

Chapter 13

Nuclei

Atom = nucleus + electrons.

The tiny nucleus carries $> 99.9\%$ of the atomic mass in a sphere of radius few fm.

It decides chemistry (via Z) & releases huge energies in fission and fusion.

* Discovery

Rutherford (1911) - alpha scattering off a thin gold foil \rightarrow most alpha pass through ; a few scatter at very large angles.

Inference : atom is mostly empty , with a tiny positive core - the nucleus.

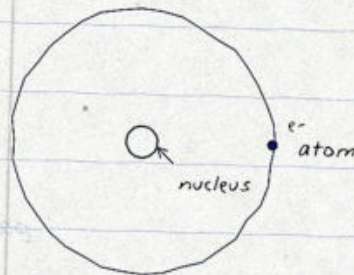


Fig. atom : nucleus + orbiting electrons

(not to scale - nucleus is 10^{-5} of atom)

Composition of the Nucleus

Nucleons : protons + neutrons

Proton : charge $+e$, $m_p = 1.00728 \text{ u}$

Neutron : charge zero , $m_n = 1.00867 \text{ u}$

(Chadwick , 1932 discovered the neutron.)

Three numbers describe a nucleus

$Z =$ atomic number $=$ no. of protons

$N =$ neutron number $=$ no. of neutrons

$A =$ mass number $= Z + N$ (nucleons)

$$A = Z + N$$

<- fundamental

<- nucleon count

*

Notation

Symbol : $A_Z X$ (e.g. ${}_{92}^{235} \text{U}$)

A on top , Z at bottom , $X =$ element.

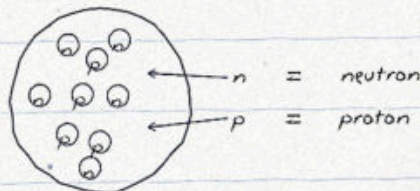


Fig. protons (p) + neutrons (n) = nucleons

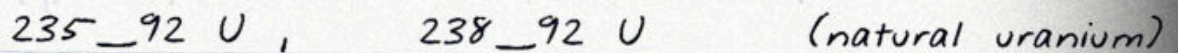
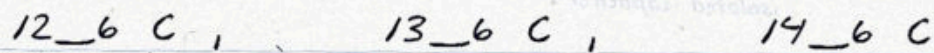
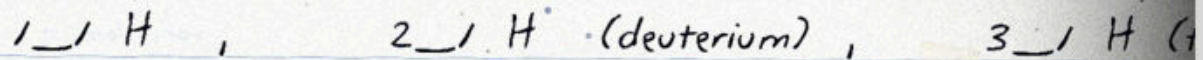
Bound by strong nuclear force (next page).

Isotopes , Isobars , Isotones

Isotopes - same Z , different A

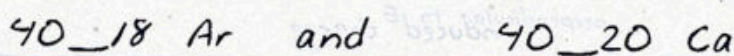
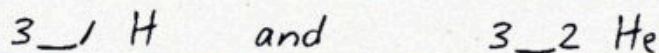
Same element (same chemistry) , different N .

Examples :



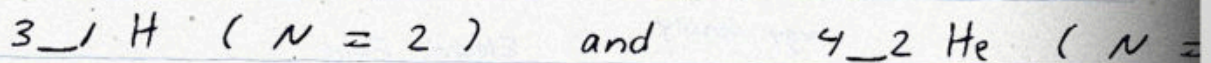
Isobars - same A , different Z

Same total nucleons ; different elements.



Isotones - same N , different Z

Same neutron number , different element.



Quick check (typo corrected)

Isotopes have same ~~A~~ Z but different A .

Isobars : same A , Z varies.

Isotones : same N , Z varies ; $A = N + Z$.

Atomic mass.

Natural samples are mixtures of isotopes \rightarrow

atomic mass = weighted avg of isotope masses.

Atomic Mass Unit (u)

Nuclear masses are tiny in kg \rightarrow use a more convenient unit, the u (or amu).

Definition

1 u = 1/12 (mass of one $^{12}_6\text{C}$ atom)

$$1 \text{ u} = 1.66054 \times 10^{-27} \text{ kg}$$

\leftarrow very small !
 \leftarrow atomic scale

Key nucleon / electron masses

$$m_p = 1.007276 \text{ u} = 938.27 \text{ MeV} / c^2$$

$$m_n = 1.008665 \text{ u} = 939.57 \text{ MeV} / c^2$$

$$m_e = 0.000549 \text{ u} = 0.511 \text{ MeV} / c^2$$

Note : m_n slightly $>$ m_p \rightarrow free neutron is unstable and decays ($T_{1/2} = 10.2 \text{ min}$).

Energy - mass conversion

$$E = mc^2$$

\leftarrow Einstein
 \leftarrow 1905

$$1 \text{ u} = 931.5 \text{ MeV} / c^2$$

\leftarrow use this !
 \leftarrow u \leftrightarrow MeV

Size of the Nucleus

Scattering experiments (high - energy electrons & alpha particles) give the nuclear radius.

Empirical radius formula

$$R = R_0 A^{(1/3)}$$

$$\begin{aligned} \leftarrow R_0 & 1.2 \text{ fm} \\ \leftarrow 1 \text{ fm} & = 10^{-15} \text{ m} \end{aligned}$$

$$\text{Volume } V = \frac{4}{3} \pi R^3 = \frac{4}{3} \pi R_0^3 A$$

V proportional to A

Nuclear density

$$\text{Mass} = A m_n \text{ approx ; } \rho = \text{mass} / V$$

$$\rho = \frac{3 m_n}{4 \pi R_0^3} \leftarrow \text{no } A \text{ ?}$$

$\leftarrow \text{constant}$

Plug numbers :

$$\rho \text{ approx } 2.3 \times 10^{17} \text{ kg / m}^3$$

(About 10^{14} times the density of water !)

Implication

Nucleons are packed almost as tight as they can be ; matter inside nucleus is incompressible.

Mass Defect

Careful mass measurements show :

Mass of a nucleus $<$ total mass of free
 Z protons + N neutrons.

The missing mass appears as binding energy.

Definition

$$\Delta m = [Z m_p + N m_n] - M_N$$

M_N ← of nucleus

If atomic mass M is given, use also :

$$\Delta m = Z m_H + N m_n - M(\text{atom})$$

m_H ← of (p + e)

Sample calc : ${}^4_2\text{He}$

$$\text{Free : } 2 m_p + 2 m_n = 4.03190 \text{ u}$$

$$\text{Actual nuclide mass} = 4.00260 \text{ u}$$

$$\Delta m = 4.03190 - 4.00260 = \cancel{0.0303} 0.02930$$

Such a small fraction (0.7 %) yields huge
 energy via $E = m c^2$. See next page.

Binding Energy

Energy needed to break a nucleus into its free constituent nucleons. Equivalently, the energy released when the nucleus is assembled from free protons & neutrons.

Formula

$$BE = \Delta m c^2$$

← energy from
← mass defect

$$BE = \Delta m (\text{u}) \times 931.5 \text{ MeV}$$

← if Δm
← in u

Example : BE of ${}^4_2\text{He}$

$$\Delta m = 0.02930 \text{ u} \quad (\text{from prev page})$$

$$BE = 0.02930 \times 931.5 \text{ MeV}$$

$$= 27.3 \text{ MeV} \quad (28 \text{ MeV})$$

Compare : chemical bond energies few eV ?

Nuclear binding is 10^6 times stronger.

Binding energy per nucleon

$$BE / A \quad (\text{MeV per nucleon})$$

← stability
← measure

BE / A vs A Curve

Plot binding energy per nucleon BE / A against mass number A for stable nuclei.

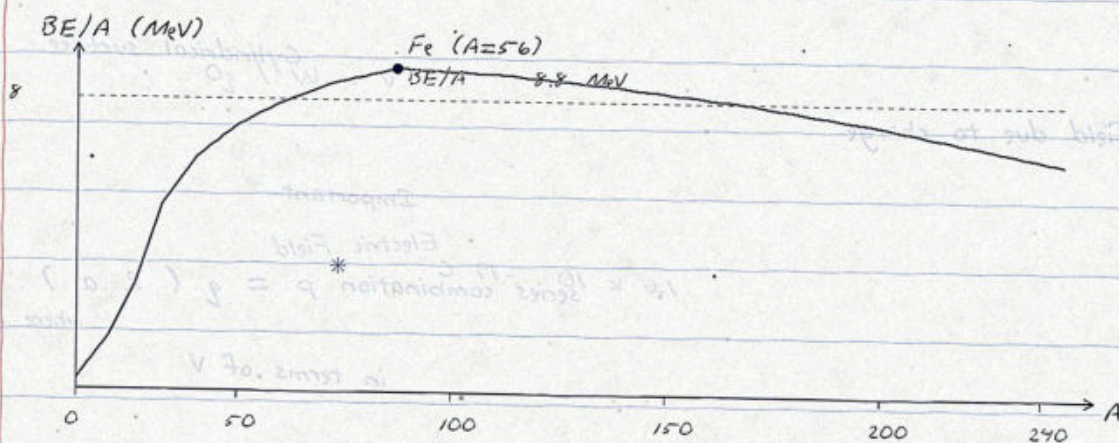


Fig. binding energy per nucleon vs A

Key features

1. Peak around $A = 56$ (Fe) at 8.8 MeV/A .
2. For $A = 30 - 170$: BE/A is fairly flat at 8 MeV (nuclear force saturation).
3. Low A nuclei (H, He, Li) - low BE/A .
4. High A nuclei (U, Pu) - falls again.

Stable nuclei prefer the middle of the curve.

Implications of BE/A Curve

Why fission releases energy

Heavy nucleus (e.g. U, $A = 240$) has $BE/A = 7.6$ MeV. When it splits into two medium fragments ($A = 120$ each) with $BE/A = 8.5$ MeV, each nucleon becomes more tightly bound \rightarrow energy released.

$$Q \approx 240 \times (8.5 - 7.6) = 216 \text{ MeV}$$

\leftarrow per fission
 \leftarrow of 23

Why fusion releases energy

Two light nuclei (H, H) have $BE/A = 1.1$ MeV. When they fuse to ${}^4\text{He}$, BE/A jumps to 7.07 MeV. Each nucleon is far more tightly bound \rightarrow huge release per unit mass.

Per kg, fusion releases $4 \times$ more energy than fission of ${}^{235}\text{U}$.

Why Fe is the most stable

BE/A maximum at $A = 56 \rightarrow$ any reaction moving away from Fe (either way) costs energy. Stellar fusion stops once Fe is built up; core then collapses \rightarrow supernova!

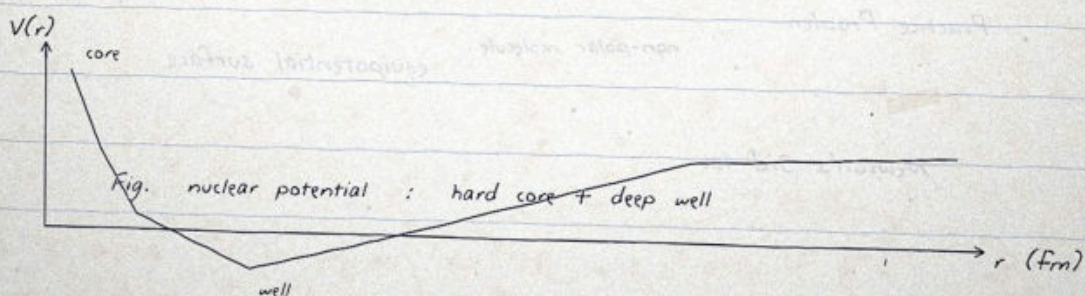
Nuclear Forces

Inside a nucleus , protons repel via Coulomb force - yet the nucleus holds together !

So another , stronger , attractive force must act * - the strong nuclear force.

Key properties

- ① Short ranged : a few fm only.
- ② Stronger than Coulomb at fm scale ;
100 x electromagnetic force.
- ③ Charge independent : acts on
p - p , n - n & p - n identically.
- ④ Saturation : each nucleon binds only
to its nearest neighbours.
- ⑤ Has a repulsive core at $r < 0.7$ fm.



Radioactivity

Discovery

Henri ~~Becquerel~~ Becquerel (1896) :

uranium salts spontaneously emitted radiation that fogged a photographic plate.

M & P Curie (1898) : isolated polonium & radium. Rutherford later identified 3 types. *

Three kinds of radioactive emission

alpha : ${}^4_2\text{He}$ nucleus (2 p + 2 n)

beta- : fast electron (from neutron decay)

betat+ : positron (from proton decay)

gamma : high - energy photon (no mass, no Q)

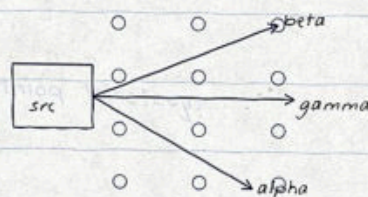


Fig. separation in B field - 3 distinct paths

Key observation

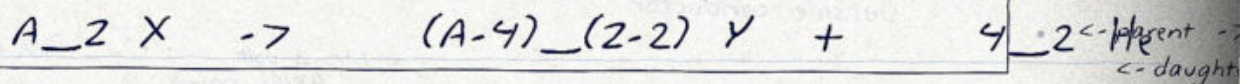
All three are emitted by an unstable nucleus.

Alpha Decay

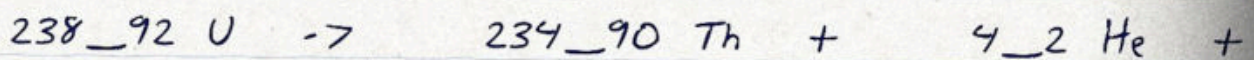
Heavy unstable nucleus emits an alpha particle

(= 4_2He nucleus). Z falls by 2, A by 4.

General equation



Example : 238_{92}U



Q - value = energy released

$$Q = (M_X - M_Y - M_{\alpha}) c^2$$

$\leftarrow Q > 0$
 \leftarrow for decay

Q for $\text{U} - 238 \rightarrow \text{Th} - 234$: 4.27 MeV

(Shared between alpha + daughter as KE.)

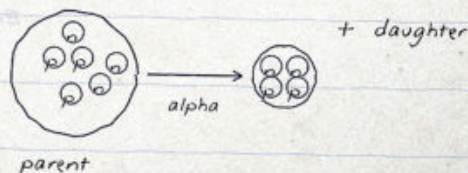


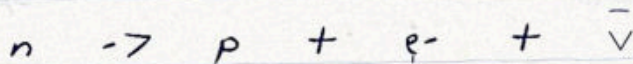
Fig. alpha emission - daughter is lighter.

Beta Decay

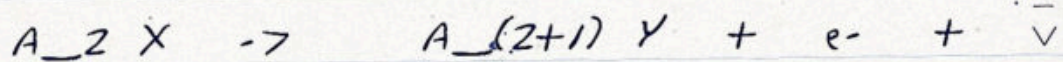
Beta - minus (n \rightarrow p)

A neutron in the nucleus converts to a proton.

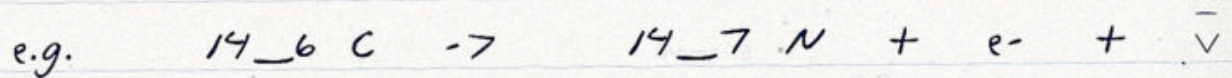
An electron (e^-) and an anti - neutrino are emitted.



\leftarrow neutron \rightarrow proton
 \leftarrow Z up by 1



\leftarrow A
 \leftarrow

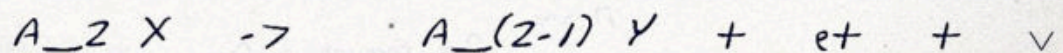


Beta - plus (p \rightarrow n)

A proton converts to a neutron ; emits a positron (e^+) and a neutrino.



\leftarrow proton \rightarrow neutron
 \leftarrow Z down by 1



Pauli (1930) predicted the neutrino to save energy, momentum & spin conservation.

Gamma Emission

After alpha or beta decay, the daughter nucleus is often left in an excited state. It de-excites by emitting one (or more) gamma photons.

Properties

- ① No change in Z or A .
- ② Photons - no charge, no mass.
- ③ Energies in keV - MeV range.
- ④ Highly penetrating ; shielded by Pb.



$\leftarrow *$ = excited
 \leftarrow state

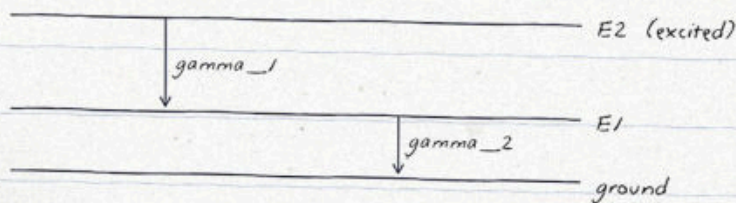


Fig. nuclear levels - gamma photon emitted

Law of Radioactive Decay

Each unstable nucleus has a fixed probability of decaying per unit time. For a sample of N (large) nuclei :

Differential form

$$\boxed{dN / dt = - \lambda N}$$

← λ = decay constant

Integrate from N_0 at $t = 0$ to N at time t :

$$\boxed{N(t) = N_0 e^{-\lambda t}}$$

← exp decay of nuclei

Activity

Number of decays per second :

$$\boxed{A = - dN/dt = \lambda N}$$

← in becquerel
← (decays / s)

$$A(t) = A_0 e^{-\lambda t} ; A_0 = \lambda N_0$$

Units

$$1 \text{ Bq} = 1 \text{ decay / s} ; 1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}$$

Half - life & Mean Life

Half - life $T_{1/2}$

Time in which half of the original nuclei decay.

$$\text{Set } N(T_{1/2}) = N_0 / 2 \text{ in } N = N_0 e^{-\lambda t}$$

$$T_{1/2} = \ln 2 / \lambda = 0.693 / \lambda$$

use this ?
← lambda

Mean life τ

Average lifetime of a nucleus :

$$\tau = 1 / \lambda$$

← no log ?

$$\tau = T_{1/2} / \ln 2 = 1.443 T_{1/2}$$

← tau > T_{1/2}

After n half - lives

$$N = N_0 / 2^n$$

← fast check

← n = t / T_{1/2}

After 1 half - life : 50 % left

After 2 half - lives : 25 % left

After 10 half - lives : 0.1 % left.

Worked Example - Half life

Problem

$T_{1/2}$ of a sample is 20 min. Find activity left after 1 hour, starting from A_0 .

Step 1 : count half - lives

$$t = 60 \text{ min} ; T_{1/2} = 20 \text{ min}$$

$$n = t / T_{1/2} = 60 / 20 = 3$$

Step 2 : apply $A = A_0 / 2^n$

$$A = A_0 / 2^3 = A_0 / 8 \text{ (12.5 \% of } A_0 \text{)}$$

Cross check via lambda

$$\lambda = 0.693 / 20 = 0.0347 \text{ per min}$$

$$\lambda t = 0.0347 \times 60 = 2.08$$

$$A / A_0 = e^{-2.08} = 0.125 \text{ - same !}$$

Quick estimates

After 4 half - lives : 6.25 % left.

After 5 half - lives : 3.125 % left.

Rule of thumb : N decreases by ~~to~~ 2x for every half - life that passes.

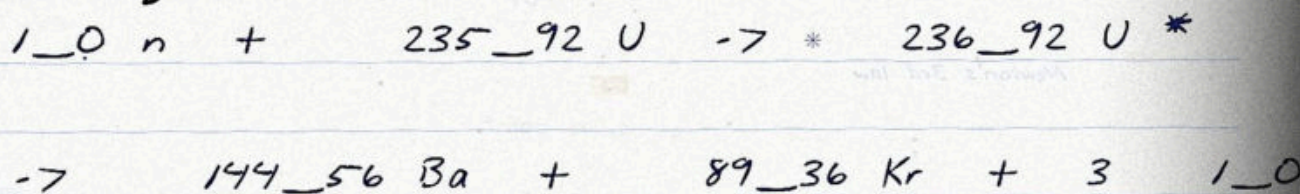
Nuclear Fission

Discovery

Hahn & Strassmann (Germany, 1939) found Ba in U targets after neutron bombardment.

Meitner & Frisch interpreted it as splitting of the U nucleus \rightarrow 'nuclear fission'.

Typical equation



Q approx 200 MeV per fission

\leftarrow huge!
 \leftarrow 10⁶ x

Where the energy goes

- ① KE of fragments : 165 MeV
- ② KE of neutrons : 5 MeV
- ③ Gammas : 7 MeV
- ④ Beta and neutrinos : 23 MeV

Other fragment pairs are possible (Cs, Xe, ...).

Chain Reaction

* Each fission releases 2 - 3 neutrons . If even one of these triggers another fission \rightarrow chain.

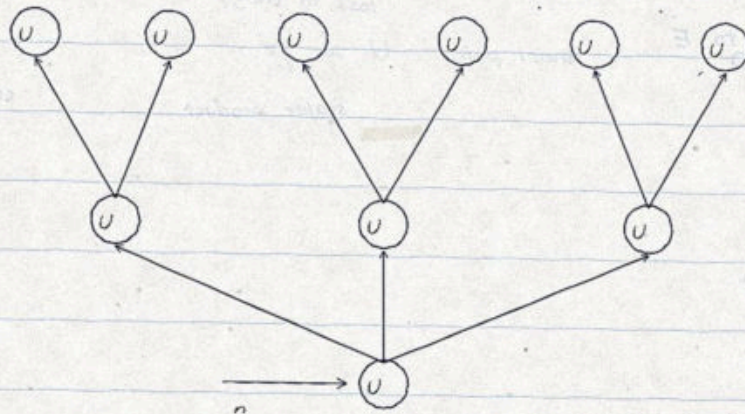


Fig. uncontrolled chain - doubles each step

Critical mass

If sample is too small , neutrons escape without causing fission \rightarrow no chain.

Minimum mass for self - sustaining reaction is the critical mass (50 kg for pure ^{235}U).

Below it - sub - critical ; above - super - critical.

Nuclear Reactor

A device to harness a controlled chain reaction to generate heat \rightarrow steam. \rightarrow electricity.

Essential parts

- ① Fuel : ^{235}U , ^{239}Pu (fissile)
- ② Moderator : slows fast neutrons to thermal speed - water , D_2O , graphite.
- ③ Control rods: absorb extra n - B , Cd.
- ④ Coolant : water or liquid Na - carries heat to a steam generator.
- ⑤ Shielding : concrete + lead - stops gamma & neutron leakage.

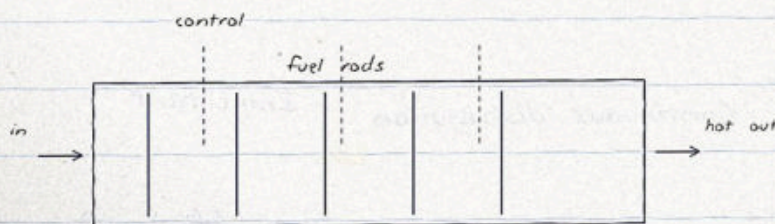


Fig. schematic of a fission reactor

Multiplication Factor k

Let $k =$ ratio of neutrons in one generation to those in the previous generation.

$$k = N_{\text{next}} / N_{\text{prev}}$$

\leftarrow neutron
 \leftarrow multiplier

Three regimes

- ① $k < 1$: sub-critical ; reaction dies.
- ② $k = 1$: critical ; steady reactor.
- ③ $k > 1$: super-critical ; growing power
or - if uncontrolled - a bomb.

Controlling k

Insert control rods \rightarrow absorb neutrons
 \rightarrow reduces k back to 1.

Withdraw rods slightly \rightarrow k slightly > 1
 \rightarrow reactor power rises smoothly.

Indian power reactors

Tarapur, Kaiga, Kudankulam, Kakrapar -
PHWR (heavy water) and BWR types.

Nuclear Fusion

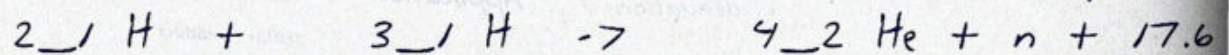
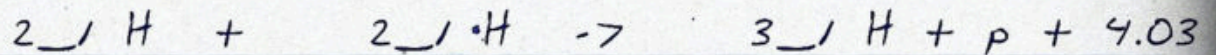
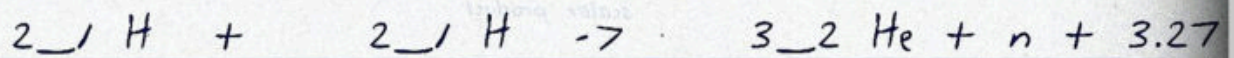
Two light nuclei combine \rightarrow one heavier nucleus + energy. Powers the stars.

Why difficult on Earth

Both nuclei carry +ve charges \rightarrow Coulomb repulsion is enormous at fm distances.

Need $T \approx 10^7 - 10^8$ K to overcome the barrier \rightarrow 'thermonuclear fusion'.

Simple example : D - D fusion



Per - kg energy

Per kg of fuel : fusion \gg fission \gg chemical combustion
 4×10^7 x respectively.

Tokamak & ITER

Plasma confined by magnetic bottle (tokamak).

ITER (France, international) - D - T fusion at 150 million degrees ; India is a partner.

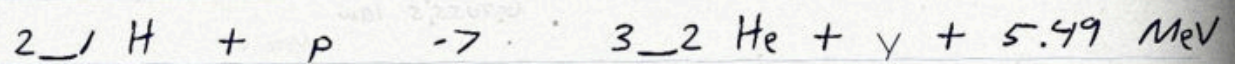
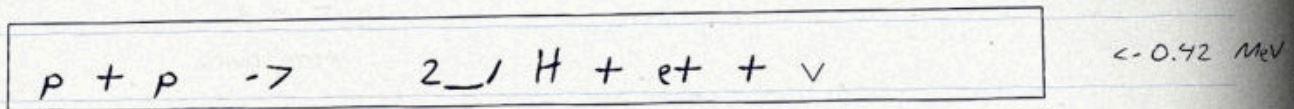
Cleaner than fission - no long lived waste,

fuel (D) is essentially limitless in seawater.

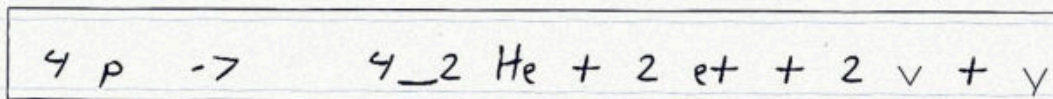
Sun - the p p Cycle

Stars like the Sun burn hydrogen via the proton - proton chain (pp - I) :

Three steps



Net reaction



Energy released approx 26.7 MeV

\leftarrow per cycle
 \leftarrow of $4p \rightarrow$

Lifetime of the Sun

Mass of Sun $M = 2 \times 10^{30} \text{ kg}$. If 10% of H undergoes fusion \rightarrow Sun shines for 10 billion years. We are half way through.

Fission vs Fusion

Property	Fission	Fusion
Process	splits A	joins A
Fuel	U, Pu (heavy)	H, D (light)
Trigger	thermal n	very high T
E / event	200 MeV	17 - 26 MeV
E / kg	smaller	4 x more *
Waste	long lived	almost none

Fig. fission vs fusion at a glance

Both arise from the BE / A curve

Heavy nuclei \rightarrow split toward Fe.

Light nuclei \rightarrow fuse toward Fe.

Either direction • - more BE / A, so energy out.

Conservation laws (always hold)

① Charge Z is conserved.

② Nucleon number A is conserved.

Also E, p, L (with neutrinos counted).

Worked Example - Fission Energy

Problem

Find energy released when 1 kg of ^{235}U undergoes complete fission. Take 200 MeV per fission.

Step 1 : number of nuclei

$$1 \text{ mol of } ^{235}\text{U} = 235 \text{ g} ; N_A = 6.022 \times 10^{23}$$

$$N = (1000 / 235) \times 6.022 \times 10^{23}$$

approx 2.56×10^{24} nuclei

Step 2 : total energy

$$E = N \times 200 \text{ MeV}$$

$$= 2.56 \times 10^{24} \times 200 \text{ MeV}$$

$$= 5.12 \times 10^{26} \text{ MeV}$$

$E \text{ approx } 8.2 \times 10^{13} \text{ J}$
--

$$1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$$

Comparison

Burning 1 kg coal $3 \times 10^7 \text{ J}$.

Fission of 1 kg U $10^7 \times$ more energy ?

1 kg U - city power for ~~a day~~ a month.

Hence reactors only need tiny fuel loads.

Worked Example - BE of ${}^4_2\text{He}$

Given data

$$m_p = 1.00728 \text{ u} ; \quad m_n = 1.00867 \text{ u}$$

$$M ({}^4_2\text{He nucleus}) = 4.00150 \text{ u}$$

Find BE and BE / A. *

Step 1 : mass defect

$$\Delta m = 2 m_p + 2 m_n - M$$

$$= 2 (1.00728) + 2 (1.00867) - 4.00150$$

$$= 2.01456 + 2.01734 - 4.00150$$

$$= 0.03040 \text{ u}$$

Step 2 : binding energy

$$\text{BE} = \Delta m \times 931.5 \text{ MeV}$$

$$= 0.03040 \times 931.5$$

BE approx 28.3 MeV	← for ${}^4_2\text{He}$
--------------------	-------------------------

Step 3 : BE / nucleon

$$\text{BE} / A = 28.3 / 4 = 7.07 \text{ MeV / nucleon}$$

Comments

${}^4_2\text{He}$ is exceptionally stable for a light nucleus.

Hence stars favour it as an end-product.

Also explains why alpha decay is common.

Summary - Key Formulae

Composition & size

$$A = Z + N \quad ; \quad \text{notation } A_Z X$$

$$R = R_0 A^{(1/3)} \quad ; \quad R_0 = 1.2 \text{ fm}$$

$$\rho \text{ approx } 2.3 \times 10^{17} \text{ kg / m}^3$$

Mass - energy

$$1 \text{ u} = 931.5 \text{ MeV / } c^2 = 1.66 \times 10^{-27} \text{ kg}$$

$$\Delta m = 2 m_p + N m_n - M_N$$

$$BE = \Delta m c^2 \quad ; \quad BE/A = \text{stability}$$

Radioactive decay

$$N(t) = N_0 e^{(-\lambda t)} \quad ; \quad A = \lambda N$$

$$T_{1/2} = 0.693 / \lambda \quad ; \quad \tau = 1 / \lambda$$

$$N = N_0 / 2^n \quad (\text{after } n \text{ half-lives})$$

Decay types

$$\alpha : Z \rightarrow Z - 2, A \rightarrow A - 4$$

$$\beta^- : n \rightarrow p + e^- + \bar{\nu}$$

$$\beta^+ : p \rightarrow n + e^+ + \nu$$

$$\gamma : Z, A \text{ unchanged} \quad ; \quad \text{photon only}$$

Fission & fusion

