# Collegedunia NCERT Formula Sheet

*The Ultimate Formula Reference for Class 12 Physics*

## Chapter 12: Atoms

| Constant / Quantity | Value |
|---|---|
| Bohr radius, $a_0$ | $0.529$ Å = $5.29 \times 10^{-11}$ m |
| Hydrogen ground-state energy | $-13.6$ eV |
| Rydberg constant, $R$ | $1.097 \times 10^{7}$ m$^{-1}$ |
| Rydberg energy, $Rhc$ | $13.6$ eV |
| $hc$ | $1240$ eV·nm |

## 1 Rutherford Model & $\alpha$-Scattering

The nuclear model of the atom emerged from Geiger and Marsden's $\alpha$-scattering experiment, which Rutherford analysed in 1911. Most $\alpha$-particles passed through gold foil undeflected; a tiny fraction bounced back (NCERT 12.2–12.3).

**Rutherford's nuclear atom**

Most of the atom is empty space; the entire positive charge and almost all the mass are concentrated in a tiny dense **nucleus** ($\sim 10^{-15}$ m), with electrons orbiting at distances $\sim 10^{-10}$ m. **Failure**: classical orbiting electrons must radiate and spiral into the nucleus — the model can't explain why atoms are stable.

**Distance of closest approach**

$$r_0 = \frac{1}{4\pi\epsilon_0}\frac{2Ze^2}{K}$$

where $K$ = kinetic energy of $\alpha$-particle; $Z$ = atomic number of target.

Set the initial KE equal to electrostatic PE at closest approach. Gives an upper bound on the nuclear radius.

**Impact parameter & scattering angle**

$$b = \frac{Ze^2 \cot(\theta/2)}{4\pi\epsilon_0 K}$$

Smaller $b$ (more head-on) $\Rightarrow$ larger scattering angle. Backscatters ($\theta \to 180°$) need $b \to 0$ — direct hits on the nucleus.

## 2 Bohr's Model of Hydrogen

Bohr postulated that electrons orbit only in discrete circular paths where angular momentum is quantised — and that energy is emitted only when an electron jumps between these allowed orbits (NCERT 12.4–12.5).

**Bohr's three postulates**

(i) Electrons move in **circular orbits** without radiating.
(ii) Allowed orbits have **quantised angular momentum**: $L = n\hbar = nh/(2\pi)$.
(iii) Photons are emitted when the electron **jumps** between allowed orbits: $h\nu = E_i - E_f$.

**Bohr radius (orbit $n$)**

$$r_n = \frac{n^2h^2\epsilon_0}{\pi m_e e^2 Z} = \frac{n^2}{Z}a_0$$

where $a_0 = 0.529$ Å is the Bohr radius (H, $n = 1$).

Orbit radius scales as $n^2$. For H ($Z = 1$): $r_2 = 4a_0$, $r_3 = 9a_0$, etc. For He$^+$ ($Z = 2$): all radii are halved.

**Velocity in orbit $n$**

$$v_n = \frac{Ze^2}{2\epsilon_0 nh} = \frac{Zc}{137n}$$

$1/137$ is the **fine-structure constant**. Velocity decreases as $1/n$ — outer orbits are slower.

**Energy in orbit $n$**

$$E_n = -\frac{m_e e^4 Z^2}{8\epsilon_0^2 n^2 h^2} = -\frac{13.6\,Z^2}{n^2}\ \text{eV}$$

For hydrogen ($Z = 1$): $E_n = -\dfrac{13.6}{n^2}$ eV

Negative because the electron is **bound**. Ground state ($n = 1$): $E_1 = -13.6$ eV. Ionisation energy: 13.6 eV (the work to free the electron).

**KE, PE, total energy relations**

$KE = -E,\quad PE = 2E,\quad E_{\text{total}} = KE + PE.$
For H ground state: $KE = +13.6$ eV, $PE = -27.2$ eV, $E = -13.6$ eV.
Total energy is half the magnitude of PE (**virial theorem** for $1/r$ potentials).

## 3 Hydrogen Spectrum

Transitions between allowed levels emit (or absorb) photons of definite frequencies, organised into spectral series named after their discoverers (NCERT 12.6).

**Frequency & wavelength of emitted photon**

$$h\nu = E_i - E_f = 13.6\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)\ \text{eV}$$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

where $R = 1.097 \times 10^7$ m$^{-1}$ is the Rydberg constant; $n_i > n_f$ for emission.

Observed line spectra of H match this exactly — the great triumph of Bohr's model.

**Spectral series (final level $n_f$)**

**Lyman** ($n_f = 1$): UV.
**Balmer** ($n_f = 2$): visible.
**Paschen** ($n_f = 3$): IR.
**Brackett** ($n_f = 4$): far IR.
**Pfund** ($n_f = 5$): far IR.

Each series has a **shortest** $\lambda$ (when $n_i = \infty$) and a **longest** $\lambda$ (when $n_i = n_f + 1$).

**Series limit & first line**

Series limit (shortest $\lambda$, $n_i = \infty$): $\dfrac{1}{\lambda} = \dfrac{R}{n_f^2}$

First line (longest $\lambda$, $n_i = n_f + 1$):

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{(n_f+1)^2}\right)$$

For Lyman: limit is at 91 nm, first ($\alpha$) line at 122 nm. For Balmer: H$\alpha$ at 656 nm (red), limit at 365 nm.

**Number of spectral lines**

Maximum lines from level $n$: $\frac{n(n-1)}{2}$

Lines from level $n_i$ to level $n_f$: $\frac{(n_i - n_f)(n_i - n_f + 1)}{2}$

From $n = 4$ to ground state: $\binom{4}{2} = 6$ possible spectral lines (covers Lyman, Balmer, Paschen contributions).

**Excitation & ionisation energies**

**Excitation energy**: energy required to lift the electron from ground state to an excited state. For H, $1 \to 2$: $E_2 - E_1 = 10.2$ eV.

**Ionisation energy**: energy to free the electron entirely ($n = \infty$). For H: 13.6 eV.

**Binding energy** of state $n$: $|E_n|$ — the amount of energy that holds the electron in that state.

## 4 Hydrogen-like Atoms & Limitations

Bohr's model extends to any single-electron ion ($He^+$, $Li^{2+}$, etc.) with $Z$ in place of 1. It fails for multi-electron atoms (NCERT 12.5 cont.).

**Hydrogen-like atom (single electron, charge**

$$E_n = -\frac{13.6\,Z^2}{n^2}\ \text{eV}$$

$$r_n = \frac{n^2}{Z} a_0$$

$$v_n = \frac{Z}{n} \cdot \frac{c}{137}$$

$He^+$ ($Z = 2$): $E_1 = -54.4$ eV. $Li^{2+}$ ($Z = 3$): $E_1 = -122.4$ eV. Energies scale as $Z^2$, radii as $1/Z$.

**Limitations of Bohr's model**

(i) Works only for **one-electron** systems.
(ii) Cannot explain **relative intensities** of spectral lines.

(iii) Cannot explain **fine structure** (closely spaced lines).
(iv) Cannot account for **Zeeman effect** (lines splitting in $\vec{B}$).
(v) Postulates of quantisation are **ad hoc** — with no underlying derivation.
**Quantum mechanics** (Schrödinger, 1926) replaced it.

**JEE/NEET Extension: de Broglie justification**

The angular-momentum quantisation $L = n\hbar$ falls out naturally if you require the electron's orbit to fit a **whole number of de Broglie wavelengths**: $2\pi r_n = n\lambda = nh/(m_e v_n)$. The mysterious Bohr postulate becomes a standing-wave condition.

**Spectral series order**

**L**azy **B**oys **P**lay **B**aseball **P**oorly: Lyman ($n_f = 1$), Balmer ($n_f = 2$), Paschen ($n_f = 3$), Brackett ($n_f = 4$), Pfund ($n_f = 5$).
Or just: **1, 2, 3, 4, 5** = **L, B, P, B, P**.

**Sign of energy**

Bound-state energies are **negative** ($E_n < 0$). **Higher** states (larger $n$) have **less negative** energies — they are **higher** (closer to zero). The electron falls **down** the energy ladder during emission, but its $n$-value goes **down** too.

## Quick Reference — Atoms

| **Quantity / Concept** | **Expression** | **Notes** |
|---|---|---|
| Closest approach | $\frac{2Ze^2}{4\pi\epsilon_0 K}$ | Bound on nuclear size |
| Angular momentum (Bohr) | $n\hbar$ | Quantised |
| Bohr radius (orbit $n$) | $n^2 a_0/Z$ | $a_0 = 0.529$ Å |
| Velocity (orbit $n$) | $Zc/(137\,n)$ | Slower for outer orbits |
| Energy (level $n$) | $-13.6Z^2/n^2$ eV | Ground state: $-13.6$ eV |
| KE / PE / total | $-E_n$ / $2E_n$ / $E_n$ | Virial relation |
| Rydberg formula | $R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ | Gives $1/\lambda$ |
| Photon energy | $E_i - E_f$ | Emission: $n_i > n_f$ |
| Series limit | $R/n_f^2$ | Shortest $\lambda$ |
| Lines from level $n$ | $n(n-1)/2$ | All transitions down |
| Ionisation energy (H) | 13.6 eV | $n = 1 \to \infty$ |
| He$^+$ ground state | $-54.4$ eV | $Z^2$ scaling |
| de Broglie ($e^-$ in orbit) | $2\pi r_n = n\lambda$ | Standing-wave picture |