The Collegedunia NCERT Notes

The Ultimate NCERT Guide for Class 12 Physics

Chapter 12: Atoms

1 Introduction and Early Atomic Models

The journey into the atom begins with the fundamental question: What is the structure of matter? This chapter delves into the experiments and models that revealed the atom's inner structure, leading to the birth of modern physics.

1.1 Thomson's Plum Pudding Model

The first modern atomic model was proposed by J.J. Thomson, the discoverer of the electron.

J.J. Thomson's Atomic Model (1898)

- The atom is a sphere of uniform **positive charge** with a radius of about 10^{-10} m.
- **Electrons** (negatively charged) are embedded within this positive sphere like seeds in a watermelon or plums in a pudding.
- The total positive charge equals the total negative charge, making the atom **electrically neutral** as a whole.

Uniform Positive Sphere

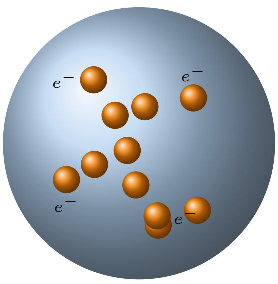


Figure 1: The 'Plum Pudding' model: Electrons (Orange) in a positive matrix (Blue).

Expected Outcome of Alpha-Scattering

According to Thomson's model, if a beam of energetic α-particles (positively charged) were fired at a thin gold foil, the prediction was:

All particles should pass through with only small, uniform deflections.
No large-angle scattering was expected because the positive charge and mass were thought to be uniformly distributed.

1.2 Rutherford's Alpha-Particle Scattering Experiment

The experiment that shattered the plum pudding model. Ernest Rutherford, along with his assistants Geiger and Marsden, performed this landmark experiment in 1911.

Experimental Setup

- **Source:** A radioactive source (Radon) emits a narrow, collimated beam of high-energy α-particles.
- **Target:** A very thin gold foil (about 1000 atoms thick).
- **Detector:** A movable screen coated with zinc sulfide (ZnS) that produces a tiny flash of light (scintillation) when struck by an α-particle. This could be rotated to detect particles at various angles (θ).

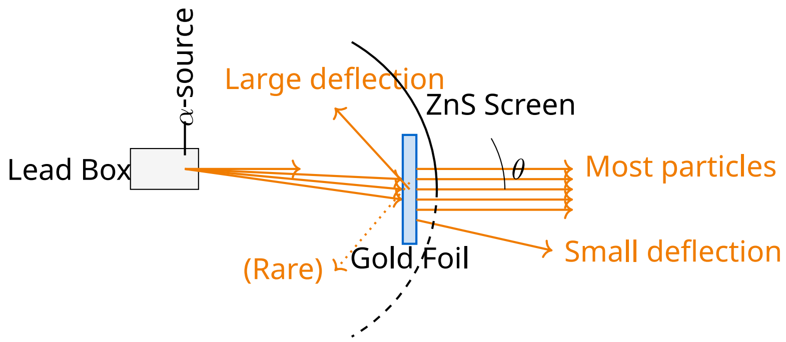


Figure 2: Schematic of Rutherford's α-particle scattering experiment.

1.3 Observations and Conclusions

The results were startling and contradicted Thomson's model completely.

Key Observations of the Experiment

1. **Most** of the α**-particles passed straight through** the gold foil without any deflection.
2. **A few** particles were deflected through **small angles** (θ).
3. **Very few** (about 1 in 8000) were deflected through **large angles** $(> 90°)$. Some even bounced back (scattering angle $\approx 180°$).

In Rutherford's own words: *"It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you."*

These observations led to a revolutionary new model of the atom.

Rutherford's Nuclear Model of the Atom (Postulates)

1. **The Nucleus:** The entire positive charge and almost all the mass of the atom is concentrated in a tiny central core called the **nucleus**. Its size is about 10^{-15} m, which is 10^{-5} times the size of an atom (10^{-10} m).
2. **Electron Orbits:** The electrons revolve around the nucleus in circular orbits at very high speeds, much like planets around the sun. The required centripetal force is provided by the electrostatic force of attraction between the negatively charged electrons and the positively charged nucleus.
3. **Empty Space:** Most of the atom is empty space. This explains why most α-particles pass straight through.

1.4 Connecting the Dots: How Observations Led to the Nuclear Model

Experimental Observation	Conclusion / Inference
Most α-particles pass through undeflected.	Most of the atom is **empty space**.
A few α-particles are deflected by small angles.	The positive charge is not uniformly distributed. The nucleus is **positively charged** and repels the α-particle.
Very few (1 in 8000) are deflected back ($\theta \geq 90°$).	The positive charge and mass are concentrated in an extremely small **dense nucleus**. A head-on collision can cause a full rebound.

Table 2: Mapping experimental observations to the nuclear model.

Rutherford's Scattering Formula (Understanding the Math)

Rutherford derived a formula that relates the number of scattered particles (N) to the scattering angle (θ). While the full derivation is beyond the scope of NCERT, the formula and its implications are crucial.

The number of α-particles scattered at an angle θ is given by:

$$N(\theta) \propto \frac{1}{\sin^4(\theta/2)}$$

Key Implications of the Formula:

- As θ increases, $\sin^4(\theta/2)$ increases, so $N(\theta)$ decreases rapidly. This perfectly explained why large-angle scattering is so rare.
- The formula was experimentally verified by Geiger and Marsden, providing strong evidence for the nuclear model.

1.5 Drawbacks of Rutherford's Model

Despite its success, the nuclear model had a fatal flaw based on classical physics.

The Fundamental Limitation

1. **Instability of the Atom:** An electron revolving in a circular orbit is constantly accelerating (centripetal acceleration). According to Maxwell's electromagnetic theory, an accelerated charged particle must radiate energy in the form of electromagnetic waves.
2. **Spiral into the Nucleus:** As the electron loses energy, its orbit radius would continuously decrease, and it would follow a spiral path, eventually collapsing into the nucleus in about 10^{-8} seconds.
3. **Continuous Spectrum:** The frequency of the emitted radiation would change as the electron spirals in, resulting in a **continuous spectrum**. However, atoms are known to emit **line spectra** (discrete wavelengths).

Conclusion: Rutherford's model could not explain the **stability of the atom** and the origin of **line spectra**. This crisis paved the way for the Bohr model.

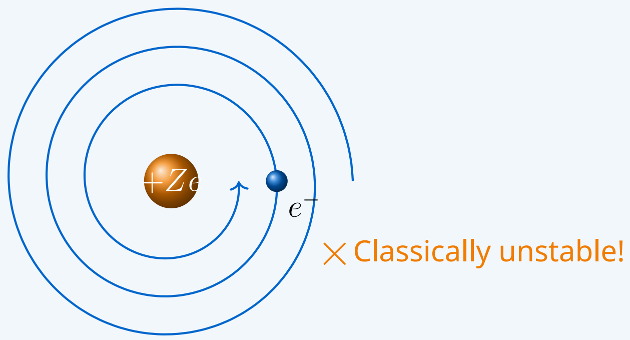


Figure 3: According to classical physics, an orbiting electron should spiral into the nucleus, making the atom unstable.

Key Takeaways for Exams

- Thomson's model: Atom is a uniform positive sphere with embedded electrons.
- Rutherford's experiment: Bombarded a thin gold foil with α-particles.
- Key Observation: Most passed through, a few deflected at small angles, and very few rebounded.
- Nuclear Model Postulates: (1) Tiny, dense, positive nucleus, (2) Electrons orbit the nucleus, (3) Atom is mostly empty space.
- Estimate of nuclear size: $\sim 10^{-15}$ m, which is 10^5 times smaller than the atom ($\sim 10^{-10}$ m).
- Scattering formula: $N(\theta) \propto 1/\sin^4(\theta/2)$.
- Main Drawback: Could not explain atomic stability or line spectra (classical instability).

2 Bohr's Model of the Hydrogen Atom

To overcome the fatal drawbacks of Rutherford's model, Niels Bohr, in 1913, proposed a new model for the hydrogen atom by applying the nascent quantum theory to classical mechanics. His model was a brilliant synthesis of Planck's quantum idea, Einstein's photon concept, and Rutherford's nuclear model.

2.1 Bohr's Postulates

Bohr's model is built upon three fundamental postulates that were radical departures from classical physics.

Bohr's Three Postulates for the Hydrogen Atom

Postulate 1: Stationary Orbits
An electron in an atom can revolve in certain stable, circular orbits without emitting any radiant energy. These non-radiating orbits are called **stationary orbits** or **allowed energy levels**. While in these orbits, the atom is stable.
Postulate 2: Quantization of Angular Momentum
The angular momentum (L) of an electron in a stationary orbit is an integral multiple of $h/2\pi$, where h is Planck's constant.

$$L = mvr = n\left(\frac{h}{2\pi}\right) = n\hbar$$

Here, n is an integer ($n = 1, 2, 3, \ldots$) called the **principal quantum number**. This postulate introduces quantization into the atomic model.
Postulate 3: Emission/Absorption of Radiation

An electron can make a transition from one stationary orbit to another only by absorbing or emitting a photon whose energy is exactly equal to the energy difference between the two states.

$$\Delta E = E_f - E_i = h\nu = \frac{hc}{\lambda}$$

- If $E_f < E_i$, a photon is **emitted**.
- If $E_f > E_i$, a photon is **absorbed**.

2.2 Derivation of Key Quantities for a Stationary Orbit

Consider a hydrogen-like atom with atomic number Z (for hydrogen, $Z = 1$). Let the electron of mass m and charge e revolve in a circular orbit of radius r with velocity v, around a nucleus of charge $+Ze$.

Derivation of Radius, Velocity, and Energy

Step 1: Balance of Forces
The centripetal force required for circular motion is provided by the Coulomb's electrostatic force of attraction.

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2} \tag{1}$$

Step 2: Apply Bohr's Quantization Rule
From Postulate 2, angular momentum is quantized:

$$mvr = n\frac{h}{2\pi} = n\hbar \tag{2}$$

Step 3: Derive Radius of n^{th} orbit (r_n)
From (2), $v = \frac{n\hbar}{mr}$. Substituting into (1) and solving for r:

$$\frac{m}{r}\left(\frac{n\hbar}{mr}\right)^2 = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$$

$$\frac{n^2\hbar^2}{mr} = \frac{Ze^2}{4\pi\varepsilon_0} \implies r = \frac{n^2\hbar^2 4\pi\varepsilon_0}{mZe^2}$$

Thus, the radius of the n^{th} orbit is:

$$\boxed{r_n = \left(\frac{4\pi\varepsilon_0\hbar^2}{me^2}\right)\frac{n^2}{Z} = a_0\frac{n^2}{Z}}$$

where $a_0 = \frac{4\pi\varepsilon_0\hbar^2}{me^2} \approx 5.29 \times 10^{-11}$ m is the **Bohr radius** (radius of the first orbit of hydrogen, $n = 1, Z = 1$).

Step 4: Derive Velocity of electron in n^{th} orbit (v_n)

From (2), $v = \frac{n\hbar}{mr_n}$. Substituting r_n:

$$v_n = \frac{n\hbar}{m\left(a_0 \frac{n^2}{Z}\right)} = \frac{Z\hbar}{ma_0}\frac{1}{n}$$

Using the expression for a_0, this simplifies to:

$$\boxed{v_n = \left(\frac{e^2}{4\pi\varepsilon_0\hbar}\right)\frac{Z}{n} = v_0\frac{Z}{n}}$$

where $v_0 = \frac{e^2}{4\pi\varepsilon_0\hbar} \approx 2.18 \times 10^6$ m/s is the velocity in the first Bohr orbit of hydrogen. A crucial note is that $v_0 \approx c/137$, where the dimensionless constant $\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137}$ is the **fine-structure constant**.

Step 5: Derive Total Energy of electron in n^{th} orbit (E_n)

Total energy $E =$ Kinetic Energy (K) + Potential Energy (U).

$$K = \frac{1}{2}mv^2; \qquad U = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r} \quad \text{(Coulomb potential)}$$

From (1), we know $\frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}$. So, $E = K + U = \frac{Ze^2}{8\pi\varepsilon_0 r} - \frac{Ze^2}{4\pi\varepsilon_0 r} = -\frac{Ze^2}{8\pi\varepsilon_0 r}$. Substituting the expression for r_n:

$$E_n = -\frac{Ze^2}{8\pi\varepsilon_0} \cdot \frac{1}{a_0\frac{n^2}{Z}} = -\frac{Z^2e^2}{8\pi\varepsilon_0 a_0}\frac{1}{n^2}$$

Substituting a_0, we get the final expression for energy:

$$\boxed{E_n = -\left(\frac{me^4}{8\varepsilon_0^2h^2}\right)\frac{Z^2}{n^2}}$$

For hydrogen ($Z = 1$), the energy in the ground state ($n = 1$) is $E_1 = -13.6$ eV. This is called the **ground state energy**.

$$\boxed{E_n(\text{in eV}) = -\frac{13.6}{n^2} \quad \text{(for Hydrogen)}}$$

Key Points on Energy Levels

- The total energy E_n is **negative**, signifying that the electron is bound to the nucleus. Positive energy means the electron is free (ionized).
- As n increases, the energy becomes less negative (i.e., increases). The state with $n = 1$ has the lowest (most negative) energy and is the **ground state**.
- States with $n = 2, 3, 4\ldots$ are **excited states**. The $n \to \infty$ state corresponds to $E_\infty = 0$, which is the ionization limit.

- The **ionization energy** is the minimum energy required to remove an electron from the ground state. For hydrogen, it is $|E_1| = 13.6$ eV.

2.3 Origin of Spectral Lines: The Hydrogen Spectrum

Bohr's third postulate explains the beautiful line spectrum of hydrogen. When an electron jumps from a higher energy level n_i to a lower energy level n_f, it emits a photon of a specific wavelength.

Derivation of the Rydberg Formula

The energy of the emitted photon is:

$$h\nu = \frac{hc}{\lambda} = E_{n_i} - E_{n_f}$$

For hydrogen ($Z = 1$), substituting the expression for E_n:

$$\frac{hc}{\lambda} = \left(-\frac{me^4}{8\varepsilon_0^2 h^2}\frac{1}{n_i^2}\right) - \left(-\frac{me^4}{8\varepsilon_0^2 h^2}\frac{1}{n_f^2}\right)$$

$$\frac{1}{\lambda} = \frac{me^4}{8\varepsilon_0^2 h^3 c}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

The constant term is the **Rydberg constant**, R:

$$\boxed{R = \frac{me^4}{8\varepsilon_0^2 h^3 c} \approx 1.097 \times 10^7 \text{ m}^{-1}}$$

Thus, the wavelength of any emitted line in the hydrogen spectrum is given by the Rydberg formula:

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

where $n_f < n_i$ and both are integers.

2.4 Naming the Spectral Series

Transitions ending at a specific lower level n_f form a **spectral series**. Each series is named after its discoverer.

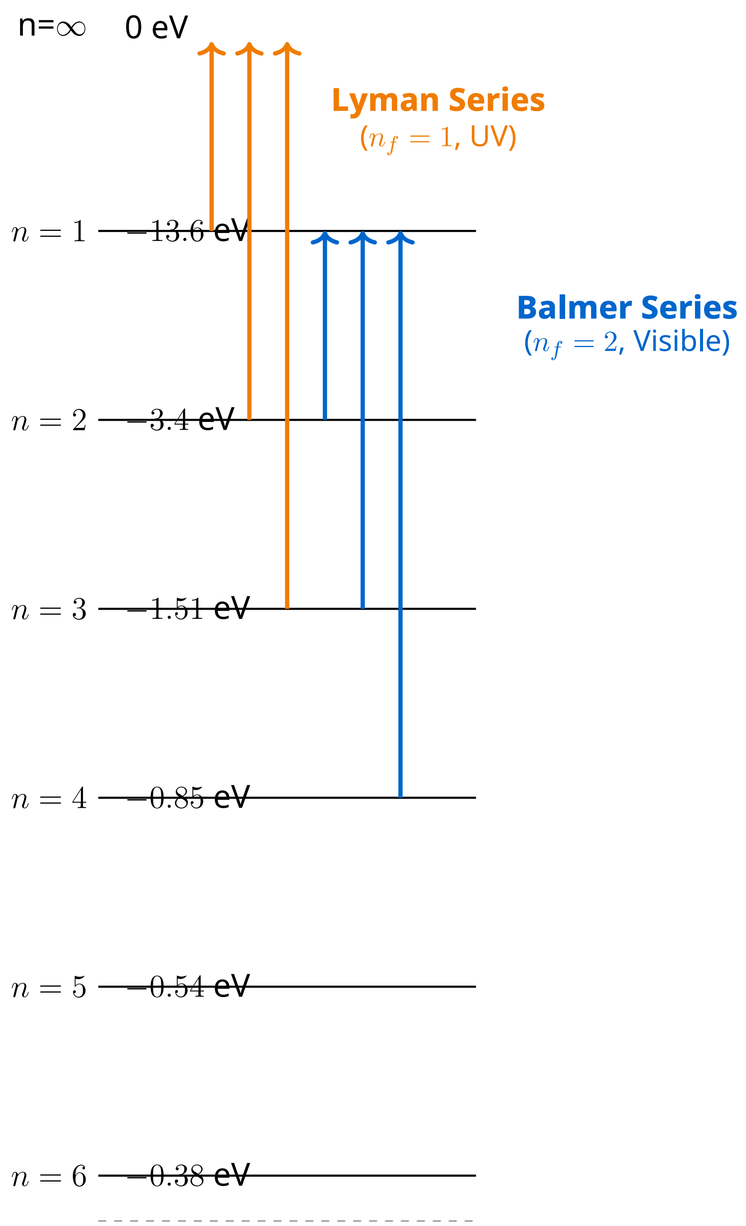


Figure 4: Energy level diagram for hydrogen, showing the formation of the Lyman and Balmer series. (Energies not to exact vertical scale).

The Hydrogen Spectral Series

Series Name	Final Orbit (n_f)	Region of Spectrum	First Line (n_i)
Lyman	1	Ultraviolet	2
Balmer	2	Visible	3
Paschen	3	Infrared	4
Brackett	4	Infrared	5

Pfund	5	Far Infrared	6

The members of a series are named α, β, γ, etc., corresponding to the first, second, third line in the series (i.e., $n_i = n_f + 1, n_f + 2, \ldots$).

Example: First Line of Balmer Series

The H_α line is the first line of the Balmer series, corresponding to the transition from $n_i = 3$ to $n_f = 2$.

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{36}$$

This gives a wavelength of $\lambda = 656.3$ nm, which is in the red region of the visible spectrum.

2.5 Limitations of Bohr's Model

Despite its stunning success for hydrogen, Bohr's model was not the final word.

Failures of Bohr's Atomic Model

1. **Multi-Electron Atoms:** It failed to explain the spectrum of atoms with more than one electron (like helium), even qualitatively.
2. **Fine Structure:** High-resolution spectroscopes revealed that many spectral lines are actually composed of two or more closely spaced lines (fine structure). Bohr's model could not explain this.
3. **Intensity of Spectral Lines:** The model could not predict the relative intensities (brightness) of different spectral lines.
4. **Orbit Shape:** It assumed orbits were circular, while later models showed they could be elliptical.
5. **Validity of Postulates:** The model was a hybrid of classical and quantum ideas. The postulates themselves (especially stationary states and the quantization rule) had no theoretical justification within Bohr's framework. They were "rules of thumb" that worked.

Conclusion: Bohr's model provided a crucial stepping stone to a more complete quantum mechanical model of the atom (Schrödinger's model), which we will study next.

3 De Broglie's Explanation and the Quantum Mechanical Model

Bohr's model, while phenomenally successful for hydrogen, left a crucial question unanswered: *Why* is the angular momentum of an electron quantized? The answer came from a brilliant insight by Louis de Broglie, which ultimately paved the way for the modern quantum mechanical model of the atom.

3.1 De Broglie's Matter Wave Hypothesis

In 1924, de Broglie proposed that just as light exhibits wave-particle duality, material particles like electrons should also have an associated wavelength.

De Broglie Wavelength

Any moving particle with momentum $p = mv$ has an associated wavelength λ, given by:

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

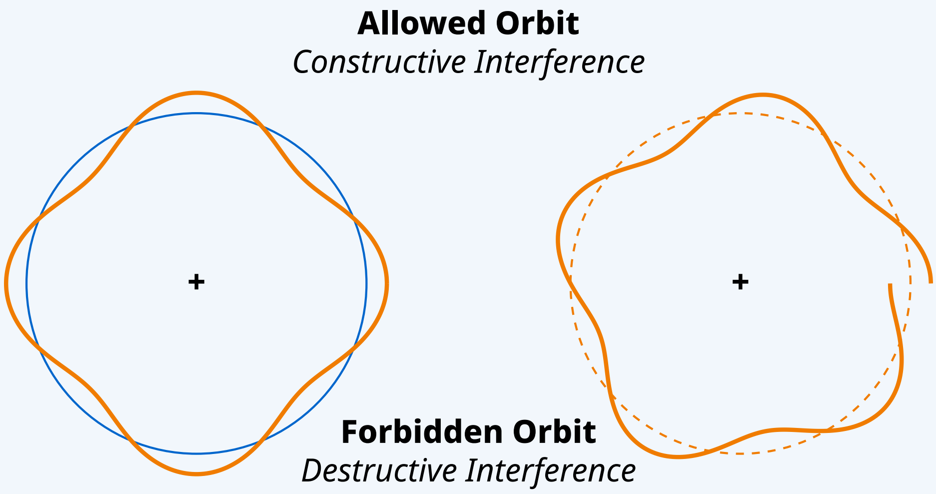
where h is Planck's constant. These waves are called **matter waves** or **de Broglie waves**.

3.2 De Broglie's Explanation of Bohr's Quantization Rule

De Broglie provided a beautiful physical picture for Bohr's second postulate. He argued that an electron in a circular orbit is a standing matter wave.

The Standing Wave Condition

For a wave to be stable and non-radiating, it must be a **standing wave** that closes on itself in phase. This means the circumference of the orbit must contain an exact integer number of electron wavelengths.



A standing wave for $n = 4$ | Destructive interference (not allowed)

Figure 5: Visualizing Bohr orbits as standing electron waves. Only orbits with integer number of wavelengths survive.

Mathematically, this condition is:

$$2\pi r_n = n\lambda, \quad \text{where } n = 1, 2, 3, \ldots$$

Deriving Bohr's Quantization from de Broglie's Hypothesis

Starting from the standing wave condition and using de Broglie's formula:

$$2\pi r_n = n\lambda = n\left(\frac{h}{mv}\right)$$

$$2\pi r_n(mv) = nh$$

$$mvr_n = n\frac{h}{2\pi} = n\hbar$$

And there it is! The condition for a stable electron wave is exactly **Bohr's quantization of angular momentum**. This provided a profound physical justification for what was previously just a rule of thumb.

3.3 Transition to the Quantum Mechanical Model

De Broglie's idea was the spark that ignited the development of full-fledged quantum mechanics. The atom needed a radical new framework, not just a patch to classical physics.

The Heisenberg Uncertainty Principle (A Fundamental Limit)

Werner Heisenberg proposed a principle that sets a fundamental limit to the precision with which certain pairs of physical properties can be known simultaneously. For position (x) and momentum (p):

$$\boxed{\Delta x \cdot \Delta p \geq \frac{\hbar}{2}}$$

Implication for the Atom: Bohr's model spoke of a well-defined electron *orbit* with a precise radius and velocity. But this is fundamentally impossible! If an electron is confined to a tiny space (Δx is small), the uncertainty in its momentum (Δp) must be large. We cannot define a neat, trajectory-like orbit for an electron.

This idea completely dismantles the concept of fixed circular orbits. We must replace the "orbit" with something more probabilistic.

3.4 The Schrödinger Equation and the Modern View

In 1926, Erwin Schrödinger proposed a wave equation for matter, analogous to the wave equation for light. This is the cornerstone of Quantum Mechanics.

Key Concepts of the Quantum Mechanical Model

1. **Wave Function (ψ):** The state of an electron in an atom is described by a mathematical function called the **wave function**, denoted by $\psi(x, y, z, t)$. ψ itself has no direct physical meaning.

2. **Probability Density ($|\psi|^2$):** The square of the absolute magnitude of the wave function, $|\psi|^2$, at any point gives the **probability density** of finding the electron at that point. This is Max Born's probabilistic interpretation.

$$|\psi|^2 dV = \text{Probability of finding the electron in volume } dV$$

3. **Orbitals, not Orbits:** The idea of a fixed circular **orbit** is replaced by an **orbital**. An orbital is a three-dimensional region of space around the nucleus where the probability of finding the electron is high (e.g., 90%).

4. **Quantization Arises Naturally:** In Schrödinger's model, quantized energy levels (E_n) and other quantum numbers emerge naturally from the mathematical boundary conditions of solving the wave equation for a bound electron. Quantization is not an external "postulate" but a result of the wave nature.

Quantum Numbers in Schrödinger's Model

The solution to the Schrödinger equation for a hydrogen atom yields three quantum numbers that completely describe the state of an electron. These match the patterns seen in spectra.

Quantum Number	Symbol	Possible Values	Physical Significance
Principal	n	$1, 2, 3, \ldots$	Determines the **energy** and size of the shell.
Orbital (Azimuthal)	l	$0, \ldots, (n-1)$	Determines the **angular momentum** and orbital shape.
Magnetic	m_l	$-l, \ldots, 0, \ldots, l$	Determines the **orientation** of the orbital in space.

Notation: For a given l, we use letters.

$l =$	0	1	2	3
Orbital letter	s	p	d	f

So, an electron with $n = 2, l = 1$ is in a 2p orbital. Later, a fourth quantum number (spin, m_s) was introduced to explain fine structure.

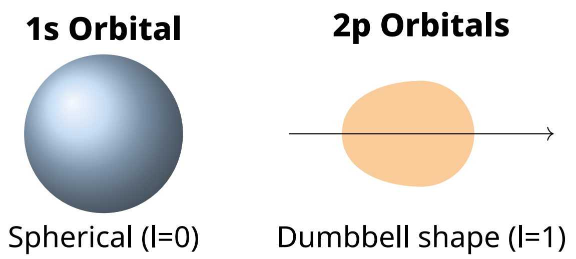


Figure 6: A very schematic representation of electron probability clouds for s and p orbitals.

Key Takeaways: Bohr vs. Quantum Mechanical Model

Bohr's Model	Quantum Mechanical Model
Particle model with wave-like aspects forcefully added as a postulate.	The electron is treated as a wave-particle dual entity from the start.

Well-defined circular **orbits** with fixed radius and velocity.	**Orbitals**: 3D probability clouds where $\|\psi\|^2$ is high. No definite path.
Angular momentum $L = n\hbar$ (only integer multiples).	Angular momentum $L = \sqrt{l(l+1)}\hbar$, which can be zero ($L = 0$ for $l = 0$).
Quantization is an **external postulate**.	Quantization of energy and angular momentum **emerges naturally** from solving the Schrödinger equation.
Described by one quantum number (n).	Described by three quantum numbers: n, l, m_l.
Could not explain fine structure or spectra of multi-electron atoms.	Can (with spin m_s) explain fine structure and provides a framework for all atoms.

Final Thought: De Broglie's wave-particle duality and the Heisenberg uncertainty principle form the conceptual bedrock upon which Schrödinger built his revolutionary quantum mechanical model of the atom. This model, with its probabilistic orbitals, is our most accurate and complete description of the atomic world.

4 Atomic Spectra and Numerical Applications

A spectrum is the fingerprint of an element. The study of atomic spectra provided the crucial experimental data that led to Bohr's model and remains a powerful tool for understanding atomic structure. This topic consolidates the spectral series of hydrogen and equips you with problem-solving skills for numerical questions.

4.1 Types of Spectra

Before diving deep into hydrogen, it's vital to understand the two fundamental types of spectra.

Emission and Absorption Spectra

1. Emission Spectrum:

- Produced when atoms in an excited state fall to lower energy states, emitting photons of specific frequencies.
- Appears as **bright colored lines** on a dark background.
- Each element has a unique emission spectrum, acting as its atomic fingerprint.

2. Absorption Spectrum:

- Produced when white light (continuous spectrum) passes through a cool, dilute gas. The atoms absorb photons of specific frequencies to jump to higher energy states.
- Appears as **dark lines** (missing frequencies) on a continuous bright background.
- The dark lines in the absorption spectrum of a gas correspond exactly in wavelength to the bright lines in its emission spectrum. This is a cornerstone of spectral analysis.

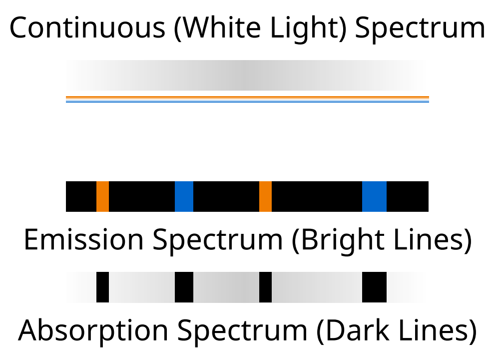


Figure 7: Visual comparison of emission and absorption line spectra.

4.2 The Hydrogen Spectral Series in Detail

Recall the Rydberg formula, the master equation for the hydrogen spectrum:

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

where $R = 1.097 \times 10^7$ m^{-1} is the Rydberg constant, n_f is the final orbit, and n_i is the initial orbit ($n_i > n_f$).

Complete Overview of the Hydrogen Spectral Series

Series	n_f	n_i	Region	First Line (λ)	Series Limit (λ)
Lyman	1	$2, 3, \ldots$	UV	$n_i = 2 \to 1$ 121.6 nm	$n_i = \infty \to 1$ 91.2 nm
Balmer	2	$3, 4, \ldots$	Visible	$n_i = 3 \to 2$ (H_α) 656.3 nm	$n_i = \infty \to 2$ 364.6 nm
Paschen	3	$4, 5, \ldots$	IR	$n_i = 4 \to 3$ 1875 nm	$n_i = \infty \to 3$ 820.4 nm
Brackett	4	$5, 6, \ldots$	IR	$n_i = 5 \to 4$ 4051 nm	$n_i = \infty \to 4$ 1458 nm
Pfund	5	$6, 7, \ldots$	Far IR	$n_i = 6 \to 5$ 7458 nm	$n_i = \infty \to 5$ 2279 nm

4.3 Understanding the Series Limit

A crucial concept is the **series limit**, which corresponds to the shortest wavelength (highest energy) line in a given series.

Series Limit

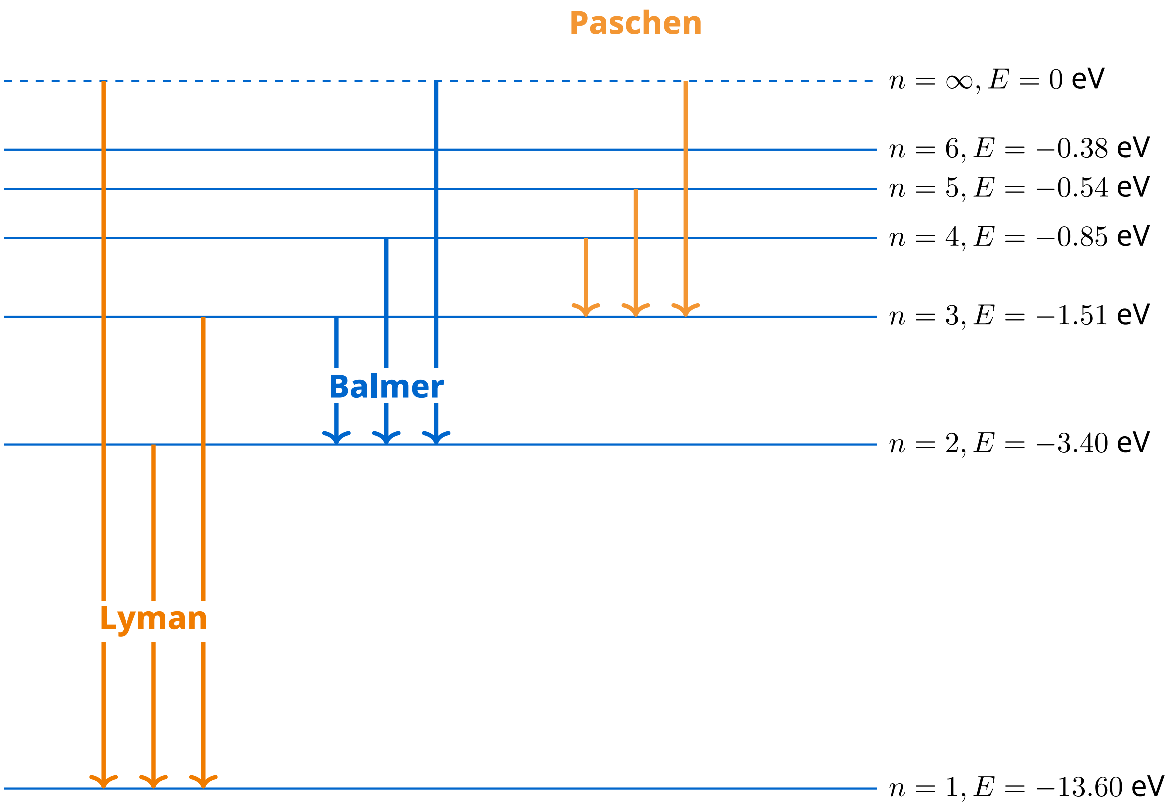
The series limit occurs when the electron makes a transition from $n_i = \infty$ (a free electron, zero energy) to a given n_f. The wavelength is given by:

$$\frac{1}{\lambda_{\text{limit}}} = R\left(\frac{1}{n_f^2} - \frac{1}{\infty^2}\right) = \frac{R}{n_f^2}$$

$$\boxed{\lambda_{\text{limit}} = \frac{n_f^2}{R}}$$

The energy corresponding to the series limit is the ionization energy required to remove the electron from that energy level n_f to infinity.

$$\boxed{E_{\text{ionization}} = \frac{13.6}{n_f^2}\ \text{eV}}$$



Note: Energy scale is not linear; spacing is adjusted for clarity.

Figure 8: High-clarity energy level diagram with expanded spacing for upper levels.

4.4 Mastering Numerical Problems

The most common numerical problems revolve around the Rydberg formula, energy level transitions, and ionization energies. Let's methodically go through solved examples.

Solved Example 1: Wavelength of a Balmer Line

Question: Determine the wavelength of the H_β line in the Balmer series of the hydrogen spectrum. **(Hint:** H_β is the second line of the Balmer series).
Solution:

1. **Identify the transition:** The Balmer series corresponds to $n_f = 2$. The first line, H_α, is from $n_i = 3$ to 2. Therefore, the second line, H_β, is from $n_i = 4$ to 2.

2. **Apply the Rydberg formula:**

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = R\left(\frac{1}{4} - \frac{1}{16}\right) = R\left(\frac{4-1}{16}\right) = \frac{3R}{16}$$

3. **Solve for λ:**

$$\lambda = \frac{16}{3R} = \frac{16}{3 \times (1.097 \times 10^7 \text{ m}^{-1})} = 4.861 \times 10^{-7} \text{ m} = 486.1 \text{ nm}$$

Solved Example 2: Finding Quantum Numbers

Question: The wavelength of the first line of the Lyman series for hydrogen is identical to that of the second line of the Balmer series for some hydrogen-like ion X (with a single electron). What is the atomic number Z of ion X?
Solution:

1. **Write the modified Rydberg formula for a hydrogen-like ion:** For an ion with atomic number Z,

$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

2. **Define the transitions:**
   - For Hydrogen ($Z = 1$), first Lyman line: $n_f = 1, n_i = 2$.
   - For Ion X, second Balmer line: $n_f = 2, n_i = 4$.
3. **Equate their wave numbers:**

$$R(1)^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = RZ^2 \left(\frac{1}{2^2} - \frac{1}{4^2} \right)$$

$$\left(1 - \frac{1}{4} \right) = Z^2 \left(\frac{1}{4} - \frac{1}{16} \right)$$

$$\frac{3}{4} = Z^2 \left(\frac{4-1}{16} \right) = Z^2 \left(\frac{3}{16} \right)$$

4. **Solve for Z:**

$$Z^2 = \frac{3}{4} \times \frac{16}{3} = 4 \implies Z = 2$$

   The ion is He^+ (singly ionized helium).

Solved Example 3: Ionization Energy

Question: The ground state energy of hydrogen is -13.6 eV. What is the kinetic energy and potential energy of the electron in this state? Also, find the wavelength of the photon emitted when an electron in the third excited state of Li^{2+} ($Z = 3$) makes a transition to the ground state.
Solution (Part 1):

1. We know that Total Energy $E = K + U$, and from the virial theorem for Coulomb forces, $U = -2K$.

2. Therefore, $E = K + (-2K) = -K$. So, $K = -E$.
3. For ground state: $K = -(-13.6$ eV$) =$ **13.6 eV**.
4. Potential Energy: $U = -2K =$ **-27.2 eV**.

Solution (Part 2):

1. The energy of an electron in a hydrogen-like species is $E_n = -13.6\frac{Z^2}{n^2}$ eV.
2. For Li^{2+} ($Z = 3$), the third excited state corresponds to $n = 4$. The ground state is $n = 1$.
3. Energy difference:

$$\Delta E = E_4 - E_1 = -13.6 \times 3^2 \left(\frac{1}{4^2} - \frac{1}{1^2}\right) = -122.4\left(\frac{1}{16} - 1\right) = -122.4\left(-\frac{15}{16}\right)$$

$$\Delta E = 122.4 \times \frac{15}{16} = 114.75 \text{ eV}$$

4. Convert energy (Joules) to wavelength (meters): $\Delta E = \frac{hc}{\lambda}$.

$$\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} \cdot \text{nm}}{114.75 \text{ eV}} \approx 10.8 \text{ nm}$$

This falls in the extreme ultraviolet (EUV) region.

4.5 Common Problem-Solving Shortcuts

Quick Calculation Tricks for Exams

- **Energy in eV:** $E_n = -13.6\frac{Z^2}{n^2}$ eV. Memorize this. It's your starting point for 90% of energy problems.
- **Photon Energy-Wavelength Relation:** $\lambda(\text{in nm}) = \frac{1240}{E(\text{in eV})}$. This conversion is incredibly useful and saves time.
- **Rydberg Quick Formula:** $\Delta E = 13.6Z^2\left|\frac{1}{n_f^2} - \frac{1}{n_i^2}\right|$ eV. Then convert to wavelength using $1240/\Delta E$.
- **Ionization from Ground:** The energy required to ionize an atom from its ground state ($n = 1$) is simply $|E_1| = 13.6Z^2$ eV.
- **Angular Momentum:** $L = n\hbar = n\frac{h}{2\pi}$. For $n = 3$, $L = 3\hbar$, and so on.
- **Radius:** $r_n = 0.529\frac{n^2}{Z}$ Å (where 1 Å = 10^{-10} m). The Bohr radius a_0 is 0.529 Å.
- **Velocity:** $v_n = 2.18 \times 10^6\frac{Z}{n}$ m/s. The fine-structure constant $\alpha = v_0/c \approx 1/137$.

Key Conceptual Points for Exam Questions

- A **line spectrum** is characteristic of atoms in the gaseous state. Solids and dense gases emit continuous spectra.
- The **first line** in a series has the longest wavelength (lowest energy) in that series.
- The **series limit** has the shortest wavelength (highest energy) in that series.
- The Balmer series is the only series in the **visible region** of the electromagnetic spectrum.
- In a discharge tube, the emission spectrum is observed. For absorption, you need a background source of continuous white light.
- **Isoelectronic species** (like H, He^+, Li^{2+}) have the same number of electrons. Their spectral lines obey the formula with the Z^2 factor.

5 Quick Revision: Formula Sheet and NCERT Solved Examples

This section is your final checkpoint. It consolidates every critical formula, constant, and concept from the chapter into one place. This is followed by meticulously solved NCERT examples, which are the gold standard for CBSE board exam preparation.

5.1 The Ultimate Formula Sheet for Atoms

Bohr's Postulates and Derived Quantities

Fundamental Postulates:

1. Stationary orbits: Electrons revolve without radiating energy.
2. Quantization of Angular Momentum: $L = mvr = n\frac{h}{2\pi} = n\hbar$
3. Energy Transition: $\Delta E = E_f - E_i = h\nu = \frac{hc}{\lambda}$

For Hydrogen-like Species (atomic number Z):

$$\text{Radius of } n^{th} \text{ orbit: } \boxed{r_n = \frac{a_0 n^2}{Z} = 0.529\frac{n^2}{Z}\ \text{Å}} \quad (1)$$

$$\text{Velocity in } n^{th} \text{ orbit: } \boxed{v_n = \frac{v_0 Z}{n} = 2.18 \times 10^6 \frac{Z}{n}\ \text{m/s}} \quad (2)$$

$$\text{Energy of } n^{th} \text{ orbit: } \boxed{E_n = -13.6\frac{Z^2}{n^2}\ \text{eV} = -2.18 \times 10^{-18}\frac{Z^2}{n^2}\ \text{J}} \quad (3)$$

Spectral Series and the Rydberg Formula

Rydberg Formula (Wave Number):

$$\boxed{\bar{\nu} = \frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

where $R = 1.097 \times 10^7$ m^{-1} is the Rydberg constant.
Energy Form:

$$\boxed{\Delta E = 13.6Z^2\left|\frac{1}{n_f^2} - \frac{1}{n_i^2}\right|\ \text{eV}}$$

Photon Wavelength-Energy Conversion:

$$\boxed{\lambda(\text{nm}) = \frac{1240}{E(\text{eV})}}$$

Hydrogen Spectral Series:

Series	n_f	Region	First Member
Lyman	1	Ultraviolet	$n_i = 2$ (121.6 nm)
Balmer	2	Visible	$n_i = 3, H_\alpha$ (656.3 nm)
Paschen	3	Infrared	$n_i = 4$ (1875 nm)
Brackett	4	Infrared	$n_i = 5$ (4051 nm)
Pfund	5	Far Infrared	$n_i = 6$ (7458 nm)

Important Constants and Values to Memorize

Physical Constant	Symbol	Value
Bohr Radius	a_0	5.29×10^{-11} m = 0.529 Å
Rydberg Constant	R	1.097×10^{7} m^{-1}
Ground State Energy of Hydrogen	E_1	-13.6 eV
Ionization Energy of Hydrogen	$E_\infty - E_1$	13.6 eV
Velocity in First Bohr Orbit	v_0	2.18×10^{6} m/s $\approx c/137$
Fine Structure Constant	α	$\frac{e^2}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137}$
Planck's Constant	h	6.63×10^{-34} J s
hc (for energy-wavelength)	hc	1240 eV·nm
Electron Volt	1 eV	1.6×10^{-19} J

Key Conceptual Facts and Comparisons

Thomson vs. Rutherford:

- Thomson: Uniform positive sphere ($\sim 10^{-10}$ m), electrons embedded. Failed to explain large-angle α-scattering.
- Rutherford: Dense positive nucleus ($\sim 10^{-15}$ m), electrons orbit like planets. Failed to explain stability and line spectra.

Bohr vs. Quantum Mechanical Model:

- Bohr: Well-defined circular orbits, one quantum number (n), quantization by postulate. Works for H, fails for He.
- Quantum: Probability clouds (orbitals), three quantum numbers (n, l, m_l), quantization emerges naturally. Works for all atoms.

De Broglie's Insight:

- Electron is a standing wave around the nucleus. $2\pi r_n = n\lambda$ leads to $L = n\hbar$.

Crucial Mnemonics:

- **LY**man is in **U**V (Ultraviolet). $\rightarrow$ **LYU**
- **BA**lmer is **V**isible. $\rightarrow$ **BAV**
- **PAS**chen is **IR** (Infrared). $\rightarrow$ **PASIR**
- Size of nucleus to atom: 1 : 100,000 (like a cricket ball in a stadium).

5.2 NCERT Textbook Solved Examples

These examples are directly from the NCERT Class 12 Physics textbook and are frequently adapted for board exam questions. Each step is explained clearly for your understanding.

NCERT Example 12.1: Energy Level Calculation

Question: The ground state energy of hydrogen atom is -13.6 eV. What are the kinetic and potential energies of the electron in this state?
Solution:

1. We know that for an electron in a Coulomb field (like the hydrogen atom), the total energy E is related to the kinetic energy K and potential energy U by the virial theorem: $U = -2K$.
2. The total energy is the sum: $E = K + U = K + (-2K) = -K$.
3. Therefore, $K = -E = -(-13.6 \text{ eV}) =$ **13.6 eV**.
4. The potential energy: $U = E - K = -13.6$ eV $- 13.6$ eV $=$ **-27.2 eV**.

Check: $K + U = 13.6 + (-27.2) = -13.6$ eV, which matches the total energy.

NCERT Example 12.2: Wavelength of a Transition

Question: An electron in a hydrogen atom makes a transition from $n = 2$ to $n = 1$. Calculate the wavelength of the photon emitted. Given that the ground state energy is -13.6 eV.
Solution:

1. The energy of the n^{th} level is $E_n = -13.6/n^2$ eV.
2. Energy of initial state ($n = 2$): $E_2 = -13.6/4 = -3.4$ eV.
3. Energy of final state ($n = 1$): $E_1 = -13.6$ eV.
4. Energy of emitted photon: $\Delta E = E_2 - E_1 = -3.4 - (-13.6) = 10.2$ eV.
5. Use the relation $\lambda(\text{nm}) = 1240/E(\text{eV})$:

$$\lambda = \frac{1240 \text{ eV} \cdot \text{nm}}{10.2 \text{ eV}} \approx \textbf{121.6 nm}$$

This is the first line of the Lyman series, in the ultraviolet region.

NCERT Example 12.3: Balmer Series and Visible Light

Question: Using the Rydberg formula, calculate the wavelengths of the first four spectral lines in the Balmer series of the hydrogen spectrum. In which regions do these lines lie?
Solution:

1. For the Balmer series, $n_f = 2$. The first four lines correspond to $n_i = 3, 4, 5, 6$.
2. Recall the Rydberg formula: $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right) = R\left(\frac{1}{4} - \frac{1}{n_i^2}\right)$.
   (a) H_α **(**$n_i = 3$**):** $\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{9}\right) = 1.097 \times 10^7 \times \frac{5}{36} \implies \lambda = 656.3$ nm (Red).
   (b) H_β **(**$n_i = 4$**):** $\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{16}\right) = 1.097 \times 10^7 \times \frac{3}{16} \implies \lambda = 486.1$ nm (Blue-Green).
   (c) H_γ **(**$n_i = 5$**):** $\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{25}\right) = 1.097 \times 10^7 \times \frac{21}{100} \implies \lambda = 434.0$ nm (Violet).
   (d) H_δ **(**$n_i = 6$**):** $\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{36}\right) = 1.097 \times 10^7 \times \frac{8}{36} \implies \lambda = 410.2$ nm (Violet).
3. All four lines lie in the **visible region** of the electromagnetic spectrum. As n_i increases, the spectral lines come closer together and converge at the series limit of 364.6 nm (UV).

NCERT Example 12.4: Hydrogen-like Ion

Question: A singly ionized helium atom (He^+) is hydrogen-like. Calculate the wavelength of the photon required to excite the electron in He^+ from the first Bohr orbit to the third Bohr orbit.
Solution:

1. For a hydrogen-like ion with atomic number Z, the energy is $E_n = -13.6\frac{Z^2}{n^2}$ eV. For He^+, $Z = 2$.
2. Energy of the first orbit ($n = 1$): $E_1 = -13.6 \times \frac{2^2}{1^2} = -54.4$ eV.
3. Energy of the third orbit ($n = 3$): $E_3 = -13.6 \times \frac{2^2}{3^2} = -\frac{54.4}{9} \approx -6.04$ eV.
4. The energy required for excitation: $\Delta E = E_3 - E_1 = -6.04 - (-54.4) = 48.36$ eV.
5. Convert to wavelength: $\lambda(\text{nm}) = \frac{1240}{\Delta E(\text{eV})} = \frac{1240}{48.36} \approx$ **25.6 nm**.
6. This wavelength lies in the extreme ultraviolet (EUV) region.

NCERT Example 12.5: De Broglie Wavelength of an Orbital Electron

Question: Using de Broglie's hypothesis, explain with the help of a suitable diagram, why Bohr's second postulate of quantization of orbital angular momentum is a consequence of the wave nature of the electron. Also, find the de Broglie wavelength of an electron in the $n = 3$ orbit of hydrogen.
Solution (Part 1 - Explanation):

1. An electron moving in a circular orbit behaves as a matter wave.

2. For the electron wave to be stable (a standing wave) and not radiate energy, the circumference of the orbit must contain an integral number of wavelengths.

3. Mathematically: $2\pi r_n = n\lambda$, where $n = 1, 2, 3, \dots$

4. Substituting de Broglie's relation $\lambda = h/p = h/mv$:

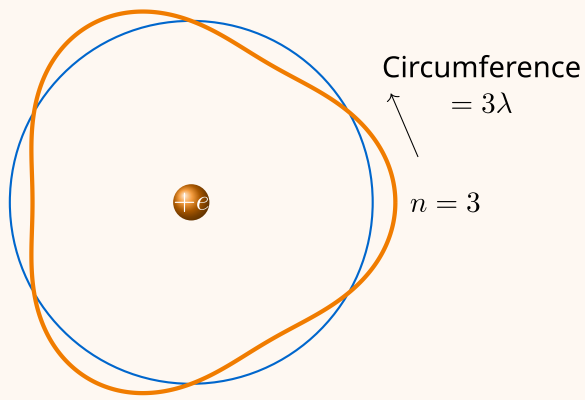
$$2\pi r_n = n\frac{h}{mv} \implies mvr_n = n\frac{h}{2\pi} = n\hbar$$

This is exactly Bohr's quantization condition, thus proving it is a natural consequence of the electron's wave nature.

Solution (Part 2 - Calculation):

1. For hydrogen ($Z = 1$), the radius of the n^{th} orbit is $r_n = a_0 n^2$.
2. For $n = 3$, $r_3 = a_0 \times 3^2 = 9a_0$.
3. From the standing wave condition: $2\pi r_3 = 3\lambda$.
4. $\lambda = \frac{2\pi r_3}{3} = \frac{2\pi(9a_0)}{3} = 6\pi a_0$.
5. Substitute $a_0 = 5.29 \times 10^{-11}$ m:

$\lambda = 6\times3.14\times5.29\times10^{-11}$ m $\approx$ **9.97** $\times10^{-10}m$**9.97** $\times10^{-10}m$**9.97** $\times10^{-10}m$**9.97** $\times10^{-10}m$



Standing Electron Wave
(De Broglie's picture of a stable orbit)

Figure 9: The standing wave pattern for an electron in the $n = 3$ orbit. The circumference contains exactly three de Broglie wavelengths.

NCERT Example 12.6: Absorption Spectrum Analysis

Question: The absorption spectrum of hydrogen shows a dark line at 434 nm. Between which two energy levels did the electron jump to cause this absorption? Also, determine the initial energy level of the electron.

Solution:

1. An absorption line means the electron jumped from a lower energy level (n_f) to a higher energy level (n_i), absorbing a photon of exactly that wavelength. We know $\lambda = 434$ nm.
2. Energy of absorbed photon: $\Delta E = \frac{1240 \text{ eV·nm}}{434 \text{ nm}} \approx 2.86$ eV.
3. The hydrogen atom absorbs this energy. It must start from a stationary state. Given that 434 nm is in the visible region, it is a line in the Balmer series. For the Balmer series, the lower level is always $n_f = 2$.
4. So, the electron jumped from $n_f = 2$. Let's find n_i.
5. Energy difference: $E_{n_i} - E_2 = 2.86$ eV.
6. $E_{n_i} = E_2 + 2.86 = (-3.4 \text{ eV}) + 2.86 = -0.54$ eV.
7. Since $E_n = -13.6/n_i^2$, we have $-13.6/n_i^2 = -0.54 \implies n_i^2 = 13.6/0.54 \approx 25.2 \approx 25$.
8. Thus, $n_i = 5$.
9. **Answer:** The electron jumped from $n = 2$ to $n = 5$, absorbing a violet photon of 434 nm (this is the H_γ line in absorption).

6 Chapter at a Glance: Mind Map

The Complete Story of "Atoms" for Class 12

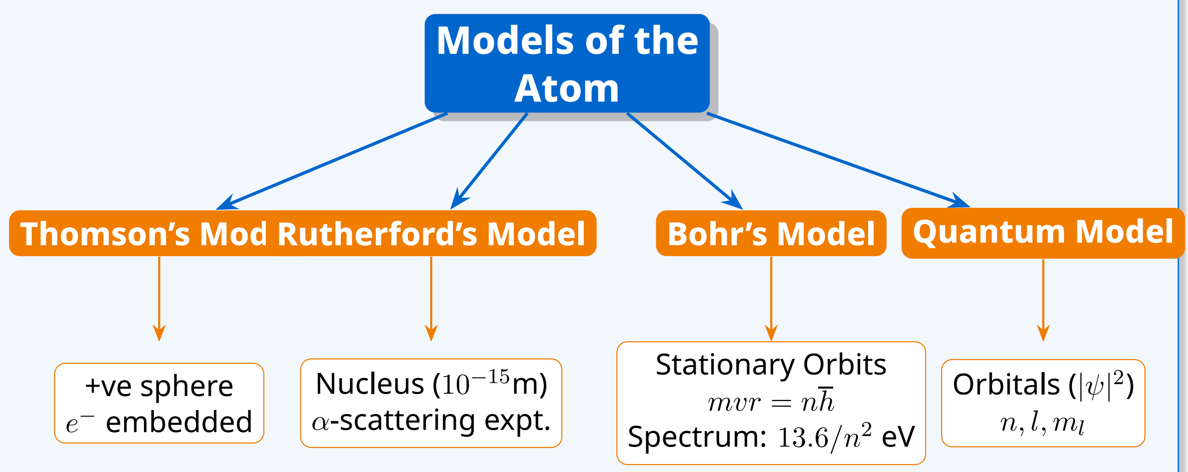


Figure 10: A visual summary of the evolution of atomic models.

— End of Chapter: Atoms —