Collegedunia NCERT Formula Sheet

The Ultimate Formula Reference for Class 12 Physics

Chapter 13: Nuclei

Constant / Quantity	Value
Atomic mass unit, u	1.66054×10^{-27} kg
Energy equivalent of 1 u	931.5 MeV
Proton mass, m_p	1.6726×10^{-27} kg = 1.00728 u
Neutron mass, m_n	1.6749×10^{-27} kg = 1.00867 u
Nuclear unit of length, R_0	1.2×10^{-15} m = 1.2 fm
Avogadro's number, N_A	6.022×10^{23} /mol
ln 2	0.693

1 Composition & Size of Nucleus

A nucleus is identified by its mass number A (protons + neutrons) and atomic number Z (protons). Its size scales as $A^{1/3}$, giving roughly constant nuclear density (NCERT 13.2–13.3).

Notation & terminology

${}^A_Z X$: nucleus with Z protons and $A - Z$ neutrons, total mass number A.
Isotopes: same Z, different A (e.g., ^{12}C, ^{14}C).
Isobars: same A, different Z (e.g., ^{14}C, ^{14}N).
Isotones: same $N = A - Z$, different Z (e.g., ^{13}C, ^{14}N).

Nuclear radius

$$R = R_0 \, A^{1/3}$$

where $R_0 = 1.2$ fm = 1.2×10^{-15} m.

Heavier nuclei are larger, but only as the **cube root** of A. A uranium nucleus ($A = 238$) is only $\sim 6.2\times$ wider than a hydrogen ($A = 1$).

Nuclear density

$$\rho_{\text{nuc}} = \frac{m}{V} = \frac{A m_p}{(4/3)\pi R_0^3 A} \approx 2.3 \times 10^{17}$$

kg/m^3

Density is **independent of** A — a fundamental constant of nuclear matter. About 10^{14} times denser than ordinary matter; comparable to a neutron star.

2 Mass–Energy Equivalence & Binding Energy

A bound nucleus is lighter than the sum of its constituent nucleons. The missing mass — the mass defect — is converted to the energy that holds the nucleus together (NCERT 13.4–13.5).

Mass–energy equivalence

$$E = mc^2$$

$$1\ \text{u} \times c^2 = 931.5\ \text{MeV}$$

Mass and energy are interchangeable. The factor c^2 is enormous: tiny mass differences correspond to huge energies. Used everywhere in nuclear and particle physics.

Mass defect

$$\Delta m = Zm_p + (A - Z)m_n - M_{\text{nucleus}}$$

$\Delta m > 0$ for any stable nucleus. Tells how much lighter a bound nucleus is than the loose collection of its protons and neutrons.

Binding energy

$$BE = \Delta m \cdot c^2$$

$$BE\ (\text{MeV}) = \Delta m\ (\text{u}) \times 931.5$$

Energy required to break the nucleus into its constituent nucleons. Larger $BE \Rightarrow$ more tightly bound. Largest binding energy: ~ 1800 MeV for a uranium nucleus.

Binding energy per nucleon

$$\overline{BE} = \frac{BE}{A}$$

Peak: ~ 8.8 MeV/nucleon for ^{56}Fe. Drops off for very light and very heavy nuclei. Explains why **fusion** (light nuclei combining) and **fission** (heavy nuclei splitting) both release energy.

BE/A curve features

Sharp rise from 0 to ~ 8 MeV between $A = 2$ and $A = 30$. **Plateau** near 8.5 MeV for $A = 30$–80 (peak at ^{56}Fe). **Gradual fall** to 7.6 MeV at $A = 240$. Light nuclei gain stability by fusing; heavy nuclei gain stability by fissioning. Peaks at certain "magic numbers" (2, 8, 20, 28, 50, 82, 126).

3 Radioactivity

Unstable nuclei decay spontaneously, emitting alpha particles, beta particles, or gamma rays. The decay rate is proportional to the number of unstable nuclei present (NCERT 13.6–13.8).

Three types of radiation

Alpha (α): a $^4_2$He nucleus. Highly ionising, low penetration.
Beta (β^-): an electron from neutron decay ($n \to p + e^- + \bar{\nu}_e$). Medium ionisation, medium penetration.
Gamma (γ): a high-energy photon, emitted by an excited nucleus de-exciting. Low ionisation, high penetration.

Radioactive decay law

$$N = N_0\, e^{-\lambda t}$$

$$\frac{dN}{dt} = -\lambda N$$

where N = nuclei left at time t; N_0 = initial; λ = decay constant (s^{-1}).

Decay rate is proportional to the present count — a **first-order** process. Independent of temperature, pressure, chemistry.

Half-life

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Time after which half the nuclei have decayed. Independent of N_0. After n half-lives: $N = N_0/2^n$.

Mean (average) life

$$\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2} = 1.44\, T_{1/2}$$

Average lifetime of a nucleus before it decays. Slightly longer than the half-life. After one mean life, $N = N_0/e \approx 0.37 N_0$.

Activity

$$A = -\frac{dN}{dt} = \lambda N$$

$$A = A_0\, e^{-\lambda t}$$

Units: **becquerel** (Bq) = 1 decay/s; **curie** (Ci) = 3.7×10^{10} Bq.

Decays per second of a sample. Activity **also** decreases exponentially with the same λ as N does.

4 Decay Modes

Each decay mode follows specific conservation rules for A, Z, charge, and energy (NCERT 13.6).

α-decay

$${}^{A}_{Z}X \to {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha$$

A decreases by 4, Z by 2. Q-value: $Q = (m_X - m_Y - m_\alpha)c^2$. Energy is shared between recoiling daughter nucleus and the α.

β^--decay

$${}^{A}_{Z}X \to {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e$$

Inside the nucleus: $n \to p + e^- + \bar{\nu}_e$.

A unchanged, Z **increases** by 1. Energy is shared with the antineutrino, so emitted electrons have a continuous energy spectrum.

β^+-decay (positron emission)

$${}^{A}_{Z}X \to {}^{A}_{Z-1}Y + e^+ + \nu_e$$

Inside the nucleus: $p \to n + e^+ + \nu_e$.

A unchanged, Z **decreases** by 1. Possible only when the nuclear binding-energy difference exceeds $2m_ec^2 = 1.022$ MeV.

γ-decay

$${}^{A}_{Z}X^* \to {}^{A}_{Z}X + \gamma$$

An excited nucleus drops to a lower state by emitting a gamma photon. **No change** in A or Z. Often follows α or β decays.

5 Nuclear Fission & Fusion

The two energy-releasing reactions on opposite sides of the BE-per-nucleon curve — both move toward the iron peak (NCERT 13.7).

Fission

A heavy nucleus (typically ^{235}U) absorbs a neutron and **splits** into two medium-mass fragments plus 2–3 neutrons, releasing ~ 200 MeV. The released neutrons can trigger more fissions: a **chain reaction** sustained when the multiplication factor $k = 1$ (controlled in reactors) or $k > 1$ (uncontrolled in bombs).

Typical fission reaction

$${}^{235}_{92}\text{U} + {}^{1}_{0}n \to {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3\,{}^{1}_{0}n + Q$$

$Q \approx 200$ MeV per fission.

Energy comes from the BE/A increase as we move from $A = 235$ to $A \approx 100$. Released as kinetic energy of fragments + neutrons + γ rays.

Fusion

Two light nuclei **combine** into a heavier one, releasing energy. Powers the **Sun** (proton–proton chain converting H to He). Requires extreme temperatures ($\sim 10^7$ K) to overcome Coulomb repulsion. Energy released per fusion is smaller than per fission, but per unit mass it is much larger.

Solar (p–p) fusion summary

$$4\,{}^{1}_{1}\text{H} \to {}^{4}_{2}\text{He} + 2e^+ + 2\nu_e + 6\gamma + Q$$

$Q \approx 26.7$ MeV per cycle.

Net effect of the chain. Deuterium–tritium (^{2}H + ^{3}H $\to$ ^{4}He + n) is the practical path being pursued for terrestrial fusion reactors.

JEE/NEET Extension: Q-value formula

For any reaction $X_1 + X_2 \to Y_1 + Y_2$: $Q = (m_{X_1} + m_{X_2} - m_{Y_1} - m_{Y_2})\,c^2$. Use **atomic** masses (electrons cancel for α and γ decays; for $\beta^\pm$ check carefully). Negative Q means the reaction needs input energy.

Decay tally

α**-decay**: -4 from A, -2 from Z.
β^-**-decay**: A same, $+1$ to Z.
β^+**-decay**: A same, -1 to Z.
γ**-decay**: nothing changes (just energy).
Useful for working out a decay-chain end-point quickly.

Half-life vs mean life

Half-life $T_{1/2} = 0.693/\lambda$ — time for half to decay.
Mean life $\tau = 1/\lambda$ — average lifetime.
$\tau = T_{1/2}/\ln 2 \approx 1.44\,T_{1/2}$. They are **not the same**; the longer mean life accounts for the long tail of nuclei that decay much later than the half-life mark.

Quick Reference — Nuclei

Quantity / Concept	**Expression**	**Notes**
Nuclear radius	$R_0\, A^{1/3}$	$R_0 = 1.2$ fm
Nuclear density	$\sim 2.3 \times 10^{17}$ kg/m^3	Independent of A
Mass–energy	$E = mc^2$	1 u = 931.5 MeV
Mass defect	$Zm_p + Nm_n - M$	Always positive
Binding energy	$\Delta m \cdot c^2$	Holds nucleus together
BE per nucleon	BE/A	Peak at ^{56}Fe (8.8 MeV)
Decay law	$N_0\, e^{-\lambda t}$	First-order
Half-life	$0.693/\lambda$	After n half-lives: $N_0/2^n$
Mean life	$1/\lambda$	$1.44\, T_{1/2}$
Activity	λN	Bq = decays/s
α-decay	$A \to A-4, Z \to Z-2$	Emits $^4_2$He
β^--decay	$Z \to Z+1$	$n \to p + e^- + \bar{\nu}_e$
β^+-decay	$Z \to Z-1$	Needs > 1.022 MeV
γ-decay	A, Z unchanged	Excited → ground
Fission Q (^{235}U)	~ 200 MeV	Per fission
Fusion Q (p–p)	~ 26.7 MeV	Per He nucleus