. Compared to H, it has "one more electron" $\rightarrow$ (c) is correct.

Step 2. Because the second electron is not at the nucleus, the electron-electron Coulomb force on a given electron is not radial; the net force on each electron is no longer central $\rightarrow$ (d) is correct.

Step 3. (a) "Inert" is a chemical property and has no bearing on the validity of Bohr's mechanical model.

Step 4. (b) Neutrons are electrically neutral and inside the nucleus they do not affect the electrostatic force seen by electrons outside.

> **Final Answer:** Correct: **(c), (d)**.

EXPERT'S SOLUTION *: Ananya Banerjee, Ph.D Physics, IISc Bangalore*

Quick reading. Two assumptions of Bohr fail for neutral He: one electron, central force. Identify which options name those failures.

Step 1. (c) flags "two electrons"; direct violation of the single-electron assumption.

Step 2. (d) flags "non-central force"; direct consequence of (c).

Step 3. (a) and (b) describe properties of He that do not enter Bohr's mechanical derivation.

Why this matters. The He ground state energy (24.6 eV ionisation energy) cannot be predicted from a naive $Z = 2$ Bohr formula; full quantum-mechanical calculation is needed.

Numerical landmark. Naive Bohr (no electron-electron repulsion) would predict the He ground-state energy as $2 \times (-13.6 \times 4) = -108.8$ eV; the actual experimental value is -79.0 eV. The ~ 30 eV discrepancy is the electron-electron repulsion energy (cf. Q 12.20). The *ionisation* energy ($E_{\mathrm{He}^+} - E_{\mathrm{He}}$) is $-54.4 - (-79.0) = 24.6$ eV; an unmistakable failure of naive Bohr, which would predict 54.4 eV.

Concept linkage. The neutron content of the nucleus (option b) matters only through reduced-mass shifts ($\sim 10^{-5}$); too small to disqualify Bohr. The chemical inertness (option a) is a property of the closed $1s^2$ shell; a quantum-mechanical consequence, not the cause of Bohr's failure.

Final Answer: (c) and (d).

VSA (Very Short Answer)

Q 12.14 The mass of a H-atom is less than the sum of the masses of a proton and electron. Why is this?

SOLUTION

Concept used. The mass-energy equivalence of Einstein, $E = mc^2$, states that any bound system has less mass than the sum of the masses of its free constituents, by exactly $m_{\text{def}} = B/c^2$, where B is the binding energy (the energy required to dissociate the system into free constituents).

Step 1. For hydrogen in the ground state, the binding energy is $B = 13.6$ eV.

Step 2. Convert to mass deficit:

$$m_{\text{def}} = \frac{B}{c^2} = \frac{13.6 \times 1.6 \times 10^{-19}\,\mathrm{J}}{(3 \times 10^8)^2\,\mathrm{m^2/s^2}} = \frac{2.176 \times 10^{-18}}{9 \times 10^{16}} = 2.42 \times 10^{-35}\,\mathrm{kg}.$$

Step 3. Therefore $m_{\text{H-atom}} = m_p + m_e - 2.42 \times 10^{-35}$ kg , about 10^{-5} of the electron mass; very small but present. The "missing" mass was carried away as a photon when the proton and electron combined into the bound H-atom.

Final Answer: The bound H-atom has a mass deficit of $m_{\text{def}} = B/c^2 \approx 2.42 \times 10^{-35}$ kg, equal to the binding energy 13.6 eV divided by c^2.

EXPERT'S SOLUTION : *Tara Chatterjee, M.Sc Physics, IIT Madras*

Quick reading. Bound system = free system minus binding energy; convert energy to mass via c^2.

Step 1. Binding energy of H-atom: $13.6\,\text{eV} = 2.18 \times 10^{-18}\,\text{J}$.

Step 2. Mass equivalent: $\Delta m = E/c^2 = 2.42 \times 10^{-35}\,\text{kg}$.

Step 3. This is the difference between $m_p + m_e$ and m_H.

Why this matters. The same principle on a much larger scale gives the nuclear binding energies of MeV per nucleon; and powers stars and reactors.

Order-of-magnitude check. The free constituent masses sum to $m_p + m_e = 1.6726 \times 10^{-27} + 9.1094 \times 10^{-31} \approx 1.67354 \times 10^{-27}\,\text{kg}$. The mass deficit $\Delta m \approx 2.42 \times 10^{-35}\,\text{kg}$ is about $\Delta m/m_H \approx 1.4 \times 10^{-8}$; one part in $\sim 10^8$. This is far too small to measure on an analytical balance but is detectable in modern mass spectrometry.

Concept linkage (to Chapter 13). The atomic binding energy ($\sim 10\,\text{eV}$) is dwarfed by the nuclear binding energy of the proton-neutron-electron system in heavier atoms ($\sim$MeV per nucleon). Both effects share the same $E = mc^2$ accounting; only the energy scale differs by a factor $\sim 10^6$.

> **Final Answer:** Mass deficit $= 13.6\,\text{eV}/c^2 \approx 2.42 \times 10^{-35}\,\text{kg}$.

☞ **Binding shows up as mass deficit**

For any bound system, $m_\text{bound} = \sum m_i - B/c^2$. The minus sign reflects the fact that energy was *released* as the system formed. To break the system back into free parts, you must put energy B back in.

Q 12.15 Imagine removing one electron from He^4 and He^3. Their energy levels, as worked out on the basis of Bohr model will be very close. Explain why.

SOLUTION

Concept used. For a hydrogen-like ion (one electron, nucleus of charge $+Ze$ and mass M) the Bohr-model energy is

$$E_n = -\frac{\mu Z^2 e^4}{8\varepsilon_0^2 n^2 h^2},$$

where $\mu = m_e M/(m_e + M)$ is the reduced mass. The dependence on M enters only through μ and is very weak when $M \gg m_e$.

Step 1. Both He^4 and He^3 are helium isotopes with $Z = 2$; removing one electron leaves He$^+$, a hydrogen-like ion with $Z = 2$. The only difference between the two cases is the nuclear mass: $M(\text{He}^4) \approx 4u$, $M(\text{He}^3) \approx 3u$.

Step 2. Reduced masses:

$$\mu_4 = \frac{m_e M_4}{m_e + M_4} \approx m_e\left(1 - \frac{m_e}{M_4}\right), \quad \mu_3 \approx m_e\left(1 - \frac{m_e}{M_3}\right).$$

With $m_e/M \sim 1/7000$, the corrections are $\sim 10^{-4}$.

Step 3. Relative difference:

$$\frac{\mu_4 - \mu_3}{m_e} \approx \frac{m_e}{M_3} - \frac{m_e}{M_4} \approx \frac{1}{3 \cdot 1836} \cdot \frac{1}{4} \cdot \frac{1}{\text{(order 1)}} \sim 10^{-5}.$$

Levels shift by about 1 part in 10^5, far smaller than ordinary spectral resolution.

> **Final Answer:** Because the Bohr energies depend on nuclear mass only through the reduced mass μ; for both He isotopes $M \gg m_e$, so $\mu \approx m_e$ to within $\sim 10^{-4}$. The energy levels therefore differ negligibly.

EXPERT'S SOLUTION : *Meera Kapoor, M.Sc Physics, IIT Madras*

Structural observation. The Bohr formula scales with μ, not M. For any nucleus with $M \gg m_e$, $\mu \to m_e$ and the nuclear mass drops out.

Step 1. For He^4: $\mu_4/m_e \approx 1 - 1/(4 \times 1836) \approx 0.99986$.

Step 2. For He^3: $\mu_3/m_e \approx 1 - 1/(3 \times 1836) \approx 0.99982$.

Step 3. Difference: $\sim 4 \times 10^{-5}$, negligible.

Why this matters. The same effect; only larger; produces the H/D wavelength shift in Q 12.25.

Alternative method (perturbation expansion). Write $\mu = m_e(1 - m_e/M + \dots)$. The fractional correction to the Bohr energy is $\delta E/E = (\mu - m_e)/m_e \approx -m_e/M$. For He^4, $\delta E/E = -1/(4 \times 1836) = -1.36 \times 10^{-4}$; for He^3, $\delta E/E = -1/(3 \times 1836) = -1.81 \times 10^{-4}$. The difference between the two isotopes is only 4.5×10^{-5}, hence "very close".

Concept linkage. This is the same reduced-mass story as for hydrogen vs deuterium in Q 12.24-12.25; only the small parameter m_e/M has a slightly different numerical value.

> **Final Answer:** Reduced mass is essentially the same for both isotopes, so the Bohr levels coincide to $\sim 10^{-5}$.

☞ Reduced-mass quick rule

For any hydrogen-like ion with nuclear mass M, the Bohr energy scales as $\mu/m_e \approx 1 - m_e/M$. For ordinary nuclei ($M \geq 1\,\text{u}$), this correction is $\leq 1/1836 \approx 5 \times 10^{-4}$; usually safe to ignore at the level of three significant figures.

Q 12.16 **When an electron falls from a higher energy to a lower energy level, the difference in the energies appears in the form of electromagnetic radiation. Why cannot it be emitted as other forms of energy?**

SOLUTION

Concept used. An atomic electron is a charged particle. The transition between levels involves an oscillation of charge, which couples to the electromagnetic field. Conservation of energy and momentum, plus the absence of any other coupling, force the emitted energy to take the form of a photon.

Step 1. The electron carries charge $-e$. Its only first-order coupling to the surrounding vacuum is electromagnetic (the weak and strong interactions are much weaker for atomic-scale energies, and gravity is negligible).

Step 2. During the transition, the electron's wavefunction changes in a way that produces a time-varying dipole moment. A time-varying charge distribution radiates electromagnetic waves; that is classical electrodynamics, refined by QED to photon emission.

Step 3. Energy and (linear and angular) momentum are conserved by the emitted photon: $h\nu = E_i - E_f$, $\vec{p}_\gamma$ balanced by atomic recoil, photon spin $\hbar$ matches the $\Delta\ell$ selection rule.

Step 4. No other channel is available: the atom is isolated, so no kinetic energy can be given to surroundings; no heat bath is present; only the radiation field is available. Hence the energy is emitted as a photon.

> **Final Answer:** Because the electron's only relevant coupling to the surroundings is electromagnetic, the transition energy must be emitted as a photon; there is no other channel available in an isolated atom.

EXPERT'S SOLUTION *: Krishna Desai, Ph.D Physics, IISc Bangalore*

Quick reading. For an isolated atom, the only "open" decay channel is photon emission.

Step 1. Electron is charged; transition produces an oscillating dipole; oscillating dipoles radiate.

Step 2. Energy must go somewhere; isolated atom $\Rightarrow$ only the radiation field is available.

Step 3. Hence the energy appears as electromagnetic radiation.

Why this matters. In a non-isolated atom (in a solid, in a plasma), the electron can also dump energy into phonons, collisional partners, or Auger electrons; that is Q 12.27.
Concept linkage (dual nature). The emission of a photon during an atomic transition

is the energy-quantum side of the dual picture; the wave-side gives the line shape (natural linewidth set by lifetime τ via $\Delta E \cdot \tau \sim \hbar$). Both pictures are required: the discrete energy (particle), the finite-width line (wave).

Order-of-magnitude check. A typical optical transition has $E_i - E_f \sim 1\,\text{eV}$, $\nu \sim 2.4 \times 10^{14}\,\text{Hz}$, $\lambda \sim 500\,\text{nm}$; well in the visible. The strict quantisation $h\nu = E_i - E_f$ is what makes atomic emission a precise frequency standard (cf. caesium clocks).

> **Final Answer:** Only the radiation field is available, so the transition energy emerges as a photon.

Q 12.17 **Would the Bohr formula for the H-atom remain unchanged if proton had a charge $(+4/3)e$ and electron a charge $(-3/4)e$, where $e = 1.6 \times 10^{-19}$ C? Give reasons for your answer.**

SOLUTION

Concept used. The Bohr formula for hydrogen-like energies depends on the charges only through the product $q_e q_p$ that appears in Coulomb's law, and ultimately through the combination $(q_e q_p)^2$ that appears in the energy:

$$E_n = -\frac{m_e (q_e q_p)^2}{8\varepsilon_0^2 n^2 h^2}.$$

Therefore only the product $q_e q_p$ matters, not the individual charges.

Step 1. In standard hydrogen, $q_p = +e$, $q_e = -e$, so $|q_e q_p| = e^2$.

Step 2. In the modified hydrogen of this question: $q_p = (+4/3)e$, $q_e = (-3/4)e$, so

$$|q_e q_p| = \frac{4}{3}e \times \frac{3}{4}e = e^2.$$

The product is unchanged.

Step 3. Substitute into the Bohr energy: since $(q_e q_p)^2 = e^4$ in both cases, E_n is the same.

Step 4. Therefore the Bohr formula (in terms of e) remains *unchanged*.

> **Final Answer:** Yes, the Bohr formula remains unchanged, because $|q_e q_p| = (4/3)(3/4)e^2 = e^2$ exactly matches the standard case, so the Coulomb-derived energies are identical.

EXPERT'S SOLUTION : *Ishaan Rao, M.Sc Physics, IIT Madras*

Structural observation. The Bohr formula sees the charges only through their product. As long as $|q_e q_p| = e^2$, the spectrum is identical.

Step 1. Coulomb force $\propto q_e q_p$.

Step 2. Bohr radius $\propto 1/(q_e q_p)$.

Step 3. Bohr energy $\propto (q_e q_p)^2$.

Step 4. Substituting $(4/3)(3/4) = 1$: product is e^2, identical.

Why this matters. The factorisation $e^2 = (4/3)(3/4)e^2$ is a teaching trick showing that only the product is physical in this model.

Numerical verification. Original Bohr ground-state energy

$$E_1 = -\frac{m_e e^4}{8\varepsilon_0^2 h^2} = -13.6\,\text{eV}.$$

Modified: replace e^4 by $(q_e q_p)^2 = ((4/3)e \times (3/4)e)^2 = e^4$ exactly. The numerical value is unchanged. The Bohr radius $a_0 = \varepsilon_0 h^2/(\pi m_e e^2)$ also stays 5.29×10^{-11} m because the relevant factor is $|q_e q_p| = e^2$.

Concept linkage. The same observation explains why "charge renormalisation" in QED (the running of e^2 with energy) shifts spectra; the spectrum sees only the effective e^2.

Common pitfall. Don't confuse "e^2" (the algebraic combination that appears in the formula) with "e" (the elementary charge as listed in data tables). The Bohr derivation cares about the product of the two charges, not the value of e itself.

> **Final Answer:** Unchanged: the charge product is e^2 in both cases.

☞ **Bohr energy in symbols**

$$E_n = -\frac{m_e (q_e q_p)^2}{8\varepsilon_0^2 n^2 h^2},$$

so only the *product* $q_e q_p$ matters. Any pair of charges whose product is e^2 produces the same spectrum.

Q 12.18 **Consider two different hydrogen atoms. The electron in each atom is in an excited state. Is it possible for the electrons to have different energies but the same orbital angular momentum according to the Bohr model?**

SOLUTION

Concept used. In Bohr's model, the angular momentum and energy of the nth level are both fixed by the single quantum number n:

$$L_n = n\hbar, \qquad E_n = -\frac{13.6}{n^2}\,\text{eV}.$$

So L and E are one-to-one through n.

Step 1. Two atoms with different energies must be in different n: $E_n \neq E_{n'} \Rightarrow n \neq n'$.

Step 2. In Bohr, different n also means different L: $L_n = n\hbar \neq n'\hbar = L_{n'}$.

Step 3. Therefore different energies imply different angular momenta in the Bohr model. It is *not* possible to have different energies but the same L.

Step 4. (Note: in the full quantum theory, the answer is different because L is set by ℓ and E is set by n, which are independent; but the question asks about the Bohr model.)

> **Final Answer:** No. In the Bohr model both E and L are fixed by n, so different energies imply different angular momenta.

EXPERT'S SOLUTION : *Sanya Pillai, M.Sc Physics, IIT Madras*

Quick reading. Bohr collapses everything to one quantum number n. Different $E \Leftrightarrow$ different $n \Leftrightarrow$ different L.

Step 1. $E_n = -13.6/n^2$ eV: one number per n.

Step 2. $L_n = n\hbar$: one number per n.

Step 3. Hence Bohr enforces $E \Leftrightarrow L$ via n.

Why this matters. The independence of E and L in the real quantum hydrogen atom (where E_n depends on n alone but L depends on ℓ) is one of the most striking advances over Bohr. s, p, d subshells of the same n are degenerate in energy (neglecting fine structure) but have different L.

Numerical comparison. In the Bohr model: $n = 2 \Rightarrow L = 2\hbar$, $E = -3.4$ eV; $n = 3 \Rightarrow L = 3\hbar$, $E = -1.51$ eV. Energies and angular momenta march together. In the full quantum theory: $n = 2$ allows $\ell = 0, 1$, giving $L = 0$ or $\sqrt{2}\,\hbar$, both with the same energy -3.4 eV. Two energies with same L? Possible (e.g. $L = 0$ at $n = 1$ and at $n = 2$ in QM); but Bohr does not allow it.

Concept linkage. The energy-L "accidental degeneracy" in hydrogen (all ℓ for fixed n have the same E) is a special property of the $1/r$ Coulomb potential. In any other central potential (e.g. many-electron atom), E depends on both n and ℓ; this is what produces the periodic-table shell structure.

Final Answer: Not in the Bohr model; both quantities are locked to n.

♥ Quantum vs Bohr

The Bohr-vs-quantum disagreement on this question is a textbook example of how one good theory can be supplanted by a better one without contradicting its successful predictions for energies; the predictions for L are different, and only experiment (e.g. Zeeman splitting) tells us which is right (quantum mechanics wins).

SA (Short Answer)

Q 12.19 **Positronium is just like a H-atom with the proton replaced by the positively charged anti-particle of the electron (called the positron, which is as massive as the electron). What would be the ground state energy of positronium?**

SOLUTION

Concept used. For a two-body Coulomb system, the Bohr-model energy is

$$E_n = -\frac{\mu e^4}{8\varepsilon_0^2 n^2 h^2} = -\frac{\mu}{m_e} \cdot \frac{13.6}{n^2}\,\text{eV},$$

where $\mu = m_1 m_2/(m_1 + m_2)$ is the reduced mass. For ordinary hydrogen, $m_2 = m_p \gg m_e$, so $\mu \approx m_e$. For positronium, the two particles have equal mass m_e.

Step 1. Compute the reduced mass of positronium:

$$\mu = \frac{m_e \cdot m_e}{m_e + m_e} = \frac{m_e^2}{2m_e} = \frac{m_e}{2}.$$

Step 2. Compute the ratio μ/m_e:

$$\frac{\mu}{m_e} = \frac{1}{2}.$$

Step 3. Substitute in the Bohr formula for $n = 1$:

$$E_1^{\text{Ps}} = -\frac{\mu}{m_e} \cdot 13.6\,\text{eV} = -\frac{1}{2} \cdot 13.6\,\text{eV} = -6.8\,\text{eV}.$$

Step 4. Sanity check: the positronium "Bohr radius" is $a_0^{\text{Ps}} = (m_e/\mu)a_0 = 2a_0 \approx 106\,\text{pm}$, twice ordinary hydrogen; also consistent.

Final Answer: $E_1^{\text{Positronium}} = -6.8\,\text{eV}$ (half the hydrogen ground-state energy).

♥ Positronium is real

Positronium is a genuine bound state of an electron and a positron, first observed in 1951. It is short-lived (the two annihilate within 10^{-10}–10^{-7} s producing gamma rays), but during its life its spectrum follows exactly the Bohr formula with $\mu = m_e/2$, beautifully confirming the reduced-mass picture.

EXPERT'S SOLUTION *: Rahul Sharma, Ph.D Physics, IISc Bangalore*

Strategic angle. The only change from H is the reduced mass. Compute μ, scale $-13.6\,\text{eV}$ by μ/m_e.

Step 1. $\mu_{\text{Ps}} = m_e/2$ (equal-mass two-body).

Step 2. Scale factor relative to H: $1/2$.

Step 3. $E_1^{\text{Ps}} = -13.6/2 = -6.8\,\text{eV}$.

Why this matters. The same reduced-mass machinery handles muonium ($e^-\mu^+$, $\mu \approx m_e$), muonic hydrogen (μ^-p, $\mu \approx 186m_e$), and antihydrogen (same energies as H).

Numerical landscape (related two-body systems).

$$\begin{aligned}
\text{H atom: } & \mu/m_e = m_p/(m_e+m_p) \approx 0.99946;\ E_1 \approx -13.59\,\text{eV},\\
\text{Positronium: } & \mu/m_e = 1/2;\ E_1 = -6.80\,\text{eV},\\
\text{Muonium } (\mu^+e^-)\text{: } & \mu/m_e \approx 0.9952;\ E_1 \approx -13.54\,\text{eV},\\
\text{Muonic hydrogen } (\mu^-p)\text{: } & \mu/m_\mu \approx 0.881;\ \text{deeper, bound } \mu^- \text{ at small } r.
\end{aligned}$$

The Bohr radius scales as $a_0 \cdot (m_e/\mu)$: positronium has $\approx 106\,\text{pm}$; muonic hydrogen has $\approx 285\,\text{fm}$.

Concept linkage. Both the energy and the radius are determined by a single ratio μ/m_e; this is the unifying parameter for all hydrogen-like two-body Coulomb systems.

Final Answer: $E_1 = -6.8\,\text{eV}$.

☞ Reduced mass

For two particles of masses m_1, m_2: $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$. Limiting cases: if $m_1 = m_2 = m$, $\mu = m/2$ (positronium); if $m_2 \gg m_1$, $\mu \to m_1$ (ordinary hydrogen).

Q 12.20 **Assume that there is no repulsive force between the electrons in an atom but the force between positive and negative charges is given by Coulomb's law as usual. Under such circumstances, calculate the ground state energy of a He-atom.**

SOLUTION

Concept used. Without electron-electron repulsion, the two electrons of helium are independent. Each electron moves in the central Coulomb field of the $Z = 2$ nucleus, so each is governed by the hydrogen-like Bohr formula with $Z = 2$. The total ground-state energy is then twice the single-electron ground-state energy.

Step 1. Single-electron energy in the field of the He nucleus ($Z = 2$, $n = 1$):

$$E_1(Z = 2) = -13.6 \times \frac{Z^2}{n^2}\,\text{eV} = -13.6 \times 4\,\text{eV} = -54.4\,\text{eV}.$$

Step 2. Both electrons sit in this $n = 1$ level (allowed because they are non-interacting; in real He they would also fit, since electrons are fermions and the two have opposite spins).

Step 3. Total ground-state energy:

$$E_{\text{He, ground}} = 2 \times E_1(Z = 2) = 2 \times (-54.4)\,\text{eV} = -108.8\,\text{eV}.$$

Step 4. Sanity check: the experimental He ground-state energy is -79.0 eV; the difference $-108.8 - (-79.0) = -29.8$ eV is the true electron-electron repulsion energy, which we set to zero in this idealisation.

Final Answer: $E_{\text{He, no repulsion}} = -108.8$ eV.

✗ Don't forget the factor 2

A common slip is to compute one electron's energy and stop. With electron-electron repulsion turned off, both electrons contribute independently, so the total is $2 \times (-54.4) = -108.8$ eV, not -54.4 eV.

EXPERT'S SOLUTION *: Aditi Joshi, M.Sc Physics, IIT Madras*

Strategic angle. "No repulsion" means each electron is hydrogen-like; sum two independent contributions.

Step 1. Each electron sees $Z = 2$, $n = 1$: $E = -13.6 \times 4 = -54.4$ eV.

Step 2. Two electrons, no interaction: $E_{\text{total}} = 2 \times (-54.4) = -108.8$ eV.

Step 3. Compare to the real He energy (-79.0 eV): the missing 29.8 eV is the suppressed Coulomb repulsion.

Why this matters. The exercise quantifies electron-electron repulsion as a $\sim 30\%$ correction to the He energy; a useful landmark before tackling multi-electron quantum chemistry.

Concept linkage (to many-electron atoms). The exact solution of helium with electron-electron repulsion is the historical first triumph of quantum-mechanical variational methods (Hylleraas, 1929). Modern density functional theory (DFT) extends this to any neutral atom. The Bohr-style no-repulsion estimate is the zeroth-order starting point for all such calculations.

Order-of-magnitude check. The mean electron-electron distance in helium is comparable to the Bohr radius for $Z = 2$: $a_0/Z \approx 0.265\,\text{Å}$. The classical Coulomb energy at that separation is

$$U_{ee} \sim \frac{e^2}{4\pi\varepsilon_0 \cdot a_0/Z} = 27.2 \times Z\,\text{eV} = 54.4\,\text{eV} \quad (Z = 2).$$

Quantum-mechanical averaging reduces this to $\sim 30\,\text{eV}$, the observed correction. Order of magnitude matches.

Final Answer: $E = -108.8\,\text{eV}$.

Q 12.21 Using Bohr model, calculate the electric current created by the electron when the H-atom is in the ground state.

SOLUTION

Concept used. An electron moving in a circular Bohr orbit is equivalent to a tiny current loop. The **current** carried by a single charge $-e$ orbiting with period T is

$$I = \frac{e}{T} = e\nu = \frac{e\omega}{2\pi},$$

where $\nu = 1/T$ is the orbital frequency and ω the angular frequency.

Step 1. For the ground state of H ($n = 1$), the orbital radius is the Bohr radius:

$$r = a_0 = 5.29 \times 10^{-11}\,\text{m}.$$

Step 2. The orbital speed comes from Bohr's quantisation $m_e v r = n\hbar$ with $n = 1$:

$$v = \frac{\hbar}{m_e a_0}.$$

Substitute $\hbar = 1.055 \times 10^{-34}\,\text{J s}$, $m_e = 9.11 \times 10^{-31}\,\text{kg}$, $a_0 = 5.29 \times 10^{-11}\,\text{m}$:

$$v = \frac{1.055 \times 10^{-34}}{9.11 \times 10^{-31} \times 5.29 \times 10^{-11}} = \frac{1.055 \times 10^{-34}}{4.819 \times 10^{-41}} = 2.19 \times 10^6\,\text{m/s}.$$

Step 3. Period of one revolution:

$$T = \frac{2\pi r}{v} = \frac{2\pi \times 5.29 \times 10^{-11}}{2.19 \times 10^6} = \frac{3.324 \times 10^{-10}}{2.19 \times 10^6} = 1.518 \times 10^{-16}\,\text{s}.$$

Step 4. Current:

$$I = \frac{e}{T} = \frac{1.6 \times 10^{-19}}{1.518 \times 10^{-16}} = 1.054 \times 10^{-3}\,\text{A} \approx 1.05\,\text{mA}.$$

> **Final Answer:** $I \approx 1.05 \times 10^{-3}\,\text{A} \approx 1.05\,\text{mA}$.

EXPERT'S SOLUTION *: Neha Bhat, M.Sc Astrophysics, IIT Kanpur*

Quick reading. Current = charge per period. Period = circumference ÷ speed.

Step 1. $v = \hbar/(m_e a_0) = 2.19 \times 10^6$ m/s.

Step 2. $T = 2\pi a_0/v = 1.52 \times 10^{-16}$ s.

Step 3. $I = e/T = 1.05\,\text{mA}$.

Why this matters. The associated orbital magnetic moment $\mu = IA = I\pi a_0^2 = 9.27 \times 10^{-24}\,\text{A m}^2$ is exactly the Bohr magneton.

Alternative method (energy-based). The orbital frequency follows from $E_n = -\frac{1}{2}\hbar\omega_n$ (the virial theorem for $1/r$ potentials), so for $n = 1$

$$\omega_1 = \frac{2|E_1|}{\hbar} = \frac{2 \times 13.6 \times 1.6 \times 10^{-19}}{1.055 \times 10^{-34}} = 4.13 \times 10^{16}\,\text{rad/s}.$$

Then $\nu_1 = \omega_1/2\pi = 6.57 \times 10^{15}$ Hz, and $I = e\nu_1 = 1.05 \times 10^{-3}$ A; identical to the direct calculation.

Order-of-magnitude check. A typical wire-loop current in a household circuit is ~ 1 A; the single-electron Bohr current is $\sim 1000\times$ smaller, but the loop is 10^{20} times smaller in area. The magnetic moment $\sim 10^{-23}\,\text{A m}^2$ is exactly the right scale for atomic-scale magnetism.

Concept linkage. The Bohr magneton $\mu_B = e\hbar/(2m_e)$ sets the scale for atomic and electronic magnetism; for the hydrogen ground state, the orbital magnetic moment is exactly μ_B.

> **Final Answer:** $I \approx 1.05\,\text{mA}$.

☞ **Period vs frequency**

A common slip is to write $I = eT$ instead of $I = e/T$. Dimensionally: I = charge/time = $e \times$ (rev per second) $= e\nu = e/T$. Always check the units!

Q 12.22 **Show that the first few frequencies of light that is emitted when electrons fall to the nth level from levels higher than n, are approximate harmonics (i.e. in the ratio $1 : 2 : 3 \ldots$) when $n \gg 1$.**

SOLUTION

Concept used. The Rydberg formula for transitions $n+p \to n$ gives the emission frequency

$$\nu_p = cR\left(\frac{1}{n^2} - \frac{1}{(n+p)^2}\right), \quad p = 1, 2, 3, \ldots$$

For large n we Taylor-expand $1/(n+p)^2$ in powers of p/n to extract the dominant behaviour.

Step 1. Expand $\dfrac{1}{(n+p)^2}$ for $p \ll n$:

$$\frac{1}{(n+p)^2} = \frac{1}{n^2}\left(1+\frac{p}{n}\right)^{-2} = \frac{1}{n^2}\left(1 - \frac{2p}{n} + \frac{3p^2}{n^2} - \ldots\right).$$

Step 2. Substitute into the Rydberg formula:

$$\nu_p = cR\left[\frac{1}{n^2} - \frac{1}{n^2}\left(1 - \frac{2p}{n} + \frac{3p^2}{n^2} - \ldots\right)\right].$$

Step 3. Simplify by cancelling $1/n^2$:

$$\nu_p = cR\left[\frac{2p}{n^3} - \frac{3p^2}{n^4} + \ldots\right].$$

Step 4. Keep the leading term (valid when $n \gg p$):

$$\nu_p \approx \frac{2cRp}{n^3}.$$

Hence

$$\frac{\nu_1}{1} = \frac{\nu_2}{2} = \frac{\nu_3}{3} = \cdots = \frac{2cR}{n^3},$$

showing $\nu_p \propto p$, i.e. the first few frequencies are in the ratio $1:2:3:\ldots$; they are approximate harmonics.

> **Final Answer:** For $n \gg 1$, $\nu_p \approx (2cR/n^3)\,p$, so successive emission frequencies $\nu_1 : \nu_2 : \nu_3 = 1 : 2 : 3$, i.e. approximate harmonics.

☞ Correspondence principle

This is a special case of Bohr's **correspondence principle**: for large quantum numbers, the quantum spectrum approaches the classical one. A classical electron in a circular orbit radiates at the fundamental frequency and its multiples (harmonics); exactly what we just derived from the Rydberg formula.

EXPERT'S SOLUTION : *Pranav Verma, Ph.D Physics, IISc Bangalore*

Strategic angle. The classical orbit radiates at ω and its integer multiples. So the high-n quantum spectrum must agree.

Step 1. Classical orbital frequency $\nu_{\text{cl}} = (2cR)/n^3$ for large n (this also follows from $\omega = v/r$ with Bohr-model values).

Step 2. Quantum result: $\nu_p = (2cR/n^3)p$ from the expansion above.

Step 3. The pth line is at exactly the pth harmonic of ν_{cl}.

Why this matters. The correspondence principle was Bohr's own bridge between the new quantum theory and classical radiation theory.
Numerical demonstration. Take $n = 100$. Exact:

$$\begin{aligned}\nu_1 &= cR(1/100^2 - 1/101^2) = cR(2.94 \times 10^{-6}),\\ \nu_2 &= cR(1/100^2 - 1/102^2) = cR(5.81 \times 10^{-6}),\\ \nu_3 &= cR(1/100^2 - 1/103^2) = cR(8.62 \times 10^{-6}).\end{aligned}$$

Ratios: $\nu_2/\nu_1 = 1.975 \approx 2$, $\nu_3/\nu_1 = 2.929 \approx 3$. The harmonic ratio is approximate but very close; deviation is $\sim 3p/(2n) \sim$ few percent for $p \sim 3$, $n = 100$.
Concept linkage (semiconductors). The classical radiation limit (the "high-n" Rydberg regime) is also the regime where atoms behave most like Drude conductors; the spectral pile-up at the ionisation edge is a precursor of the conduction-band continuum in solids.

> **Final Answer:** $\nu_p \propto p$ for $n \gg 1$; harmonic series.

Q 12.23 **What is the minimum energy that must be given to a H atom in ground state so that it can emit an H_γ line in Balmer series? If the angular momentum of the system is conserved, what would be the angular momentum of such H_γ photon?**

SOLUTION

Concept used. The H_γ line is the third line of the Balmer series, corresponding to the transition $n = 5 \to n = 2$ (Balmer lines: H_α is $3 \to 2$, H_β is $4 \to 2$, H_γ is $5 \to 2$). For a ground-state atom to emit H_γ on its way down, it must first be excited to $n = 5$. The minimum energy required is the excitation energy $E_5 - E_1$. The photon emitted in the $5 \to 2$ transition then carries angular momentum equal to the difference of the atomic angular momenta (in the Bohr picture $L_n = n\hbar$), or, equivalently, the photon itself carries $\hbar$ of intrinsic spin.

Step 1. Bohr energies (in eV): $E_n = -13.6/n^2$. $E_1 = -13.6,\ E_5 = -13.6/25 = -0.544$.

Step 2. Minimum energy to raise the atom to $n = 5$:

$$\Delta E = E_5 - E_1 = -0.544 - (-13.6) = 13.056\,\text{eV} \approx 13.06\,\text{eV}.$$

Step 3. Angular momentum of the photon. In the Bohr model the atom's orbital L changes from $L_5 = 5\hbar$ to $L_2 = 2\hbar$ in the emission, so the photon must carry

$$\Delta L_{\text{atom}} = L_5 - L_2 = 5\hbar - 2\hbar = 3\hbar.$$

(In the full quantum theory each photon carries spin $\hbar$, and a single $5 \to 2$ transition is allowed only when $\Delta\ell = \pm 1$. The Bohr-style answer expected by the Exemplar is $3\hbar$.)

Final Answer: Minimum excitation energy ≈ 13.06 eV; angular momentum of H_γ photon $= 3\hbar$ (Bohr-model estimate).

EXPERT'S SOLUTION *: Ankit Patel, M.Sc Physics, IIT Madras*

Strategic angle. Two numbers needed: (i) excitation energy ground $\to n_{\text{upper}}$, (ii) Bohr-model ΔL across the H_γ transition.

Step 1. Identify H_γ: third Balmer line $= 5 \to 2$.

Step 2. Excitation energy: $E_5 - E_1 = 13.6(1 - 1/25) = 13.6 \times 24/25 = 13.056\,\text{eV}$.

Step 3. Bohr angular momentum lost in emission: $5\hbar - 2\hbar = 3\hbar$.

Why this matters. The number tells you the minimum "investment" of energy to access a specific spectral line; useful for designing discharge tubes or laser pumps.

Alternative method (cascade routing). Once the atom is in $n = 5$, it can de-excite by several routes:

$$5 \to 4 \to 3 \to 2 \to 1, \quad 5 \to 2 \to 1, \quad 5 \to 1, \ldots$$

The H_γ line $(5 \to 2)$ is emitted along any cascade that includes the $5 \to 2$ step. The angular momentum balance is strict: $\Delta L_{\text{atom}} = 5\hbar - 2\hbar = 3\hbar$ goes to the photon (in the Bohr model). In the full quantum treatment, only $\Delta\ell = \pm 1$ photons are allowed; the $5 \to 2$ jump proceeds via the $5p \to 2s$ or $5d \to 2p$ sub-transitions.

Wavelength of H_γ.

$$\frac{1}{\lambda} = R(1/4 - 1/25) = R \times 21/100, \quad \lambda = \frac{100}{21R} = 4340\,\text{Å},$$

in the violet, consistent with the observed colour.

Concept linkage. The "minimum energy to make line X" problem is the basis of *spectroscopic notation* of an atom's term diagram; it generalises easily to multi-electron atoms where L, S, J replace the simple n label.

Final Answer: 13.06 eV; $L_\gamma = 3\hbar$.

✗ H_γ is the third Balmer line, not the third spectral line

The Greek-letter ordering $H_\alpha, H_\beta, H_\gamma, H_\delta, \ldots$ counts *within* the Balmer series only. Don't confuse it with the overall Lyman/Balmer/Paschen ordering or with line counts from $n = 1$. H_γ corresponds to $n_i = 5 \to n_f = 2$ specifically.

LA (Long Answer)

Q 12.24 The first four spectral lines in the Lyman series of a H-atom are $\lambda = 1218$ Å, 1028 Å, 974.3 Å, and 951.4 Å. If instead of Hydrogen, we consider Deuterium, calculate the shift in the wavelength of these lines.

SOLUTION

Concept used. The Rydberg constant for an atom is

$$R = R_\infty \cdot \frac{\mu}{m_e},$$

where $\mu = m_e M/(m_e + M)$ and M is the nuclear mass. The wavelength of a Rydberg transition is $\lambda \propto 1/R$, so

$$\frac{\lambda_D}{\lambda_H} = \frac{R_H}{R_D} = \frac{\mu_H}{\mu_D}.$$

Step 1. Reduced masses, using $m_e = 9.109 \times 10^{-31}$ kg, $M_H = 1.6725 \times 10^{-27}$ kg, $M_D = 3.3374 \times 10^{-27}$ kg:

$$\mu_H = \frac{m_e M_H}{m_e + M_H} = \frac{(9.109 \times 10^{-31})(1.6725 \times 10^{-27})}{(9.109 \times 10^{-31}) + (1.6725 \times 10^{-27})}.$$

Denominator $\approx 1.67341 \times 10^{-27}$ kg. Numerator $\approx 1.5234 \times 10^{-57}$. So $\mu_H \approx 9.104 \times 10^{-31}$ kg.

Step 2. Similarly for deuterium:

$$\mu_D = \frac{(9.109 \times 10^{-31})(3.3374 \times 10^{-27})}{(9.109 \times 10^{-31}) + (3.3374 \times 10^{-27})} \approx 9.106 \times 10^{-31}\ \text{kg}.$$

Step 3. Ratio:

$$\frac{\mu_H}{\mu_D} = \frac{9.104}{9.106} \approx 0.99973.$$

Equivalently, $\lambda_D = 0.99973 \times \lambda_H$, i.e. the deuterium lines are shorter by about 0.0273%.

Step 4. Shift for each line $\Delta\lambda = \lambda_H - \lambda_D = \lambda_H(1 - 0.99973) = 0.00027\lambda_H$:

$$\begin{aligned}
\lambda_H &= 1218\,\text{Å}: && \Delta\lambda \approx 0.00027 \times 1218 \approx 0.33\,\text{Å},\\
\lambda_H &= 1028\,\text{Å}: && \Delta\lambda \approx 0.00027 \times 1028 \approx 0.28\,\text{Å},\\
\lambda_H &= 974.3\,\text{Å}: && \Delta\lambda \approx 0.00027 \times 974.3 \approx 0.26\,\text{Å},\\
\lambda_H &= 951.4\,\text{Å}: && \Delta\lambda \approx 0.00027 \times 951.4 \approx 0.26\,\text{Å}.
\end{aligned}$$

Step 5. Direction: deuterium lines are slightly *shorter* than hydrogen lines (blueshift), because $\mu_D > \mu_H$ gives a slightly larger Rydberg constant.

> **Final Answer:** Wavelength shifts $\Delta\lambda \approx 0.33,\ 0.28,\ 0.26,\ 0.26\,\text{Å}$ for the first four Lyman lines (deuterium lines blueshifted relative to hydrogen).

EXPERT'S SOLUTION *: Yash Gupta, Ph.D Physics, IISc Bangalore*

Strategic angle. The whole isotope shift collapses to $\Delta\lambda/\lambda = 1 - \mu_H/\mu_D \approx (m_e/M_H)(M_D - M_H)/M_D$.

Step 1. Define $\delta = 1 - \mu_H/\mu_D$. For $M \gg m_e$,

$$\frac{\mu}{m_e} \approx 1 - \frac{m_e}{M},$$

so

$$\delta \approx \frac{m_e}{M_H} - \frac{m_e}{M_D} = m_e\left(\frac{1}{M_H} - \frac{1}{M_D}\right).$$

Plug in values:

$$\delta = 9.109 \times 10^{-31}\left(\frac{1}{1.6725 \times 10^{-27}} - \frac{1}{3.3374 \times 10^{-27}}\right).$$

$$\frac{1}{1.6725 \times 10^{-27}} = 5.979 \times 10^{26}, \quad \frac{1}{3.3374 \times 10^{-27}} = 2.996 \times 10^{26}.$$

$\delta = 9.109\times10^{-31}(5.979-2.996)\times10^{26} = 9.109\times10^{-31}\times2.983\times10^{26} \approx 2.72\times10^{-4}$.

Step 2. Multiply δ by each line's wavelength to get the shift. For $1218\,\text{Å}$: $\Delta\lambda = 2.72 \times 10^{-4} \times 1218 = 0.33\,\text{Å}$. Likewise 0.28, 0.26, $0.26\,\text{Å}$.

Why this matters. This 0.027% shift was the experimental signature Urey detected (Q 12.25), winning a Nobel.

Direction check (blueshift vs redshift). Deuterium has a heavier nucleus ($M_D > M_H$), so $\mu_D > \mu_H$, so $R_D > R_H$, so $\lambda_D < \lambda_H$; the deuterium line is blueshifted by a fraction $\delta \approx m_e(1/M_H - 1/M_D)$. Sign and magnitude both verified.

Order-of-magnitude check. $m_e/M_H \approx 1/1836 \approx 5.4 \times 10^{-4}$, and the relative-mass term $(M_D - M_H)/M_D \approx 1/2$, giving $\delta \sim 2.7 \times 10^{-4}$; matches the exact calculation. The numerical "trick" is the small parameter m_e/M_H that drives the whole isotope shift.

Concept linkage (dual nature). The fact that the Rydberg constant depends on the nuclear mass via μ is consistent with the wave-particle picture: the de Broglie wavelength of the "reduced particle" sets the orbital quantisation, not the wavelength of either particle separately.

Final Answer: $\Delta\lambda$ (in Å) $\approx 0.33,\ 0.28,\ 0.26,\ 0.26.$

♥ Isotope shifts as a discovery tool

The same machinery that gives the H/D shift is used today to distinguish isotopes of any element via optical or laser spectroscopy; the field of *laser isotope separation* (e.g. for uranium enrichment, for medical-isotope production) exists precisely because μ depends on nuclear mass.

Q 12.25 **Deuterium was discovered in 1932 by Harold Urey by measuring the small change in wavelength for a particular transition in ^{1}H and ^{2}H. This is because, the wavelength of transition depends to a certain extent on the nuclear mass. If nuclear motion is taken into account then the electrons and nucleus revolve around their common centre of mass. Such a system is equivalent to a single particle with a reduced mass μ, revolving around the nucleus at a distance equal to the electron-nucleus separation. Here $\mu = m_eM/(m_e + M)$ where M is the nuclear mass and m_e is the electronic mass. Estimate the percentage difference in wavelength for the 1st line of the Lyman series in ^{1}H and ^{2}H. (Mass of ^{1}H nucleus is 1.6725×10^{-27} kg, Mass of ^{2}H nucleus is 3.3374×10^{-27} kg, Mass of electron $= 9.109 \times 10^{-31}$ kg.)**

SOLUTION

Concept used. The Rydberg-formula wavelength scales inversely with the reduced mass:

$$\frac{1}{\lambda} = R_\infty \cdot \frac{\mu}{m_e}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right).$$

For the same transition (same n_1, n_2),

$$\frac{\lambda_H}{\lambda_D} = \frac{\mu_D}{\mu_H},$$

so the percentage difference $(\lambda_H - \lambda_D)/\lambda_H \times 100\%$ is the same for every line.

Step 1. Compute μ_H. With $m_e = 9.109 \times 10^{-31}$, $M_H = 1.6725 \times 10^{-27}$:

$$\mu_H = \frac{m_eM_H}{m_e + M_H} = \frac{(9.109 \times 10^{-31})(1.6725 \times 10^{-27})}{9.109 \times 10^{-31} + 1.6725 \times 10^{-27}}.$$

Denominator $= 1.67341 \times 10^{-27}$ kg. Numerator $= 1.5234 \times 10^{-57}$.

$$\mu_H = \frac{1.5234 \times 10^{-57}}{1.67341 \times 10^{-27}} = 9.1041 \times 10^{-31}\,\text{kg}.$$

Step 2. Compute μ_D:

$$\mu_D = \frac{(9.109 \times 10^{-31})(3.3374 \times 10^{-27})}{9.109 \times 10^{-31} + 3.3374 \times 10^{-27}} = 9.1065 \times 10^{-31}\,\text{kg}.$$

(Denominator $= 3.33831 \times 10^{-27}$.)

Step 3. Ratio:

$$\frac{\mu_D}{\mu_H} = \frac{9.1065}{9.1041} = 1.000264.$$

Therefore $\lambda_H/\lambda_D = 1.000264$, i.e. the deuterium line is shorter by a factor $1/1.000264 = 0.999736$.

Step 4. Percentage difference:

$$\frac{\lambda_H - \lambda_D}{\lambda_H} = 1 - \frac{1}{1.000264} = 1 - 0.999736 = 2.64 \times 10^{-4} = 0.0264\%.$$

Some textbooks round to 0.0273%; the small variation is in the round-off of μ.

Final Answer: Percentage difference in wavelength $\approx 0.026\%$ (deuterium line slightly shorter than hydrogen line).

♥ Why Urey won the Nobel

The shift Urey measured is tiny; a few parts in 10^4. Detecting it required exquisite high-resolution spectroscopy and a sample enriched in deuterium. Once detected, it confirmed the existence of a heavy hydrogen isotope, opening the field of nuclear isotopes and ultimately enabling deuterated solvents in NMR, heavy-water reactors, and biological tracer studies.

EXPERT'S SOLUTION *: Vivaan Reddy, M.Tech Applied Physics, IIT Delhi*

Strategic angle. The fractional shift is governed by the small parameter $m_e/M \ll 1$.

Step 1. Approximate $\mu/m_e \approx 1 - m_e/M$.

Step 2. Then

$$\frac{\mu_D}{\mu_H} \approx \frac{1 - m_e/M_D}{1 - m_e/M_H} \approx 1 + m_e\left(\frac{1}{M_H} - \frac{1}{M_D}\right).$$

Plug in:

$$\begin{aligned} m_e\left(\frac{1}{M_H}-\frac{1}{M_D}\right) &= 9.109\times10^{-31}(5.979-2.996)\times10^{26} \\ &= 9.109\times10^{-31}\times2.983\times10^{26} \\ &= 2.72\times10^{-4}. \end{aligned}$$

Step 3. So the wavelength shift is $\sim 2.7\times10^{-4}$, or $\sim 0.027\%$. This matches the exact calculation.

Why this matters. An order-of-magnitude estimate via m_e/M would have flagged the size of the effect Urey was looking for, justifying his investment in high-resolution spectroscopy.

Numerical landscape (other isotope shifts).

- ^{1}H vs ^{2}H (this problem): $\sim 2.7\times10^{-4}$ (large).
- ^{3}He vs ^{4}He: $\sim 4.5\times10^{-5}$ (cf. Q 12.15).
- ^{12}C vs ^{13}C: $\sim 4.3\times10^{-6}$ (much smaller).
- Heavy elements: dominated by nuclear volume effect, not reduced mass.

The lighter the nucleus, the bigger the relative shift; this is why Urey chose hydrogen, the element with the largest possible isotope shift in the periodic table.

Concept linkage (Chapter 11 dual nature). The same fractional shift would appear in Rydberg-formula predictions of emission frequencies $\nu = c/\lambda$; the discrete-photon picture inherits the reduced-mass correction directly.

> **Final Answer:** Percentage difference $\approx 0.027\%$.

☞ **The fractional shift is universal**

For any Rydberg transition (Lyman, Balmer, Paschen, . . .), the fractional H/D wavelength shift is the same $\sim 0.027\%$. The *absolute* shift in Å scales with λ, so longer wavelengths show larger absolute shifts.

Q 12.26 **If a proton had a radius R and the charge was uniformly distributed, calculate using Bohr theory, the ground state energy of a H-atom when (i) $R = 0.1\,\text{Å}$ and (ii) $R = 10\,\text{Å}$.**

SOLUTION

Concept used. Inside a uniformly charged sphere of radius R and total charge $+e$, Gauss's law gives the field as growing linearly with distance from the centre:

$$E(r) = \frac{e}{4\pi\varepsilon_0} \cdot \frac{r}{R^3} \quad (r < R),$$

while outside ($r > R$) the field is the usual Coulomb field of a point charge.
The corresponding potential energy of an electron at distance r is

$$U(r) = \begin{cases} \frac{e^2}{8\pi\varepsilon_0 R}\left(\frac{r^2}{R^2} - 3\right), & r < R, \\ -\frac{e^2}{4\pi\varepsilon_0 r}, & r \geq R, \end{cases}$$

(reference: $U(\infty) = 0$, U continuous at $r = R$).
We compare the Bohr ground-state radius $a_0 = 0.53\,\text{Å}$ with each R. If $a_0 > R$, the electron orbits *outside* the proton and the usual Bohr formula applies. If $a_0 < R$, the electron orbits *inside* the proton and a new derivation is needed.

Case (i): $R = 0.1\,\text{Å} < a_0 = 0.53\,\text{Å}$

Step 1. Since the Bohr radius $a_0 = 0.53\,\text{Å}$ is much larger than $R = 0.1\,\text{Å}$, the electron in the ground state lies *outside* the charge distribution. The proton looks point-like to the electron.

Step 2. Therefore the standard Bohr formula applies:

$$E_1 = -13.6\,\text{eV}.$$

Step 3. There is essentially no change from the point-proton case because the electron never penetrates the charge sphere.

Case (ii): $R = 10\,\text{Å} \gg a_0$

Now the electron orbits well *inside* the proton's charge distribution. We redo the Bohr derivation using the interior potential.

Step 1. Inside the sphere ($r < R$), the field on the electron is $E(r) = (e/4\pi\varepsilon_0)r/R^3$. The Coulomb force on the electron has magnitude

$$F(r) = \frac{e^2}{4\pi\varepsilon_0} \cdot \frac{r}{R^3}.$$

This is a *linear-restoring* (Hooke-like) force, just like a 3D harmonic oscillator.

Step 2. Circular-orbit condition (centripetal = Coulomb):

$$\frac{m_e v^2}{r} = \frac{e^2 r}{4\pi\varepsilon_0 R^3} \Rightarrow v^2 = \frac{e^2 r^2}{4\pi\varepsilon_0 m_e R^3}.$$

Step 3. Bohr quantisation $m_e v r = n\hbar$ with $n = 1$:

$$v = \frac{\hbar}{m_e r}.$$

Equate with v^2 from step 2:

$$\frac{\hbar^2}{m_e^2 r^2} = \frac{e^2 r^2}{4\pi\varepsilon_0 m_e R^3}.$$

Step 4. Solve for r^4:

$$r^4 = \frac{4\pi\varepsilon_0 \hbar^2 R^3}{m_e e^2} = a_0 R^3,$$

using $a_0 = 4\pi\varepsilon_0\hbar^2/(m_e e^2)$. So

$$r = (a_0 R^3)^{1/4}.$$

Plug in $a_0 = 0.53\,\text{Å}$, $R = 10\,\text{Å}$:

$$r = (0.53 \times 10^3)^{1/4} = (530)^{1/4} \approx 4.80\,\text{Å}.$$

Since $r = 4.80 < R = 10\,\text{Å}$, the assumption "$r < R$" is self-consistent.

Step 5. Compute the orbital kinetic energy:

$$KE = \tfrac{1}{2} m_e v^2 = \frac{\hbar^2}{2 m_e r^2}.$$

Numerically, with $r = 4.80 \times 10^{-10}$ m:

$$\begin{aligned} KE &= \frac{(1.055 \times 10^{-34})^2}{2 \times 9.11 \times 10^{-31} \times (4.80 \times 10^{-10})^2} \\ &= \frac{1.113 \times 10^{-68}}{4.199 \times 10^{-49}} = 2.65 \times 10^{-20}\,\text{J}. \end{aligned}$$

Convert: $2.65 \times 10^{-20}/1.6 \times 10^{-19} = 0.166\,\text{eV}$.

Step 6. Compute the potential energy inside the sphere at $r = 4.80\,\text{Å}$:

$$U = \frac{e^2}{8\pi\varepsilon_0 R}\left(\frac{r^2}{R^2} - 3\right).$$

Numerator prefactor:

$$\begin{aligned} \frac{e^2}{8\pi\varepsilon_0 R} &= \frac{1}{2} \cdot \frac{e^2}{4\pi\varepsilon_0 R} \\ &= \frac{1}{2} \cdot \frac{(1.6 \times 10^{-19})^2 \times 9 \times 10^9}{10 \times 10^{-10}} \\ &= \frac{1}{2} \cdot 2.304 \times 10^{-19} \\ &= 1.152 \times 10^{-19}\,\text{J} = 0.72\,\text{eV}. \end{aligned}$$

Bracket: $r^2/R^2 - 3 = (4.80/10)^2 - 3 = 0.2304 - 3 = -2.770$. So $U = 0.72 \times (-2.770) = -1.99\,\text{eV}$.

Step 7. Total energy:

$$E = KE + U = 0.166 - 1.99 = -1.83\,\text{eV}.$$

(Bohr ground-state energy is dramatically less negative; the atom is much less bound; because the linear restoring force is weaker than the $1/r^2$ Coulomb force at these distances.)

Final Answer: **(i)** For $R = 0.1\,\text{Å}$: standard Bohr value, $E_1 = -13.6\,\text{eV}$. **(ii)** For $R = 10\,\text{Å}$: $r \approx 4.80\,\text{Å}$, $E_1 \approx -1.83\,\text{eV}$.

EXPERT'S SOLUTION : *Riya Kapoor, Ph.D Physics, IISc Bangalore*

Picture-first. Compare R to a_0. If $R \ll a_0$, the electron stays outside, business as usual. If $R \gg a_0$, the electron sits inside a uniformly charged ball; a 3D harmonic oscillator.

Step 1. Case (i): $R = 0.1\,\text{Å}$, $a_0 = 0.53\,\text{Å}$, electron orbits outside: $E_1 = -13.6\,\text{eV}$.

Step 2. Case (ii): $R = 10\,\text{Å}$. Inside the sphere, the force is linear in r. Bohr quantisation gives $r^4 = a_0 R^3$.

Step 3. Compute: $r = (0.53 \times 1000)^{1/4} = 530^{1/4} \approx 4.8\,\text{Å}$.

Step 4. Energy from $KE + U$: about $-1.83\,\text{eV}$.

Why this matters. The hypothetical reflects on how *sensitive* the Bohr binding is to the assumption of a point nucleus. Real protons have $R \sim 0.84\,\text{fm}$, far smaller than a_0, so the point approximation is excellent.

Alternative method (3D harmonic oscillator). Inside the uniformly charged sphere, the linear restoring force corresponds to a 3D harmonic oscillator with

$$k = \frac{e^2}{4\pi\varepsilon_0 R^3}, \qquad \omega = \sqrt{\frac{k}{m_e}}.$$

The Bohr-quantised circular orbit for this oscillator gives the ground-state radius via $\hbar = m_e \omega r^2$, i.e. $r = \sqrt{\hbar/(m_e\omega)}$. Numerically for $R = 10\,\text{Å}$: $\omega = \sqrt{e^2/(4\pi\varepsilon_0 R^3 m_e)}$. Plug in and you recover $r \approx 4.8\,\text{Å}$. The energy is $E = \hbar\omega$ in the classical-orbit picture ($KE + PE$ each contribute $\frac{1}{2}$); agrees with the explicit calculation up to the constant from $U(R_0)$ at the surface.

Order-of-magnitude check. Case (ii) gives binding $\sim 1.8\,\text{eV}$; nearly an order of magnitude smaller than the standard $13.6\,\text{eV}$. This is consistent with the much weaker interior force ("linear" instead of "$1/r^2$").

Concept linkage (nucleus structure). Real protons have finite size $\sim 0.84\,\text{fm}$; this produces a tiny but measurable shift in atomic spectra known as the *nuclear volume effect*. Precision spectroscopy of muonic hydrogen ($\mu^- p$, $r_{\text{Bohr}} \sim 285\,\text{fm}$ so the muon is much closer to the proton) is sensitive to this effect and has triggered the modern "proton radius puzzle".

Final Answer: (i) $-13.6\,\text{eV}$; (ii) $\approx -1.83\,\text{eV}$.

✗ Compare R to a_0 first

The first move in this kind of problem is always to compare R with the Bohr radius $a_0 = 0.53\,\text{Å}$. If $R \ll a_0$, the electron orbits outside and the unmodified Bohr formula holds. If $R \gg a_0$, the electron lives inside and you must redo the derivation with the new force law. Skipping this comparison wastes time on the wrong calculation.

Q 12.27 In the Auger process an atom makes a transition to a lower state without emitting a photon. The excess energy is transferred to an outer electron which may be ejected by the atom. (This is called an Auger electron.) Assuming the nucleus to be massive, calculate the kinetic energy of an $n = 4$ Auger electron emitted by Chromium by absorbing the energy from a $n = 2$ to $n = 1$ transition.

SOLUTION

Concept used. In the Auger process an inner-shell vacancy is filled by an electron from a higher level, and the energy released is given to another bound electron (the Auger electron), which is then ejected. Energy conservation: the energy released in the inner transition equals the binding energy of the Auger electron plus its kinetic energy after ejection.

For chromium $Z = 24$. In the simplest hydrogen-like (Bohr-model) approximation, energy levels scale as

$$E_n = -\frac{13.6\,Z^2}{n^2}\,\text{eV}.$$

Step 1. Compute the energy released in the $n = 2 \to n = 1$ transition of Cr in the hydrogen-like approximation:

$$\Delta E_{21} = E_1 - E_2 = -13.6Z^2\left(1 - \frac{1}{4}\right) = 13.6 \times 24^2 \times \frac{3}{4}\,\text{eV}.$$

Compute step by step: $24^2 = 576$. $13.6 \times 576 = 7833.6$. $7833.6 \times 3/4 = 5875.2\,\text{eV}$. So $\Delta E_{21} = 5875.2\,\text{eV}$.

Step 2. This energy is delivered to the $n = 4$ Auger electron. The binding energy of an $n = 4$ electron in the Cr field (still hydrogen-like approximation) is

$$|E_4| = \frac{13.6\,Z^2}{n^2} = \frac{13.6 \times 576}{16} = \frac{7833.6}{16} = 489.6\,\text{eV}.$$

Step 3. Energy conservation: the released energy goes partly into unbinding the $n = 4$ electron and partly into its kinetic energy:

$$\Delta E_{21} = |E_4| + KE \Rightarrow KE = \Delta E_{21} - |E_4| = 5875.2 - 489.6 = 5385.6\,\text{eV}.$$

Final Answer: $KE_{\text{Auger}} = \Delta E_{21} - |E_4| = 5875.2 - 489.6 \approx 5385.6\,\text{eV} \approx 5.39\,\text{keV}$.

☞ **Auger spectroscopy**

The energy of Auger electrons is element-specific (because $E_n \propto Z^2$), making it a powerful surface-analytical probe (Auger Electron Spectroscopy, AES) used in materials science and nanotechnology.

EXPERT'S SOLUTION *: Karan Joshi, M.Sc Physics, IIT Madras*

Strategic angle. Two energies, one subtraction.

Step 1. Inner-shell drop (energy released):

$$\Delta E_{21} = 13.6Z^2(1 - 1/4) = 13.6 \cdot 576 \cdot 3/4 = 5875.2\,\text{eV}.$$

Step 2. Outer-electron binding: $|E_4| = 13.6Z^2/16 = 489.6\,\text{eV}$.

Step 3. $KE = 5875.2 - 489.6 = 5385.6\,\text{eV}$.

Why this matters. The huge Z^2 factor explains why Auger electrons from heavy elements have keV energies, far larger than photoemission electrons from valence shells.

Concept linkage (X-ray spectra). The same $n = 2 \to n = 1$ transition emits the chromium K_α X-ray photon when the energy escapes radiatively (instead of being captured by the Auger electron). The competition between radiative (K_α) and non-radiative (Auger) decay is governed by the *fluorescence yield* ω_K, which is small for light elements (favouring Auger) and approaches 1 for heavy elements (favouring X-ray emission).

Numerical comparison. For Cr the K_α X-ray energy is $5.41\,\text{keV}$ experimentally; the Bohr-model estimate $\Delta E_{21} = 5875\,\text{eV}$ is too large by $\sim 9\%$ because of shielding of the $n = 2 \to n = 1$ transition by the other inner-shell electrons (a "screened" Z of roughly 22 instead of 24 gives $\sim 5.4\,\text{keV}$). This is the famous *Moseley's law*.

Order-of-magnitude check. For hydrogen ($Z = 1$), ΔE_{21} would be $10.2\,\text{eV}$; for Cr ($Z = 24$), the scaling factor $Z^2 = 576$ gives $\sim 5.9\,\text{keV}$; correct ratio.

Final Answer: $\approx 5.39\,\text{keV}$.

☞ **Auger process in one line**

$\text{Atom}^* \to \text{Atom}^+ + e^-_{\text{Auger}}$. An inner-shell vacancy is filled by an outer electron, with the released energy ejecting another electron instead of producing a photon. Energy conservation: $\Delta E_{\text{inner}} = |E_{\text{outer}}| + KE_{\text{Auger}}$.

Q 12.28 **The inverse square law in electrostatics is $|F| = \dfrac{e^2}{(4\pi\varepsilon_0)r^2}$ for the force between an electron and a proton. The $\dfrac{1}{r}$ dependence of $|F|$ can be understood in quantum theory as being due to the fact that the 'particle' of light (photon) is massless. If photons had a mass m_p, the force would be modified to $|F| = \dfrac{e^2}{(4\pi\varepsilon_0)r^2}\left[\dfrac{1}{r^2} + \dfrac{\lambda}{r}\right] e^{-\lambda r}$, where $\lambda = m_p c/\hbar$ and $\hbar = h/2\pi$. Estimate the change in the ground state energy of a H-atom if m_p were 10^{-6} times the mass of an electron.**

SOLUTION

Concept used. A massive-photon (Yukawa-type) modification of Coulomb's law replaces $1/r$ by an exponentially screened potential

$$U(r) = -\frac{e^2}{4\pi\varepsilon_0} \cdot \frac{e^{-\lambda r}}{r}, \qquad \lambda = \frac{m_p c}{\hbar}.$$

For $\lambda r \ll 1$ the modification is a tiny perturbation; we can estimate the change in the ground-state energy by computing $\langle \Delta U \rangle$ on the unperturbed Bohr wavefunction (equivalently, evaluating ΔU at the Bohr radius).

Step 1. Compute λ for $m_p = 10^{-6} m_e$:

$$\lambda = \frac{(10^{-6})(9.11 \times 10^{-31})(3 \times 10^8)}{1.055 \times 10^{-34}}.$$

Numerator
$= 10^{-6} \times 9.11 \times 10^{-31} \times 3 \times 10^8 = 10^{-6} \times 2.733 \times 10^{-22} = 2.733 \times 10^{-28}$ kg m/s.
Therefore

$$\lambda = \frac{2.733 \times 10^{-28}}{1.055 \times 10^{-34}} = 2.591 \times 10^6\,\mathrm{m}^{-1}.$$

i.e. $\lambda \approx 2.6 \times 10^6\,\mathrm{m}^{-1}$, with $1/\lambda \approx 3.86 \times 10^{-7}$ m.

Step 2. Compare λr at the Bohr radius $r = a_0 = 5.29 \times 10^{-11}$ m:

$$\lambda a_0 = 2.591 \times 10^6 \times 5.29 \times 10^{-11} = 1.37 \times 10^{-4}.$$

Very small: the exponential $e^{-\lambda r} \approx 1 - \lambda r$.

Step 3. Expand the modified potential to first order in λr:

$$U(r) = -\frac{e^2}{4\pi\varepsilon_0} \cdot \frac{e^{-\lambda r}}{r} \approx -\frac{e^2}{4\pi\varepsilon_0} \cdot \frac{1 - \lambda r}{r} = -\frac{e^2}{4\pi\varepsilon_0 r} + \frac{e^2\lambda}{4\pi\varepsilon_0}.$$

The first term is the usual Coulomb potential; the second term is a constant shift $\Delta U_0 = e^2\lambda/(4\pi\varepsilon_0)$.

Step 4. Compute ΔU_0:

$$\Delta U_0 = \frac{e^2\lambda}{4\pi\varepsilon_0} = \left(\frac{e^2}{4\pi\varepsilon_0 a_0}\right) \times (\lambda a_0) = (27.2\,\mathrm{eV}) \times (1.37 \times 10^{-4}) \approx 3.7 \times 10^{-3}\,\mathrm{eV}.$$

Here $e^2/(4\pi\varepsilon_0 a_0) = 2 \times 13.6\,\mathrm{eV} = 27.2\,\mathrm{eV}$ is twice the H ionisation energy.

Step 5. Since the modification is a (nearly) constant positive shift in U over the region where the electron lives, the ground-state energy shifts up by roughly the same amount:

$$\Delta E_1 \approx +3.7 \times 10^{-3}\,\text{eV} \approx +3.7\,\text{meV}.$$

The atom is therefore less tightly bound, but by less than a ten-thousandth of the original binding energy.

Final Answer: $\Delta E_1 \approx +3.7\,\text{meV}$; the modification raises the ground-state energy by about 4×10^{-3} eV.

♥ Limits on photon mass

Experimental upper bounds on the photon mass from such spectroscopic-style tests (and from larger-scale tests like the solar-wind magnetic field) are now $m_p \lesssim 10^{-18}\,\text{eV}/c^2 \sim 10^{-24}\,m_e$; vastly more restrictive than the $10^{-6}m_e$ of this problem. The exercise illustrates that even a tiny hypothetical photon mass would have an observable spectroscopic fingerprint.

EXPERT'S SOLUTION *: Dev Verma, M.Tech Applied Physics, IIT Delhi*

Strategic angle. Small-λr expansion converts the problem to a constant shift in the potential, which translates directly into a shift in the energy.

Step 1. $\lambda = m_p c/\hbar = 2.59 \times 10^6\,\text{m}^{-1}$.

Step 2. $\lambda a_0 = 1.37 \times 10^{-4}$, so $e^{-\lambda a_0} \approx 1 - \lambda a_0$.

Step 3. Energy shift $\approx (e^2\lambda)/(4\pi\varepsilon_0) = 27.2 \times \lambda a_0 = 27.2 \times 1.37 \times 10^{-4} \approx 3.7$ meV.

Why this matters. Even a $10^{-6}m_e$ photon-mass test shifts hydrogen levels at the meV level; well within precision spectroscopy reach. Hence such a mass is firmly ruled out.

Alternative method (first-order perturbation theory). The modified potential is $U_{\text{new}}(r) - U_{\text{Coul}}(r) \approx (e^2\lambda)/(4\pi\varepsilon_0)$ plus higher-order $(\lambda r)^2$ corrections. First-order perturbation theory gives

$$\Delta E_1 = \langle\psi_{1s}|U_{\text{new}} - U_{\text{Coul}}|\psi_{1s}\rangle \approx \frac{e^2\lambda}{4\pi\varepsilon_0},$$

since the leading term is a constant. The result $\approx 3.7\,\text{meV}$ matches the direct Bohr-style estimate.

Concept linkage (Yukawa form). The exponentially screened $1/r$ potential $V(r) \propto e^{-\lambda r}/r$ is the *Yukawa potential*; it appears in (i) hypothetical massive-photon QED here, (ii) nuclear-force theory with pion exchange (Chapter 13), and (iii) plasma physics as Debye screening. One form, three contexts.

Order-of-magnitude check. A photon mass $10^{-6}m_e$ corresponds to an energy

$\sim 0.5\,\mathrm{keV}/c^2 \cdot 10^{-6} \cdot 10^3 \sim 0.5\,\mathrm{eV}/c^2$. The induced spectroscopic shift $\sim 3.7\,\mathrm{meV}$ is of order $(\lambda a_0) \times E_{\text{atomic}} \sim 10^{-4} \times 13.6\,\mathrm{eV}$; the right magnitude.

Final Answer: $\Delta E \approx +3.7\,\mathrm{meV}$.

☞ Small-perturbation reflex

When a force law is modified by a small parameter ($\lambda r \ll 1$ here, $m_e/M \ll 1$ in reduced-mass problems, $\Delta U_0 \ll |E_{\text{bind}}|$ in many tunnelling problems), expand to first order in the small parameter. The leading correction is usually a *constant shift* in U, which translates directly into a constant shift in E.

Q 12.29 The Bohr model for the H-atom relies on the Coulomb's law of electrostatics. Coulomb's law has not directly been verified for very short distances of the order of angstroms. Supposing Coulomb's law between two opposite charges $+q_1$, $-q_2$ is modified to

$$|F| = \frac{q_1 q_2}{4\pi\varepsilon_0} \cdot \frac{1}{r^2},\ r \geq R_0; \quad = \frac{q_1 q_2}{4\pi\varepsilon_0 R_0^2}\left(\frac{R_0}{r}\right)^{\varepsilon},\ r \leq R_0.$$

Calculate in such a case, the ground state energy of a H-atom, if $\varepsilon = 0.1$, $R_0 = 1\,$Å.

SOLUTION

Concept used. Inside the radius $R_0 = 1\,$Å, Coulomb's law is replaced by a power-law $F \propto r^{-\varepsilon}$ with $\varepsilon = 0.1$. The Bohr ground-state radius for normal H is $a_0 = 0.53\,$Å$< R_0$, so the electron *lives entirely inside* the modified-law region. We must redo the Bohr derivation with the new force law.

Step 1. Write the modified force on the electron at distance r ($r < R_0$), with $q_1 q_2 = e \cdot e = e^2$ and attraction directed inward:

$$|F(r)| = \frac{e^2}{4\pi\varepsilon_0 R_0^2}\left(\frac{R_0}{r}\right)^{\varepsilon} = \frac{e^2 R_0^{\varepsilon-2}}{4\pi\varepsilon_0} \cdot r^{-\varepsilon}.$$

Let $K = e^2 R_0^{\varepsilon-2}/(4\pi\varepsilon_0)$; then $|F(r)| = Kr^{-\varepsilon}$.

Step 2. Centripetal balance:

$$\frac{m_e v^2}{r} = Kr^{-\varepsilon} \Rightarrow m_e v^2 = Kr^{1-\varepsilon}.$$

Step 3. Bohr quantisation, $n = 1$:

$$m_e v r = \hbar \Rightarrow v = \frac{\hbar}{m_e r}.$$

Square: $v^2 = \hbar^2/(m_e^2 r^2)$, so $m_e v^2 = \hbar^2/(m_e r^2)$.

Step 4. Equate the two expressions for $m_e v^2$:

$$\frac{\hbar^2}{m_e r^2} = K r^{1-\varepsilon} \Rightarrow r^{3-\varepsilon} = \frac{\hbar^2}{m_e K}.$$

Solve for r:

$$r = \left(\frac{\hbar^2}{m_e K}\right)^{1/(3-\varepsilon)}.$$

Step 5. Plug in $\varepsilon = 0.1$, $R_0 = 10^{-10}$ m. Compute K:

$$K = \frac{e^2 R_0^{-1.9}}{4\pi\varepsilon_0}.$$

Use $e^2/(4\pi\varepsilon_0) = 2.307 \times 10^{-28}\,\text{N m}^2$ and $R_0^{-1.9} = (10^{-10})^{-1.9} = 10^{19}\,\text{m}^{-1.9}$:

$$K = 2.307 \times 10^{-28} \times 10^{19} = 2.307 \times 10^{-9}\,\text{N m}^{0.1}.$$

Then

$$\frac{\hbar^2}{m_e K} = \frac{(1.055 \times 10^{-34})^2}{(9.11 \times 10^{-31})(2.307 \times 10^{-9})} = \frac{1.113 \times 10^{-68}}{2.102 \times 10^{-39}} = 5.297 \times 10^{-30}\,\text{m}^{2.9}.$$

Step 6. Take the $1/2.9$ power:

$$r = (5.297 \times 10^{-30})^{1/2.9}\,\text{m}.$$

Compute the exponent in $\log_{10}$ terms:
$\log_{10}(5.297 \times 10^{-30}) = \log_{10} 5.297 - 30 = 0.724 - 30 = -29.276$. Divide by 2.9: $-29.276/2.9 = -10.095$. So $r = 10^{-10.095}\,\text{m} = 0.804 \times 10^{-10}\,\text{m} = 0.80\,\text{Å}$. (Consistent with $r < R_0 = 1\,\text{Å}$: assumption holds.)

Step 7. Velocity:

$$v = \frac{\hbar}{m_e r} = \frac{1.055 \times 10^{-34}}{(9.11 \times 10^{-31})(8.04 \times 10^{-11})} = \frac{1.055 \times 10^{-34}}{7.324 \times 10^{-41}} = 1.441 \times 10^6\,\text{m/s}.$$

Kinetic energy:

$$\begin{aligned} KE &= \tfrac{1}{2} m_e v^2 = \tfrac{1}{2}(9.11 \times 10^{-31})(1.441 \times 10^6)^2 \\ &= \tfrac{1}{2} \times 9.11 \times 10^{-31} \times 2.077 \times 10^{12} \\ &= 9.461 \times 10^{-19}\,\text{J}. \end{aligned}$$

Convert: $9.461 \times 10^{-19}/1.6 \times 10^{-19} = 5.91\,\text{eV}$.

Step 8. Potential energy. Integrate F from r to R_0 to find $U(r) - U(R_0)$ in the modified region; the value of U at R_0 is the usual Coulomb potential evaluated at R_0 (the two laws match continuously at $r = R_0$):

$$U(R_0) = -\frac{e^2}{4\pi\varepsilon_0 R_0} = -\frac{2.307 \times 10^{-28}}{10^{-10}} = -2.307 \times 10^{-18}\,\text{J} = -14.4\,\text{eV}.$$

Inside: $U(r) - U(R_0) = \int_r^{R_0} Ks^{-\varepsilon}(-1)\, ds \cdot (-1)$; sign convention: $F = -dU/dr$, so $U(r) = U(R_0) - \int_r^{R_0} F(s)\, ds$, with the attractive force having magnitude $Ks^{-\varepsilon}$:

$$U(r) = U(R_0) - \int_r^{R_0} Ks^{-\varepsilon}(-1)\, ds = U(R_0) + K \int_r^{R_0} s^{-\varepsilon}\, ds.$$

Actually, with attractive force (pointing toward origin), $F_r = -Kr^{-\varepsilon}$ (negative radial), and $U(r) = -\int_\infty^r F_r\, dr$. For $r < R_0$,

$$U(r) = U(R_0) + \int_r^{R_0} Ks^{-\varepsilon}\, ds = U(R_0) + \frac{K}{1-\varepsilon}\left(R_0^{1-\varepsilon} - r^{1-\varepsilon}\right).$$

With $\varepsilon = 0.1$: $1 - \varepsilon = 0.9$, $K/(1-\varepsilon) = K/0.9 = 2.563 \times 10^{-9}$ in SI.
$R_0^{0.9} = (10^{-10})^{0.9} = 10^{-9}\ \text{m}^{0.9}$.
$r^{0.9} = (0.804 \times 10^{-10})^{0.9} = 10^{-9} \times (0.804)^{0.9} = 10^{-9} \times 0.821 = 8.21 \times 10^{-10}$. So $R_0^{0.9} - r^{0.9} = 10^{-9} - 8.21 \times 10^{-10} = 1.79 \times 10^{-10}$. Therefore

$$U(r) - U(R_0) = 2.563 \times 10^{-9} \times 1.79 \times 10^{-10} = 4.588 \times 10^{-19}\,\text{J} = 2.87\,\text{eV}.$$

And $U(r) = -14.4 + 2.87 = -11.53\,\text{eV}$. (Wait; the modification makes the potential *less* negative inside than the pure Coulomb, since the force grows more slowly than $1/r^2$. So $U(r) > U_{\text{Coulomb}}(r)$, as expected.)

Step 9. Total energy:

$$E = KE + U = 5.91 - 11.53 = -5.62\,\text{eV}.$$

So the binding energy in the modified theory is about $5.6\,\text{eV}$; substantially less than the standard $13.6\,\text{eV}$.

Final Answer: Ground-state radius $r \approx 0.80\,\text{Å}$; ground-state energy $E_1 \approx -5.6\,\text{eV}$ (compared with $-13.6\,\text{eV}$ in unmodified Coulomb law).

EXPERT'S SOLUTION : *Ishita Kapoor, Ph.D Physics, IISc Bangalore*

Strategic angle. Inside R_0 the force law is weaker than $1/r^2$, so the orbit is larger and binding is shallower. Redo Bohr: solve $r^{3-\varepsilon} = \hbar^2/(m_e K)$ for r; then KE from quantisation, U by integrating the new force.

Step 1. Set up: $K = e^2 R_0^{\varepsilon-2}/(4\pi\varepsilon_0)$; with $\varepsilon = 0.1$, $R_0 = 1\,\text{Å}$, $K \approx 2.31 \times 10^{-9}$ SI.

Step 2. Solve $r^{2.9} = \hbar^2/(m_e K) = 5.30 \times 10^{-30}\ \text{m}^{2.9}$: $r \approx 0.80\,\text{Å}$.

Step 3. Compute $v = \hbar/(m_e r) \approx 1.44 \times 10^6$ m/s; $KE = \frac{1}{2} m_e v^2 \approx 5.91\,\text{eV}$.

Step 4. Compute U by integrating the modified force from r to R_0 and adding the standard Coulomb $U(R_0) = -14.4\,\text{eV}$. Result: $U(r) \approx -11.53\,\text{eV}$.

Step 5. Total: $E = 5.91 - 11.53 \approx -5.6\,\text{eV}$.

Why this matters. Constraints from atomic spectroscopy on deviations from Coulomb's law at angstrom scales are tight precisely because such modifications shift the ground-state energy by several eV; far more than spectroscopic precision.

Concept linkage. Tests of Coulomb's law at short distances parallel similar tests of the inverse-square law at large distances (geophysical measurements of G at km scales). Both probe whether nature deviates from the canonical $1/r^2$; both have so far found agreement to extraordinary precision.

Numerical sensitivity. The Bohr binding energy goes from $-13.6\,\text{eV}$ (Coulomb) to $-5.6\,\text{eV}$ (modified) here; an enormous fractional shift of $\sim 60\%$. Such a shift would be obvious in the H spectrum (line wavelengths would change by factor ~ 2.5); experimental hydrogen spectroscopy thus rules out any such modification at $1\,\text{Å}$ scales.

Order-of-magnitude check. The ground-state radius shifts from $0.53\,\text{Å}$ (standard) to $0.80\,\text{Å}$ (modified). This makes sense: the weaker interior force allows the electron to orbit at a larger radius, reducing binding.

Final Answer: $E_1 \approx -5.6\,\text{eV}$, $r \approx 0.80\,\text{Å}$.

♥ Coulomb's law at short distances

Direct laboratory tests of Coulomb's law have verified the $1/r^2$ form down to $\sim 10^{-15}\,\text{m}$ (in scattering experiments), and atomic spectroscopy implicitly tests it at the $\sim 10^{-10}\,\text{m}$ scale (since any deviation would shift line wavelengths measurably). Both kinds of test now constrain any departures from $1/r^2$ to be very small; well within the noise floor of even modern high-resolution spectroscopy.

Key Takeaways

- Bohr's hydrogen-like formulas $r_n = n^2 a_0/Z$, $E_n = -13.6Z^2/n^2\,\text{eV}$ apply only to one-electron systems; the loss of centrality kills the model for multi-electron atoms (Q 12.3, 12.5, 12.10, 12.13).
- The Bohr model must be derived in an inertial frame; for two-body systems use the reduced mass $\mu = mM/(m+M)$ (Q 12.2, 12.15, 12.19, 12.25).
- Spectral series are organised by their lower terminus: Lyman ($\to n = 1$), Balmer ($\to n = 2$), Paschen ($\to n = 3$). Series limits accumulate as $1/n^2 \to 0$ (Q 12.11, 12.12).
- Isotope shifts (H vs D) and small physical effects (massive photon, modified Coulomb law) appear as tiny but spectroscopically detectable energy corrections (Q 12.25, 12.28, 12.29).

- The Auger process and current/magnetic-moment calculations show how Bohr energies scale with Z^2 (Q 12.21, 12.27).
- For large n, quantum spectra approach classical radiation harmonics; Bohr's correspondence principle (Q 12.22).

End of NCERT Exemplar Problems