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NCERT SOLUTIONS

Class 12 Physics

Chapter 12: Atoms

Detailed Step-by-Step Exercise Solutions

Q1 **Choose the correct alternative from the clues given at the end of each statement:**

(a) **The size of the atom in Thomson's model is __________ the atomic size in Rutherford's model.**
(much greater than / no different from / much less than.)

(b) **In the ground state of __________ electrons are in stable equilibrium, while in __________ electrons always experience a net force.**
(Thomson's model / Rutherford's model.)

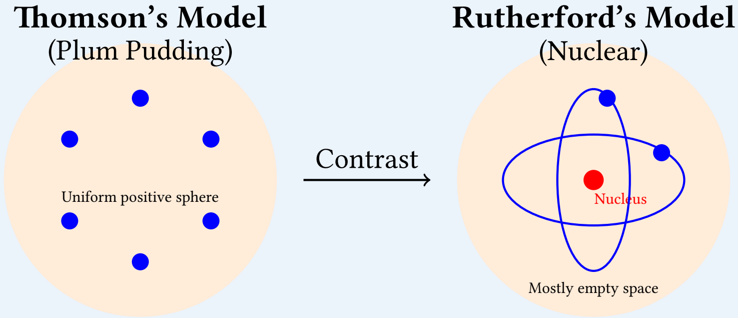
(c) **A classical atom based on __________ is doomed to collapse.**
(Thomson's model / Rutherford's model.)

(d) **An atom has a nearly continuous mass distribution in a __________ but has a highly non-uniform mass distribution in __________.**
(Thomson's model / Rutherford's model.)

(e) **The positively charged part of the atom possesses most of the mass in __________.**
(Rutherford's model / both the models.)

Solution

Understanding the Context: Thomson vs. Rutherford Atomic Models
This question tests the fundamental conceptual differences between J.J. Thomson's "plum pudding" model and Ernest Rutherford's nuclear model of the atom. Let's briefly recap the essential features of each.



Now, let's address each part systematically.

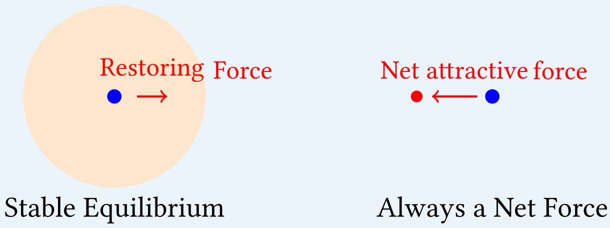
(a) Comparative Size of the Atom

- In **Thomson's model**, the positive charge is spread uniformly over a sphere of radius approximately equal to the atomic radius ($\sim 10^{-10}$ m). The entire sphere constitutes the atom.
- In **Rutherford's model**, the positive charge is concentrated in an extremely tiny central nucleus (radius $\sim 10^{-15}$ m), while the electrons orbit at distances comparable to the atomic radius ($\sim 10^{-10}$ m). The atom's overall size is defined by the electron orbits, making it roughly the same size as Thomson's atom.
- Therefore, the overall size of the atom in both models is comparable—**no different from**.

Answer (a): no different from

(b) Stable Equilibrium of Electrons

- In **Thomson's model**, the electrons are stationary and embedded within a uniform sphere of positive charge. In the *ground state*, the electrostatic forces balance perfectly, placing the electrons in **stable equilibrium** (like seeds in a watermelon). If displaced slightly, they experience a restoring force back to their equilibrium position.
- In **Rutherford's model**, the electrons are in motion around the nucleus. A stationary electron would be pulled straight into the nucleus. Even in orbit, as per classical electrodynamics, an accelerating charge radiates energy, spirals inward, and thus always experiences a **net force** toward the nucleus. There is no static stable equilibrium.



Answer (b): In the ground state of **Thomson's model** electrons are in stable equilibrium, while in **Rutherford's model** electrons always experience a net force.

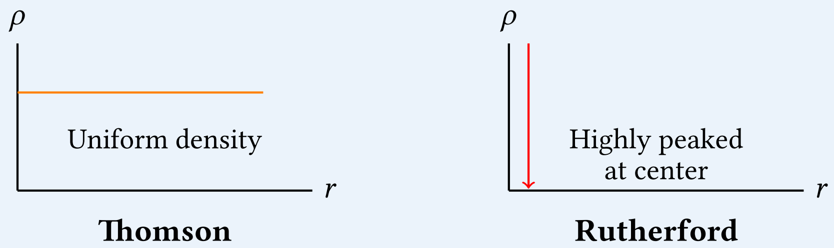
(c) The Classical Collapse

- In **Rutherford's model**, electrons orbit the nucleus much like planets orbit the sun. However, classical electromagnetic theory demands that any accelerating charged particle must radiate energy. An orbiting electron continuously loses energy, its orbit radius decreases, and it would spiral into the nucleus in about 10^{-10} seconds. Thus, a classical atom based on Rutherford's model is **doomed to collapse**.
- Thomson's model, while incorrect in other respects, did not suffer from this specific instability because the electrons were considered stationary.

Answer (c): A classical atom based on **Rutherford's model** is doomed to collapse.

(d) Mass Distribution within the Atom

- **Thomson's model:** The positive charge (and the bulk of the mass, since electrons are light) is uniformly distributed throughout the atomic sphere. This gives a **nearly continuous mass distribution**.
- **Rutherford's model:** Almost the entire mass of the atom is concentrated in the tiny, dense central nucleus. The rest of the atom is mostly empty space with a few orbiting electrons. This results in a **highly non-uniform mass distribution**.



Answer (d): An atom has a nearly continuous mass distribution in a **Thomson's model** but has a highly non-uniform mass distribution in **Rutherford's model**.

(e) Location of Mass in the Atom

- In **both models**, the positive charge (which constitutes the majority of the atomic mass) is responsible for most of the mass. In Thomson's model, it's the uniform positive sphere; in Rutherford's model, it's the dense nucleus. The electrons, being nearly 1836 times lighter than protons, contribute negligibly to the total mass.
- The question specifically asks where "the positively charged part" possesses most of the mass. Since the positive part is the dominant mass contributor in *both*, the answer is **both the models**.

Answer (e): The positively charged part of the atom possesses most of the mass in **both the models**.

Expert's Solution — Aditya Raj, B.Tech CSE, IIT Delhi

Conceptual Rosetta Stone: Thomson vs. Rutherford

To master these comparison questions, it helps to use a structured table. Let's condense the core differences:

Feature	Thomson's Model	Rutherford's Model
Nickname	Plum Pudding	Nuclear / Planetary
Positive Charge	Spread uniformly over entire atom ($\sim 10^{-10}$ m)	Concentrated in a tiny nucleus ($\sim 10^{-15}$ m)
Stability	Electrons in static, stable equilibrium	Classically unstable; electron should spiral in
Mass Distribution	Nearly uniform	Highly non-uniform, concentrated at center
Overall Size	$\sim 10^{-10}$ m	$\sim 10^{-10}$ m (defined by electron orbits)
Collapse	Not classically predicted	Doomed to collapse per classical EM theory

Memory Tip: Thomson → *Uniform, Stable, Embedded.* Rutherford → *Concentrated, Unstable (classically), Orbital.* Rutherford's model completely shattered the notion of a "filled" atom and revealed its beautiful, mostly empty structure—a miniature solar system, albeit one that violated classical physics.

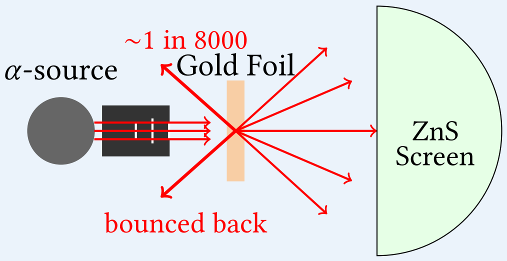
Did You Know?

The Crucial Experiment: The Geiger-Marsden experiment (under Rutherford's guidance) fired alpha particles at a thin gold foil. Most passed straight through, but a tiny fraction (~1 in 8000) bounced back. Rutherford famously said: *"It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you."* This observation was impossible under Thomson's model and directly led to the nuclear model.

Q2 **Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14 K.) What results do you expect?**

Solution

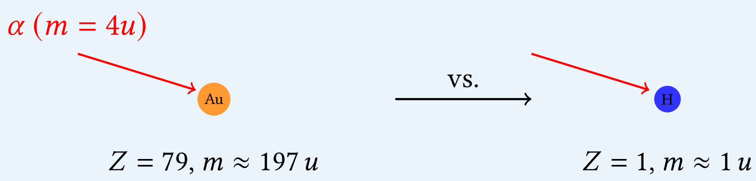
Recalling the Original Geiger-Marsden/Rutherford Experiment
Before predicting the outcome with a hydrogen target, let's briefly recap the landmark experiment that established the nuclear model.



Key observations with Gold (Au) foil:

- Most α-particles passed straight through with little or no deflection.
- A very small fraction (~0.01%) were scattered through large angles ($> 90°$), and some even bounced back.
- **Conclusion:** The atom has a tiny, dense, positively charged nucleus where most of the mass is concentrated.

Now, Analyzing the Hydrogen Target Scenario
Hydrogen is the simplest element with atomic number $Z = 1$. Let us examine the key differences between a gold target and a hydrogen target using the framework of Rutherford scattering.



Factor 1: Nuclear Charge (Z)
According to Rutherford's scattering formula, the number of particles scattered through a given angle θ is proportional to Z^2.

$$N(\theta) \propto Z^2$$

- Gold nucleus: $Z_{Au} = 79$
- Hydrogen nucleus (proton): $Z_H = 1$

Therefore, the scattering probability reduces by a factor of:

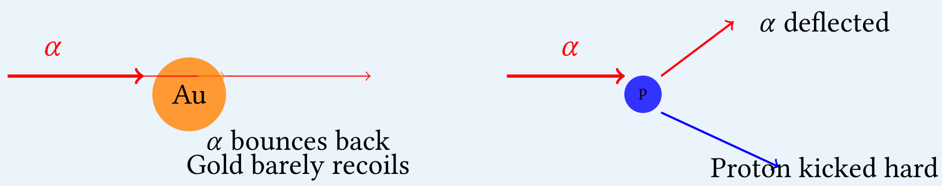
$$\frac{N_\text{H}}{N_\text{Au}} = \left(\frac{1}{79}\right)^2 = \frac{1}{6241} \approx 1.6 \times 10^{-4}$$

This means that the large-angle scattering events (the hallmark of Rutherford's experiment) will be **more than six thousand times rarer** with a hydrogen target. This alone makes the experiment dramatically less effective.

Factor 2: Nuclear Mass and Recoil
In an elastic collision between an α-particle ($m_\alpha = 4\,u$) and a target nucleus, the maximum energy transfer occurs in a head-on collision:

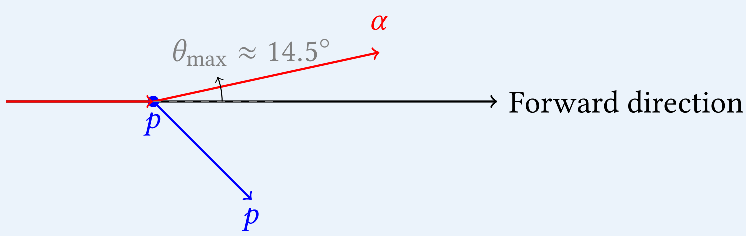
$$\Delta E_\text{max} = \frac{4 m_\alpha m_T}{(m_\alpha + m_T)^2} \times E_\alpha$$

Let us compare two extreme scenarios visually:



- **Gold:** $m_T \approx 197\,u \gg m_\alpha = 4\,u$.
  - The nucleus acts like a heavy, immovable target.
  - The α-particle retains most of its kinetic energy and can be backscattered.
- **Hydrogen:** $m_T \approx 1\,u < m_\alpha = 4\,u$.
  - The target proton is *lighter* than the projectile α-particle!
  - In a collision, the proton recoils with substantial energy. The α-particle cannot bounce back; it gets significantly deflected but continues forward.
  - Maximum scattering angle for α on a stationary proton (from classical kinematics, $m_1 > m_2$):

$$\theta_\alpha^\text{max} = \sin^{-1}\left(\frac{m_T}{m_\alpha}\right) = \sin^{-1}\left(\frac{1}{4}\right) \approx 14.5°$$

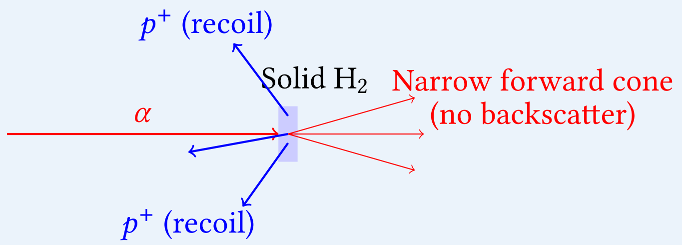


Factor 3: What Would We Actually Observe?
Combining both factors, the expected results are profoundly different from the gold foil experiment:

1. **No large-angle scattering:** Due to the low nuclear charge ($Z = 1$), the Coulomb repulsion is 79 times weaker. Backscattering (events at $\theta > 90°$) will be virtually absent—roughly $(1/79)^2\times$ fewer than gold. Simultaneously, kinematics forbids an α-particle from bouncing back from a lighter stationary proton.

2. **Predominantly forward scattering:** Most α-particles will pass through with only small deflections, confined to a narrow forward cone ($\theta \lesssim 15°$).

3. **Energetic recoil protons:** The most striking new feature would be the ejection of high-energy protons from the foil. In a head-on collision, an α-particle can transfer up to 64% of its kinetic energy to a proton.



Conclusion:

- Rutherford's key signature—rare large-angle scattering events revealing a dense nucleus—would **not** be observed with a hydrogen target.
- The experiment would instead reveal the collision kinematics of alpha particles with protons, allowing scientists to study the proton as a recoil particle. In fact, this very principle (scattering α-particles off hydrogen) was later used by Marsden and others to infer that the hydrogen nucleus was a fundamental particle—the proton.

✔ **Expected Results:** No large-angle scattering or backscattering of α-particles would be observed. Most α-particles would suffer only small deflections (within $\sim 15°$ forward cone). Instead, we would observe energetic protons ejected from the foil due to recoil in α–p collisions.

Expert's Solution — Shruti Pathak, B.Tech Engineering Physics, IIT Madras

Kinematic Insight: Why Light Targets Don't Backscatter

Many students intuitively think that any nucleus, being positive, should repel α-particles backward. However, kinematics plays an equally crucial role. Let's derive the maximum scattering angle.

For an elastic collision with m_1 (projectile) and m_2 (target initially at rest):

The scattering angle θ_1 in the lab frame is given by:

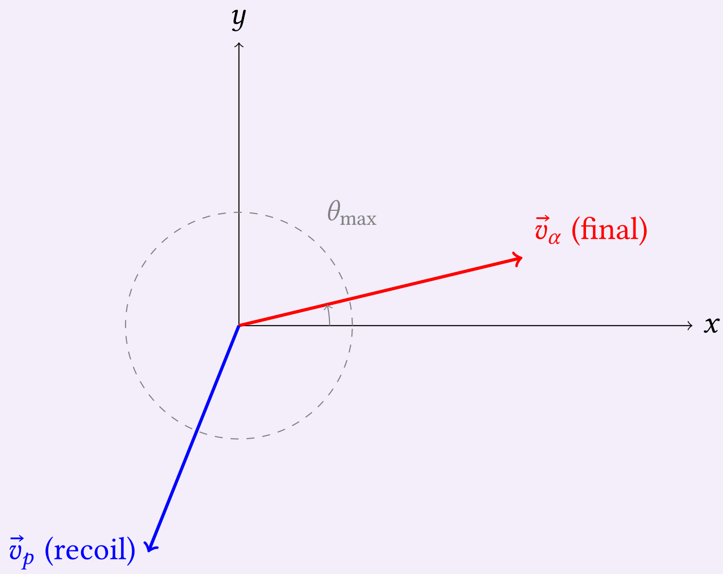
$$\tan\theta_1 = \frac{\sin\Theta}{\cos\Theta + (m_1/m_2)}$$

where Θ is the scattering angle in the centre-of-mass frame.

When $m_1 > m_2$ (as here, $4u > 1u$), the maximum lab scattering angle occurs when the derivative $d\theta_1/d\Theta = 0$, giving:

$$\theta_1^{\max} = \sin^{-1}\left(\frac{m_2}{m_1}\right)$$

$$\theta_\alpha^{\max} = \sin^{-1}\left(\frac{1}{4}\right) = \sin^{-1}(0.25) \approx 14.48^\circ$$



Historical Irony: Rutherford actually *did* fire α-particles at hydrogen (and nitrogen) shortly after the gold foil experiment. In 1919, he observed that α-particles striking nitrogen produced long-range scintillations that turned out to be high-energy protons. This was the first artificially induced nuclear reaction:

$$\alpha + {}^{14}\text{N} \rightarrow {}^{17}\text{O} + p$$

This discovery of "knocking out" protons from nitrogen marked the birth of experimental nuclear physics and earned Rutherford the title "the father of nuclear physics."

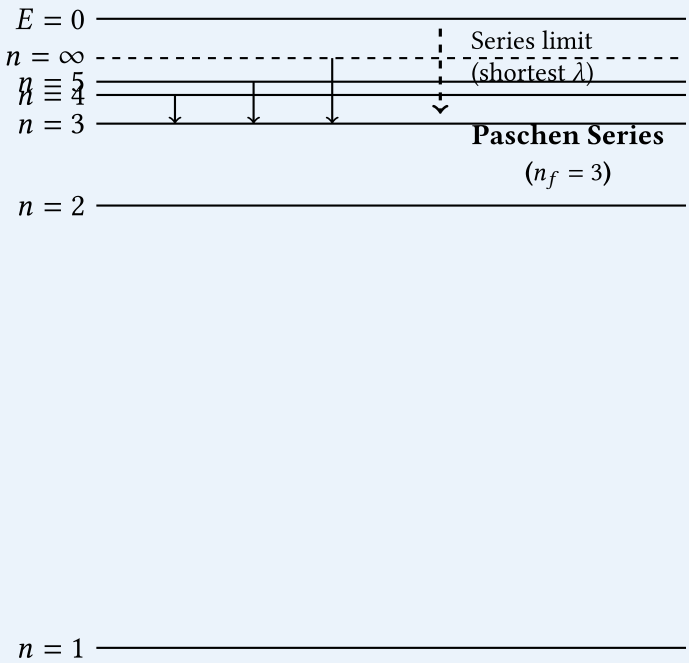
Did You Know?

Collision Intuition: Imagine throwing a heavy bowling ball (α) at a light ping-pong ball (proton). The bowling ball cannot bounce back—it just pushes the ping-pong ball forward. Conversely, throwing the ping-pong ball at a heavy bowling ball (gold nucleus) is what makes the ping-pong ball bounce backward. The mass ratio is everything in collision kinematics!

Q3 What is the shortest wavelength present in the Paschen series of spectral lines?

Solution

Understanding Spectral Series in Hydrogen
The hydrogen spectrum consists of several series of spectral lines, each corresponding to electronic transitions from higher energy levels to a specific lower energy level. The series are named after their discoverers.



The Paschen Series

- The Paschen series arises when an electron in a hydrogen atom makes a transition from a higher energy level ($n_i = 4, 5, 6, \ldots$) **down to** the third energy level ($n_f = 3$).
- These transitions produce photons in the **infrared (IR)** region of the electromagnetic spectrum.
- The wavelength λ of the emitted photon is given by the Rydberg formula:

Rydberg Formula (for Hydrogen):

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

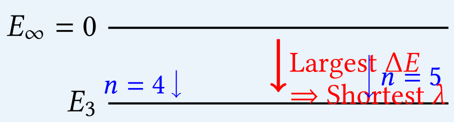
where:

- $R = 1.097 \times 10^7\ \text{m}^{-1}$ is the Rydberg constant.
- $n_f = 3$ (fixed lower level for Paschen series).
- $n_i = 4, 5, 6, \ldots$ (upper levels).

Finding the Shortest Wavelength (Series Limit)

The *shortest* wavelength in any spectral series corresponds to the transition with the *largest* energy difference. This occurs when the electron falls from the highest possible energy level—that is, from infinity ($n_i = \infty$).

Series Limit: $n_i = \infty$ to $n_f = 3$



Mathematical Derivation:
For the shortest wavelength (series limit), we set $n_i \to \infty$ and $n_f = 3$:

$$\frac{1}{\lambda_{\text{min}}} = R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right)$$

Since $\frac{1}{\infty^2} = 0$:

$$\frac{1}{\lambda_{\text{min}}} = R\left(\frac{1}{9} - 0\right) = \frac{R}{9}$$

Therefore:

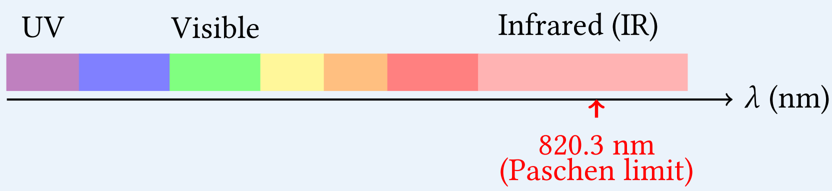
$$\lambda_{\text{min}} = \frac{9}{R}$$

Substituting the Value of R:
Given $R = 1.097 \times 10^7\ \text{m}^{-1}$:

$$\lambda_{\text{min}} = \frac{9}{1.097 \times 10^7\ \text{m}^{-1}}$$

Calculating step-by-step:

$$\begin{aligned}
\lambda_{\text{min}} &= \frac{9}{1.097} \times 10^{-7}\ \text{m} \\
&= 8.2033 \times 10^{-7}\ \text{m} \\
&= 8.2033 \times 10^{-7} \times 10^9\ \text{nm} \quad (\text{since } 1\ \text{m} = 10^9\ \text{nm}) \\
&= 820.33\ \text{nm}
\end{aligned}$$



Verification: We can also approach this conceptually. The energy of the $n = 3$ level in hydrogen is:

$$E_3 = -\frac{13.6}{3^2}\ \text{eV} = -\frac{13.6}{9}\ \text{eV} \approx -1.511\ \text{eV}$$

The ionization energy from $n = 3$ is $+1.511$ eV. Using $E = hc/\lambda$:

$$\lambda_{\text{min}} = \frac{hc}{E} = \frac{1240\ \text{eV·nm}}{1.511\ \text{eV}} \approx 820.6\ \text{nm}$$

Both methods yield consistent results.

Answer: The shortest wavelength present in the Paschen series is

$$\boxed{\lambda_{\min} = \frac{9}{R} \approx 8.20 \times 10^{-7}\ \text{m} = 820\ \text{nm}}$$

This lies in the infrared region of the electromagnetic spectrum.

Expert's Solution — Rohan Deshmukh, B.Tech Electrical Engineering, IIT Bombay

Series Limit Memorization Trick

For any hydrogen spectral series with lower level n_f, the shortest wavelength (series limit) is always:

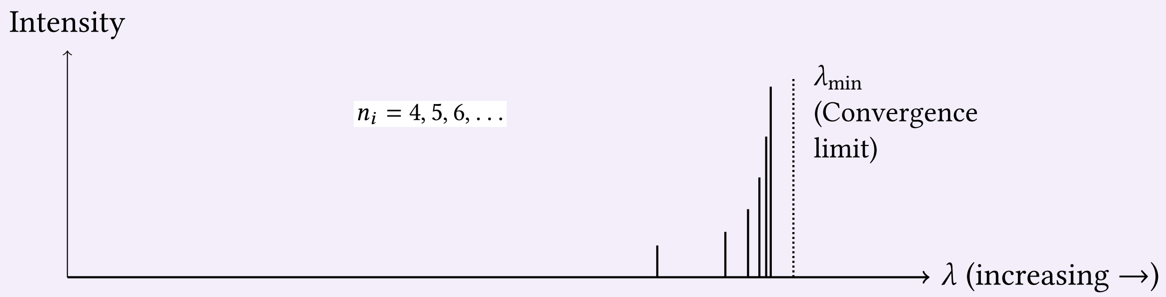
$$\lambda_{\min} = \frac{n_f^2}{R}$$

This is a powerful shortcut! Let's verify for the well-known series:

Series	n_f	$\lambda_{\min}$	Region
Lyman	1	$1/R \approx 91.2$ nm	Ultraviolet (UV)
Balmer	2	$4/R \approx 364.6$ nm	Near UV/Visible
Paschen	3	$9/R \approx 820.3$ nm	Infrared (IR)
Brackett	4	$16/R \approx 1458$ nm	Infrared (IR)
Pfund	5	$25/R \approx 2278$ nm	Far Infrared

Physical Interpretation of the Series Limit:

- The shortest wavelength photon in a series is emitted when a **free electron** ($n = \infty$, zero total energy) is captured directly into the orbit $n = n_f$.
- This is exactly the reverse of the ionization process from level n_f.
- Therefore, $\lambda_{\min}$ corresponds to the **ionization wavelength** for an electron initially in the $n = n_f$ state.
- On the longer wavelength side, successive lines crowd together and converge to this limit, creating a **convergence limit** in the spectrum.



Did You Know?

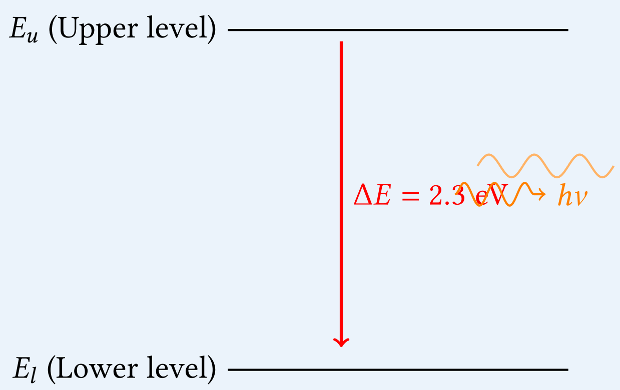
Why "Paschen" Series? Named after Friedrich Paschen, a German physicist who discovered this infrared series in 1908. The Paschen series was historically significant because its observation required the development of sensitive infrared detectors—pushing the boundaries of experimental spectroscopy beyond visible light.

Q4 A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom makes a transition from the upper level to the lower level?

Solution

Understanding the Problem

When an electron in an atom makes a transition from a higher energy level (E_u) to a lower energy level (E_l), the energy difference is emitted as a photon. The photon carries away exactly this energy difference.



The Fundamental Relation

The energy of the emitted photon is related to its frequency by Planck's equation:

Planck-Einstein Relation:

$$E = h\nu$$

where:

- E = energy of the photon
- $h = 6.626 \times 10^{-34}$ J·s (Planck's constant)
- ν = frequency of the radiation

For an atomic transition:

$$\Delta E = E_u - E_l = h\nu$$

Therefore, the frequency is:

$$\nu = \frac{\Delta E}{h}$$

Step 1: Convert Energy from eV to Joules
The energy difference is given in electron-volts (eV), but Planck's constant is in SI units (J·s). We must convert eV to Joules.
Conversion factor:

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

Therefore:

$$\Delta E = 2.3 \text{ eV} = 2.3 \times 1.602 \times 10^{-19} \text{ J}$$

Calculating:

$$\Delta E = 2.3 \times 1.602 \times 10^{-19}$$
$$= 3.6846 \times 10^{-19} \text{ J}$$

Step 2: Calculate the Frequency
Using $\nu = \Delta E/h$:

$$\nu = \frac{3.6846 \times 10^{-19} \text{ J}}{6.626 \times 10^{-34} \text{ J·s}}$$

Separating the numerical and exponential parts:

$$\nu = \frac{3.6846}{6.626} \times \frac{10^{-19}}{10^{-34}} = \frac{3.6846}{6.626} \times 10^{15}$$

Evaluating the coefficient:

$$\frac{3.6846}{6.626} \approx 0.5561$$

Therefore:

$$\nu = 0.5561 \times 10^{15} \text{ Hz}$$
$$\nu = 5.561 \times 10^{14} \text{ Hz}$$

Alternative Shortcut: Using the Useful Constant *hc*
Physicists often use a convenient combination of constants:

$$hc = 1240 \text{ eV·nm} \quad \text{(approximately)}$$

However, for frequency, there's another handy relation. Since $E = h\nu$, we can write:

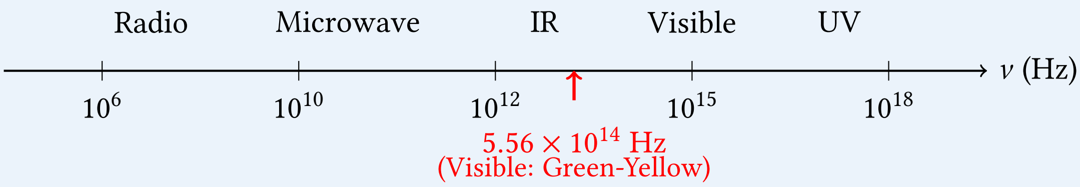
$$\nu \text{ (in Hz)} = \frac{E \text{ (in eV)} \times 1.602 \times 10^{-19}}{6.626 \times 10^{-34}}$$

Or even simpler, since 1 eV corresponds to a frequency of:

$$\nu_{1\text{eV}} = \frac{1.602 \times 10^{-19}}{6.626 \times 10^{-34}} \approx 2.418 \times 10^{14} \text{ Hz}$$

Then for 2.3 eV:

$$\nu = 2.3 \times 2.418 \times 10^{14} = 5.561 \times 10^{14} \text{ Hz}$$



Verification: We can cross-check by computing the wavelength λ and verifying it lies in the visible region:

$$\lambda = \frac{c}{\nu} = \frac{3.0 \times 10^8}{5.561 \times 10^{14}} \approx 5.39 \times 10^{-7} \text{ m} = 539 \text{ nm}$$

A wavelength of 539 nm corresponds to green-yellow light, which is consistent with an energy gap of 2.3 eV (visible photons range from ~1.6 eV to ~3.2 eV).

Answer: The frequency of the radiation emitted is

$$\boxed{\nu = 5.56 \times 10^{14} \text{ Hz}}$$

which lies in the visible region (green-yellow light, $\lambda \approx 539$ nm).

Expert's Solution — Kavya Nair, B.Tech Engineering Physics, NIT Surathkal

Smart Conversion Factors Every Physics Student Should Memorize

This problem highlights the importance of knowing your conversion constants cold. Here's a cheat sheet that will save you precious minutes in exams:

Constant	Value	Usefulness
hc	1240 eV·nm	λ (nm) from eV instantly
h	4.136×10^{-15} eV·s	ν (Hz) from eV instantly
1 eV	2.418×10^{14} Hz	Frequency per eV
$k_B T$ (at 300 K)	≈ 0.026 eV	Thermal energy scale

Using the h in eV·s shortcut:

If you memorize $h = 4.136 \times 10^{-15}$ eV·s, then:

$$\nu = \frac{\Delta E}{h} = \frac{2.3 \text{ eV}}{4.136 \times 10^{-15} \text{ eV·s}}$$

$$\nu = \frac{2.3}{4.136} \times 10^{15} = 0.556 \times 10^{15} = 5.56 \times 10^{14} \text{ Hz}$$

Done in **one step**, with no Joule conversion needed! This is a massive time-saver.

Mnemonic for Atomic Physics:

"Energy in eV times 1240 gives wavelength in nm."

$E(\text{eV}) \times \lambda(\text{nm}) \approx 1240$

For this problem: $2.3 \times \lambda \approx 1240 \implies \lambda \approx 539$ nm

Then $\nu = c/\lambda = 3 \times 10^8/539 \times 10^{-9} \approx 5.56 \times 10^{14}$ Hz

Did You Know?

What Does 2.3 eV Mean Physically? An energy gap of 2.3 eV is typical for visible light emission from semiconductors (like GaP LEDs, which emit green light). In atoms, such gaps occur in transitions between moderately excited states. For context:

- Red light: $\sim$ 1.8 eV
- Green light: $\sim$ 2.3 eV $\longleftarrow$ **This problem!**
- Blue light: $\sim$ 2.7 eV
- Violet light: $\sim$ 3.1 eV

The human eye's peak sensitivity is at ~555 nm (green), which is almost exactly at 2.23 eV. Nature optimized our vision right where the sun's spectrum peaks—an elegant fact!

Q5 The ground state energy of hydrogen atom is −13.6 eV. What are the kinetic and potential energies of the electron in this state?

Solution

Understanding the Bohr Model of Hydrogen Atom

In the Bohr model, the electron in a hydrogen atom revolves around the nucleus in a circular orbit. The total energy E of the electron is the sum of its kinetic energy K and potential energy U:

Total Energy:

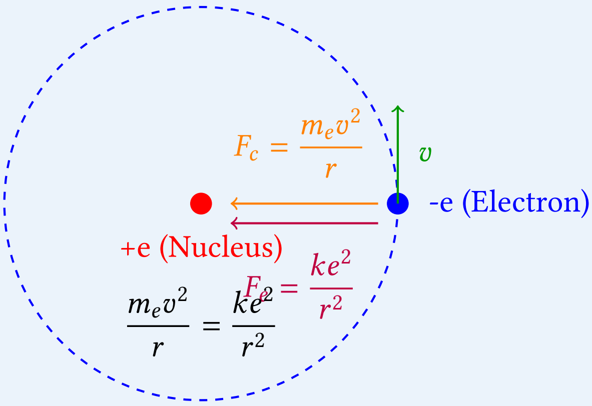
$$E = K + U$$

where:

- $K = \frac{1}{2}m_e v^2$ (Kinetic Energy, always positive)
- $U = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$ (Electrostatic Potential Energy, negative for attractive force)

The Virial Theorem Connection

For a system bound by an inverse-square law force (like the Coulomb force between the electron and nucleus), a beautiful and powerful result from classical mechanics applies: the **Virial Theorem**.



For the Coulomb potential ($U \propto -1/r$), the virial theorem states:

$$\boxed{K = -\frac{1}{2}U}$$

Or equivalently:

$$U = -2K$$

This is a **general result** for any inverse-square law central force system (including gravitational orbits!).

Step 1: Express Total Energy in Terms of Kinetic Energy

We know:

$$E = K + U$$

Substituting $U = -2K$ from the virial theorem:

$$E = K + (-2K) = -K$$

Therefore:

$$\boxed{K = -E}$$

This remarkable result tells us that the kinetic energy is the **negative** of the total energy for a Coulomb-bound system.

Step 2: Calculate Kinetic Energy
Given the ground state total energy:

$$E = -13.6 \text{ eV}$$

The kinetic energy is:

$$K = -E = -(-13.6 \text{ eV}) = +13.6 \text{ eV}$$

✔ **Kinetic Energy:** $\boxed{K = 13.6 \text{ eV}}$

Step 3: Calculate Potential Energy
Using the virial theorem relation $U = -2K$:

$$U = -2 \times 13.6 \text{ eV} = -27.2 \text{ eV}$$

Alternatively, from $E = K + U$:

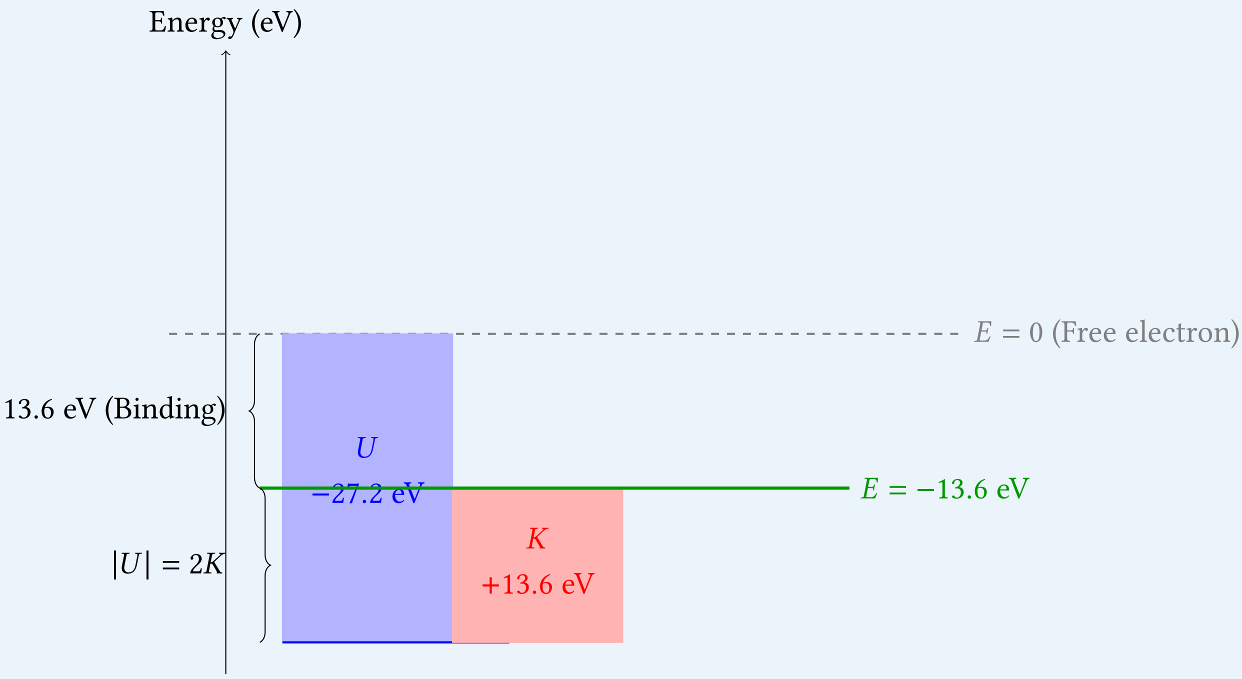
$$U = E - K = -13.6 \text{ eV} - 13.6 \text{ eV} = -27.2 \text{ eV}$$

Both methods give the same result.

✔ **Potential Energy:** $\boxed{U = -27.2 \text{ eV}}$

Verification and Physical Picture
Let's verify that these values make physical sense:

$$E = K + U = 13.6 \text{ eV} + (-27.2 \text{ eV}) = -13.6 \text{ eV} \quad \checkmark$$



The diagram beautifully illustrates:

- The electron sits in a deep potential well ($U = -27.2$ eV) created by the nuclear attraction.
- Its kinetic energy ($K = +13.6$ eV) is exactly **half the magnitude** of the potential energy.
- The total energy ($E = -13.6$ eV) represents the binding energy—the energy required to free the electron (ionize the atom).

Final Answer:

$$\boxed{\text{Kinetic Energy, } K = +13.6 \text{ eV} \quad ; \quad \text{Potential Energy, } U = -27.2 \text{ eV}}$$

Expert's Solution — Vivek Sharma, B.Tech CSE, NIT Trichy

The Virial Theorem: A Universal Shortcut
The virial theorem is arguably the most powerful shortcut in atomic physics. For **any** system bound by a potential $U \propto r^n$, the average kinetic and potential energies satisfy:

$$2\langle K \rangle = n\langle U \rangle$$

For the Coulomb potential, $U \propto r^{-1}$, so $n = -1$:

$$2\langle K \rangle = -\langle U \rangle \quad \Rightarrow \quad \langle U \rangle = -2\langle K \rangle$$

Consequence Table for Hydrogen-like Atoms:

Energy Level (n)	Total E_n	Kinetic K_n	Potential U_n
$n = 1$ (Ground)	-13.6 eV	$+13.6$ eV	-27.2 eV
$n = 2$	-3.4 eV	$+3.4$ eV	-6.8 eV
$n = 3$	-1.51 eV	$+1.51$ eV	-3.02 eV
$n \to \infty$	0 eV	0 eV	0 eV

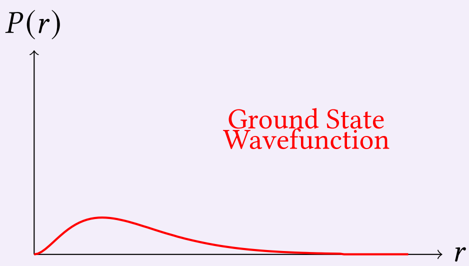
Notice the pattern: for any level n,

$$K_n = |E_n| \quad ; \quad U_n = 2E_n$$

Why This Matters in Quantum Mechanics:
Remarkably, the virial theorem holds **even in full quantum mechanical treatment** of the hydrogen atom (not just the semi-classical Bohr model). The expectation values satisfy:

$$\langle K \rangle_{nlm} = -\frac{1}{2}\langle U \rangle_{nlm}$$

This is a consequence of the fact that the hydrogen wavefunctions are eigenstates of a Hamiltonian with a $-1/r$ potential. The theorem provides a quick consistency check for any calculation involving hydrogenic atoms.



Virial theorem applies to expectation values

Did You Know?

Mnemonics for the Ground State:

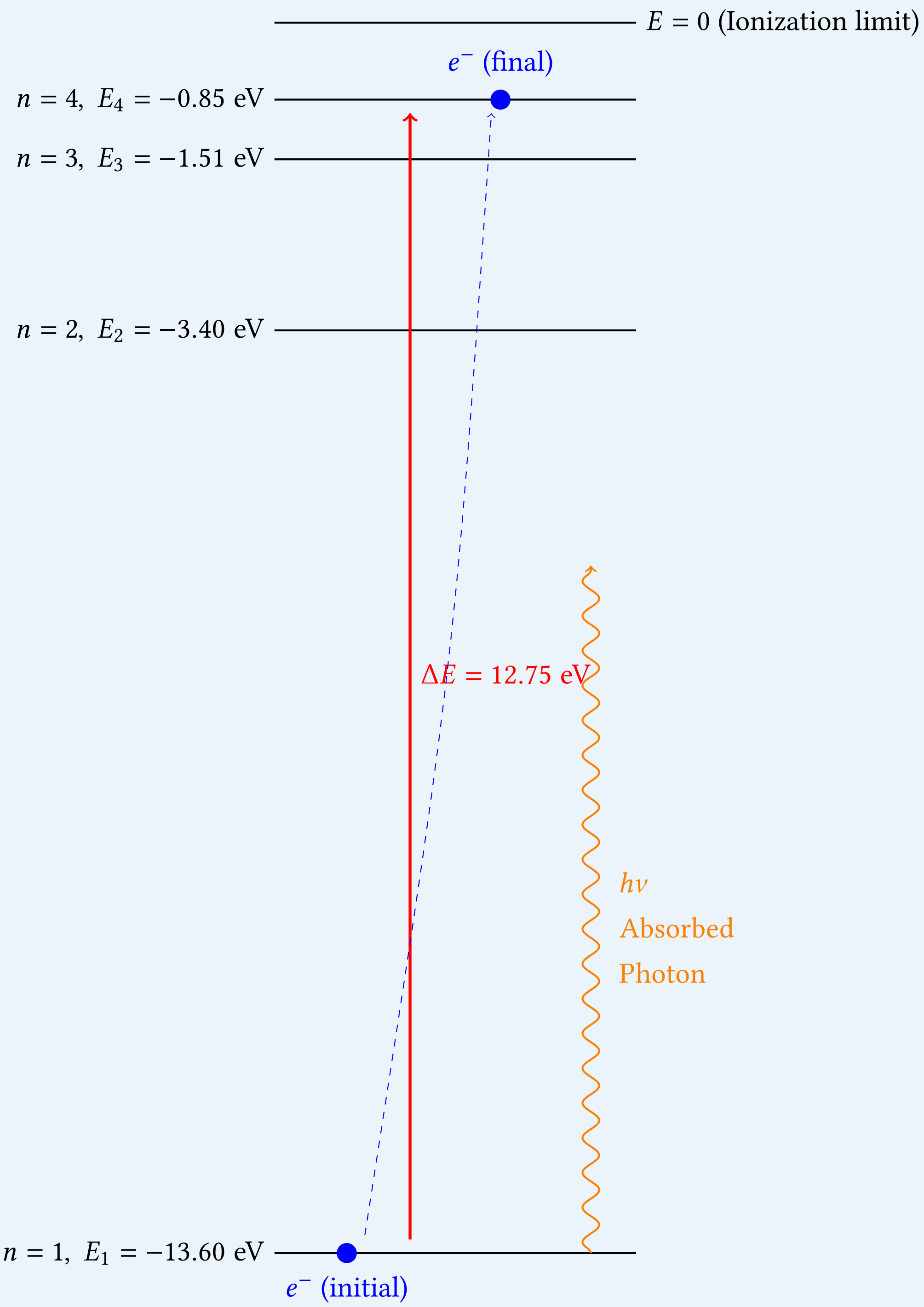
- **Kinetic Energy** K is **positive** and equals the **binding energy** (13.6 eV).
- **Potential Energy** U is **negative** and **twice** the total energy (−27.2 eV).
- Think: *"K is Kind (positive), U is Unbelievably negative, and E is Exactly in between—half of U."*

This relationship explains why the electron doesn't collapse into the nucleus despite the attractive Coulomb force: as it gets closer, U becomes more negative, but the kinetic energy rises even faster due to the uncertainty principle, maintaining the virial balance at a stable equilibrium radius (the Bohr radius, a_0).

Q6 **A hydrogen atom initially in the ground level absorbs a photon, which excites it to the $n = 4$ level. Determine the wavelength and frequency of the photon.**

Solution

Understanding the Absorption Process
When a hydrogen atom in the ground state ($n_i = 1$) absorbs a photon, the electron jumps to a higher energy level ($n_f = 4$). The energy of the absorbed photon must exactly match the energy difference between these two levels.



Step 1: Recall the Energy Level Formula for Hydrogen

The energy of an electron in the n^{th} Bohr orbit of hydrogen is:

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

Step 2: Calculate the Energies of the Two Levels

Ground state ($n_i = 1$):

$$E_1 = -\frac{13.6}{1^2} = -13.60 \text{ eV}$$

Excited state ($n_f = 4$):

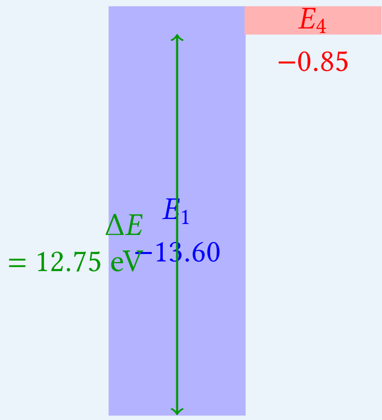
$$E_4 = -\frac{13.6}{4^2} = -\frac{13.6}{16} = -0.85 \text{ eV}$$

Step 3: Determine the Photon Energy
The energy of the absorbed photon equals the difference between the final and initial energy levels:

$$\Delta E = E_4 - E_1$$

$$\Delta E = (-0.85 \text{ eV}) - (-13.60 \text{ eV})$$

$$\Delta E = -0.85 \text{ eV} + 13.60 \text{ eV} = 12.75 \text{ eV}$$



Step 4: Calculate the Frequency of the Photon
Using Planck's relation $E = h\nu$:

$$\nu = \frac{\Delta E}{h}$$

We'll use the convenient value of Planck's constant in eV·s:

$$h = 4.136 \times 10^{-15} \text{ eV·s}$$

Therefore:

$$\nu = \frac{12.75 \text{ eV}}{4.136 \times 10^{-15} \text{ eV·s}}$$

Simplifying:

$$\nu = \frac{12.75}{4.136} \times 10^{15} \text{ Hz}$$

$$\nu = 3.082 \times 10^{15} \text{ Hz}$$

Frequency: $\boxed{\nu = 3.08 \times 10^{15} \text{ Hz}}$

Step 5: Calculate the Wavelength of the Photon
Using the relation $c = \lambda\nu$:

$$\lambda = \frac{c}{\nu}$$

Substituting $c = 3.0 \times 10^8$ m/s and the calculated frequency:

$$\lambda = \frac{3.0 \times 10^8 \text{ m/s}}{3.082 \times 10^{15} \text{ s}^{-1}}$$

$$\lambda = \frac{3.0}{3.082} \times 10^{-7} \text{ m}$$

$$\lambda = 0.9734 \times 10^{-7} \text{ m} = 9.734 \times 10^{-8} \text{ m}$$

Converting to nanometers (1 nm = 10^{-9} m):

$$\lambda = 97.34 \text{ nm}$$

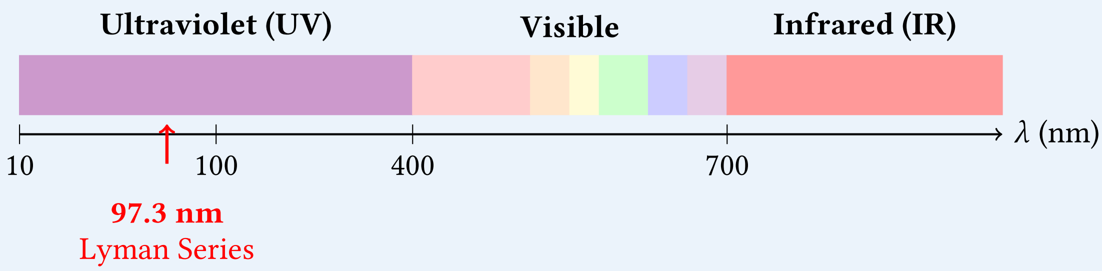
Alternative Shortcut using $hc = 1240$ eV·nm:
We can bypass the frequency calculation entirely:

$$\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV·nm}}{12.75 \text{ eV}} = 97.25 \text{ nm}$$

The small difference (~0.1 nm) is due to rounding in the intermediate steps.

✔ **Wavelength:** $\boxed{\lambda = 97.3 \text{ nm} \quad (\text{or } 9.73 \times 10^{-8} \text{ m})}$

Identifying the Spectral Series
Since the transition is **to** the ground state ($n_f = 1$) **from** a higher level ($n_i = 4$), this photon belongs to the **Lyman series**. The Lyman series lies entirely in the **ultraviolet (UV)** region of the electromagnetic spectrum.



Verification using the Rydberg Formula:
We can verify our result using the Rydberg formula:

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{4^2}\right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(1 - \frac{1}{16}\right) = 1.097 \times 10^7 \times \frac{15}{16}$$

$$\frac{1}{\lambda} = 1.028 \times 10^7 \text{ m}^{-1}$$

$$\lambda = 9.72 \times 10^{-8} \text{ m} = 97.2 \text{ nm}$$

Matches perfectly within rounding.

Final Answer:

$$\boxed{\nu = 3.08 \times 10^{15}\ \text{Hz} \quad ; \quad \lambda = 97.3\ \text{nm}}$$

This is an ultraviolet photon belonging to the Lyman series ($n_i = 4 \to n_f = 1$).

Expert's Solution — Arjun Mehta, B.Tech Engineering Physics, IIIT Hyderabad

The Lyman Series: Window to the Ground State

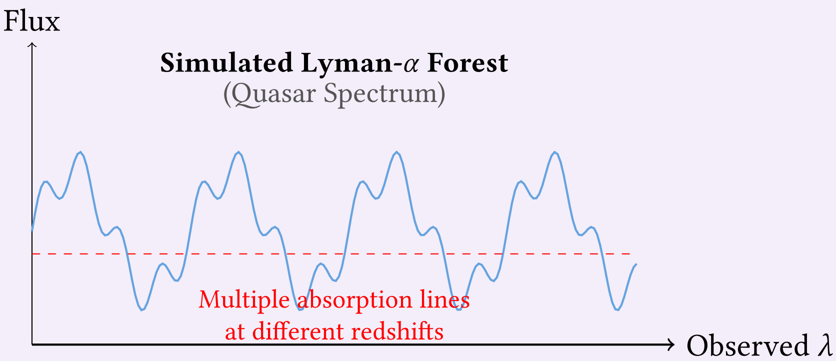
The Lyman series (transitions to $n_f = 1$) is special because it involves the ground state. Its photons carry the highest energies of all hydrogen spectral series, placing them in the ultraviolet region.

Transition ($n_i \to 1$)	Energy (eV)	Wavelength (nm)	Name
$2 \to 1$	10.20	121.6	Lyman-α
$3 \to 1$	12.09	102.6	Lyman-β
$4 \to 1$	12.75	97.3	**Lyman-γ (This problem!)**
$5 \to 1$	13.06	95.0	Lyman-δ
$\infty \to 1$	13.60	91.2	Lyman Limit

Why UV? The energy gap between the ground state and even the first excited state ($n = 2$) is already 10.2 eV, which is well beyond the visible range (1.6–3.2 eV). Thus, **all** Lyman series photons are ultraviolet.

Astrophysical Significance: The Lyman series is crucial in astronomy. Because hydrogen is the most abundant element in the universe, the Lyman-α line (121.6 nm) is one of the most important spectral features for studying:

- Intergalactic medium and cosmic structure
- Star formation regions
- Quasar absorption spectra (Lyman-α forest)



Did You Know?

Quick Check for Hydrogen Transitions: Whenever you compute a hydrogen transition energy, use this mental checklist:

1. $n_f = 1$ **(Lyman)?** Expect UV, $\lambda < 122$ nm, $\Delta E > 10.2$ eV.
2. $n_f = 2$ **(Balmer)?** Expect visible/UV, $\lambda = 365\text{–}656$ nm, $\Delta E = 1.9\text{–}3.4$ eV.
3. $n_f = 3$ **(Paschen)?** Expect IR, $\lambda > 820$ nm, $\Delta E < 1.5$ eV.

Our result ($\lambda = 97$ nm, $\Delta E = 12.75$ eV) fits perfectly in the Lyman series expectation. Always sanity-check your answers this way!

Q7 Bohr Model Dynamics

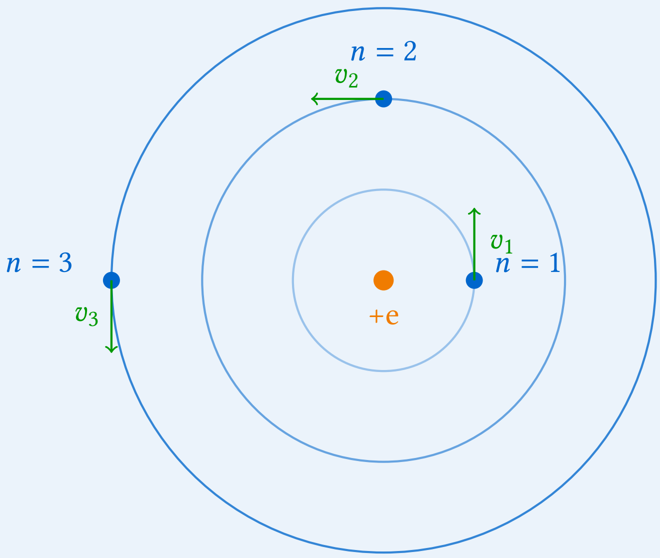
(a) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the $n = 1$, 2, and 3 levels.

(b) Calculate the orbital period in each of these levels.

Solution

Recalling Bohr's Model Fundamentals

In Bohr's model of the hydrogen atom, the electron revolves around the nucleus in circular orbits under the Coulomb force. The model is built on two key postulates:



Postulate 1: Coulomb Force provides Centripetal Force

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^2} = \frac{m_e v_n^2}{r_n}$$

Postulate 2: Quantization of Angular Momentum

$$m_e v_n r_n = n\frac{h}{2\pi} = n\hbar$$

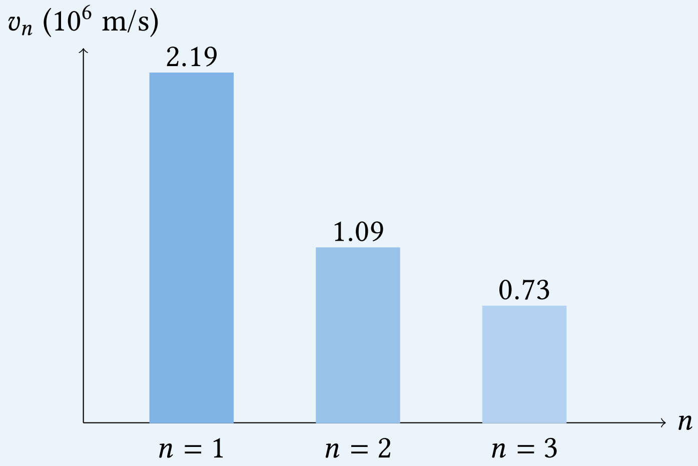
Part (a): Speed of the Electron in $n = 1, 2, 3$

Orbital Speed in Bohr Model

$$v_n = \frac{e^2}{2\varepsilon_0 h}\cdot\frac{1}{n} = \frac{v_1}{n}$$

Using $v_1 = 2.188 \times 10^6$ m/s:

- **n = 1:** $v_1 = 2.19 \times 10^6$ m/s
- **n = 2:** $v_2 = v_1/2 = 1.09 \times 10^6$ m/s
- **n = 3:** $v_3 = v_1/3 = 7.29 \times 10^5$ m/s



Part (b): Orbital Period in Each Level

Orbital Period Scaling

$$T_n = \frac{2\pi r_n}{v_n} = \frac{2\pi a_0}{v_1}\cdot n^3 = T_1 \cdot n^3$$

Using $T_1 = 1.52 \times 10^{-16}$ s:

- **n = 2:** $T_2 = T_1 \times 8 = 1.22 \times 10^{-15}$ s
- **n = 3:** $T_3 = T_1 \times 27 = 4.10 \times 10^{-15}$ s

tablehead **Level** n	**Radius** r_n **(m)**	**Speed** v_n **(m/s)**	**Period** T_n **(s)**
$n = 1$	5.29×10^{-11}	2.19×10^{6}	1.52×10^{-16}
$n = 2$	2.12×10^{-10}	1.09×10^{6}	1.22×10^{-15}
$n = 3$	4.76×10^{-10}	7.29×10^{5}	4.10×10^{-15}
lightgray **Scaling**	$\propto n^2$	$\propto 1/n$	$\propto n^3$

Expert's Solution — Ananya Krishnan, B.Tech Engineering Physics, IIT Kanpur

The Fine-Structure Constant: Nature's Magic Number $\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \approx \frac{1}{137}$ characterizes the strength of electromagnetic interaction. $v_1 = \alpha c$.
Scaling Laws in Bohr's Model — A Summary:

$$\begin{aligned} r_n &= a_0 n^2 && \propto n^2 && \text{(Orbital radius)} \\ v_n &= \frac{\alpha c}{n} && \propto 1/n && \text{(Orbital speed)} \\ E_n &= -\frac{13.6}{n^2}\text{ eV} && \propto 1/n^2 && \text{(Total energy)} \\ T_n &= T_1 n^3 && \propto n^3 && \text{(Orbital period)} \end{aligned}$$

Did You Know?

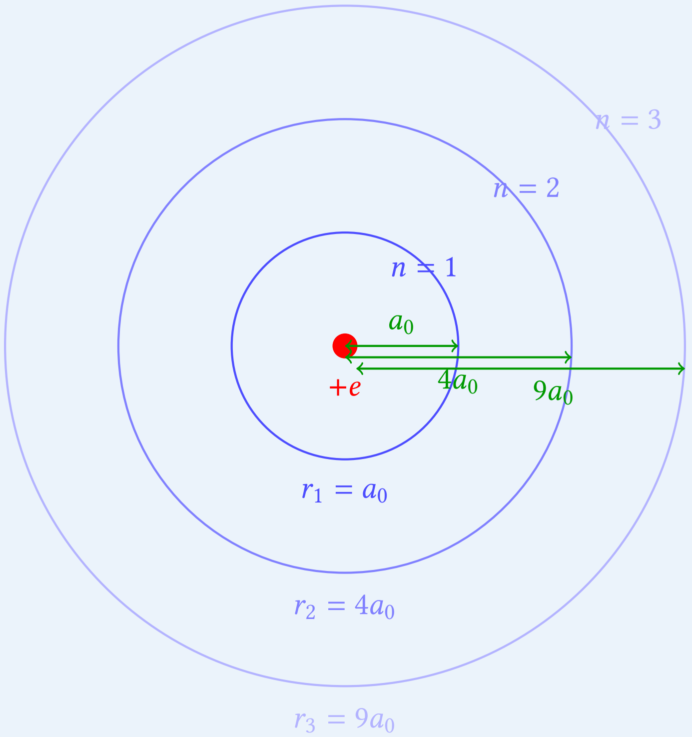
Mental Check for Exam:

- **Higher orbit → Larger radius**
- **Larger radius → Weaker Coulomb force**
- **Weaker force → Lower speed needed**

Q8 The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. What are the radii of the $n = 2$ and $n = 3$ orbits?

Solution

Understanding Bohr's Radius Quantization
In the Bohr model, the electron can only revolve in certain allowed stationary orbits. The radii of these orbits are quantized and follow a simple scaling law with the principal quantum number n.



The Bohr Radius Formula

From Bohr's postulates, the radius of the n^{th} stationary orbit is given by:

Radius of the n^{th} Bohr Orbit:

$$r_n = \frac{\varepsilon_0 h^2}{\pi m_e e^2} \cdot n^2$$

Or, in terms of the Bohr radius a_0:

$$\boxed{r_n = a_0 n^2}$$

where $a_0 = 5.3 \times 10^{-11}$ m is the radius of the innermost orbit ($n = 1$), also called the **Bohr radius**.

Part 1: Radius for $n = 2$

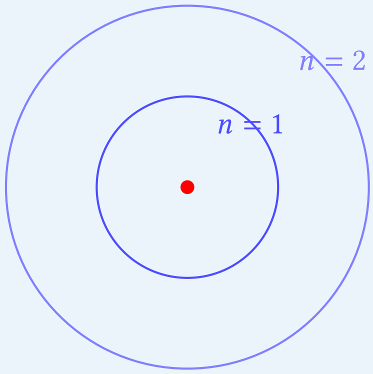
Using the scaling relation $r_n = a_0 n^2$:

$$r_2 = a_0 \times 2^2$$

$$r_2 = 5.3 \times 10^{-11} \text{ m} \times 4$$

$$r_2 = 21.2 \times 10^{-11} \text{ m}$$

$$r_2 = 2.12 \times 10^{-10} \text{ m}$$



$r_2 = 4 \times r_1$ (Four times larger)

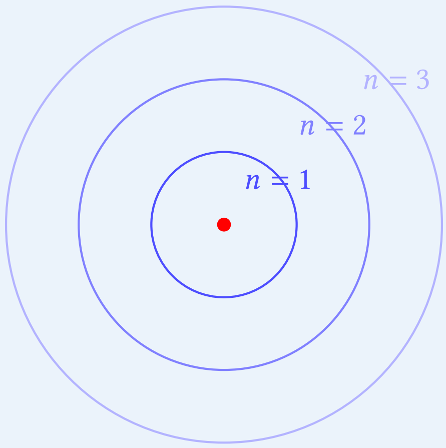
✔ **Radius for $n = 2$:**

$$\boxed{r_2 = 2.12 \times 10^{-10}\ \text{m}}$$

Part 2: Radius for $n = 3$

Similarly:

$$r_3 = a_0 \times 3^2$$
$$r_3 = 5.3 \times 10^{-11}\ \text{m} \times 9$$
$$r_3 = 47.7 \times 10^{-11}\ \text{m}$$
$$r_3 = 4.77 \times 10^{-10}\ \text{m}$$



$r_3 = 9 \times r_1$ (Nine times larger)

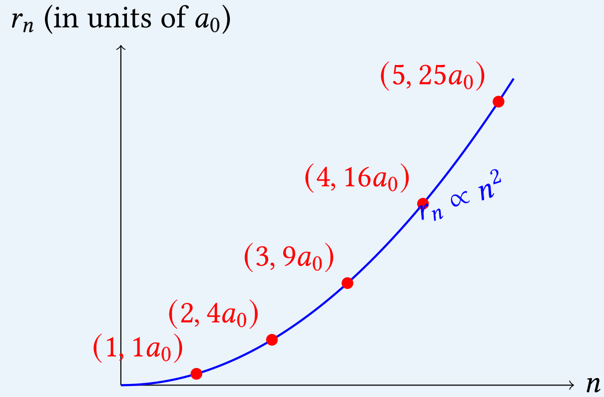
✔ **Radius for $n = 3$:**

$$\boxed{r_3 = 4.77 \times 10^{-10}\ \text{m}}$$

Summary Table:

Quantum Level n	Formula $r_n = a_0 n^2$	Value (m)	Value (Å)
$n = 1$	$a_0 \times 1$	5.30×10^{-11}	0.53 Å
$n = 2$	$a_0 \times 4$	2.12×10^{-10}	2.12 Å
$n = 3$	$a_0 \times 9$	4.77×10^{-10}	4.77 Å

Physical Insight:
The n^2 dependence means that as we move to higher quantum levels, the orbits expand very rapidly. The third orbit ($n = 3$) is already nine times larger in radius than the ground state. This rapid expansion of the orbital radius is a consequence of the quantization of angular momentum combined with the inverse-square force law.



Final Answer:

$$r_2 = 2.12 \times 10^{-10} \text{ m} \quad ; \quad r_3 = 4.77 \times 10^{-10} \text{ m}$$

The radii increase quadratically with the principal quantum number: $r_n \propto n^2$.

Expert's Solution — Ritika Sen, B.Tech CSE, NIT Durgapur

The Bohr Radius: A Fundamental Length Scale
The Bohr radius $a_0 = 5.29 \times 10^{-11}$ m (or 0.529 Å) is one of the most important length scales in all of physics. It sets the typical size of atoms and defines the scale for all atomic phenomena.

The Four Fundamental Constants that Define a_0:

h	$\longrightarrow$	Planck's constant (quantum mechanics)
m_e	$\longrightarrow$	Electron mass (particle property)
e	$\longrightarrow$	Elementary charge (EM interaction)
ε_0	$\longrightarrow$	Permittivity of free space (EM interaction)

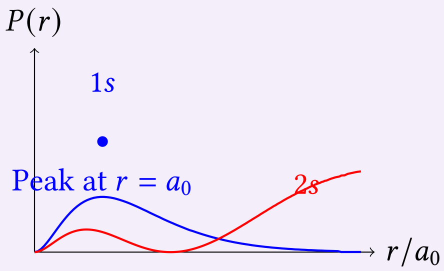
Why $r_n \propto n^2$? A Dimensional Argument
The n^2 scaling can be understood from simple dimensional analysis of Bohr's two postulates:
1. **Angular momentum quantization:** $mvr = n\hbar \implies v = n\hbar/mr$ 2. **Force balance:** $mv^2/r = ke^2/r^2 \implies v^2 = ke^2/mr$
Substituting v from (1) into (2):

$$\frac{n^2\hbar^2}{m^2r^2} = \frac{ke^2}{mr} \quad \implies \quad r = \frac{n^2\hbar^2}{mke^2} \propto n^2$$

The n^2 emerges directly because angular momentum ($\propto n$) enters squared into the kinetic energy expression.
Comparison with Real Hydrogen Atom (Quantum Mechanics):
In full quantum mechanics, the electron doesn't have a sharp orbit. Instead, we talk about the **most probable radius** from the radial wavefunction $R_{n\ell}(r)$. For the ground state (1s), the probability density is:

$$P(r) = \frac{4}{a_0^3} r^2 e^{-2r/a_0}$$

The most probable radius for the 1s state is exactly a_0, matching the Bohr model perfectly! For higher n states, the expectation value $\langle r \rangle$ scales similarly to $n^2 a_0$, though with some ℓ-dependent corrections.



★ Did You Know?

Quick Conversion: Bohr Radii to Angstroms

$$1 \text{ Å} = 10^{-10} \text{ m} = 100 \text{ pm}$$

$$a_0 \approx 0.53 \text{ Å} \approx 53 \text{ pm}$$

So the radii in more familiar units are:

- $r_1 = 0.53$ Å (about half an angstrom)
- $r_2 = 2.12$ Å
- $r_3 = 4.77$ Å

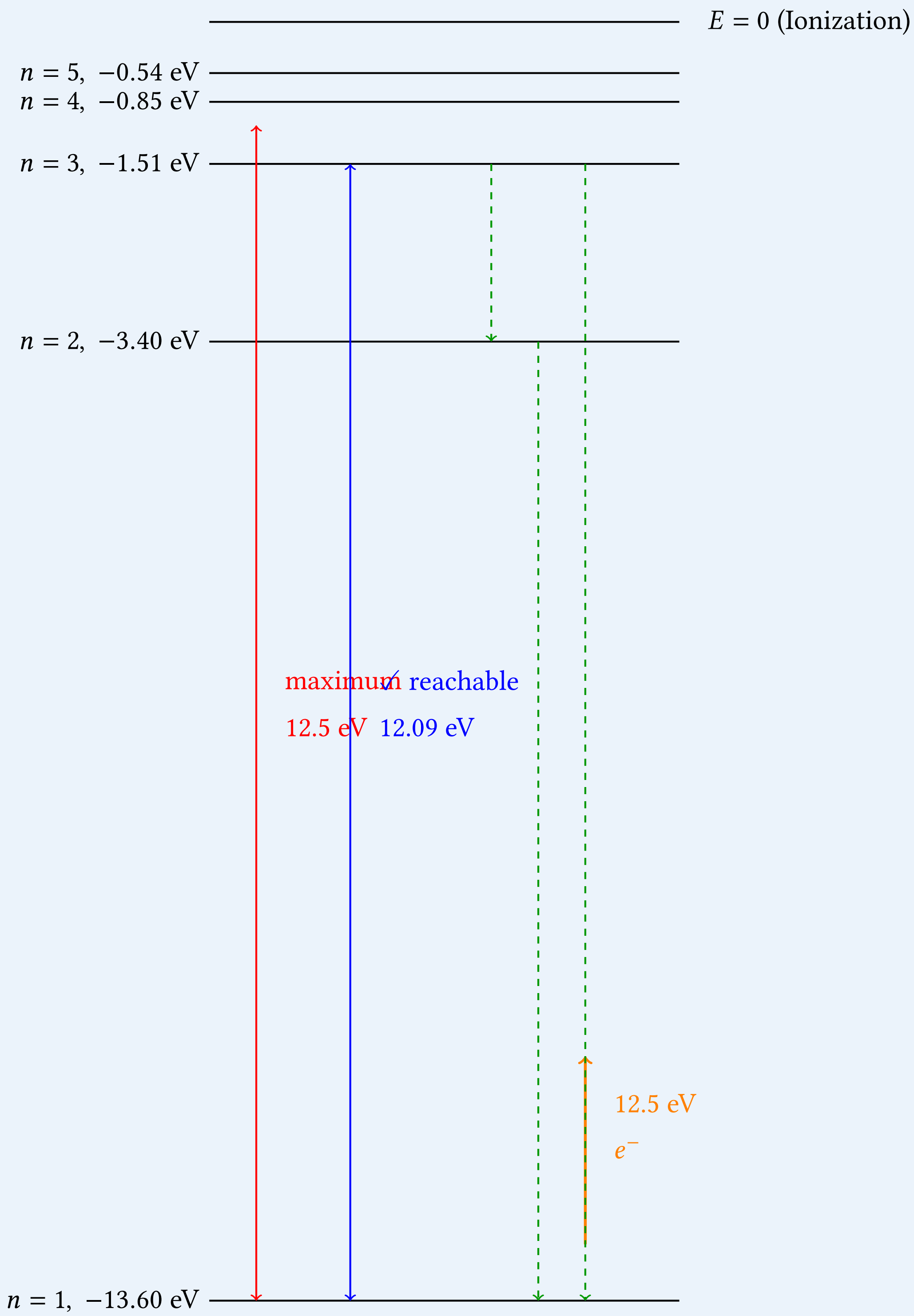
For comparison, the typical diameter of a hydrogen atom is about 1 Å (twice the Bohr radius). This is why the angstrom became the standard unit in atomic and molecular physics!

Q9 **A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?**

Solution

Understanding the Excitation Process

When electrons with kinetic energy 12.5 eV collide with hydrogen atoms in the ground state ($E_1 = -13.6$ eV), they can transfer energy to excite the atoms to higher energy levels. However, the excitation can only occur if the electron's energy matches or exceeds the energy difference between the ground state and some excited state.



Step 1: Determine the Highest Accessible Energy Level
The energy of the n^{th} level in hydrogen is:

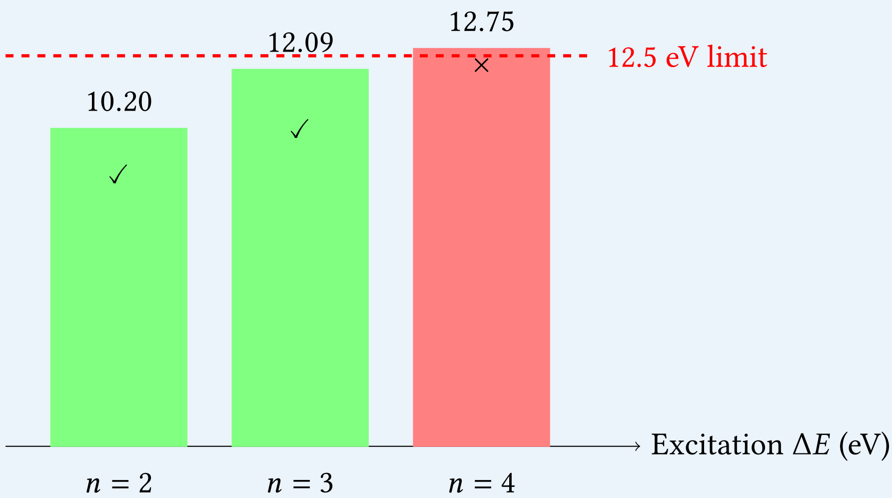
$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

The energy required to excite from ground state ($n = 1$) to level n is:

$$\Delta E = E_n - E_1 = (-13.6/n^2) - (-13.6) = 13.6\left(1 - \frac{1}{n^2}\right) \text{ eV}$$

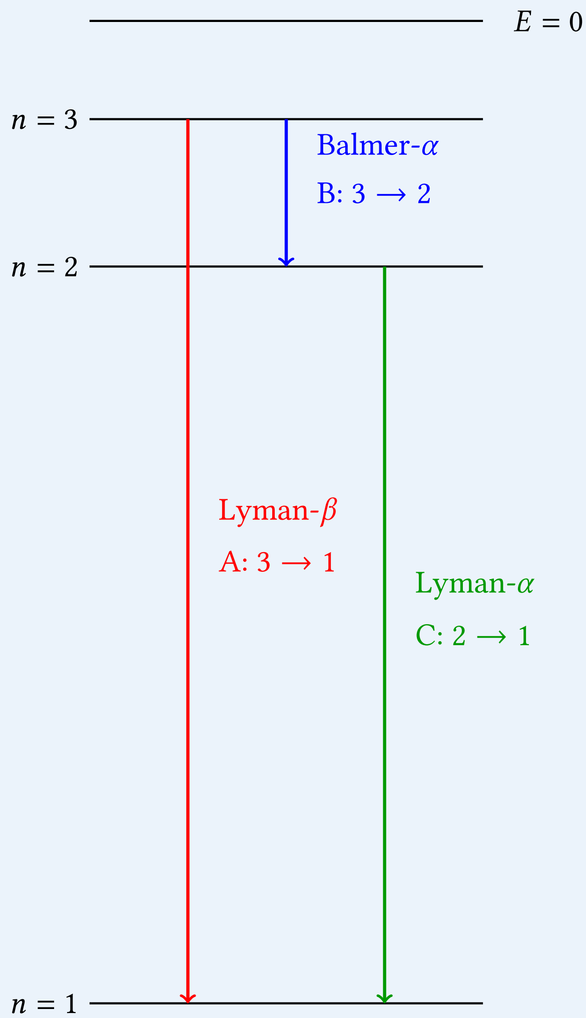
The bombarding electron has 12.5 eV. Let us calculate excitation energies to find which levels are accessible:

- $n = 2$: $\Delta E_{1\to 2} = 13.6\left(1 - \frac{1}{4}\right) = 13.6 \times 0.75 = 10.20$ eV
  $\longrightarrow$ **Accessible** ($10.20 < 12.5$ eV)
- $n = 3$: $\Delta E_{1\to 3} = 13.6\left(1 - \frac{1}{9}\right) = 13.6 \times \frac{8}{9} = 12.09$ eV
  $\longrightarrow$ **Accessible** ($12.09 < 12.5$ eV)
- $n = 4$: $\Delta E_{1\to 4} = 13.6\left(1 - \frac{1}{16}\right) = 13.6 \times \frac{15}{16} = 12.75$ eV
  $\longrightarrow$ **Not accessible!** ($12.75 > 12.5$ eV)



Conclusion: The 12.5 eV electrons can excite hydrogen atoms only up to $n = 3$. Atoms excited to $n = 2$ or $n = 3$ will subsequently de-excite, emitting photons.

Step 2: Identify All Possible Transitions
Once atoms are excited to $n = 2$ and $n = 3$, they can de-excite through various pathways:



There are **three** possible radiative transitions:

(A) $n = 3 \longrightarrow n = 1$ — Lyman series (Lyman-β)

(B) $n = 3 \longrightarrow n = 2$ — Balmer series (Balmer-α, also called H-α)

(C) $n = 2 \longrightarrow n = 1$ — Lyman series (Lyman-α)

Step 3: Calculate the Wavelength for Each Transition

We use the Rydberg formula:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

where $R = 1.097 \times 10^7\ \text{m}^{-1}$.

Alternatively, using the shortcut $hc = 1240$ eV·nm:

$$\lambda = \frac{1240}{\Delta E}\ \text{nm}$$

Transition A: $n = 3 \longrightarrow n = 1$ **(Lyman-β)**

$$\Delta E_A = E_3 - E_1 = (-1.51) - (-13.60) = 12.09 \text{ eV}$$
$$\lambda_A = \frac{1240}{12.09} \approx 102.6 \text{ nm}$$

This is in the **ultraviolet (UV)** region.

Transition B: $n = 3 \longrightarrow n = 2$ **(Balmer-α / H-α)**

$$\Delta E_B = E_3 - E_2 = (-1.51) - (-3.40) = 1.89 \text{ eV}$$
$$\lambda_B = \frac{1240}{1.89} \approx 656.1 \text{ nm}$$

This is in the **visible** region (Red light).

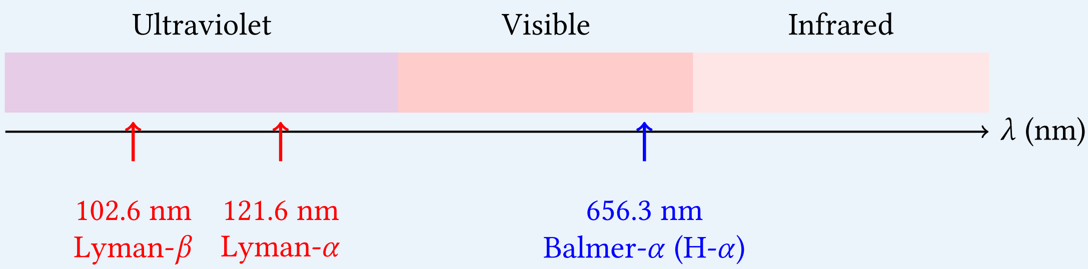
Transition C: $n = 2 \longrightarrow n = 1$ **(Lyman-α)**

$$\Delta E_C = E_2 - E_1 = (-3.40) - (-13.60) = 10.20 \text{ eV}$$
$$\lambda_C = \frac{1240}{10.20} \approx 121.6 \text{ nm}$$

This is in the **ultraviolet (UV)** region.

Summary of Emitted Radiation:

Transition	**Series**	ΔE **(eV)**	λ **(nm)**	**Region**
$n = 3 \longrightarrow n = 1$	Lyman-β	12.09	102.6	Ultraviolet
$n = 3 \longrightarrow n = 2$	Balmer-α (H-α)	1.89	656.3	Visible (Red)
$n = 2 \longrightarrow n = 1$	Lyman-α	10.20	121.6	Ultraviolet



Important Note: The excited atoms will not emit **all** possible transitions to $n = 4$ or higher because the 12.5 eV electron beam lacks sufficient energy to excite atoms to $n = 4$ ($\Delta E = 12.75$ eV). Thus, only transitions originating from $n = 2$ and $n = 3$ can occur.

Answer: Two spectral series will be emitted:

$$\boxed{\text{Lyman Series: } \lambda_1 = 121.6 \text{ nm } (2 \longrightarrow 1),\ \lambda_2 = 102.6 \text{ nm } (3 \longrightarrow 1)}$$

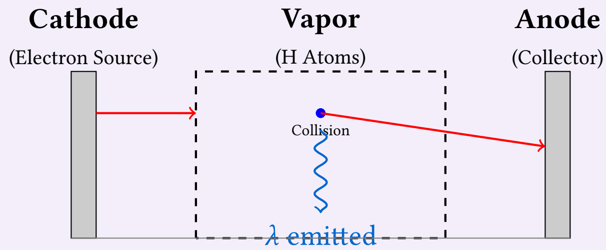
$$\boxed{\text{Balmer Series: } \lambda = 656.3 \text{ nm } (3 \longrightarrow 2)}$$

The Balmer-α line at 656.3 nm is in the visible red region. The Lyman lines are in the ultraviolet.

Expert's Solution — Siddharth Rao, B.Tech Engineering Physics, IIT Guwahati

Franck-Hertz Experiment Connection

This problem beautifully illustrates the principle behind the classic **Franck-Hertz experiment** (1914), which provided the first direct experimental evidence for Bohr's quantized energy levels. In that experiment, electrons were accelerated through mercury vapor, and it was observed that the electrons lost energy only in discrete amounts, corresponding to specific atomic transitions.



Why Only Up to $n = 3$?

The fact that 12.5 eV electrons can only reach $n = 3$ is a beautiful demonstration of energy quantization. Even though the bombarding electron has plenty of energy, it cannot transfer just any amount to the atom. The energy transfer must be exactly equal to an allowed transition energy. Since 12.75 eV (for $n = 1 \to 4$) exceeds the available 12.5 eV, the $n = 4$ state remains completely inaccessible.

Visible vs. UV Lines:

If you were to perform this experiment:

- The H-α line at 656.3 nm would be visible to the naked eye as a **red glow** from the hydrogen gas.
- The Lyman lines (102.6 nm and 121.6 nm) are in the far UV and would require specialized detectors (or photographic plates) to observe, as they are absorbed by air and not visible to the human eye.

This is exactly why the Balmer series was discovered first (by Ångström in 1853, well before quantum theory) — it lies in the visible region! The Lyman series was discovered much later (1906-1914) using vacuum spectrographs.

Did You Know?

Energy Threshold Check in Exams: When solving such problems, always ask:

1. What is the maximum energy this electron/photon can provide?
2. What is the excitation energy for each level?
3. Where does the energy "run out"?

For photons, the atom absorbs the entire photon or nothing at all (resonance absorption). For electron impact, the electron can retain some kinetic energy, so it can excite any level up to its total energy — but never exceed it. Here, 12.5 eV $<$ 12.75 eV, so $n = 4$ is **forbidden**.

Q10 **In accordance with the Bohr's model, find the quantum number that characterises the earth's revolution around the sun in an orbit of radius** 1.5×10^{11} **m with orbital speed** 3×10^4 **m/s. (Mass of earth** = 6.0×10^{24} **kg.)**

Solution

Understanding the Problem

This is a delightful application of Bohr's quantization condition to a macroscopic system! According to Bohr's second postulate, the angular momentum of an orbiting body is quantized in integer multiples of $h/2\pi$:

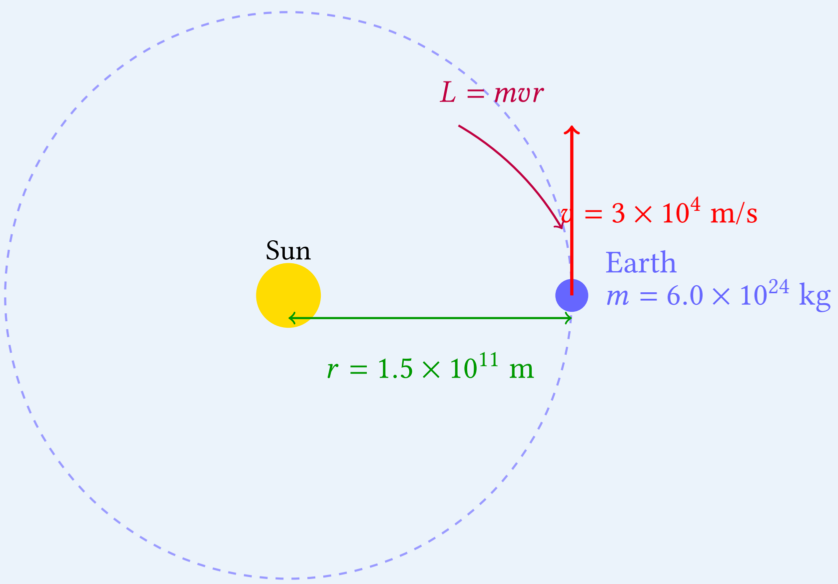
Bohr's Angular Momentum Quantization:

$$L = mvr = n\frac{h}{2\pi} = n\hbar$$

where:

- m = mass of the orbiting body (Earth)
- v = orbital speed
- r = orbital radius
- $h = 6.626 \times 10^{-34}$ J·s (Planck's constant)
- n = principal quantum number (integer)

Visualizing the Earth-Sun System through Bohr's Lens



Step 1: Write the Bohr Quantization Condition

$$mvr = n\frac{h}{2\pi}$$

Solving for the quantum number n:

$$n = \frac{2\pi mvr}{h}$$

Step 2: Substitute the Given Values

- Mass of Earth, $m = 6.0 \times 10^{24}$ kg
- Orbital speed, $v = 3.0 \times 10^{4}$ m/s
- Orbital radius, $r = 1.5 \times 10^{11}$ m
- Planck's constant, $h = 6.626 \times 10^{-34}$ J·s

$$n = \frac{2\pi \times (6.0 \times 10^{24}) \times (3.0 \times 10^{4}) \times (1.5 \times 10^{11})}{6.626 \times 10^{-34}}$$

Step 3: Calculate the Numerical Value
First, let's group the numbers and powers of 10 separately:
Numerator:

$$\begin{aligned} 2\pi \times 6.0 \times 3.0 \times 1.5 &\approx 6.283 \times 6.0 \times 3.0 \times 1.5 \\ &= 6.283 \times 27.0 \\ &\approx 169.64 \end{aligned}$$

Power of 10 in numerator:

$$10^{24} \times 10^{4} \times 10^{11} = 10^{39}$$

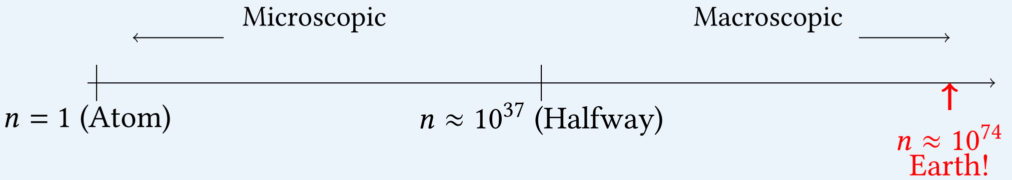
So the numerator is approximately:

$$169.64 \times 10^{39} = 1.6964 \times 10^{41}$$

Division by h:

$$n = \frac{1.6964 \times 10^{41}}{6.626 \times 10^{-34}} = \frac{1.6964}{6.626} \times 10^{41+34}$$

$$n = 0.256 \times 10^{75} = 2.56 \times 10^{74}$$



Step 4: Express the Answer Clearly
Let us perform a more precise calculation:

$$n = \frac{2 \times 3.1416 \times (6.0 \times 10^{24}) \times (3.0 \times 10^{4}) \times (1.5 \times 10^{11})}{6.626 \times 10^{-34}}$$

$$n = \frac{2 \times 3.1416 \times 6.0 \times 3.0 \times 1.5 \times 10^{24+4+11}}{6.626 \times 10^{-34}}$$

$$n = \frac{169.65 \times 10^{39}}{6.626 \times 10^{-34}} = \frac{169.65}{6.626} \times 10^{73}$$

$$n = 25.60 \times 10^{73} = 2.56 \times 10^{74}$$

Answer:

$$\boxed{n \approx 2.6 \times 10^{74}}$$

The quantum number characterizing Earth's revolution around the Sun is astronomically large — about 10^{74}.

Physical Interpretation: The Correspondence Principle

This result beautifully illustrates Bohr's **Correspondence Principle**: for very large quantum numbers, quantum mechanical predictions must match classical physics. Since $n \approx 10^{74}$ is enormously large, the quantization of angular momentum is completely imperceptible at the macroscopic scale.

Microscopic (Atom)	Macroscopic (Earth)
$n = 1, 2, 3, \ldots$	$n \approx 2.6 \times 10^{74}$
Discrete energy levels visible	Energy levels **appear continuous**
ΔE is significant	ΔE is negligible
Quantum effects dominate	Classical physics adequate

The energy difference between adjacent quantum levels for Earth would be:

$$\Delta E \approx \frac{dE}{dn}\Delta n = \frac{dE}{dn} \times 1$$

This difference is so fantastically tiny that it's completely unmeasurable, explaining why we never observe "quantum jumps" in planetary orbits!

Complete Answer:

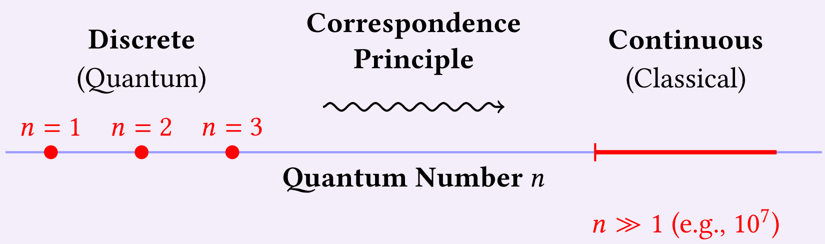
$$\boxed{n = \frac{2\pi mvr}{h} \approx 2.6 \times 10^{74}}$$

This astronomically large quantum number demonstrates why quantum effects are imperceptible at macroscopic scales, in perfect agreement with the Bohr Correspondence Principle.

Expert's Solution — Tanmay Patel, B.Tech CSE, NIT Surat

The Correspondence Principle: Bridging Quantum and Classical Worlds

This problem is perhaps the most elegant illustration of Niels Bohr's **Correspondence Principle**, which he formulated in 1923. The principle states that the predictions of quantum mechanics must reduce to those of classical physics in the limit of large quantum numbers.



What If We Treat Earth Quantum Mechanically?

For fun, let's calculate the **spacing between adjacent quantum levels**:

The orbital energy of Earth (classically) is:

$$E_n \approx -\frac{GM_\odot m}{2r} \quad \text{(virial theorem for gravity)}$$

Using $r_n \propto n^2$ (from Bohr-like quantization), we can show that:

$$E_n \propto -\frac{1}{n^2}$$

The energy difference between n and $n+1$ for large n:

$$\Delta E = E_{n+1} - E_n \approx \frac{2|E_n|}{n}$$

Earth's orbital kinetic energy is approximately:

$$K = \frac{1}{2}mv^2 = 0.5 \times 6.0 \times 10^{24} \times (3 \times 10^4)^2 \approx 2.7 \times 10^{33}\ \text{J}$$

So:

$$\Delta E \approx \frac{2 \times 2.7 \times 10^{33}}{2.6 \times 10^{74}} \approx 2 \times 10^{-41}\ \text{J}$$

This is **so small** that the corresponding photon wavelength would be:

$$\lambda = \frac{hc}{\Delta E} \approx \frac{2 \times 10^{-25}}{2 \times 10^{-41}} \approx 10^{16}\ \text{m}$$

This wavelength is **larger than the solar system**! Such a photon would be completely undetectable, explaining why Earth's orbit appears perfectly continuous.

Historical Note:

Bohr himself used this very reasoning to argue that the quantum behavior he proposed for atoms would not contradict the well-established classical behavior of macroscopic objects. He originally developed the "planetary model" of the atom precisely because of the mathematical similarity between Coulomb's law and Newton's law of gravitation — both are $1/r^2$ central forces.

Poetic Parallel: Atom as a Miniature Solar System

Atom		Solar System
Nucleus (proton)	$\longleftrightarrow$	Sun
Electron	$\longleftrightarrow$	Earth
Coulomb force ($\propto 1/r^2$)	$\longleftrightarrow$	Gravity ($\propto 1/r^2$)
$n = 1$ (ground state)	$\longleftrightarrow$	$n \approx 10^{74}$
Quantum effects visible	$\longleftrightarrow$	Classical behavior apparent

Did You Know?

Exam Shortcut for Angular Momentum Quantization: Whenever a problem asks for a quantum number using Bohr's condition, directly use:

$$n = \frac{2\pi mvr}{h}$$

For hydrogen-like microscopic systems, n is small (1, 2, 3, ...). For macroscopic objects, n will always come out **huge** ($\gg 1$). This huge value is your immediate signal that quantum effects are negligible for that system — a quick self-consistency check!