The Collegedunia NCERT Notes

The Ultimate NCERT Guide for Class 12 Physics

Chapter 13: Nuclei

1 Atomic Masses and Composition of Nucleus

1.1 The Atomic Nucleus

The Nucleus: Center of the Atom

- The existence of a nucleus was proposed by **Ernest Rutherford** in 1911 based on his α-particle scattering experiment.
- The nucleus is a small, dense, positively charged core located at the center of an atom.
- It contains almost all the mass of the atom but occupies only a tiny fraction of its volume.
- **Radius of a nucleus** is of the order of 10^{-15} m (1 Fermi), while the atomic radius is of the order of 10^{-10} m.

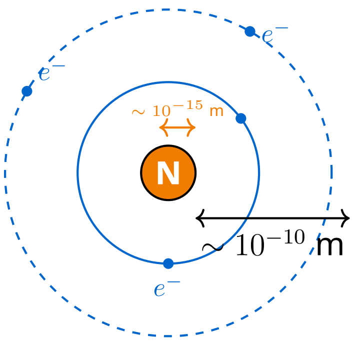


Figure 1: Rutherford's atomic model: A tiny, dense, positively charged nucleus at the center, surrounded by orbiting electrons.

1.2 Composition of Nucleus: Protons and Neutrons

After the discovery of the neutron by **James Chadwick** in 1932, the composition of the nucleus became clear. Protons (p) and neutrons (n) are the fundamental particles in a nucleus and are collectively called **nucleons**.

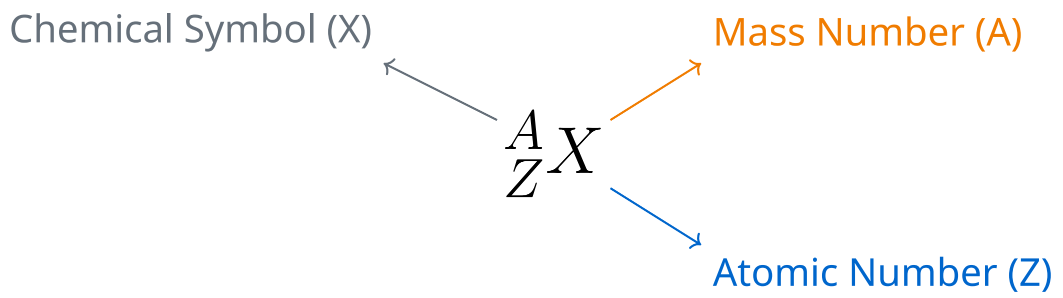
Types of Nucleons

1. **Proton (p):** A stable particle with a single positive charge ($+e = 1.602 \times 10^{-19}$ C). Its mass (m_p) is approximately 1.6726×10^{-27} kg.
2. **Neutron (n):** A neutral particle (zero charge). Its mass (m_n) is slightly larger than that of a proton, approximately 1.6749×10^{-27} kg. It is unstable outside the nucleus with a half-life of about 10 minutes.

1.3 Atomic Number, Mass Number, and Nuclide Notation

A nucleus is uniquely described by three quantities:

Nuclide Notation: $^A_Z X$



Where:

- **X** is the chemical symbol of the element.
- **Z** is the **Atomic Number** = Number of protons in the nucleus.
- **A** is the **Mass Number** = Total number of nucleons (protons + neutrons).
- **Number of neutrons (N)** = $A - Z$.

NCERT Nugget: Key Terminology

- **Isotopes:** Atoms of the same element with the same Z but different A (e.g., ^1_1H, ^2_1H, ^3_1H).
- **Isobars:** Atoms of different elements with the same A but different Z (e.g., $^{14}_6C$, $^{14}_7N$).
- **Isotones:** Atoms of different elements having the same number of neutrons N (e.g., $^{14}_6C$ ($N = 8$) and $^{15}_7N$ ($N = 8$)).

1.4 Atomic Mass Unit (amu or u)

The mass of an atom is so small that using kilograms is impractical. A new unit is defined.

Atomic Mass Unit (u)

1 atomic mass unit (u) is defined as exactly one-twelfth ($1/12$) of the mass of one atom of the carbon-12 isotope ($^{12}_{6}C$).

$$1 \text{ u} = \frac{1}{12} \times \text{mass of one } {}^{12}_{6}C \text{ atom} = 1.660539 \times 10^{-27} \text{ kg}$$

Quick Fact: Particle Masses in 'u'

Particle	Mass (u)	Mass (kg)
Proton (p)	1.007276 u	1.6726×10^{-27}
Neutron (n)	1.008665 u	1.6749×10^{-27}
Electron (e)	0.000548 u	9.1093×10^{-31}

Note: The electron mass is negligible but not zero when calculating nuclear binding energies.

1.5 Discovery of the Neutron

A brief note on the crucial experiment that completed our picture of the nucleus.

Chadwick's Experiment (1932)

Chadwick bombarded beryllium ($^{9}_{4}Be$) with alpha particles ($^{4}_{2}He$). A highly penetrating, neutral radiation was emitted, which he identified as neutrons.

$$^{9}_{4}Be +^{4}_{2} He \xrightarrow{\alpha} {}^{12}_{6}C +^{1}_{0} n$$

2 Size of the Nucleus

2.1 Rutherford's α-Scattering Experiment and Distance of Closest Approach

The size of the nucleus was first estimated from Rutherford's famous α-particle scattering experiment (Geiger-Marsden experiment).

Distance of Closest Approach

When an α-particle with kinetic energy K is directed head-on towards a nucleus of atomic number Z, it slows down due to Coulomb repulsion. At the point where its entire kinetic energy is converted into electrostatic potential energy, it momentarily stops before being repelled back.
This minimum distance r_0 is called the distance of closest approach.

$$K = \frac{1}{4\pi\varepsilon_0} \frac{(Ze)(2e)}{r_0}$$

$$r_0 = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{K}$$

Where:

- Z = atomic number of target nucleus (gold, $Z = 79$)
- $2e$ = charge of α-particle
- K = initial kinetic energy of α-particle

This gives an **upper limit** on the nuclear radius. The actual radius is smaller than r_0.

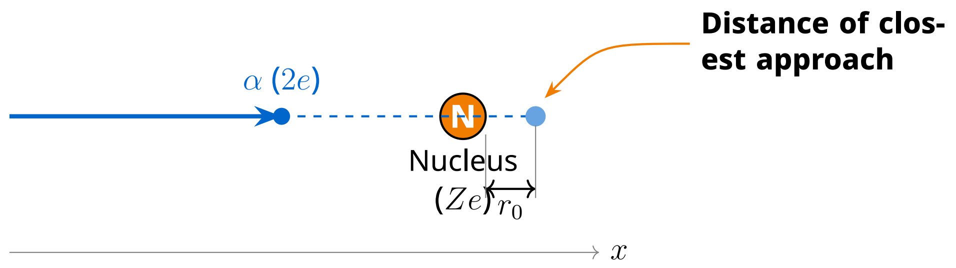


Figure 2: Head-on collision: The α-particle stops at distance r_0 where kinetic energy is fully converted to electrostatic potential energy.

2.2 Nuclear Radius Formula

From scattering experiments, it was found that the nucleus is approximately spherical and its radius R depends on its mass number A.

Nuclear Radius Formula

$$R = R_0\, A^{1/3}$$

Where:

- R = radius of the nucleus
- R_0 = empirical constant $\approx 1.2 \times 10^{-15}$ m (1.2 Fermi)
- A = mass number of the nucleus

What This Formula Tells Us

- The volume of the nucleus is directly proportional to its mass number A.
- $V \propto R^3 \propto A$, so the density of nuclear matter is **nearly constant** for all nuclei.
- Larger nuclei have larger radii, but the increase follows a cube-root re-

lationship.

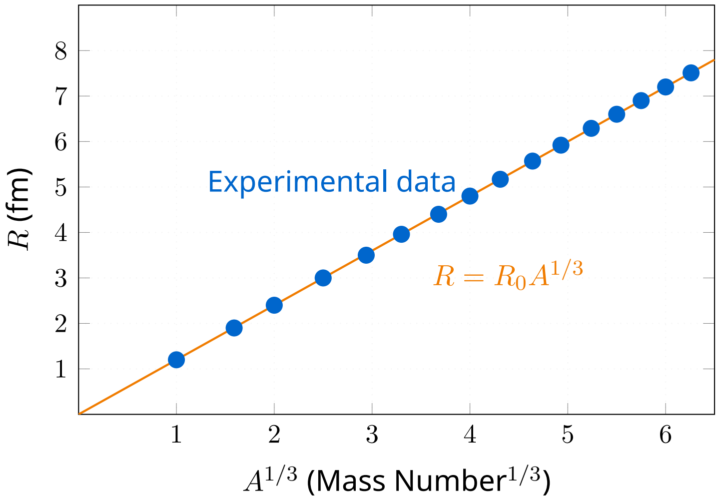


Figure 3: Graph showing nuclear radius R is proportional to $A^{1/3}$. The slope gives $R_0 \approx 1.2$ fm.

2.3 Nuclear Density

One of the most remarkable features of the nucleus is its almost constant density across all elements.

Derivation: Nuclear Density

The density of nuclear matter (ρ) is given by:

$$\begin{aligned}
\rho &= \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} \\
&= \frac{A\,m_p}{\frac{4}{3}\pi R^3} \quad \text{(approximating } m_n \approx m_p\text{)} \\
&= \frac{A\,m_p}{\frac{4}{3}\pi (R_0 A^{1/3})^3} \\
&= \frac{A\,m_p}{\frac{4}{3}\pi R_0^3\,A} \\
\rho &= \frac{3m_p}{4\pi R_0^3}
\end{aligned}$$

Substituting values: $m_p = 1.67 \times 10^{-27}$ kg, $R_0 = 1.2 \times 10^{-15}$ m:

$$\rho \approx 2.3 \times 10^{17} \text{ kg/m}^3$$

Key Takeaway: Nuclear Density

$$\rho_{\text{nuclear}} \approx 2.3 \times 10^{17}\ \text{kg/m}^3$$

- This is **independent of** A — all nuclei have roughly the same density!
- It is **enormously high**: a teaspoon of nuclear matter would weigh about a billion tonnes.
- This constant density suggests that nucleons are packed like incompressible spheres, similar to a liquid drop.

Comparison: Nuclear Density vs Atomic Density

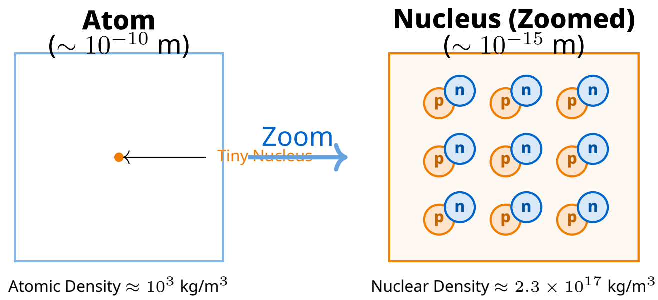


Figure 4: Visual comparison of scale and density: While the atom is mostly empty space, the nuclear material is $\sim 10^{14}$ times denser than ordinary matter.

3 Mass-Energy and Nuclear Binding Energy

3.1 Einstein's Mass-Energy Equivalence

The foundation of nuclear physics lies in Einstein's famous equation relating mass and energy.

Mass-Energy Equivalence

According to Einstein's special theory of relativity, mass and energy are interconvertible. A mass m is equivalent to an amount of energy E given by:

$$E = mc^2$$

Where:

- E = energy (in Joules)
- m = mass (in kg)
- c = speed of light in vacuum = 3×10^8 m/s

In nuclear physics units:

- 1 u of mass is equivalent to 931.5 MeV of energy.
- $1\text{ u} \times c^2 = 931.5\text{ MeV}$

Useful Energy-Mass Conversion

$$\boxed{1\text{ u} = 931.5\text{ MeV}/c^2}$$

Derivation (for reference):

$$\begin{aligned}
1\text{ u} &= 1.6605 \times 10^{-27}\text{ kg} \\
E &= mc^2 = (1.6605 \times 10^{-27}) \times (3 \times 10^8)^2 \\
&= 1.4945 \times 10^{-10}\text{ J} \\
&= \frac{1.4945 \times 10^{-10}}{1.602 \times 10^{-13}}\text{ MeV} \approx 931.5\text{ MeV}
\end{aligned}$$

3.2 Mass Defect

When we measure the mass of a nucleus, it is always found to be **slightly less** than the sum of the masses of its individual nucleons.

Mass Defect (Δm)

The **mass defect** is the difference between the total mass of the free nucleons (when separated) and the actual mass of the nucleus formed by them.

$$\boxed{\Delta m = [Zm_p + (A - Z)m_n] - M_{\text{nucleus}}}$$

Where:

- Z = number of protons
- m_p = mass of a free proton
- $(A - Z)$ = number of neutrons
- m_n = mass of a free neutron
- M_{nucleus} = actual measured mass of the nucleus

Note: In practice, we use atomic masses instead of nuclear masses. The mass defect using atomic masses is:

$$\Delta m = [Z\,M(^1_1H) + (A - Z)m_n] - M(^A_ZX)$$

3.3 Nuclear Binding Energy

The mass defect is not ”lost” — it appears as energy that holds the nucleus together.

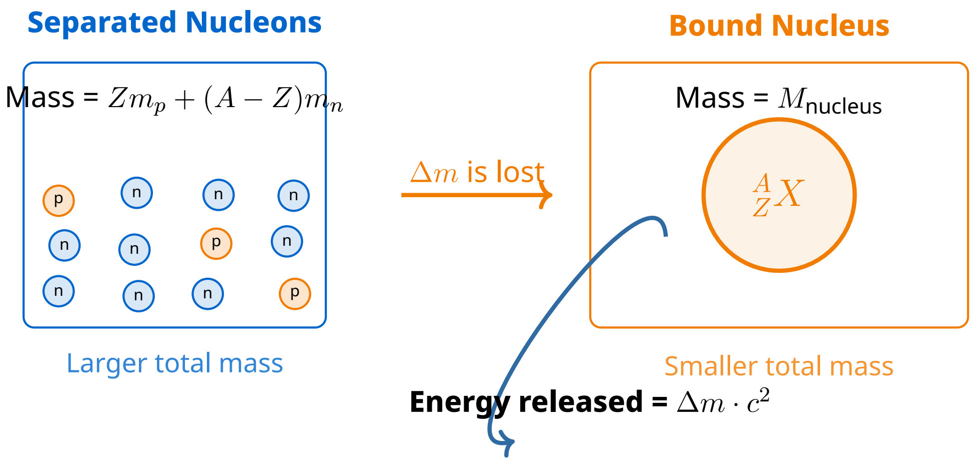


Figure 5: When nucleons come together to form a nucleus, some mass (Δm) disappears and is converted into binding energy ($\Delta m \cdot c^2$).

Binding Energy (E_b)

The **binding energy** of a nucleus is the energy required to break it apart into its constituent protons and neutrons.

$$E_b = \Delta m \cdot c^2$$

- A **higher binding energy** means a **more stable** nucleus.
- Binding energy is always a positive quantity.

NCERT Solved Example Style: Binding Energy of Deuterium

Problem: Calculate the binding energy of deuterium ${}^2_1H$. Given:

- Mass of proton, $m_p = 1.007825$ u
- Mass of neutron, $m_n = 1.008665$ u
- Mass of deuterium nucleus, $M({}^2_1H) = 2.014102$ u

Solution:

1. **Mass defect:** $\Delta m = (m_p + m_n) - M({}^2_1H) = 0.002388$ u
2. **Binding Energy:** $E_b = 0.002388 \times 931.5 \approx 2.22$ MeV

3.4 Binding Energy per Nucleon (E_{bn})

Binding Energy per Nucleon

$$E_{bn} = \frac{E_b}{A} = \frac{\Delta m \cdot c^2}{A}$$

Significance: Higher E_{bn} means greater stability.

3.5 The Binding Energy Curve

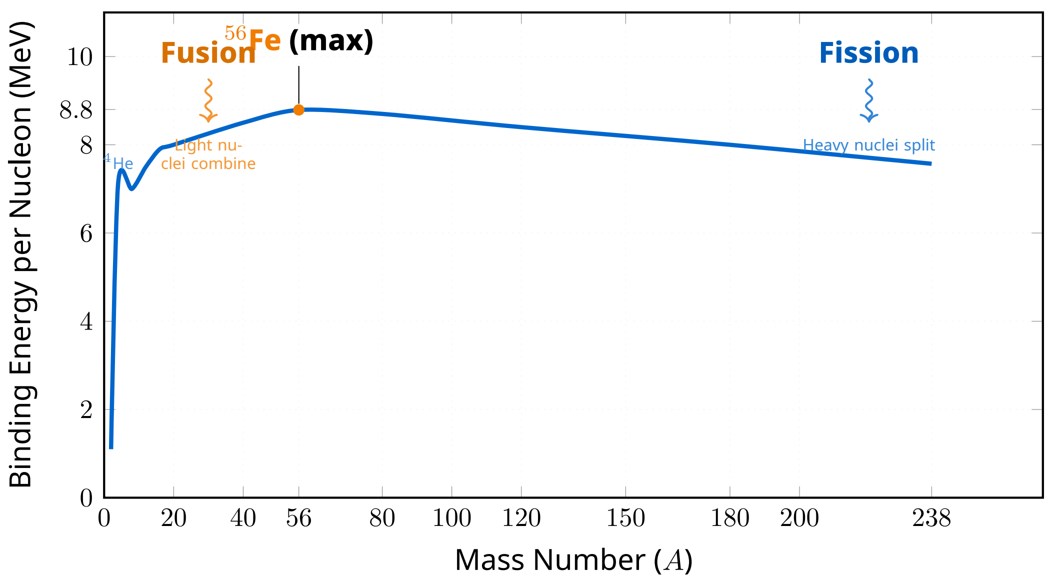


Figure 6: Refined Binding Energy Curve: Labels are moved to high-clearance areas to prevent overlap with the data plot.

Exam Must-Know: Features of Binding Energy Curve

1. **Peak:** Iron ($^{56}_{26}$Fe) with $E_{bn} \approx 8.79$ MeV.
2. **Fission:** Heavy nuclei ($A > 200$) split to increase stability.
3. **Fusion:** Light nuclei combine to release huge energy.

3.6 Liquid Drop Model

Semi-Empirical Mass Formula (Qualitative)

Binding energy is a balance of:

- **Volume Energy:** Favors binding (proportional to A).
- **Surface Energy:** Reduces binding (proportional to $A^{2/3}$).

- **Coulomb Repulsion:** Reduces binding (proportional to $Z^2/A^{1/3}$).

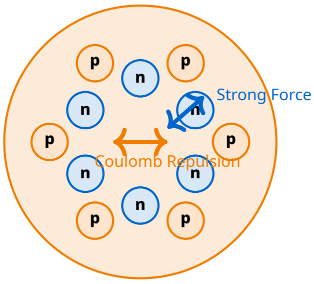


Figure 7: Internal nuclear force balance.

4 Nuclear Force

4.1 The Challenge: What Holds the Nucleus Together?

The nucleus contains protons and neutrons packed into an incredibly tiny volume. Protons experience strong Coulomb repulsion, yet the nucleus is stable. This implies the existence of a new, attractive force that overcomes this repulsion.

The Puzzle of Nuclear Stability

Consider two protons inside a nucleus separated by a distance of 2×10^{-15} m (2 fm).

- **Coulomb repulsive force:**

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \approx \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{(2 \times 10^{-15})^2} \approx 58 \text{ N}$$

  This is an **enormous** repulsive force on a nuclear scale!

- **Gravitational force:**

$$F_g = \frac{Gm_p^2}{r^2} \approx \frac{6.67 \times 10^{-11} \times (1.67 \times 10^{-27})^2}{(2 \times 10^{-15})^2} \approx 4.6 \times 10^{-35} \text{ N}$$

  Gravity is **negligible** compared to Coulomb force.

Conclusion: There must be a third fundamental force — the **strong nuclear force** — that binds nucleons together. It must be **much stronger** than Coulomb repulsion at nuclear distances.

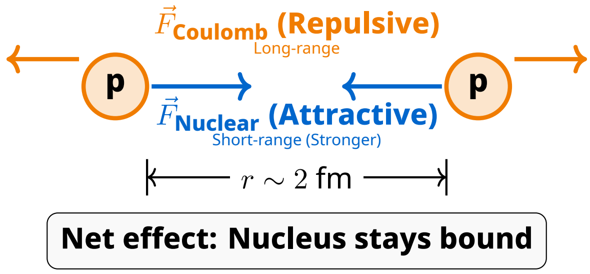


Figure 8: Balance of forces between two protons: The short-range strong nuclear force (blue) overcomes the long-range electrostatic repulsion (orange) at femtometer scales.

4.2 Characteristics of Nuclear Force

The nuclear force (also called the strong interaction or strong nuclear force) has unique properties that distinguish it from other fundamental forces.

Key Properties of Nuclear Force

1. **Strongest Fundamental Force:** It is the strongest of the four fundamental forces at nuclear distances.
   - Strength order: Strong Nuclear $>$ Electromagnetic $>$ Weak Nuclear $>$ Gravitational
2. **Short-Range Force:** It operates only within the nucleus (range $\approx$ 1–2 fm). Beyond this distance, it falls to zero rapidly.
3. **Charge Independent:** The nuclear force between any two nucleons is approximately the same.

   $$F_{pp} \approx F_{nn} \approx F_{pn}$$

   It does not depend on whether the nucleon is a proton or a neutron.
4. **Saturative Nature:** A nucleon interacts only with its immediate neighbors (nearest neighbors), not with all nucleons in the nucleus. This explains why E_{bn} eventually saturates.
5. **Spin Dependent:** The nuclear force between two nucleons is stronger when their spins are parallel (aligned) than when they are anti-parallel. This is evident from the deuteron (^2_1H) which is bound only in the triplet (parallel spin) state.
6. **Non-Central (Tensor Nature):** The nuclear force depends not only on the distance between nucleons but also on the orientation of their spins relative to the line joining them. It has a tensor component.

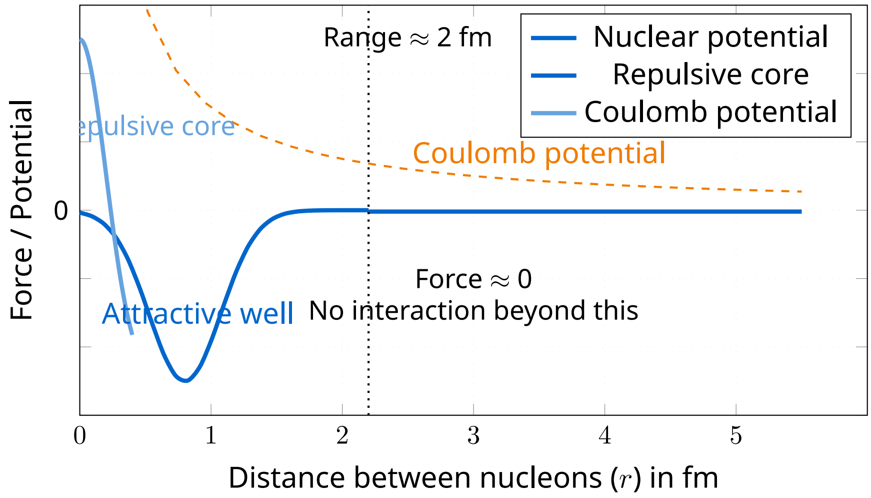


Figure 9: Variation of nuclear potential with distance between nucleons. The potential has a deep attractive well at ∼1 fm, a hard repulsive core at very short distances (< 0.5 fm), and vanishes beyond 2 fm. Coulomb potential (orange, dashed) is long-range.

Exam Alert: Key Facts about Nuclear Force

- **Range:** $\sim 1 - 2$ fm. Beyond this, nuclear force is zero.
- **Repulsive Core:** At distances $<$ 0.5 fm, the force becomes strongly repulsive, preventing nucleons from collapsing into each other.
- **Not a Central Force:** Unlike gravity or Coulomb force, nuclear force depends on spin orientation (tensor force).
- **Exchange Nature:** The nuclear force is explained by the exchange of **mesons** (π-mesons or pions) between nucleons, as proposed by **Hideki Yukawa** (1935).

4.3 Yukawa's Meson Theory of Nuclear Forces

The fundamental theory explaining the nuclear force was given by Japanese physicist Hideki Yukawa, who won the Nobel Prize for this work.

Yukawa's Meson Theory (1935)

Yukawa proposed that the nuclear force arises from the continuous exchange of virtual particles called **mesons** between nucleons.

- **The Exchange Particle:** π-meson (pion), with mass $\approx 273\, m_e$ or about 0.15 u.
- **How it works:** A nucleon continuously emits and reabsorbs virtual pions. When two nucleons are close enough, they can exchange these pions, creating an attractive force.

- **Range of force and pion mass:** Yukawa related the range of nuclear force (R) to the mass of the exchange particle (m_π) using the uncertainty principle:

$$R \approx \frac{\hbar}{m_\pi c} \approx 1.4 \text{ fm}$$

- **Prediction and Discovery:** Yukawa predicted the existence of a particle with mass $\sim 200\, m_e$. The pion (π-meson) was discovered in 1947 by **Cecil Powell** in cosmic rays.

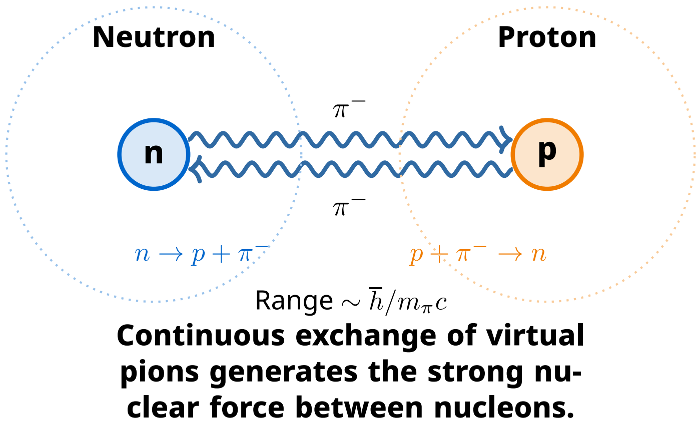


Figure 10: Yukawa's meson exchange model: Nucleons interact by exchanging virtual pions (π-mesons). The exchange transforms a neutron into a proton (or vice versa).

4.4 Comparison with Other Fundamental Forces

Quick Comparison: The Four Fundamental Forces

Force	Range	Relative Strength	Exchange Particle	Acts on
Strong Nuclear	$\sim 10^{-15}$ m	1 (Strongest)	Gluons (quarks), Pions (nucleons)	Hadrons (nucleons)
Electromagnetic	Infinite	$\sim 10^{-2}$	Photon (γ)	Charged particles
Weak Nuclear	$\sim 10^{-18}$ m	$\sim 10^{-13}$	$W^\pm, Z^0$ bosons	All particles
Gravitational	Infinite	$\sim 10^{-39}$	Graviton (hypothetical)	All masses

Note for exams: The strong nuclear force is about 10^2 times stronger than electromagnetic force, 10^{13} times stronger than weak force, and 10^{39} times stronger than gravity at nuclear scales.

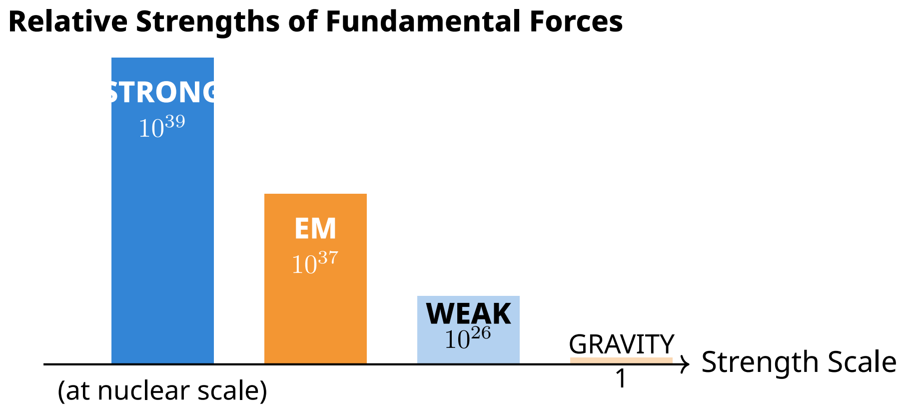


Figure 11: Visual comparison of the relative strengths of the four fundamental forces at nuclear distance scales. The strong nuclear force overwhelmingly dominates.

5 Radiocativity

5.1 Discovery and Definition

The phenomenon of radioactivity was discovered by **Henri Becquerel** in 1896, and later extensively studied by **Marie Curie** and **Pierre Curie**.

Radioactivity

Radioactivity is the spontaneous and random disintegration of unstable atomic nuclei with the emission of energetic particles or electromagnetic radiation.

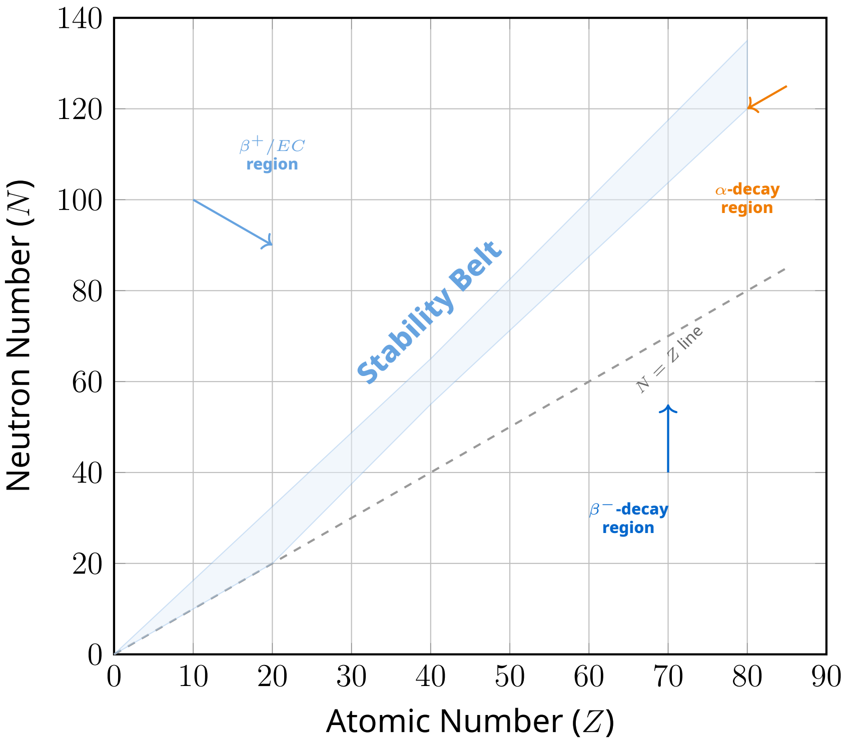
- It is a **nuclear phenomenon** — independent of chemical state, temperature, pressure, etc.
- It is a **random process** — we cannot predict which specific nucleus will decay at a given instant.
- The nucleus undergoing decay is called the **parent nucleus**, and the resulting nucleus is the **daughter nucleus**.

Key Insight: Stability and N/Z Ratio

Not all nuclei are radioactive. Stability depends on the **neutron-to-proton ratio (N/Z)**:

- **Light stable nuclei** ($A < 20$): $N/Z \approx 1$ (e.g., ${}^{12}_{6}$C: $N = 6, Z = 6$)

- **Heavy stable nuclei** ($A > 40$): $N/Z > 1$, increasing to about 1.5 for Uranium (e.g., $^{238}_{92}$U: $N = 146, Z = 92$, ratio ≈ 1.59)
- Extra neutrons are needed to provide additional nuclear force to compensate for the growing Coulomb repulsion between protons.
- If a nucleus lies outside the **stability belt**, it undergoes radioactive decay to move toward stability.



5.2 Types of Radioactive Decay

There are three primary types of radioactive decay: **alpha (α)**, **beta (β)**, and **gamma (γ)**.

5.2.1 Alpha (α) Decay

α-Decay

An α-particle is a helium nucleus ($^4_2$He), consisting of 2 protons and 2 neutrons.
General equation:

$$\boxed{{}^A_Z X \rightarrow {}^{A-4}_{Z-2}Y + {}^4_2\text{He} + Q}$$

Where Q is the energy released (disintegration energy), shared as kinetic energy between the daughter nucleus and the α-particle.
Key features:

- Occurs primarily in heavy nuclei ($Z > 82$).
- The α-particle has a discrete energy spectrum (monoenergetic).
- The daughter nucleus shifts **2 places left** in the periodic table (Soddy-Fajans displacement law).

Example: α-Decay of Uranium-238

$$^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He} + 4.27 \text{ MeV}$$

The parent uranium nucleus spontaneously emits an α-particle, transforming into thorium.

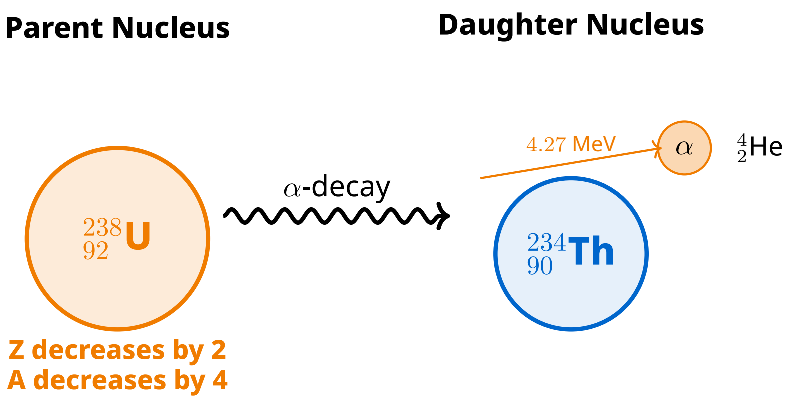


Figure 12: Schematic representation of α-decay of Uranium-238. The parent nucleus loses 2 protons and 2 neutrons, forming a daughter nucleus and an α-particle.

5.2.2 Beta (β) Decay

Beta decay is of two types: β^- (electron emission) and β^+ (positron emission).

β^- Decay (Electron Emission)

In β^- decay, a neutron inside the nucleus transforms into a proton, emitting an electron (e^-) and an antineutrino ($\bar{\nu}_e$).

General equation:

$$\boxed{^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e + Q}$$

Underlying process (at quark level):

$$n \rightarrow p^+ + e^- + \bar{\nu}_e$$

Key features:

- Occurs in neutron-rich nuclei (above the stability belt).
- The daughter nucleus shifts **1 place right** in the periodic table.

- Electrons are emitted with a **continuous energy spectrum** (not discrete) — this led to the prediction of the neutrino by **Wolfgang Pauli** in 1930.

β^+ Decay (Positron Emission)

In β^+ decay, a proton inside the nucleus transforms into a neutron, emitting a positron (e^+) and a neutrino (ν_e).
General equation:

$$\boxed{{}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^+ + \nu_e + Q}$$

Underlying process:

$$p^+ \rightarrow n + e^+ + \nu_e$$

Note: Free protons are stable; this occurs only inside the nucleus.
Electron Capture: An alternative to β^+ decay, where the nucleus captures an inner orbital electron.

$$p^+ + e^- \rightarrow n + \nu_e$$

The Neutrino Hypothesis

The continuous energy spectrum of β-decay electrons violated the principle of energy conservation. To resolve this, **Wolfgang Pauli (1930)** proposed the existence of a new, nearly massless, chargeless particle called the **neutrino** (ν).

- **Properties:** Zero charge, extremely small mass, spin $1/2$, interacts only via weak nuclear force.
- **Detection:** Experimentally confirmed in 1956 by **Clyde Cowan and Frederick Reines**.
- **Antineutrino ($\bar{\nu}_e$):** Emitted in β^- decay; neutrino (ν_e) emitted in β^+ decay.

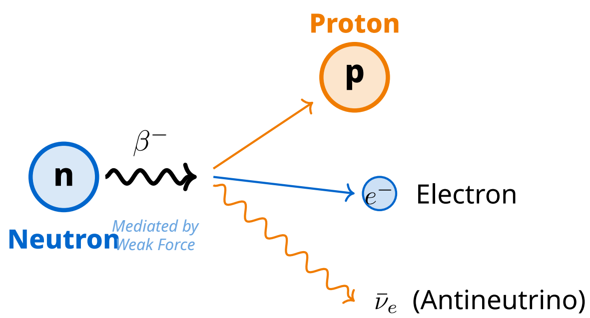


Figure 13: Beta-minus decay: The transformation of a neutron into a proton, electron, and antineutrino, ensuring charge and lepton number conservation.

5.2.3 Gamma (γ) Decay

γ-Decay

Gamma decay is the emission of high-energy electromagnetic radiation (photons) from an excited nucleus.

General equation:

$$\boxed{{}^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \gamma}$$

Where X^{*} denotes a nucleus in an excited state.

Key features:

- There is **no change** in A or Z — it is a transition between nuclear energy levels.
- Often follows α or β decay when the daughter nucleus is left in an excited state.
- γ-ray energies are typically in the range of keV to MeV.
- Similar to atomic transitions (X-rays), but involving nuclear energy levels which are $\sim 10^{6}$ times more energetic.

Comparison: Alpha, Beta, and Gamma Radiation

Property	α **(Alpha)**	β **(Beta)**	γ **(Gamma)**
Nature	Helium nucleus ($^4_2$He^{2+})	Electron or Positron	Electromagnetic wave (photon)
Charge	+2e	-e (or +e)	0
Mass	$\sim$ 4 u	$\sim$ 0.00055 u	0
Speed	$\sim 10^7$ m/s	$\sim 10^8$ m/s (up to 0.99c)	$c = 3 \times 10^8$ m/s
Ionizing Power	Highest	Intermediate	Lowest
Penetrating Power	Lowest (few cm in air)	Intermediate (few mm of Al)	Highest (several cm of Pb)
Deflection in E/B field	Yes (opposite to β)	Yes (opposite to α)	No
Energy Spectrum	Discrete (lines)	Continuous	Discrete (lines)
Change in Z	Decreases by 2	β^-: Increases by 1 β^+: Decreases by 1	No change
Change in A	Decreases by 4	No change	No change

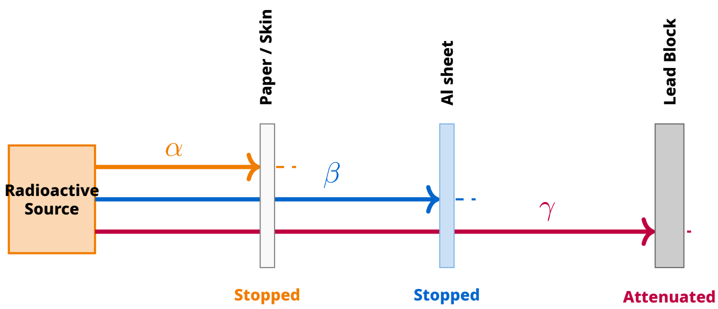


Figure 14: Penetrating power of nuclear radiations: α is stopped by paper, β by aluminum, and γ is only significantly attenuated by thick lead or concrete.

6 Radiocative Decay Law

6.1 The Statistical Nature of Radioactive Decay

Radioactive decay is a random process at the level of individual atoms. We can never predict when a specific nucleus will decay. However, for a large collection of identical nuclei, the **average decay rate** follows a precise mathematical law.

Key Idea: Probability of Decay

- Each radioactive nucleus has a **fixed probability of decaying per unit time**. This probability is called the **decay constant** (λ).
- The value of λ is characteristic of the particular radioactive species.
- λ is **independent** of external conditions like temperature, pressure, chemical state, etc.
- λ has units of s^{-1} (or min^{-1}, yr^{-1}, etc.).

6.2 The Exponential Decay Law

Derivation: Radioactive Decay Law

Let N be the number of undecayed nuclei present at time t. In a small time interval dt, the number of nuclei decaying (dN) is proportional to N and dt:

$$dN \propto -N\,dt$$

(The negative sign indicates that N decreases with time.)

$$\frac{dN}{dt} = -\lambda N$$

Where λ is the decay constant. Rearranging and integrating:

$$\frac{dN}{N} = -\lambda\,dt$$

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

$$N = N_0\, e^{-\lambda t}$$

Radioactive Decay Law

$$\boxed{N(t) = N_0\, e^{-\lambda t}}$$

Where:

- $N(t)$ = number of undecayed nuclei present at time t
- N_0 = initial number of undecayed nuclei (at $t = 0$)
- λ = decay constant (probability of decay per unit time)
- t = elapsed time

Note: The number of nuclei decayed up to time t is:

$$N_{\text{decayed}} = N_0 - N(t) = N_0(1 - e^{-\lambda t})$$

6.3 Activity of a Radioactive Sample

The **activity (**R**)** of a radioactive sample is the rate at which nuclear decays occur.

Activity (R)

$$R(t) = \left|\frac{dN}{dt}\right| = \lambda N(t) = \lambda N_0 e^{-\lambda t}$$

$$\boxed{R(t) = R_0\, e^{-\lambda t}}$$

Where $R_0 = \lambda N_0$ is the initial activity.
Units of Activity:

- **Becquerel (Bq):** 1 decay per second (SI unit)
- **Curie (Ci):** 1 Ci $= 3.7 \times 10^{10}$ Bq (activity of 1 gram of radium-226)
- **Rutherford (Rd):** 1 Rd $= 10^6$ Bq

Note: Activity also follows the same exponential decay law as N.

6.4 Half-Life ($T_{1/2}$)

Half-Life

The **half-life** ($T_{1/2}$) of a radioactive substance is the time required for **half** of the initially present nuclei to decay.

$$\text{At } t = T_{1/2}: \quad N = \frac{N_0}{2}$$

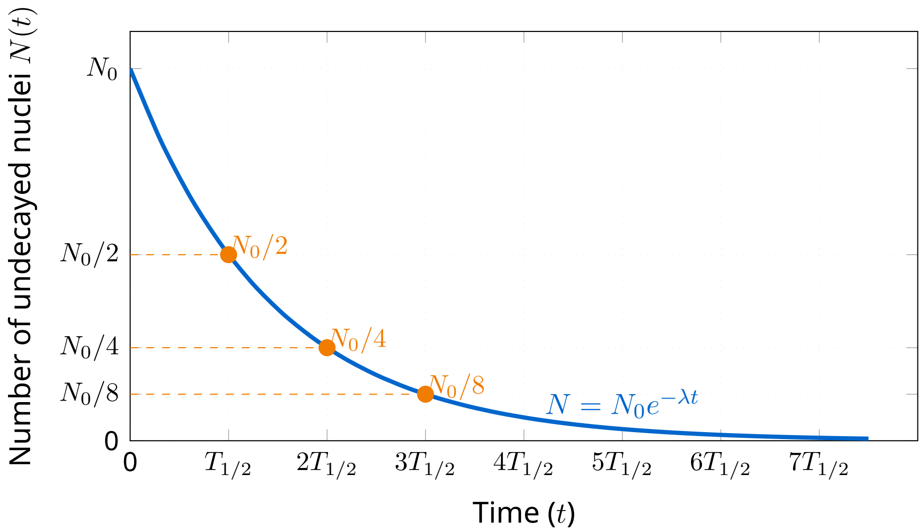


Figure 15: Exponential decay curve. The number of undecayed nuclei halves in each successive half-life period $T_{1/2}$.

Substituting into the decay law:

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$2 = e^{\lambda T_{1/2}}$$

$$\ln 2 = \lambda T_{1/2}$$

Half-Life Formula

$$\boxed{T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}}$$

After n half-lives:

$$\boxed{N = N_0 \left(\frac{1}{2}\right)^n} \quad \text{where } n = \frac{t}{T_{1/2}}$$

Also:

$$R = R_0 \left(\frac{1}{2}\right)^n$$

NCERT Style Example: Half-Life Calculation

Problem: The half-life of $^{238}_{92}$U is 4.5×10^9 years. Calculate its decay constant in s^{-1}.

Solution:

$$T_{1/2} = 4.5 \times 10^9 \text{ years}$$
$$= 4.5 \times 10^9 \times 365 \times 24 \times 3600 \text{ s}$$
$$= 1.42 \times 10^{17} \text{ s}$$

$$\lambda = \frac{0.693}{T_{1/2}}$$
$$= \frac{0.693}{1.42 \times 10^{17}}$$
$$\approx 4.9 \times 10^{-18} \text{ s}^{-1}$$

This extremely small λ explains why uranium-238 has such a long half-life and is still found on Earth.

6.5 Mean Life (τ)

Mean Life (Average Life)

The **mean life** (τ) of a radioactive substance is the average lifetime of all nuclei in a sample before they decay.

$$\tau = \frac{\text{Sum of lifetimes of all nuclei}}{\text{Total number of nuclei}}$$

Mathematically, it can be shown that:

$$\boxed{\tau = \frac{1}{\lambda}}$$

Relationship between Half-Life and Mean Life

$$T_{1/2} = \frac{\ln 2}{\lambda} = 0.693\,\tau$$

$$\tau = \frac{T_{1/2}}{\ln 2} = \frac{T_{1/2}}{0.693} \approx 1.44\,T_{1/2}$$

- Mean life is **greater** than half-life.
- After one mean life ($t = \tau$), the number of undecayed nuclei remaining is:
$$N = N_0 e^{-1} = \frac{N_0}{e} \approx 0.37 N_0$$
i.e., about 37% of the original nuclei remain (63% have decayed).

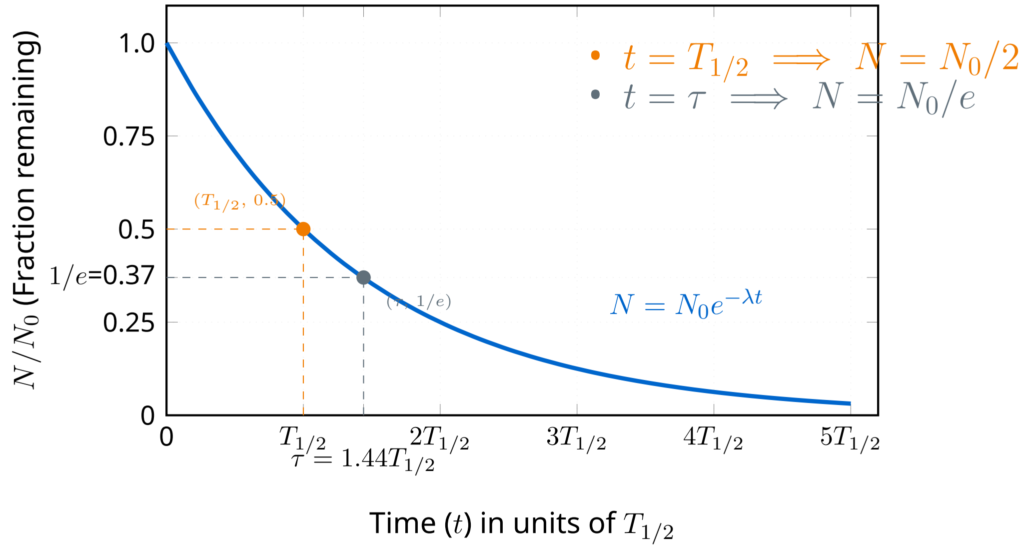


Figure 16: Exponential decay graph. Labels for $T_{1/2}$ and τ are staggered to ensure clarity on both axes.

Exam Key Points: Decay Parameters Summary

Parameter	Definition	Formula
Decay Constant (λ)	Probability of decay per unit time per nucleus	$\lambda = \frac{0.693}{T_{1/2}} = \frac{1}{\tau}$
Half-Life ($T_{1/2}$)	Time for half the nuclei to decay	$T_{1/2} = \frac{0.693}{\lambda}$
Mean Life (τ)	Average lifetime of a nucleus	$\tau = \frac{1}{\lambda} = 1.44\,T_{1/2}$
Activity (R)	Rate of decay	$R = \lambda N = R_0 e^{-\lambda t}$

Remember:

- After n half-lives: $N = N_0(1/2)^n$ and $R = R_0(1/2)^n$
- Fraction remaining after n half-lives = $(1/2)^n$
- Fraction decayed after n half-lives = $1 - (1/2)^n$

6.6 Graphical Representation

Alternative Form: Log-Linear Plot

Taking natural logarithm of the decay law:

$$\ln N = \ln N_0 - \lambda t$$

This is a straight-line equation of the form $y = c + mx$, where:

- Slope $= -\lambda$
- Intercept $= \ln N_0$

A plot of $\ln N$ vs t gives a **straight line** with negative slope, confirming the exponential nature of radioactive decay.

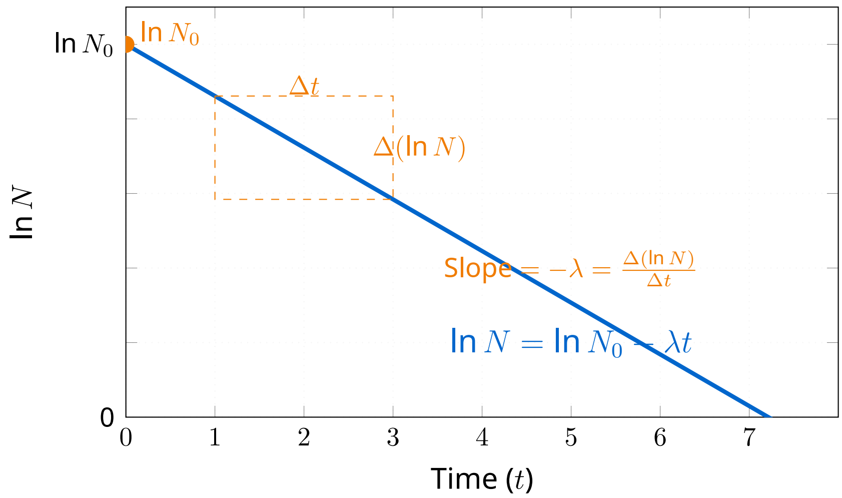


Figure 17: Semi-log plot: $\ln N$ vs t is a straight line with slope $-\lambda$, confirming the exponential decay law.

7 Nuclear Reactions

7.1 What is a Nuclear Reaction?

A nuclear reaction involves a change in the composition or energy state of a nucleus, induced by bombarding it with an energetic particle (projectile).

Nuclear Reaction

A **nuclear reaction** is a process in which a target nucleus is bombarded with a projectile particle, resulting in the formation of a different nucleus with the emission of one or more particles and release or absorption of energy.
General notation:

$$a + X \rightarrow Y + b + Q$$

Or in compact form: $X(a, b)Y$

- a = projectile (bombarding particle)
- X = target nucleus
- Y = product nucleus (daughter)
- b = emitted particle
- Q = Q-value (energy released or absorbed)

Conservation Laws in Nuclear Reactions

Nuclear reactions obey several conservation laws (same as in radioactive decay):

1. **Conservation of Charge:** Total charge before reaction = Total charge after reaction.
2. **Conservation of Mass Number (Nucleon Number):** Sum of mass numbers before = Sum of mass numbers after.
3. **Conservation of Mass-Energy:** Total relativistic mass-energy is conserved.
4. **Conservation of Linear and Angular Momentum:** Total momentum is conserved.

Note: Mass (as rest mass) is **not** conserved separately — some mass may be converted to energy (or vice versa).

7.2 Discovery of the Neutron (Chadwick's Nuclear Reaction)

A classic example of a nuclear reaction that led to a major discovery.

Chadwick's Discovery of the Neutron (1932)

James Chadwick bombarded beryllium with α-particles from polonium. A highly penetrating neutral radiation was produced, which he correctly identified as neutrons.

$$^{9}_{4}\text{Be} + ^{4}_{2}\text{He} \xrightarrow{\alpha} {}^{12}_{6}\text{C} + ^{1}_{0}n + Q$$

In compact notation: $^{9}_{4}\text{Be}(\alpha, n)^{12}_{6}\text{C}$
Conservation check:

- Mass numbers: $9 + 4 = 13 = 12 + 1$ ✓
- Charges: $4 + 2 = 6 = 6 + 0$ ✓

7.3 Discovery of the Positron (Anderson, 1932)

Another important reaction that revealed a new fundamental particle.

Discovery of Positron

Carl Anderson observed the first antimatter particle — the positron (e^+), identical to an electron but with positive charge.
A typical production reaction (pair production):

$$\gamma \rightarrow e^+ + e^-$$

This occurs when a high-energy photon passes near a heavy nucleus, converting energy into matter (pair of electron and positron).

7.4 Artificial Transmutation

Nuclear reactions can be used to artificially transform one element into another — a process called **transmutation**.

Artificial (Induced) Transmutation

The conversion of one element into another through a nuclear reaction induced by bombarding it with energetic particles.
First artificial transmutation (Rutherford, 1919):

$$^{14}_{7}\mathrm{N} + ^{4}_{2}\mathrm{He} \rightarrow ^{17}_{8}\mathrm{O} + ^{1}_{1}\mathrm{p}$$

Rutherford bombarded nitrogen gas with α-particles, producing oxygen and detecting protons. This was the first time one element was deliberately converted into another.

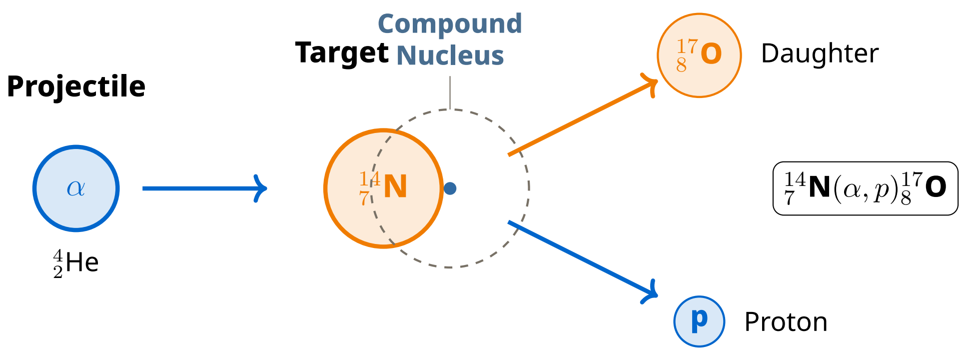


Figure 18: Rutherford's first artificial transmutation: An α-particle strikes a nitrogen nucleus to produce oxygen and a proton.

7.5 Q-Value of a Nuclear Reaction

The energy balance in a nuclear reaction is quantified by the Q-value.

Q-Value

The **Q-value** of a nuclear reaction is the difference between the total kinetic energy of the products and the total kinetic energy of the reactants, measured in the center-of-mass frame. Equivalently:

$$Q = (\text{Sum of masses of reactants} - \text{Sum of masses of products}) \times c^2$$

For reaction $a + X \rightarrow Y + b$:

$$\boxed{Q = [m(a) + m(X) - m(Y) - m(b)] \times c^2}$$

Endothermic and Exothermic Reactions

- **Exothermic (Exoergic) Reaction:** $Q > 0$
  - Mass of products is less than mass of reactants.
  - Energy is released in the form of kinetic energy of products.
  - Can occur even if the projectile has zero kinetic energy.
- **Endothermic (Endoergic) Reaction:** $Q < 0$
  - Mass of products is greater than mass of reactants.
  - Energy must be supplied (as kinetic energy of projectile) to make the reaction occur.
  - **Threshold kinetic energy** of projectile required:

$$K_{\text{th}} = |Q| \times \frac{m(a) + m(X)}{m(X)}$$

(in lab frame, where target is stationary)

NCERT Style Example: Q-Value Calculation

Problem: Calculate the Q-value for the reaction:

$${}^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{p}$$

Given masses in atomic mass units (u):

- $M({}^{14}_{7}\text{N}) = 14.003074$ u
- $M({}^{4}_{2}\text{He}) = 4.002603$ u
- $M({}^{17}_{8}\text{O}) = 16.999132$ u
- $M({}^{1}_{1}\text{H}) = 1.007825$ u

Solution:

$$\begin{aligned} \text{Total mass of reactants} &= 14.003074 + 4.002603 = 18.005677 \text{ u} \\ \text{Total mass of products} &= 16.999132 + 1.007825 = 18.006957 \text{ u} \\ \text{Mass difference} &= 18.005677 - 18.006957 = -0.001280 \text{ u} \end{aligned}$$

$$\begin{aligned} Q &= -0.001280 \times 931.5 \text{ MeV} \\ &\approx -1.19 \text{ MeV} \end{aligned}$$

Interpretation: Q is negative, so this is an **endothermic reaction**. The α-particle must have at least 1.19 MeV of kinetic energy (in center-of-mass frame) for the reaction to occur.

8 Nuclear Fission

8.1 What is Nuclear Fission?

Nuclear fission is one of the most important nuclear processes, both as a natural phenomenon and as a practical source of energy.

Nuclear Fission

Nuclear fission is the process in which a heavy nucleus splits into two (or rarely, more) lighter nuclei of comparable masses, accompanied by the release of a large amount of energy and typically some neutrons.

$$\text{Heavy nucleus} \rightarrow \text{Lighter nucleus 1} + \text{Lighter nucleus 2} + \text{neutrons} + Q$$

Key points:

- First discovered by **Otto Hahn and Fritz Strassmann** in 1938.
- Explained theoretically by **Lise Meitner and Otto Frisch** in 1939.
- The term "fission" was coined by Frisch, borrowing from biology (cell division).
- Fission can be **spontaneous** (very rare) or **induced** by neutron bombardment.

8.2 Why Fission Releases Energy

The energy release in fission can be understood from the binding energy curve.

Energy from Fission: Binding Energy Perspective

From the binding energy per nucleon (E_{bn}) curve:

- Heavy nuclei ($A \approx 240$) have $E_{bn} \approx 7.6$ MeV per nucleon.
- Medium-mass nuclei ($A \approx 120$) have $E_{bn} \approx 8.5$ MeV per nucleon.

When a heavy nucleus splits into two medium-mass nuclei, each nucleon ends up more tightly bound. The **increase in** E_{bn} corresponds to the energy released.

$$\text{Energy released} \approx A \times (\Delta E_{bn}) \approx 240 \times 0.9 \text{ MeV} \approx 200 \text{ MeV}$$

This is **enormous** compared to chemical reactions (typically few eV per atom) — about 10^8 **times greater**!

8.3 The Liquid Drop Model and Fission

The liquid drop model, proposed by **Niels Bohr and John Wheeler** (1939), provides a visual understanding of the fission process.

The Liquid Drop Model of Fission

1. A heavy nucleus behaves like a charged liquid drop held together by surface tension (nuclear force) but strained by Coulomb repulsion between protons.
2. When a neutron is absorbed, the nucleus becomes excited and starts to oscillate.
3. If the oscillation elongates the nucleus sufficiently, the Coulomb repulsion overcomes the nuclear attraction (surface tension).
4. The nucleus splits into two fragments, which fly apart due to Coulomb repulsion.
5. The fragments are neutron-rich and undergo β^- decay chains to reach stability.

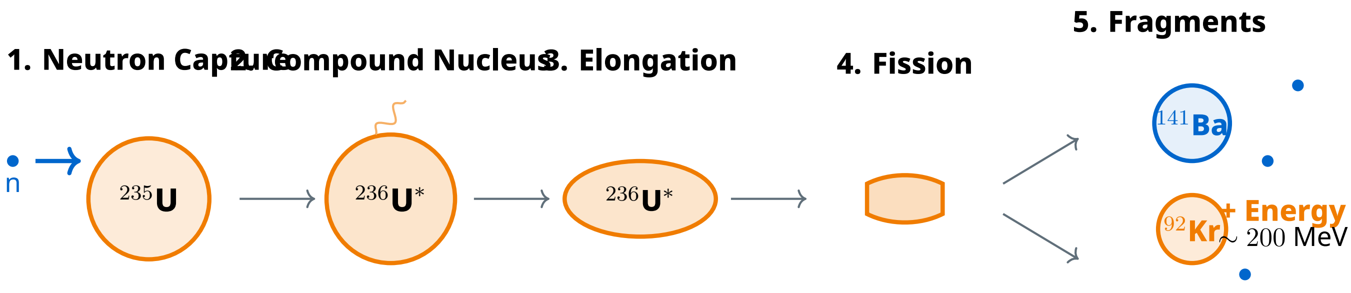


Figure 19: The fission process according to the liquid drop model: neutron capture, compound nucleus formation, elongation, splitting into fragments, and neutron emission.

8.4 Fission of Uranium-235

The most important fission reaction for nuclear energy is that of uranium-235.

Fission of $^{235}_{92}$U by Thermal Neutrons

When ^{235}U captures a slow (thermal) neutron, it forms an excited compound nucleus ^{236}U* which undergoes fission.

$$^{235}_{92}\text{U} + ^1_0 n \rightarrow ^{236}_{92}\text{U}^* \rightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3\,^1_0 n + Q$$

This is just **one possible fission channel**. The fission of ^{235}U produces many different pairs of fragments.
Typical features:

- Energy released: ~ 200 MeV per fission.
- Distribution of this energy:
  - Kinetic energy of fragments: ~ 165 MeV
  - Prompt neutrons: ~ 5 MeV
  - Prompt γ-rays: ~ 8 MeV
  - β-decay of fragments (delayed): ~ 19 MeV
  - γ-rays from decay: ~ 7 MeV
- Average number of neutrons emitted: 2.5 per fission.
- These neutrons can trigger further fissions, making a **chain reaction** possible.

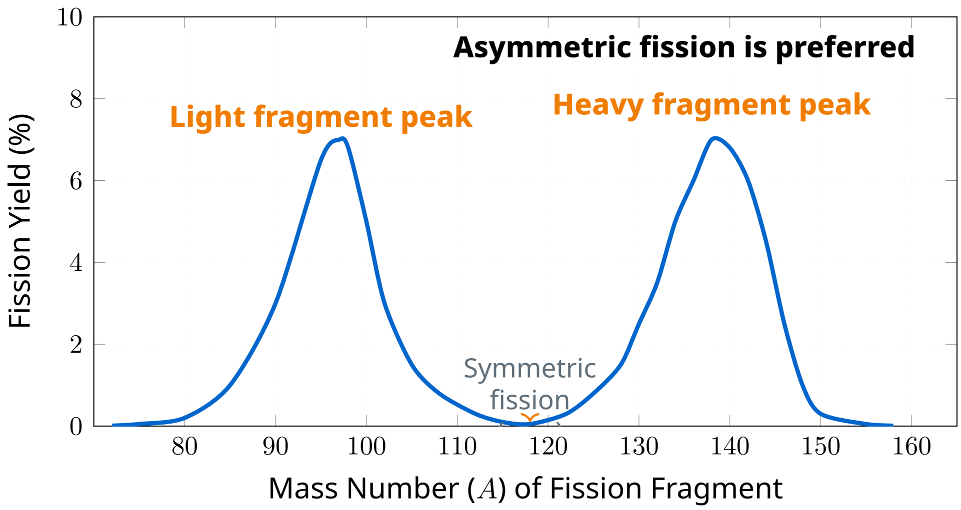


Figure 20: Distribution of fission fragments from thermal neutron-induced fission of ^{235}U. The double-hump curve shows that asymmetric fission (producing fragments of unequal masses) is far more probable than symmetric fission.

8.5 Chain Reaction

The neutrons emitted in fission can induce further fission events, creating a self-sustaining chain reaction.

Nuclear Chain Reaction

A **nuclear chain reaction** is a sequence of nuclear fission reactions in which the neutrons released from one fission event go on to trigger further fission events, leading to a self-sustaining cascade.

Multiplication factor (k):

$$k = \frac{\text{Number of neutrons in one generation}}{\text{Number of neutrons in the previous generation}}$$

- $k < 1$: Sub-critical — reaction dies out.
- $k = 1$: Critical — steady, sustained reaction (nuclear reactor).
- $k > 1$: Super-critical — exponential growth (nuclear explosion).

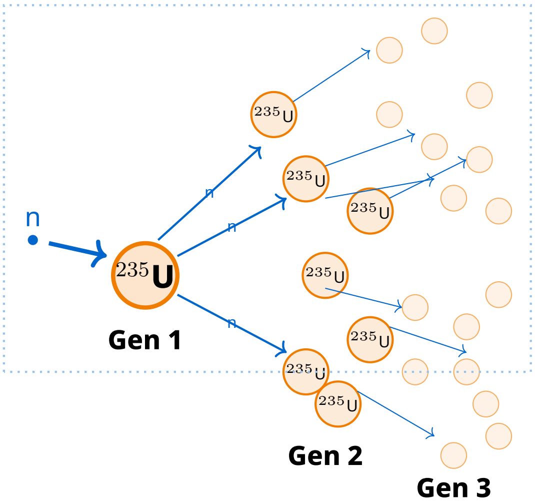


Figure 21: A schematic of a nuclear chain reaction. One neutron triggers fission of a ^{235}U nucleus, releasing more neutrons (typically 2-3) which cause further fissions, leading to exponential growth.

8.6 Critical Mass

Critical Mass

The **critical mass** is the minimum amount of fissile material required to sustain a self-sustaining nuclear chain reaction ($k = 1$).

- **Why it's needed:** In a small mass, too many neutrons escape from the surface without causing fission. As mass increases, the volume-to-surface ratio increases, reducing the fractional neutron leakage.
- **For ^{235}U:** Critical mass ≈ 50 kg (bare sphere).
- **For ^{239}Pu:** Critical mass ≈ 10 kg (bare sphere).
- Factors affecting critical mass: shape (sphere is optimal), purity, presence of moderators/reflectors, and density.

8.7 Nuclear Reactor (Brief Overview)

A nuclear reactor is a device designed to maintain a controlled, self-sustaining chain reaction and extract the energy released.

Essential Components of a Nuclear Reactor

1. **Fuel:** Fissile material like enriched uranium (^{235}U) or plutonium (^{239}Pu).
2. **Moderator:** Slows down fast neutrons to thermal energies to increase fission probability. Examples: heavy water (D_2O), graphite, ordinary water.
3. **Control Rods:** Made of neutron-absorbing materials (cadmium, boron). Inserted/withdrawn to control k and power level.
4. **Coolant:** Removes heat generated by fission. Examples: water, heavy water, liquid sodium, CO_2 gas.
5. **Shielding:** Thick concrete/lead walls to absorb dangerous radiation and protect personnel.

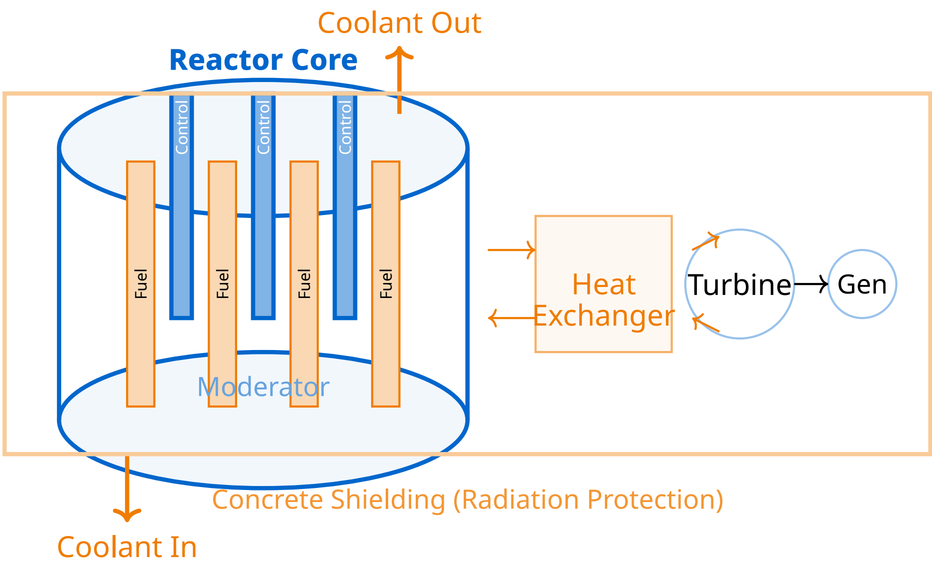


Figure 22: Simplified schematic of a nuclear reactor showing the core with fuel rods and control rods within the moderator, coolant circulation, heat exchanger, turbine, and generator.

Exam Must-Know: Fission Facts

- **Energy per fission:** ~ 200 MeV (for ^{235}U)
- **Neutrons per fission:** 2-3 (average $\sim$ 2.5)
- **Fissile vs Fissionable:**
  - **Fissile:** Nuclei that undergo fission with *thermal* (slow) neutrons: ^{235}U, ^{239}Pu, ^{233}U.
  - **Fissionable:** Nuclei that require *fast* neutrons to fission: ^{238}U, ^{232}Th.

- **Energy density:** 1 kg of ^{235}U undergoing complete fission releases $\sim 8.2 \times 10^{13}$ J, equivalent to burning ~ 3000 tonnes of coal!

9 Nuclear Fusion

9.1 What is Nuclear Fusion?

Nuclear fusion is the energy source of stars and holds promise for future energy production on Earth.

Nuclear Fusion

Nuclear fusion is the process in which two or more light atomic nuclei combine (fuse) to form a heavier nucleus, accompanied by the release of a large amount of energy.

$$\text{Light nucleus 1} + \text{Light nucleus 2} \rightarrow \text{Heavier nucleus} + Q$$

Key points:

- Fusion is the **opposite** of fission — light nuclei combine rather than heavy nuclei splitting.
- It releases energy for nuclei lighter than iron ($A < 56$), as the E_{bn} curve rises steeply in this region.
- The energy released per unit mass of fuel is much **greater** than fission.
- Fusion is the process that powers the **Sun** and other stars.

9.2 Why Fusion Releases Energy

From the binding energy curve, light nuclei have relatively low E_{bn}. When they fuse, the product nucleus has a significantly higher E_{bn}.

Energy from Fusion: The Numbers

Consider the fusion of deuterium and tritium:

- E_{bn} of ^{2}H ≈ 1.1 MeV per nucleon
- E_{bn} of ^{3}H ≈ 2.8 MeV per nucleon
- E_{bn} of ^{4}He ≈ 7.1 MeV per nucleon

When ^{2}H and ^{3}H fuse to form ^{4}He (plus a neutron), the total binding energy increases dramatically:

$$\text{Initial total } E_b = (2 \times 1.1) + (3 \times 2.8) = 2.2 + 8.4 = 10.6 \text{ MeV}$$
$$\text{Final total } E_b = 4 \times 7.1 = 28.4 \text{ MeV}$$
$$\text{Energy released} = 28.4 - 10.6 \approx 17.6 \text{ MeV}$$

This 17.6 MeV from just 5 nucleons is enormous on a *per nucleon* basis compared to fission ($\sim$200 MeV from 236 nucleons $\approx$ 0.85 MeV per nucleon).

9.3 The Coulomb Barrier: The Challenge of Fusion

Overcoming the Coulomb Barrier

Fusion is difficult to achieve because both nuclei are positively charged and strongly repel each other via the Coulomb force.

- **Coulomb Barrier:** The electrostatic potential energy barrier that must be overcome before two nuclei can come close enough ($\sim$ 1 fm) for the strong nuclear force to take over and cause fusion.
- **Height of the barrier:** For two protons:

$$U = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \approx \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{10^{-15}} \approx 2.3 \times 10^{-13}\ \text{J} \approx 1.4 \text{ MeV}$$

How to overcome the barrier:

1. **High Temperature (Thermonuclear Fusion):** Heat the fuel to millions of degrees so that nuclei have sufficient kinetic energy to overcome Coulomb repulsion despite the average energy being lower (quantum tunneling helps).
2. **High Density/Pressure:** Increase the probability of collisions (as in stellar cores).
3. **Confinement:** Keep the hot plasma confined long enough for fusion to occur.

9.4 Important Fusion Reactions

Several fusion reactions are important for energy production and stellar processes.

Key Fusion Reactions

1. **Deuterium-Deuterium (D-D) Reactions:**

$$^2_1\text{H} + ^2_1\text{H} \rightarrow\ ^3_1\text{H} + ^1_1\text{p} + 4.0 \text{ MeV}$$
$$^2_1\text{H} + ^2_1\text{H} \rightarrow\ ^3_2\text{He} + ^1_0 n + 3.3 \text{ MeV}$$

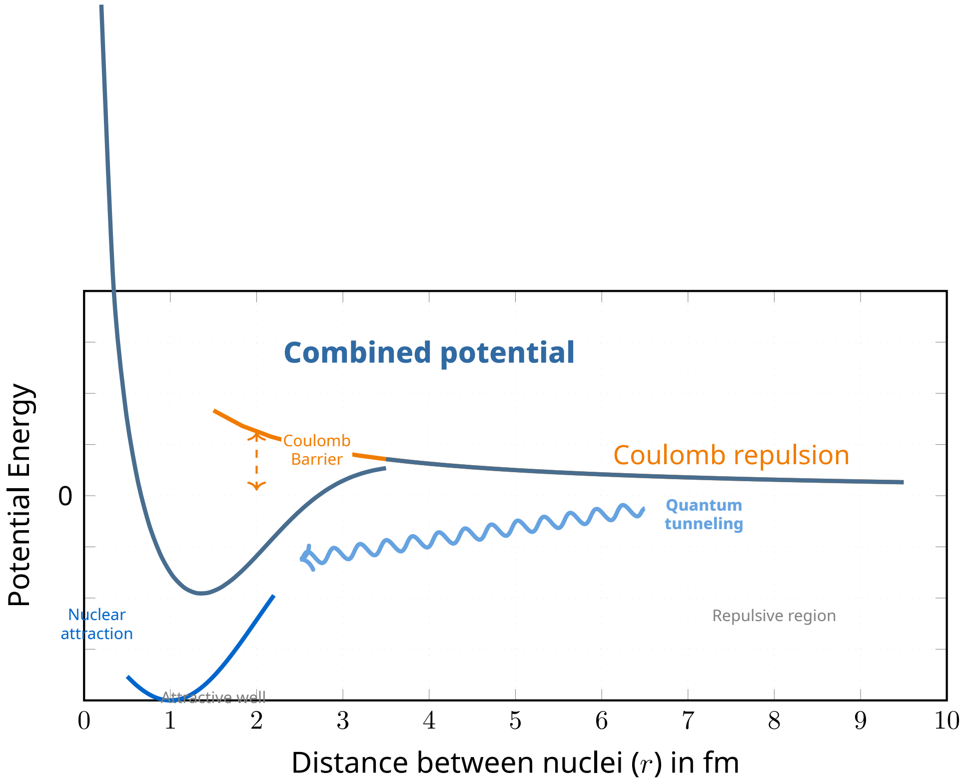


Figure 23: The combined nuclear and Coulomb potential. Labels are repositioned to avoid overlap between the repulsive region and the attractive well.

2. **Deuterium-Tritium (D-T) Reaction (most promising for reactors):**

$$^2_1\text{H} + ^3_1\text{H} \rightarrow ^4_2\text{He} + ^1_0 n + 17.6 \text{ MeV}$$

3. **Proton-Proton (p-p) Chain (in the Sun):**

$$^1_1\text{p} + ^1_1\text{p} \rightarrow ^2_1\text{H} + e^+ + \nu_e + 0.42 \text{ MeV}$$
$$^2_1\text{H} + ^1_1\text{p} \rightarrow ^3_2\text{He} + \gamma + 5.49 \text{ MeV}$$
$$^3_2\text{He} + ^3_2\text{He} \rightarrow ^4_2\text{He} + 2\,^1_1\text{p} + 12.86 \text{ MeV}$$

Net result of p-p chain: 4 protons $\rightarrow ^4_2$He + $2e^+$ + $2\nu_e$ + 2γ + 26.7 MeV

9.5 Thermonuclear Fusion: The Source of Stellar Energy

Fusion in Stars

Stars are natural fusion reactors. The energy from stars comes from thermonuclear fusion reactions occurring in their cores.

- **The Sun's Core:** Temperature $\sim 1.5 \times 10^7$ K, Density ~ 150 g/cm^3.
- **The Proton-Proton (p-p) Chain:** The dominant energy source in stars

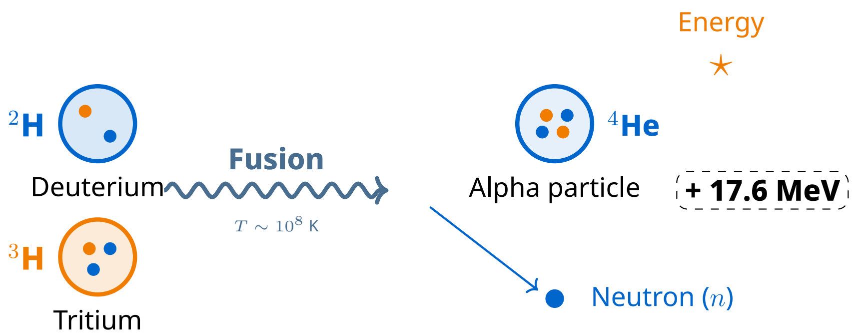


Figure 24: D-T Fusion Reaction: Moving symbols outside the circles provides a clearer view of the internal nucleon arrangements.

like the Sun (smaller stars). Net effect: $4p \rightarrow^4$ He $+$ energy.

- **CNO Cycle (Carbon-Nitrogen-Oxygen):** Dominant in more massive stars (> 1.3 solar masses). Carbon acts as a catalyst to fuse 4 protons into helium.
- **Nucleosynthesis:** Fusion in stars produces elements up to iron (^{56}Fe). Heavier elements are produced in supernova explosions (via neutron capture processes).

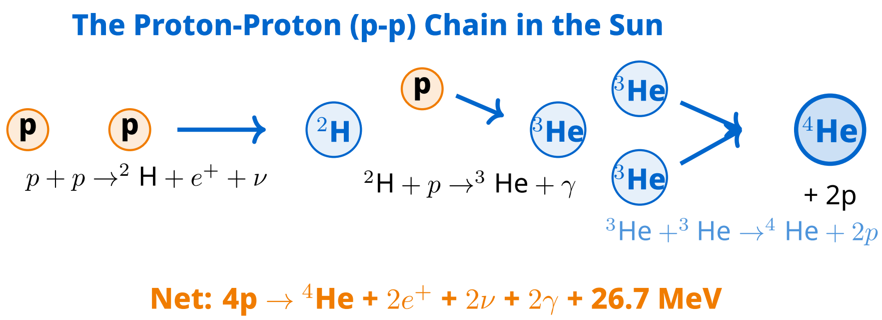


Figure 25: The three stages of the proton-proton chain that powers the Sun. Four protons ultimately fuse to form one helium nucleus, releasing 26.7 MeV of energy.

9.6 Controlled Thermonuclear Fusion (Confining the Plasma)

Achieving controlled fusion on Earth requires confining a plasma at extremely high temperatures.

Methods of Plasma Confinement

1. **Magnetic Confinement:**
   - Uses strong magnetic fields to confine the hot plasma (charged particles spiral along field lines).
   - **Tokamak:** A toroidal (doughnut-shaped) device using magnetic

fields. Most advanced design (e.g., ITER project).

- **Stellarator:** Similar to tokamak but uses twisted magnetic field coils.

2. **Inertial Confinement:**

- Uses intense laser beams or ion beams to rapidly compress and heat a small fuel pellet.
- The pellet's own inertia keeps it confined long enough for fusion to occur.
- Essentially a miniature, controlled thermonuclear explosion.

Lawson Criterion: For a self-sustaining fusion reaction, the product of plasma density (n) and confinement time (τ) must exceed a critical value:

$$n\tau \geq 10^{14} \text{ s/cm}^3 \quad \text{(for D-T fusion at } \sim 10^8 \text{ K)}$$

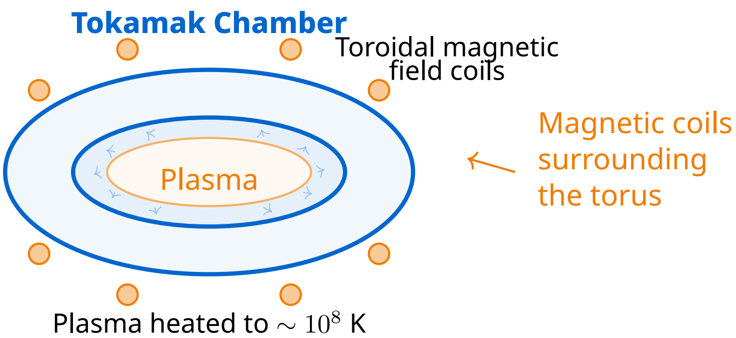


Figure 26: Conceptual diagram of a Tokamak fusion reactor. The hot plasma is confined in a toroidal (doughnut) shape by strong magnetic fields, keeping it away from the vessel walls.

Exam Key Points: Fusion vs Fission

Feature	Nuclear Fission	Nuclear Fusion
Process	Heavy nucleus splits	Light nuclei combine
Energy source	$A > 100$ (heavy nuclei)	$A < 56$ (light nuclei)
Energy per nucleon	~ 0.85 MeV/nucleon	Up to ~ 6 MeV/nucleon (D-T)
Condition needed	Thermal neutrons	Extremely high temperature ($\sim 10^8$ K)
By-products	Radioactive fission fragments	Mostly non-radioactive (helium)
Fuel availability	Limited (^{235}U is rare)	Abundant (deuterium from seawater)
Chain reaction	Possible (self-sustaining)	Not possible (requires continuous heating)
Current status	Commercially operational	Experimental stage (ITER, etc.)
Neutron production	Significant	In D-T fusion; D-D and p-p produce fewer
Environmental hazard	Radioactive waste disposal issue	Much cleaner (low radioactivity)

Remember: The Sun fuses about 6×10^{11} kg of hydrogen per second, converting about 4×10^9 kg of mass into energy ($E = mc^2$) every second!

10 NCERT Solved Examples

This section contains solved examples based on the NCERT textbook, covering key numerical problems from the chapter.

Example 13.1: Binding Energy of Iron Nucleus

Problem: Find the binding energy per nucleon for $^{56}_{26}$Fe. Given:

- Mass of $^{56}_{26}$Fe atom = 55.934939 u
- Mass of $^{1}_{1}$H atom = 1.007825 u
- Mass of neutron = 1.008665 u

Solution:

1. Number of protons, $Z = 26$
2. Number of neutrons, $N = A - Z = 56 - 26 = 30$
3. Mass defect:

$$\begin{aligned}\Delta m &= [Z \times M(^1_1\text{H}) + N \times m_n] - M(^{56}_{26}\text{Fe}) \\ &= [26 \times 1.007825 + 30 \times 1.008665] - 55.934939 \\ &= [26.203450 + 30.259950] - 55.934939 \\ &= 56.463400 - 55.934939 \\ &= 0.528461 \text{ u}\end{aligned}$$

4. Binding energy:

$$\begin{aligned}E_b &= \Delta m \times 931.5 \text{ MeV/u} \\ &= 0.528461 \times 931.5 \\ &\approx 492.26 \text{ MeV}\end{aligned}$$

5. Binding energy per nucleon:

$$E_{bn} = \frac{E_b}{A} = \frac{492.26}{56} \approx 8.79 \text{ MeV/nucleon}$$

Note: This is the highest binding energy per nucleon of any naturally occurring nucleus, confirming iron's position at the peak of the binding energy curve.

Example 13.2: Half-Life and Decay Constant

Problem: The half-life of $^{238}_{92}$U undergoing α-decay is 4.5×10^9 years.

(a) What is the decay constant?

(b) What fraction of a sample of pure ^{238}U will decay in 1 billion years?

Solution:

(a) Decay constant:

$$\begin{aligned}T_{1/2} &= 4.5 \times 10^9 \text{ years} \\ \lambda &= \frac{0.693}{T_{1/2}} \\ &= \frac{0.693}{4.5 \times 10^9} \\ &\approx 1.54 \times 10^{-10} \text{ year}^{-1}\end{aligned}$$

(b) Fraction remaining after $t = 10^9$ years:

$$\begin{aligned} \frac{N}{N_0} &= e^{-\lambda t} \\ &= e^{-(1.54\times10^{-10})\times(10^9)} \\ &= e^{-0.154} \\ &\approx 0.857 \end{aligned}$$

So, fraction decayed = $1 - 0.857 = 0.143$ or **14.3%**.

Example 13.3: Activity of a Radioactive Sample

Problem: The decay constant of a radioactive substance is 4.33×10^{-4} per year. Calculate its half-life and the time taken for 75% of it to decay.
Solution:

1. Half-life:

$$\begin{aligned} T_{1/2} &= \frac{0.693}{\lambda} \\ &= \frac{0.693}{4.33 \times 10^{-4}} \\ &\approx 1600 \text{ years} \end{aligned}$$

2. For 75% decay, the fraction remaining is 25% or 0.25:

$$\frac{N}{N_0} = 0.25 = \frac{1}{4} = \left(\frac{1}{2}\right)^2$$

This means $n = 2$ half-lives have elapsed.

$$t = n \times T_{1/2} = 2 \times 1600 = 3200 \text{ years}$$

Alternatively, using the decay law:

$$\begin{aligned} \frac{N}{N_0} &= e^{-\lambda t} \\ 0.25 &= e^{-\lambda t} \\ \ln(0.25) &= -\lambda t \\ -1.386 &= -(4.33 \times 10^{-4})\, t \\ t &= \frac{1.386}{4.33 \times 10^{-4}} \approx 3200 \text{ years} \end{aligned}$$

Example 13.4: Q-Value of a Nuclear Reaction

Problem: Calculate the Q-value for the nuclear reaction:

$${}^{10}_{5}\text{B} + {}^{4}_{2}\text{He} \rightarrow {}^{13}_{7}\text{N} + {}^{1}_{0}n$$

Given atomic masses:

- $M(^{10}_{5}\text{B}) = 10.012937$ u
- $M(^{4}_{2}\text{He}) = 4.002603$ u
- $M(^{13}_{7}\text{N}) = 13.005739$ u
- $m_n = 1.008665$ u

Solution:

$$\begin{aligned}
\text{Mass of reactants} &= 10.012937 + 4.002603 = 14.015540 \text{ u} \\
\text{Mass of products} &= 13.005739 + 1.008665 = 14.014404 \text{ u} \\
\Delta m &= 14.015540 - 14.014404 = 0.001136 \text{ u}
\end{aligned}$$

$$\begin{aligned}
Q &= \Delta m \times 931.5 \text{ MeV/u} \\
&= 0.001136 \times 931.5 \\
&\approx 1.06 \text{ MeV}
\end{aligned}$$

Interpretation: $Q > 0$, so this is an **exothermic reaction**. Energy is released in the form of kinetic energy of the products.

Example 13.5: Energy from Fission

Problem: The fission of one ^{235}U nucleus releases approximately 200 MeV of energy. Calculate:

(a) The energy released by the fission of 1 gram of ^{235}U.

(b) The mass of coal (calorific value 30 kJ/g) required to produce the same energy.

Solution:

(a) Number of atoms in 1 gram of ^{235}U:

$$\begin{aligned}
N &= \frac{\text{mass}}{\text{mass of one atom}} \\
&= \frac{1 \text{ g}}{235 \text{ u}} \\
&= \frac{1}{235 \times 1.66 \times 10^{-24}} \\
&\approx \frac{1}{3.9 \times 10^{-22}} \approx 2.56 \times 10^{21} \text{ atoms}
\end{aligned}$$

Energy released:

$$\begin{aligned}
E &= N \times 200 \text{ MeV} \\
&= 2.56 \times 10^{21} \times 200 \times 1.6 \times 10^{-13} \text{ J} \\
&\approx 8.2 \times 10^{10} \text{ J}
\end{aligned}$$

(b) Mass of coal required:

$$m_{\text{coal}} = \frac{8.2 \times 10^{10}\ \text{J}}{30 \times 10^3\ \text{J/g}} = \frac{8.2 \times 10^{10}}{3.0 \times 10^4} \approx 2.7 \times 10^6\ \text{g} = 2700\ \text{kg}$$

So, 1 gram of ^{235}U releases energy equivalent to burning about **2.7 tonnes** of coal!

Example 13.6: Radiocarbon Dating

Problem: The half-life of ^{14}C is 5730 years. An archaeological specimen of wood contains only 75% of the ^{14}C activity found in a living tree of the same type. Estimate the age of the specimen.

Solution:

$$\text{Activity ratio: } \frac{R}{R_0} = 0.75$$

Using the radioactive decay law:

$$\begin{aligned} R &= R_0 e^{-\lambda t} \\ 0.75 &= e^{-\lambda t} \\ \ln(0.75) &= -\lambda t \\ -0.2877 &= -\lambda t \\ t &= \frac{0.2877}{\lambda} \end{aligned}$$

Now, $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5730} \approx 1.21 \times 10^{-4}\ \text{year}^{-1}$

$$t = \frac{0.2877}{1.21 \times 10^{-4}} \approx 2378\ \text{years}$$

The specimen is approximately 2400 years old.

Alternative method (using fraction approach):

$$\frac{R}{R_0} = 0.75 = \left(\frac{1}{2}\right)^n$$

Taking log on both sides:

$$\ln(0.75) = n \ln(0.5)$$

$$n = \frac{\ln(0.75)}{\ln(0.5)} = \frac{-0.2877}{-0.6931} \approx 0.415$$

$$t = n \times T_{1/2} = 0.415 \times 5730 \approx 2378\ \text{years}$$

Example 13.7: Energy from D-T Fusion

Problem: In the D-T fusion reaction:

$$^2_1\text{H} + ^3_1\text{H} \rightarrow ^4_2\text{He} + n + 17.6\text{ MeV}$$

Calculate the energy released when 1 kg of deuterium fuses completely with an appropriate amount of tritium.
Given: Atomic mass of ^{2}H $= 2.014102$ u.
Solution:

1. Number of deuterium atoms in 1 kg:

$$\begin{aligned} N &= \frac{1000\text{ g}}{2.014102 \times 1.66 \times 10^{-24}\text{ g}} \\ &\approx \frac{1000}{3.34 \times 10^{-24}} \\ &\approx 3.0 \times 10^{26}\text{ atoms} \end{aligned}$$

2. Energy released:

$$\begin{aligned} E &= N \times 17.6\text{ MeV} \\ &= 3.0 \times 10^{26} \times 17.6 \times 1.6 \times 10^{-13}\text{ J} \\ &\approx 8.45 \times 10^{14}\text{ J} \end{aligned}$$

Comparison: This is about 10,000 times more energy than the fission of 1 gram of ^{235}U (which gave $\sim 8.2 \times 10^{10}$ J)!
However, achieving controlled fusion remains a challenge due to the extreme temperatures required.

NCERT Example Summary: Key Formulas Used

- **Mass defect:** $\Delta m = [Zm_p + (A - Z)m_n] - M_{\text{nucleus}}$
- **Binding energy:** $E_b = \Delta m \times 931.5$ MeV
- **Decay constant:** $\lambda = \dfrac{0.693}{T_{1/2}}$
- **Decay law:** $N = N_0 e^{-\lambda t} = N_0 \left(\dfrac{1}{2}\right)^n$
- **Q-value:** $Q = [m_{\text{reactants}} - m_{\text{products}}] \times 931.5$ MeV
- **Number of atoms:** $N = \dfrac{\text{mass}}{\text{mass of one atom}}$