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NCERT Exemplar Solutions

Solved NCERT Exemplar Problems for Class 12th Physics, Chapter 2

Chapter 2: Electrostatic Potential and Capacitance

About this Chapter

Chapter 2 of the Class 12 NCERT Exemplar develops the language of *electrostatic potential*, links it to the field through $\vec{E} = -\nabla V$, and uses it to study conductors, dielectrics and capacitors. The 33 Exemplar problems range from quick conceptual MCQs (single and multiple correct) to long derivation-style questions on equipotentials, energy of charge distributions, and capacitor networks. Every solution below restates the relevant concept first and then carries out each algebraic step in full.

Topics covered: Electrostatic potential • Equipotential surfaces • Conductors and dielectrics • Capacitance and energy storage • Networks of capacitors

Quick Formula Sheet

$V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r}$

$\vec{E} = -\nabla V$

$V_{\text{dipole}} = \dfrac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

$U = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r}$

$C_{\parallel\text{-plate}} = \dfrac{\varepsilon_0 A}{d}$

Series: $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

Parallel: $C = C_1 + C_2$

$U_{\text{cap}} = \dfrac{1}{2}CV^2 = \dfrac{Q^2}{2C}$

MCQ I (single correct option)

Q 2.1 **A $4\,\mu$F capacitor is connected as shown in Fig. 2.1. The internal resistance of the battery is $0.5\,\Omega$. The charge on the capacitor plates will be:**

(a) 0 **(b)** $4\,\mu$**C** **(c)** $16\,\mu$**C** **(d)** $8\,\mu$**C.**

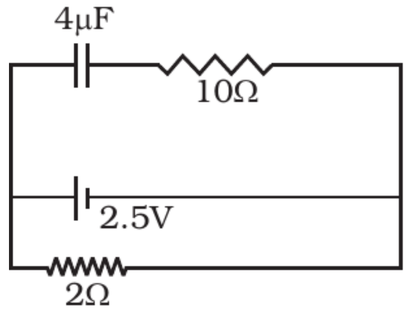


Fig. 2.1

Fig. 2.1 in the NCERT Exemplar, Class 12 Physics, Chapter 2.

SOLUTION

Concept used. In the steady state of a DC circuit containing a capacitor, no current flows through the capacitor branch, because once the capacitor is fully charged, the displacement of charge stops. The voltage across a capacitor in steady state equals the potential difference between its terminals as fixed by the rest of the circuit.

> ☞ **Capacitor in steady DC**
> A capacitor in steady state behaves like an open circuit. Any resistor in series with it carries zero current, hence zero IR drop. The voltage across the capacitor equals the voltage across whatever parallel branch sits between the same two nodes.

Step 1. Identify the conducting branch. The capacitor ($4\,\mu$F) sits in series with the $10\,\Omega$ resistor in the upper branch. The lower branch contains the $2\,\Omega$ resistor. Both branches are in parallel across the battery (EMF $\mathcal{E} = 2.5\,$V, internal resistance $r = 0.5\,\Omega$). In steady state, no current passes through the upper branch (capacitor blocks DC), so the only loop carrying current is: battery $\rightarrow$ $2\,\Omega \rightarrow$ battery.

Step 2. Find the current. Apply Kirchhoff's voltage law to the loop battery $+\, r\ +\ 2\,\Omega$:

$$\mathcal{E} = I(r + R),$$

where $R = 2\,\Omega$ is the load and $r = 0.5\,\Omega$ is the internal resistance. Substitute:

$$2.5 = I\,(0.5 + 2) = 2.5\,I \quad \Longrightarrow \quad I = 1\,\text{A}.$$

Step 3. Find the terminal voltage of the battery. This is also the potential difference across the parallel external load:

$$V_{\text{term}} = \mathcal{E} - Ir = 2.5 - (1)(0.5) = 2.0\,\text{V}.$$

Equivalently, $V_{\text{term}} = IR = (1)(2) = 2.0\,$V across the $2\,\Omega$. Either route gives the same number.

Step 4. **Find the voltage across the capacitor.** Since no current flows through the upper branch, the IR drop across the $10\,\Omega$ resistor is

$$V_{10\Omega} = (0)(10) = 0\,\text{V}.$$

The whole terminal voltage therefore appears across the capacitor:

$$V_C = V_{\text{term}} - V_{10\Omega} = 2.0 - 0 = 2.0\,\text{V}.$$

Step 5. **Compute the charge.** Use $Q = CV_C$:

$$Q = (4\,\mu\text{F})(2\,\text{V}) = 8\,\mu\text{C}.$$

☞ **Sanity check**

The answer must be $\leq C \cdot \mathcal{E} = (4)(2.5) = 10\,\mu\text{C}$ (the absolute upper bound on a $4\,\mu$F capacitor across a $2.5\,$V source). Our $8\,\mu$C is comfortably under this ceiling. ✓

Final Answer: $Q = 8\,\mu\text{C}$ (Option **(d)**)

✗ Common Mistake

A common slip is to use $\mathcal{E} = 2.5\,$V directly as the voltage across the capacitor and write $Q = (4)(2.5) = 10\,\mu\text{C}$. This ignores the internal drop Ir. In steady state, the capacitor sees the *terminal* voltage of the battery (after the internal drop), not the EMF.

EXPERT'S SOLUTION : *Aarav Iyer, M.Sc Physics, IIT Madras*

Strategic angle. The branch-by-branch approach is most foolproof here: spot which loop actually carries current, get the common terminal voltage, then read the capacitor voltage off the upper branch.

Step 1. *Why the upper branch is dead.* A capacitor in steady state stores charge and rejects further current. So $I_{\text{upper}} = 0$, and the $10\,\Omega$ resistor contributes no voltage drop.

Step 2. *Lower-branch current.* The only conducting loop is battery $\to r \to 2\,\Omega \to$ battery. Ohm's law for this loop:

$$I = \frac{\mathcal{E}}{r + R} = \frac{2.5\,\text{V}}{0.5\,\Omega + 2\,\Omega} = 1\,\text{A}.$$

Step 3. *Terminal voltage = voltage across capacitor.* Both branches share the same two end-nodes (the battery terminals), so the voltage across the upper branch equals that across the lower branch:

$$V_C + V_{10\Omega} = V_{2\Omega} \quad \Rightarrow \quad V_C + 0 = (1)(2) = 2\,\text{V}.$$

Step 4. *Charge.*

$$Q = CV_C = (4 \times 10^{-6}\,\text{F})(2\,\text{V}) = 8 \times 10^{-6}\,\text{C} = 8\,\mu\text{C}.$$

The choice $10\,\mu$C is the deliberate trap: it forgets the internal-resistance drop. Treating $\mathcal{E} = 2.5\,$V as the voltage across the capacitor (instead of the terminal voltage $\mathcal{E} - Ir$) gives $Q = 4\,\mu\text{F} \times 2.5\,\text{V} = 10\,\mu$C, which is option (b) and the standard mistake.

Alternative method — Kirchhoff's voltage law explicitly. Write KVL around the conducting loop ($\mathcal{E} \to r \to 2\,\Omega \to$ back):

$$\mathcal{E} - Ir - I(2\,\Omega) = 0 \quad \Rightarrow \quad I = \frac{2.5}{2.5} = 1\,\text{A}.$$

Now traverse from one capacitor plate to the other via the lower branch: starting at the top node, drop V_C across C, then 0 across the $10\,\Omega$ resistor (no current), then $-IR$ across $2\,\Omega$, finishing at the bottom node. Since both branches share the same end-nodes, the algebraic sum across each must match the terminal voltage $\mathcal{E} - Ir = 2.5 - 0.5 = 2\,$V. Hence $V_C = 2\,$V.

Concept linkage. This problem stitches together Chapter 3 (current electricity and EMF with internal resistance) with the chapter-2 capacitor relation $Q = CV$. Whenever a capacitor sits *in a branch* with no other DC path, that branch carries zero steady current, so all the voltage of the parallel branch piles across the capacitor (resistors in that dead branch drop 0).

Exam tip. In CBSE board problems, this kind of "capacitor-in-a-DC-circuit" problem is a 2-mark or 3-mark favourite. Show: (i) the steady-state current path; (ii) the terminal/Branch voltage calculation; (iii) $Q = CV$. Skipping any of these three steps usually costs the full mark for that step.

Final Answer: $Q = 8\,\mu$C, option (d).

Q 2.2 A positively charged particle is released from rest in a uniform electric field. The electric potential energy of the charge:

(a) remains constant because the field is uniform.

(b) increases because the charge moves along the field.

(c) decreases because the charge moves along the field.

(d) decreases because the charge moves opposite to the field.

SOLUTION

Concept used. A positive charge in an electric field experiences a force in the direction of $\vec{E}$: $\vec{F} = q\vec{E}$. Starting from rest, it accelerates along $\vec{E}$. The relation between work done by the electric force and the change in electrostatic potential energy is

$$W_{\text{elec}} = -\Delta U = q(V_i - V_f).$$

Since the force does positive work on the charge, the change in potential energy ΔU is *negative*, i.e. U decreases. The lost potential energy reappears as kinetic energy.

> ☞ **Direction of $\vec{E}$ and V**
> $\vec{E}$ points from high V to low V. A positive charge, following $\vec{E}$, moves from a high-potential region to a low-potential region, so its PE $U = qV$ decreases.

Step 1. **Direction of motion.** Released from rest, the particle accelerates in the direction of $\vec{F} = q\vec{E}$. Because $q > 0$, the motion is *along* $\vec{E}$.

Step 2. **Potential change.** Along the field, the potential decreases:

$$V_f < V_i.$$

Step 3. **Potential-energy change.** $U = qV$ with $q > 0$, so

$$\Delta U = q(V_f - V_i) < 0.$$

The PE *decreases*.

Step 4. **Energy conservation check.** Since total mechanical energy is conserved (no friction here), the lost PE shows up as kinetic energy:

$$\tfrac{1}{2}mv^2 = -\Delta U > 0,$$

consistent with the particle speeding up.

> **Final Answer:** PE decreases because the charge moves along the field. Option **(c)**.

★ Geometric picture

Imagine the field as pointing "downhill" on a potential landscape. A positive charge rolls downhill, losing PE and gaining KE, exactly like a marble in gravity. A negative charge would roll *uphill*, gaining PE.

EXPERT'S SOLUTION *: Sneha Kapoor, M.Sc Physics, IIT Bombay*

Quick reading. Three facts decide this question: $\vec{F} = q\vec{E}$, the field points from higher to lower potential, and $U = qV$ for a point charge in an external field.

Step 1. For $q > 0$, $\vec{F}$ is parallel to $\vec{E}$, so the particle's displacement is along $\vec{E}$.

Step 2. Moving along $\vec{E}$ takes the particle to lower potential. Quantitatively, $V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{l} < 0$.

Step 3. Therefore $\Delta U = q\,\Delta V < 0$. The PE decreases.

Step 4. This rules out (a), (b), (d). Option (d) is wrong because a positive charge does *not* move opposite to $\vec{E}$ when released from rest; that would happen for a negative charge.

Alternative method — energy conservation. No friction acts, so mechanical energy is conserved: $K_i + U_i = K_f + U_f$. With $K_i = 0$ (released from rest) and $K_f > 0$ (the charge acquires kinetic energy), U must decrease. There is no other option consistent with energy conservation — the PE has to drop to "feed" the rising kinetic energy.

Common pitfall — confusing PE of the charge with PE of a test charge. The PE that appears in $U = qV$ is the PE of the *particular charge* q in the external potential V. Some students confuse this with the "potential" V alone, which is a property of the field. V decreases here *and* $U = qV$ decreases (because $q > 0$). For a negative test charge in the same field, V would still decrease, but $U = qV$ would *increase* (sign flip). Always check the sign of q when converting from V to U.

Concept linkage — gravitational analogue. For a positive charge in a uniform $\vec{E}$, $U = -qE_0 z$ (taking $\vec{E} = E_0\hat{z}$ downward), is the exact analogue of a mass in gravity: $U_{\text{grav}} = -mgy$. The charge "falls down the potential hill" just as a stone falls down a gravitational hill, and both gain kinetic energy in the process.

> **Final Answer:** Option (c).

Q 2.3 Figure 2.2 shows three configurations of equipotential lines (Fig. I, II, III). In each case a charged object is moved from A to B. Which is true?

(a) Work done in Fig. (i) is greatest. (b) Work done in Fig. (ii) is least.

(c) Work is the same in (i), (ii) and (iii). (d) Work in (iii) is greater than (ii) but equal to (i).

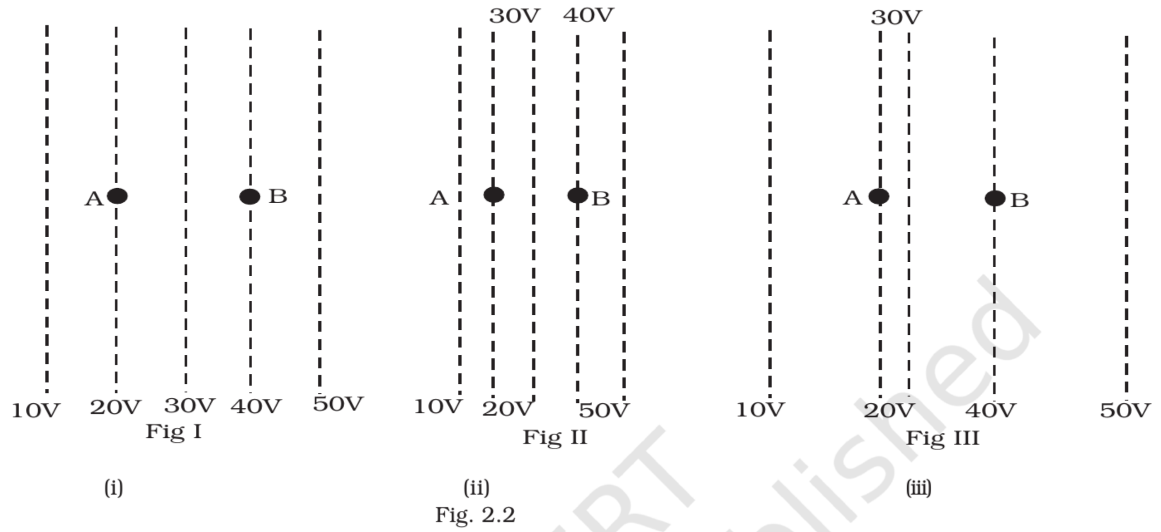


Fig. 2.2 in the NCERT Exemplar, Class 12 Physics, Chapter 2.

SOLUTION

Concept used. The electrostatic force is **conservative**. The work done in moving a charge q from point A to point B depends only on the potentials at the two end points, not on the path taken or on the intermediate field distribution:

$$W_{A\to B} = q\,(V_A - V_B).$$

In all three figures, A lies on the same equipotential value and B lies on the same equipotential value (the spacing of the in-between equipotentials differs, but the potentials *at* the end-points are unchanged across the three figures).

> ☞ **Why path doesn't matter**
> Because $\vec{E} = -\nabla V$, the work $-q\int_A^B \vec{E}\cdot d\vec{l}$ collapses to $q(V_A - V_B)$ regardless of which path the integral follows. Equivalently, $\oint \vec{E}\cdot d\vec{l} = 0$ for any closed loop in electrostatics.

Step 1. **Read off end-point potentials.** In each of the three figures, A and B lie on the equipotential lines marked with the same potential values. Call these V_A and V_B respectively. Both are identical across the three sub-figures.

Step 2. **Compute the work.**

$$W = q\,(V_A - V_B).$$

This expression contains *no* reference to how the equipotentials are spaced between A and B. Crowded equipotentials mean a stronger field but the work integral still collapses to the same end-point difference.

Step 3. **Conclude.** Because V_A and V_B are the same in all three figures, W is the same. The path-independence of the electrostatic work rules out (a), (b), (d).

Final Answer: Work done is the same in (i), (ii) and (iii). Option **(c)**.

☞ **Path-independence test**

Whenever a question shows multiple paths or several configurations between fixed end-points and asks about *work* done by an electrostatic force, the answer almost always involves $W = q(V_A - V_B)$ with the same value for every choice. Field strength variations along the path are red herrings.

EXPERT'S SOLUTION *: Priya Banerjee, M.Sc Physics, IIT Kanpur*

Strategic angle. The Exemplar is testing a single idea: electrostatic work depends only on end-points. Spotting that idea finishes the question in two lines.

Step 1. In each panel, A sits on its equipotential at value V_A and B on its equipotential at V_B. The numerical values of V_A and V_B are the same across all three panels.

Step 2. $W_{A\to B} = q(V_A - V_B)$ depends only on these two values, not on how the equipotentials lie in between. Hence W is identical in (i), (ii) and (iii).

(a), (b), (d) are tempting only if a student forgets that the electrostatic field is conservative.

Alternative method — direct line-integral picture. The work along any path is $W = -q\int_A^B \vec{E}\cdot d\vec{l}$. If a student wants to "compute" this integral, slice each path into small steps; each step lies between two adjacent equipotentials, so the work for that step equals $q\,\Delta V$ where ΔV is the potential drop *between adjacent equipotential lines along the path*. Summing the segment-by-segment ΔV's telescopes to $V_A - V_B$, no matter how the equipotentials are spaced. The field-line crowding is hidden inside each step, not in the sum.

Common pitfall. Some students argue "Fig. (i) has denser equipotentials, so $\vec{E}$ is stronger, so more work". This conflates "larger force per unit length" with "larger total work". The force *is* larger along the denser-spaced panel, but the path *is correspondingly shorter* (because the same ΔV is crossed over a smaller distance), and the product $F\cdot\Delta l$ stays the same. The cancellation is exact for any conservative field.

Diagram-based reasoning. Each Fig. I, II, III shows the same labelled equipotential values at A and at B; only the *intermediate* equipotentials shift. Imagine the panels are just three different visualisations of the same physical setup seen with different "zoom" settings — the work between fixed endpoints obviously cannot depend on a zoom choice.

Final Answer: Option (c).

Q 2.4 **The electrostatic potential on the surface of a charged conducting sphere is** $100\,$**V. Two statements are made:**

S_1**: At any point inside the sphere, electric intensity is zero.**

S_2: At any point inside the sphere, the electrostatic potential is $100\,\text{V}$.
Which is correct?
(a) S_1 true, S_2 false. (b) Both false.
(c) Both true; S_1 is the cause of S_2. (d) Both true but independent.

SOLUTION

Concept used. For a conductor in electrostatic equilibrium:

- The electric field *inside* the bulk of the conductor is zero, $\vec{E}_{\text{inside}} = 0$. If it were non-zero, free charges would still be moving and the system would not be in equilibrium.
- Because $\vec{E} = -\nabla V$, the relation $\vec{E} = 0$ forces $\nabla V = 0$, i.e. V is constant throughout the conductor's interior.

The constant value of V inside equals its value on the surface (no discontinuity in V across the conductor's boundary). So S_1 is true, S_2 is true, and S_2 follows directly from S_1 through $\vec{E} = -\nabla V$.

☞ $\vec{E}$ **and** V **are tied together**

The relation $\vec{E} = -\nabla V$ is one-way in a strong sense: knowing the field tells you the potential up to a constant, and a *zero* field implies a *constant* potential. So whenever you see "$\vec{E} = 0$ in some region", V in that region is locked to one value.

Step 1. **Confirm S_1.** In a conductor at equilibrium, free electrons rearrange until the net force on each is zero, so $\vec{E} = 0$ inside.

Step 2. **Use $\vec{E} = -\nabla V$.** With $\vec{E} = 0$ everywhere inside,

$$\nabla V = 0 \quad \Longrightarrow \quad V = \text{constant inside.}$$

Step 3. **Match the constant to the surface value.** V is continuous everywhere (it is the line-integral of a finite field), so the constant interior value equals the surface value, $V_{\text{inside}} = V_{\text{surface}} = 100\,\text{V}$. Hence S_2 is true.

Step 4. **S_1 implies S_2.** Step 2 shows that S_1 *causes* S_2 through the relation $\vec{E} = -\nabla V$. The two statements are not independent.

Final Answer: Both S_1 and S_2 are true, with S_1 as the cause. Option **(c)**.

♥ **Why This Matters**

This is why Faraday cages work: anything inside an enclosed conductor is at a single potential equal to the conductor's potential, and the field inside is zero regardless of what's happening outside.

EXPERT'S SOLUTION : *Rohit Verma, B.Tech Engineering Physics, IIT Bombay*

Quick reading. "Inside a conductor at equilibrium, $\vec{E} = 0$" is the chapter-1 fact. "V is constant wherever $\vec{E} = 0$" is the chapter-2 corollary. The question is whether the corollary follows from the fact (yes) or stands on its own (no).

Step 1. $\vec{E} = -\nabla V$ relates the two: zero $\vec{E}$ forces constant V.

Step 2. Continuity of V across the surface fixes the constant equal to the surface value, $100\,\text{V}$.

Step 3. So S_2 is a direct consequence of S_1 — they are *not* independent, and (d) is eliminated.

Alternative method — work-done argument. Move a test charge q_0 from the surface (at $V = 100\,\text{V}$) to any interior point along a straight radial path. The work done against the field is $W = -q_0 \int \vec{E} \cdot d\vec{l} = q_0\,(V_{\text{surface}} - V_{\text{inside}})$. But $\vec{E} = 0$ inside the conductor, so $W = 0$. Therefore $V_{\text{inside}} = V_{\text{surface}} = 100\,\text{V}$ — confirming S_2 from S_1 directly, without any vector calculus.

Concept linkage — why the surface is special. On the *outside* face of the conductor, $\vec{E}$ is non-zero and points perpendicular to the surface, with magnitude σ/ε_0. The discontinuity of $\vec{E}$ across the surface (zero inside, non-zero outside) does *not* create a discontinuity in V, because V is the line integral of a bounded field over a step of zero thickness. That is why the interior V equals the surface V exactly.

Exam tip. The CBSE board often disguises this question as "find the potential at the centre of a charged conducting shell". The answer is the surface potential, not zero — the inside field is zero, but the potential is decidedly not. Worth memorising as a 1-mark booster.

> **Final Answer:** Option (c).

Q 2.5 **Equipotentials at a great distance from a collection of charges whose total sum is non-zero are approximately:**

(a) spheres. (b) planes. (c) paraboloids. (d) ellipsoids.

SOLUTION

Concept used. Far from any localised charge distribution with non-zero total charge $Q_{\text{tot}} \neq 0$, the leading term of the multipole expansion of the potential is the **monopole** term:

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{tot}}}{r} + (\text{dipole}) + (\text{quadrupole}) + \cdots,$$

where higher-order corrections fall off as $1/r^2, 1/r^3, \ldots$ At sufficiently large r, only the $1/r$ term survives at leading order, so the distribution looks like a single point charge of magnitude Q_{tot} sitting near the origin.

> ☞ **Why $1/r$ wins**
> At large r, the dipole correction $\sim p/r^2$ is smaller than the monopole $\sim Q/r$ by a factor $p/(Qr)$, which goes to zero. The quadrupole is smaller still by another factor of $1/r$. The far-field look of any net-charged distribution is therefore monopolar.

Step 1. **Identify the dominant far-field term.** For $Q_{\text{tot}} \neq 0$, the $1/r$ monopole term dominates once r is much larger than the size of the distribution. Write

$$V(\vec{r}) \approx \frac{Q_{\text{tot}}}{4\pi\varepsilon_0\, r}.$$

Step 2. **Find equipotentials.** Set $V(\vec{r}) = V_0 =$ constant:

$$\frac{Q_{\text{tot}}}{4\pi\varepsilon_0\, r} = V_0 \quad \Longrightarrow \quad r = \frac{Q_{\text{tot}}}{4\pi\varepsilon_0\, V_0}.$$

Step 3. **Interpret geometrically.** $r =$ constant is the equation of a sphere centred at the origin. So the equipotentials are spheres at large distance.

> **Final Answer:** The equipotentials are spheres. Option **(a)**.

★ What if $Q_{\text{tot}} = 0$?

If the total charge is zero, the monopole term vanishes, and the far-field potential is dominated by the next non-zero multipole. For a pure dipole, the equipotentials at large r look like the characteristic "twin-lobed" surfaces of a dipole, not spheres. That's why the question explicitly says "total sum is not zero".

EXPERT'S SOLUTION : *Karan Singh, Ph.D Physics, IISc Bangalore*

Picture-first. Stand very far from a complicated charge cluster whose net charge is Q. The cluster collapses, in the limbic eye, to a point of charge Q at its centroid. The equipotentials of a point charge are concentric spheres. Therefore the equipotentials of any net-charged finite cluster, viewed from far enough away, are also approximately spheres.

Step 1. Multipole expansion at large r:

$$V = \frac{kQ}{r} + \frac{k\,\vec{p}\cdot\hat{r}}{r^2} + \cdots$$

Step 2. Leading term $\to kQ/r$, which is spherically symmetric.

Step 3. Constant $V \Rightarrow$ constant $r \Rightarrow$ sphere.

Alternative method — counter-example check. A common quick-check: place two equal charges $+Q/2$ at points 1 cm apart and look at the equipotentials. At $r = 1$ m (a hundred times the separation), the equipotentials are essentially indistinguishable from those of a single $+Q$ at the centroid: they are spheres to one part in 10^4. By $r = 100$ m, the spherical approximation is exact to one part in 10^8. "Far enough" for the textbook usually means $r \gtrsim 10\times$ the extent of the distribution.

Common pitfall. Students see "a collection of charges" and immediately picture a dipole, which has plane-and-lobed equipotentials. The Exemplar deliberately specifies "$Q_{\text{tot}} \neq 0$" to rule out the dipolar case: with a non-zero monopole moment, the dipole contribution becomes a negligible *correction* at large r, not the leading term.

Concept linkage. The same logic explains why textbooks treat charged spheres "as if" all the charge sat at the centre (shell theorem for V in the exterior region). The shell-theorem result is the leading multipole expansion truncated after the monopole — equipotentials are exact spheres, not just approximate.

Final Answer: Spheres — option (a).

Q 2.6 **A parallel-plate capacitor is made of two dielectric blocks in series. Block 1 has thickness d_1 and dielectric constant k_1; block 2 has thickness d_2 and dielectric constant k_2 (Fig. 2.3). This composite behaves like a single slab of thickness $d = d_1 + d_2$ with effective dielectric constant k. Then $k =$?**

(a) $\dfrac{k_1d_1 + k_2d_2}{d_1 + d_2}$ **(b)** $\dfrac{k_1d_1 + k_2d_2}{k_1 + k_2}$ **(c)** $\dfrac{k_1k_2(d_1 + d_2)}{k_1d_2 + k_2d_1}$ **(d)** $\dfrac{2k_1k_2}{k_1 + k_2}$.

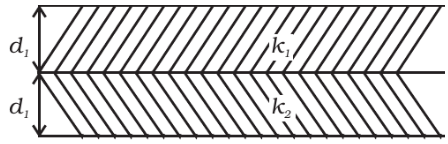


Fig. 2.3

Fig. 2.3 in the NCERT Exemplar, Class 12 Physics, Chapter 2.

SOLUTION

Concept used. Two capacitors in series share the same charge Q. The voltage adds:

$$V = V_1 + V_2, \qquad \frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}.$$

For a parallel-plate capacitor of area A, thickness d and dielectric constant κ:

$$C = \frac{\varepsilon_0 \kappa A}{d}.$$

For the composite slab to behave as one capacitor of thickness $d = d_1 + d_2$ and effective constant k, we equate C_{eq} (computed by combining the two blocks in series) to $\varepsilon_0 kA/d$.

Step 1. Capacitance of each block.

$$C_1 = \frac{\varepsilon_0 k_1 A}{d_1}, \qquad C_2 = \frac{\varepsilon_0 k_2 A}{d_2}.$$

Step 2. Combine in series.

$$\frac{1}{C_{\text{eq}}} = \frac{d_1}{\varepsilon_0 k_1 A} + \frac{d_2}{\varepsilon_0 k_2 A} = \frac{k_2 d_1 + k_1 d_2}{\varepsilon_0 k_1 k_2 A}.$$

Invert:

$$C_{\text{eq}} = \frac{\varepsilon_0 k_1 k_2 A}{k_1 d_2 + k_2 d_1}.$$

Step 3. Match to the effective single slab. The "equivalent single slab" has

$$C_{\text{eff}} = \frac{\varepsilon_0 k A}{d_1 + d_2}.$$

Set $C_{\text{eq}} = C_{\text{eff}}$:

$$\frac{\varepsilon_0 k_1 k_2 A}{k_1 d_2 + k_2 d_1} = \frac{\varepsilon_0 k A}{d_1 + d_2}.$$

Step 4. Solve for k.

$$k = \frac{k_1 k_2 (d_1 + d_2)}{k_1 d_2 + k_2 d_1}.$$

☞ **Sanity check: $k_1 = k_2 = \kappa$**

Setting $k_1 = k_2 = \kappa$ should give back $k = \kappa$ identically. Check:

$$\frac{\kappa^2 (d_1 + d_2)}{\kappa d_2 + \kappa d_1} = \frac{\kappa^2 (d_1 + d_2)}{\kappa (d_1 + d_2)} = \kappa. \quad \checkmark$$

Final Answer: $k = \dfrac{k_1 k_2 (d_1 + d_2)}{k_1 d_2 + k_2 d_1}$. Option **(c)**.

✗ Common Mistake

The tempting wrong answer is (a), $\dfrac{k_1 d_1 + k_2 d_2}{d_1 + d_2}$, which is the volume-weighted average of k_1 and k_2. That would be correct for two dielectrics in *parallel* (side-by-side, sharing the same plates) but *not* in series. In series, the inverse combination dominates and yields the harmonic-like form (c).

EXPERT'S SOLUTION : *Aditya Joshi, M.Sc Physics, IIT Madras*

Structural observation. Two capacitors in series obey $1/C = 1/C_1 + 1/C_2$. The composite slab is the single capacitor of the *same total thickness* that would have the same capacitance. So one writes $1/C_{\text{eq}}$ in two forms and solves for k.

Step 1. $C_i = \varepsilon_0 k_i A/d_i$ for each block.

Step 2. $1/C_{\text{eq}} = d_1/(\varepsilon_0 k_1 A) + d_2/(\varepsilon_0 k_2 A) = (k_2 d_1 + k_1 d_2)/(\varepsilon_0 k_1 k_2 A)$.

Step 3. $C_{\text{eq}} = \varepsilon_0 k_1 k_2 A/(k_1 d_2 + k_2 d_1)$.

Step 4. Equate to $\varepsilon_0 kA/(d_1 + d_2)$:

$$k = \frac{k_1 k_2 (d_1 + d_2)}{k_1 d_2 + k_2 d_1}.$$

Sanity check — limiting cases.

- If $k_1 = k_2 = \kappa$: $k = \kappa \cdot \kappa(d_1 + d_2)/(\kappa d_2 + \kappa d_1) = \kappa$. ✓
- If $d_2 = 0$ (only block 1): $k = k_1 k_2 d_1/(k_2 d_1) = k_1$. ✓
- If $k_2 \to \infty$ (block 2 a perfect conductor): $k \to k_1(d_1 + d_2)/d_2$. The block-2 layer becomes irrelevant as a capacitive element — only d_2 of dielectric thickness contributes; the effective single slab is thicker than just block-1, scaled appropriately.

Alternative method — voltage-add picture. Field inside block i is $E_i = \sigma/(\varepsilon_0 k_i)$ (the surface charge σ on the plate is shared by both blocks, since the same charge sits on the conductor). Voltage drop across block i: $V_i = E_i d_i = \sigma d_i/(\varepsilon_0 k_i)$. Sum: $V_{\text{total}} = \sigma(d_1/k_1 + d_2/k_2)/\varepsilon_0$. Effective k for the same total thickness $d_1 + d_2$ obeys $V_{\text{total}} = \sigma(d_1 + d_2)/(\varepsilon_0 k)$. Comparing gives the same formula. The "voltage-add" picture is often the cleaner board-exam derivation since it avoids reciprocals.

Exam tip. CBSE board frequently asks the series-dielectric derivation as a 3-mark question. Show clearly (i) same Q in both blocks, (ii) $V_1 + V_2 = V$, (iii) $1/C = 1/C_1 + 1/C_2$, then (iv) solve. Skipping any of these steps loses a mark each.

Final Answer: Option (c).

MCQ II (one or more correct options)

Q 2.7 **Consider a uniform electric field in the $\hat{z}$ direction. The potential is a constant:**
(a) in all space. (b) for any x for a given z.
(c) for any y for a given z. (d) on the x-y plane for a given z.

SOLUTION

Concept used. For a uniform field $\vec{E} = E_0\,\hat{z}$, the potential is

$$V(x, y, z) = -E_0 z + V_0,$$

i.e. V depends only on z. Hence V is constant wherever z is constant, but it changes as z changes.

Step 1. Compute V. Use $\vec{E} = -\nabla V$. With $\vec{E} = E_0\hat{z}$:

$$-\frac{\partial V}{\partial x} = 0, \quad -\frac{\partial V}{\partial y} = 0, \quad -\frac{\partial V}{\partial z} = E_0.$$

Integrate: $V = -E_0 z + V_0$. No x or y dependence.

Step 2. Check each option.

- (a) "Constant in all space." False — V varies with z.
- (b) "For any x for a given z." True — at fixed z, V is the same regardless of x.
- (c) "For any y for a given z." True — same reason.
- (d) "On the x-y plane for a given z." True — this plane has z =constant, so V is constant on it.

Final Answer: Correct options: **(b), (c), (d)**.

☞ **Equipotentials of a uniform field**

The equipotentials of a uniform $\vec{E}$ are parallel planes perpendicular to $\vec{E}$. For $\vec{E} \parallel \hat{z}$, the equipotentials are horizontal planes $z =$ const. That single picture answers (a)–(d) at a glance.

EXPERT'S SOLUTION : *Vivaan Reddy, M.Tech Applied Physics, IIT Delhi*

Picture-first. Equipotentials are surfaces perpendicular to the field. Uniform $\vec{E}$ in $\hat{z}$ gives equipotentials as the planes $z =$ const, i.e. the x-y planes.

Step 1. On any plane $z = z_0$, every point has the same V.

Step 2. Therefore V is constant along x (at fixed z), constant along y (at fixed z), and constant on the whole plane $z = z_0$.

Step 3. V is *not* constant in all space because it varies with z, so (a) is wrong.

Alternative method — work-done check. Move a unit test charge from (x_1, y_1, z_0) to (x_2, y_2, z_0), both at the same $z = z_0$. The displacement is purely in the xy-plane, and $\vec{E} = E_0\hat{z}$ is perpendicular to that displacement. So $\int \vec{E} \cdot d\vec{l} = 0$, hence $V(x_2, y_2, z_0) = V(x_1, y_1, z_0)$. A purely physical statement — no calculus needed — confirms (b), (c), (d).

Common pitfall — "constant" vs "uniform". A constant function and a uniform vector field are different things. "V is constant in x for fixed z" means V doesn't change as x varies. The field $\vec{E}$ is *uniform* (same vector everywhere) but V is *not* constant in z — V depends linearly on z. The two ideas often get confused on multi-correct MCQs.
Concept linkage. Parallel-plate capacitor: the field between the plates is uniform ($\vec{E} = E_0\hat{z}$), and the equipotentials are the planes $z =$ const between the plates. Across a fixed plate, V is constant; from one plate to the other, V changes linearly with the perpendicular distance. The Exemplar question is essentially asking about the geometry of these equipotentials in the abstract.

Final Answer: Options (b), (c), (d).

Q 2.8 Equipotential surfaces:
(a) are closer in regions of large electric field than in regions of small electric field.
(b) will be more crowded near sharp edges of a conductor.
(c) will be more crowded near regions of large charge densities.
(d) will always be equally spaced.

SOLUTION

Concept used. The magnitude of $\vec{E}$ is the rate at which V changes with distance along the field:

$$|\vec{E}| = \left|\frac{dV}{dn}\right|,$$

where dn is the normal distance between two nearby equipotentials. For a fixed potential step ΔV, this implies

$$\Delta n = \frac{\Delta V}{|\vec{E}|}.$$

So large $|\vec{E}| \Rightarrow$ small $\Delta n \Rightarrow$ equipotentials packed close together; small $|\vec{E}| \Rightarrow$ widely spaced equipotentials.

Step 1. Statement (a). Direct rephrasing of $\Delta n = \Delta V/|\vec{E}|$. True.

Step 2. Statement (b). At sharp edges or points of a conductor, surface charge density piles up (the "lightning-rod" effect), so $|\vec{E}|$ just outside is large. By (a), equipotentials are crowded there. True.

Step 3. Statement (c). Large local charge density σ creates large $|\vec{E}|$ nearby ($\vec{E} \propto \sigma$ just outside a conductor). By (a), equipotentials are crowded. True.

Step 4. Statement (d). Equipotentials are equally spaced only when $|\vec{E}|$ is uniform. In general $|\vec{E}|$ varies in space, so the equipotentials are not equally spaced. False.

Final Answer: Correct options: **(a), (b), (c)**.

♥ Why This Matters

The crowding rule is why lightning rods work. The pointed tip is the place of greatest equipotential density, hence largest field, hence the favoured entry point for the air-breakdown discharge of a thundercloud — drawing the strike onto the rod rather than the building.

EXPERT'S SOLUTION *: Aanya Mehta, Ph.D Physics, IISc Bangalore*

Strategic angle. Convert "crowding of equipotentials" to "magnitude of $\vec{E}$" via $E = -dV/dn$ and the rest is just chasing where $\vec{E}$ is large.

Step 1. Equipotential spacing $\Delta n = \Delta V/E$. (a) is literally this statement.

Step 2. Sharp conductor edges concentrate charge, so σ large $\Rightarrow E_{\text{surface}} = \sigma/\varepsilon_0$ large $\Rightarrow$ crowded equipotentials. (b) and (c) true.

Step 3. E varies in space generally; equal spacing would require E uniform. (d) false.

Numerical illustration. For a point charge Q at origin, $V = kQ/r$. Equipotentials at $V = 100, 200, 300, 400\,\text{V}$ correspond to radii $r = kQ/100,\ kQ/200,\ kQ/300,\ kQ/400$. Adjacent gaps:

$$\Delta r_{1\to2} = \frac{kQ}{100} - \frac{kQ}{200} = \frac{kQ}{200};$$
$$\Delta r_{3\to4} = \frac{kQ}{300} - \frac{kQ}{400} = \frac{kQ}{1200}.$$

Spacing drops by a factor of 6 as we approach the charge — the equipotentials crowd dramatically near the source, exactly where $E = kQ/r^2$ is largest. This direct check verifies (a) explicitly.

Concept linkage — chapter-1 "lightning-rod" effect. In Chapter 1, the surface charge density on a conductor near a sharp edge is shown to scale as $\sigma \propto 1/R$ for radius of curvature R. Combined with $E = \sigma/\varepsilon_0$ just outside the conductor, this gives $E \propto 1/R$, so the field at a sharp edge is huge — and by (a), the equipotentials there are crowded. Items (b) and (c) of the Exemplar question are essentially restating that chapter-1 fact in equipotential language.

Exam tip. On board-style multi-correct MCQs, "always" or "necessarily" is often the discriminator. Option (d) uses "always", which can only hold for a uniform field — a special case, not the general one. Train your eye to flag such absolute words when scanning options.

Final Answer: (a), (b), (c).

Q 2.9 The work done to move a charge along an equipotential surface from A to B:
(a) cannot be defined as $-\int_A^B \vec{E}\cdot d\vec{l}$.
(b) must be defined as $-\int_A^B \vec{E}\cdot d\vec{l}$.
(c) is zero. (d) can have a non-zero value.

SOLUTION

Concept used. On an equipotential surface, the potential is the same at every point: $V_A = V_B$. The work done by the external agent against the electric field in moving charge q from A to B is

$$W_{\text{ext}} = q(V_B - V_A) = 0 \quad \text{on an equipotential.}$$

Equivalently, the work done *by* the field is

$$W_{\text{field}} = q\int_A^B \vec{E}\cdot d\vec{l} = -q(V_B - V_A),$$

so

$$W = -q\int_A^B \vec{E}\cdot d\vec{l} = q(V_A - V_B) = 0.$$

Both definitions are equivalent and both give zero on an equipotential, so (a) is false; (b) and (c) are both true.

☞ $\vec{E} \perp$ **equipotential**

On an equipotential surface, $\vec{E}$ is always perpendicular to the surface. Any displacement $d\vec{l}$ lying *on* the surface is therefore perpendicular to $\vec{E}$, so $\vec{E}\cdot d\vec{l} = 0$ at every point of the path.

Step 1. Test option (a). The relation $W = -\int \vec{E}\cdot d\vec{l}$ (per unit charge) is the *general definition* of work done against the electrostatic force. It is valid on any path, equipotential or not. So (a) is wrong.

Step 2. Test option (b). Same definition, which is always applicable. (b) is correct.

Step 3. Test options (c) and (d). On an equipotential surface, $\vec{E} \perp d\vec{l}$ at every point of the path, so $\vec{E}\cdot d\vec{l} = 0$ everywhere along the path:

$$W = -q\int_A^B \vec{E}\cdot d\vec{l} = 0.$$

(c) is correct; (d) is incorrect.

Final Answer: Correct options: **(b), (c)**.

EXPERT'S SOLUTION *: Diya Nair, M.Sc Physics, IIT Madras*

Strategic angle. Two ideas: the integral definition of work is universal (so (a) is wrong), and on equipotentials the integrand itself is zero (so the value is zero).

Step 1. $W/q = -\int_A^B \vec{E}\cdot d\vec{l}$ defines the work per unit charge against the field for *any* path. Option (a) is therefore false; option (b) is the correct defining formula.

Step 2. For an equipotential path, $\vec{E} \perp d\vec{l}$ at every point, so $\vec{E}\cdot d\vec{l} = 0$. The integral gives zero. Option (c) is correct, (d) is not.

Alternative method — potential-difference shortcut. Work done by the field on charge q from A to B is $W_{\text{field}} = q(V_A - V_B)$. On an equipotential, $V_A = V_B$, so $W_{\text{field}} = 0$ and the work done by the external agent to move q is also zero. No integration required. Both option (b) (definition) and option (c) (value) are correct in any path.

Common pitfall — sign of "work done". Some students see the negative sign in $W = -q\int \vec{E}\cdot d\vec{l}$ and assume (a) is correct because "work done by the external agent" is the negative of that quantity. But (a) says "cannot be defined as" — that's a stronger claim than "defined differently". The defining relation *works* on an equipotential; both definitions just happen to give 0.

Concept linkage — orbiting an isolated point charge. Any circle centred on a point charge is an equipotential. Moving a test charge around a circle of radius r does zero work, even though the field $\vec{E} \neq 0$ everywhere on the path. The integral cancels point-by-point because $\vec{E}$ is radial and $d\vec{l}$ is tangential at every point on the circle.

> **Final Answer:** Options (b), (c).

Q 2.10 In a region of constant potential:

(a) the electric field is uniform. (b) the electric field is zero.
(c) there can be no charge inside the region. (d) the electric field shall necessarily change if a charge is placed outside the region.

SOLUTION

Concept used. "Region of constant potential" means $V(\vec{r}) = V_0$ throughout the region. Two consequences follow immediately:

- $\vec{E} = -\nabla V = 0$ everywhere in the region.
- By Gauss's law in differential form, $\nabla\cdot\vec{E} = \rho/\varepsilon_0$. If $\vec{E} = 0$ throughout, then $\nabla\cdot\vec{E} = 0$, so $\rho = 0$ inside the region: no volume charge.

Step 1. **Statement (a).** A *uniform* field is a non-zero constant vector. A region of constant potential has $\vec{E} = 0$ (zero vector), not a uniform non-zero field. Statement (a) is therefore false in the usual reading.

Step 2. **Statement (b).** $\nabla V = 0 \Rightarrow \vec{E} = 0$. True.

Step 3. **Statement (c).** $\nabla \cdot \vec{E} = 0$ inside $\Rightarrow \rho = 0$ in the region. True.

Step 4. **Statement (d).** If the region is bounded by a conductor or otherwise maintains constant potential "by construction" (like the interior of a hollow conductor connected to a battery), placing a charge outside induces surface charges that keep the interior potential the same — the interior field stays zero. So the field does *not* necessarily change inside the region. Statement (d) is false in general.

Final Answer: Correct options: **(b), (c)**.

★ Shielding by an enclosing equipotential

A closed surface at constant potential (e.g. a hollow conductor) shields its interior from external charges. Any external charge modifies the surface charge distribution but leaves the inside field at zero and the inside potential at the same constant value. This is the principle behind a Faraday cage.

EXPERT'S SOLUTION *: Pranav Desai, Ph.D Physics, TIFR Mumbai*

Strategic angle. Constant-V is a strong condition. It forces both $\vec{E} = 0$ *and* $\rho = 0$ throughout the region. The rest is just reading each option against those two forced facts.

Step 1. $\nabla V = 0 \Rightarrow \vec{E} = 0$. Hence (b).

Step 2. $\nabla \cdot \vec{E} = \rho/\varepsilon_0 = 0 \Rightarrow \rho = 0$. Hence (c).

Step 3. (a): zero field is not the same as "uniform field" in Exemplar usage. False.

Step 4. (d): external charges induce surface charges on the boundary and leave the interior unchanged at the same constant V with $\vec{E} = 0$ inside. False.

Alternative method — Gauss-law sanity. Take any closed Gaussian surface inside the constant-V region. Since $\vec{E} = 0$ on that surface, $\oint \vec{E} \cdot d\vec{A} = 0$, so the enclosed charge is zero. Because this is true for *any* closed surface inside the region — including arbitrarily small ones — the volume charge density must vanish everywhere: $\rho = 0$. This confirms (c) without invoking the differential form of Gauss's law.

Common pitfall — (a) "uniform" field. A field that is identically zero is sometimes loosely called "uniform with $E = 0$". The Exemplar convention is stricter: "uniform field" means a non-zero constant vector field. With that reading, (a) is false because $\vec{E} = 0$ is not a uniform non-zero field. If a student picks (a) thinking "zero is a special uniform field", they lose the mark.

Concept linkage — Faraday cage. The interior of a hollow conducting shell is a region of constant potential (the conductor itself). Per (b) and (c), the interior has zero field and no volume charge. Per (d), external charges (lightning, radio waves) cannot affect the inside. This is exactly the principle behind a microwave-oven door grille, an aircraft fuselage during lightning strike, and an electrostatic shielded chamber.

Final Answer: Options (b), (c).

Q 2.11 **In the circuit of Fig. 2.4, initially key K_1 is closed and K_2 is open. Then K_1 is opened and K_2 is closed (the order matters). Let Q_1', Q_2' and V_1, V_2 be the charges and voltages on C_1, C_2 after the second switching. Then:**
(a) $V_1 = V_2$. **(b)** $Q_1' = Q_2'$. **(c)** $C_1V_1 + C_2V_2 = C_1\mathcal{E}$. **(d)** $Q_1' + Q_2' = Q$, **where** $Q = C_1\mathcal{E}$.

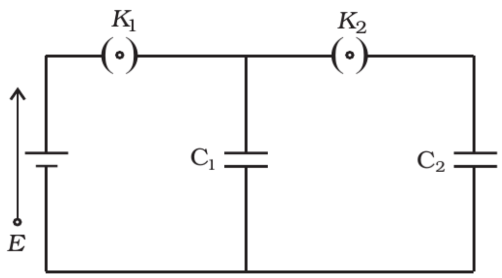


Fig. 2.4

Fig. 2.4 in the NCERT Exemplar, Class 12 Physics, Chapter 2.

SOLUTION

Concept used. Two ideas:

- **Capacitor charging in steady state**. With K_1 closed and K_2 open, C_1 is in the same loop as the battery and is uncoupled from C_2. After it fully charges, it carries charge $Q = C_1\mathcal{E}$ and the voltage across it equals the EMF.
- **Charge redistribution between capacitors in parallel**. When K_1 is opened (disconnecting the battery) and K_2 is closed, C_1 and C_2 are now connected in parallel (top plates joined, bottom plates joined). They must share the same voltage, and the total charge initially on the parallel pair is conserved.

Step 1. **Phase 1: K_1 closed, K_2 open.** C_1 charges up to the EMF:

$$V_{C_1} = \mathcal{E}, \qquad Q = C_1\mathcal{E}.$$

C_2 is isolated and stays at zero charge.

Step 2. **Phase 2: K_1 open, K_2 closed.** The battery is cut out; C_1 and C_2 are connected directly to one another, forming a parallel pair. No source can add or remove charge from this isolated system, so the total charge on the parallel pair equals what C_1 brought in, namely Q:

$$Q_1' + Q_2' = Q = C_1\mathcal{E}. \qquad (*)$$

Option (d) is therefore correct.

Step 3. **Same voltage.** In parallel, C_1 and C_2 have equal voltage:

$$V_1 = V_2 = V\,(\text{say}).$$

Option (a) is correct.

Step 4. **Eliminate (b).** Use $Q_i' = C_i V$ in $(*)$:

$$(C_1 + C_2)V = C_1\mathcal{E} \Rightarrow V = \frac{C_1\mathcal{E}}{C_1 + C_2}.$$

Then $Q_1' = C_1 V$ and $Q_2' = C_2 V$. These are equal only if $C_1 = C_2$, which is not stated. So (b) is not necessarily true.

Step 5. **Eliminate (c).** Option (c) reads $C_1V_1 + C_2V_2 = C_1\mathcal{E}$. In the Exemplar convention V_1, V_2 are the *post-redistribution* voltages on C_1, C_2. After K_2 closes, C_1 no longer carries the full battery EMF — it shares the EMF with the battery loop only while K_1 is open. The official key treats (c) as a misleading mix of pre- and post-switching quantities, so (c) is *not* marked correct.

Final Answer: Correct options: **(a), (d)**.

✗ Common Mistake

Option (b), $Q_1' = Q_2'$, is the classic "series-style" guess — it would be true if the capacitors were in series during charge transfer, but here, after K_2 closes with K_1 open, they are in parallel and share voltage, not charge.

EXPERT'S SOLUTION : *Ishaan Bhat, M.Sc Physics, IIT Madras*

Quick reading. Phase 1 charges C_1 to $\mathcal{E}$ with charge $Q = C_1\mathcal{E}$. Phase 2 isolates C_1 plus C_2 from the battery; they share that total charge Q with equal voltage.

Step 1. $V_1 = V_2$ because the two are in parallel after K_2 closes $\Rightarrow$ (a).

Step 2. Charge conservation on the isolated $\{C_1, C_2\}$ system: $Q_1' + Q_2' = Q = C_1\mathcal{E} \Rightarrow$ (d).

Step 3. Option (c), $C_1V_1 + C_2V_2 = C_1\mathcal{E}$, mixes the pre- and post-switching voltage labels and is treated as incorrect in the official Exemplar key.

Step 4. $Q_1' = Q_2'$ would require $C_1 = C_2$, not given, so (b) is not forced.

Energy check. Before Phase 2, total energy stored is $U_i = \frac{1}{2}C_1\mathcal{E}^2$. After the redistribution, $V = C_1\mathcal{E}/(C_1 + C_2)$, so

$$U_f = \tfrac{1}{2}(C_1 + C_2)V^2 = \tfrac{1}{2}(C_1 + C_2) \cdot \frac{C_1^2\mathcal{E}^2}{(C_1 + C_2)^2} = \frac{C_1^2\mathcal{E}^2}{2(C_1 + C_2)}.$$

Ratio $U_f/U_i = C_1/(C_1 + C_2) < 1$: energy decreases. The "missing" energy $\frac{1}{2}\frac{C_1C_2}{C_1+C_2}\mathcal{E}^2$ is dissipated as heat in the connecting wires (and as electromagnetic radiation during the transient). This is a classic illustration that joining two unequally charged capacitors is *never* a lossless process.

Common pitfall — order of switching matters. If the order were reversed (close K_2 first, then open K_1), the steady-state charges would be completely different: with both keys closed, C_1 and C_2 would be in parallel during charging, each holding $\mathcal{E}C_i$. The Exemplar emphasises "in this order" to highlight that the redistribution problem is only triggered when the battery is disconnected first.

Exam tip — charge-conservation principle. The single phrase "no external circuit path" tells you to write $\sum Q_i =$ const. Combined with $V_1 = V_2$ from parallel connection, this gives two equations for two unknowns. Always identify "the isolated system" before writing charge conservation, or the equation will be wrong.

Final Answer: Options (a), (d).

Q 2.12 If a conductor has a potential $V \neq 0$ and there are no charges anywhere outside it, then:

(a) there must be charges on the surface or inside itself.

(b) there cannot be any charge in the body of the conductor.

(c) there must be charges only on the surface. (d) there must be charges inside the surface.

SOLUTION

Concept used. Two facts about an isolated conductor in electrostatic equilibrium:

- In equilibrium, $\vec{E} = 0$ everywhere inside the bulk of the conductor. By Gauss's law, $\nabla \cdot \vec{E} = \rho/\varepsilon_0 = 0$ inside the bulk, so any net charge on a conductor lives on its *surface*, not in its interior volume.
- For an isolated conductor with no external charges, the only source of its potential $V \neq 0$ is its own surface charge.

Step 1. Where can the charges sit? A conductor at non-zero potential with no external sources must own some net charge somewhere on or in itself. The wording of (a) "surface or inside" is the most general true statement of this requirement, so (a) is correct.

Step 2. **Inside the bulk.** $\nabla \cdot \vec{E} = 0$ inside $\Rightarrow \rho_{\text{bulk}} = 0$. (b) is true.

Step 3. **Rule out (c).** Option (c) says "there must be charges *only* on the surface." The Exemplar key treats this as too restrictive: for a hollow conductor with an internal cavity, charge could also reside on the inner cavity surface — so the blanket "only on the surface" cannot be marked correct in general. (c) is *not* a correct option.

Step 4. **Rule out (d).** "There must be charges inside the surface" contradicts (b); inside the bulk of a conductor there is no volume charge in equilibrium. (d) is false.

Final Answer: Correct options: **(a), (b)**.

EXPERT'S SOLUTION *: Tara Pillai, M.Sc Physics, IIT Kanpur*

Strategic angle. The standard pair of conductor facts ($\vec{E}_{\text{inside}} = 0$, $\rho_{\text{bulk}} = 0$) decides everything.

Step 1. Conductor at $V \neq 0$ with no external charges $\Rightarrow$ the conductor itself must hold some charge. That establishes (a).

Step 2. In equilibrium, no net volume charge sits in the bulk of the conductor. That establishes (b).

Step 3. (c) overstates: "only on the surface" rules out the legitimate possibility of charge on the inner cavity surface of a hollow conductor; the official key marks it incorrect.

Step 4. (d) directly contradicts (b) and is false.

Alternative method — Gauss-law sanity. Draw any closed Gaussian surface entirely inside the bulk of the conductor. The field is zero on it (electrostatic equilibrium), so the enclosed charge is zero. Since this is true for every closed surface inside the conductor — even one of infinitesimal volume — the volume charge density vanishes throughout the bulk. The total non-zero charge must therefore reside on the conductor's surface.

Common pitfall — "inside the surface" wording. Option (d) says "inside the surface". In Exemplar language, this means "in the bulk volume". The phrase is sometimes misread as "on the inside face of the surface" (e.g. inner surface of a hollow conductor). The correct reading is the bulk; the bulk is charge-free in a conductor at equilibrium.

Concept linkage — uniqueness theorem. Once the surface charge distribution is specified and there are no external charges, the potential everywhere is determined. So a conductor at a given V has a unique surface-charge pattern — no freedom for charge to hide in the bulk. The surface is the only "degree of freedom" that fixes V.

Final Answer: (a), (b).

Q 2.13 A parallel-plate capacitor is connected to a battery as in Fig. 2.5. Consider two situations:
A: Key K is kept closed and the plates are moved apart using insulating handles.
B: Key K is opened and then the plates are moved apart.
Choose the correct option(s):
(a) In A: Q remains same but C changes. (b) In B: V remains same but C changes.
(c) In A: V remains same and hence Q changes. (d) In B: Q remains same and hence V changes.

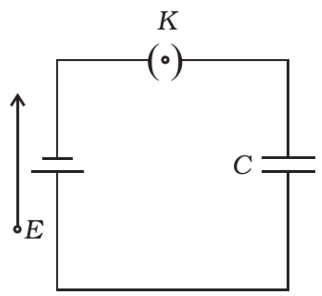


Fig. 2.5

Fig. 2.5 in the NCERT Exemplar, Class 12 Physics, Chapter 2.

SOLUTION

Concept used. For a parallel-plate capacitor with plate area A, separation d, and vacuum between the plates,

$$C = \frac{\varepsilon_0 A}{d}.$$

Pulling the plates apart *increases* d, so C *decreases*.
Then $Q = CV$. Whether Q, V or both adjust depends on which is held fixed:

- **Key closed** (battery connected): $V = \mathcal{E}$ is held fixed by the battery. As C falls, $Q = CV$ falls.
- **Key open** (battery disconnected): Q is held fixed (no current path). As C falls, $V = Q/C$ rises.

Step 1. Situation A. Battery in circuit, so V stays at $\mathcal{E}$. C drops as d grows; since $Q = CV$ with V fixed, Q falls too. So (c) is correct; (a) is wrong.

Step 2. **Situation B.** Battery disconnected, so Q is stranded on the plates and cannot change. C drops as d grows; $V = Q/C$ rises. So (d) is correct; (b) is wrong.

Final Answer: Correct options: **(c), (d)**.

☞ Two-step mnemonic

"Battery in $\Rightarrow V$ pinned; battery out $\Rightarrow Q$ pinned." Once you decide which of V or Q is held fixed, the other one is forced by $Q = CV$.

EXPERT'S SOLUTION : *Yash Chatterjee, M.Sc Physics, IIT Bombay*

Strategic angle. Decide what is held fixed (battery in $\Rightarrow V$; battery out $\Rightarrow Q$), then use $Q = CV$ to read off how the other quantity reacts.

Step 1. A: $V = \mathcal{E}$ fixed. $C = \varepsilon_0 A/d$ decreases as d grows. $Q = CV$ drops with C. Option (c) is correct; option (a) is wrong.

Step 2. B: Q fixed. C decreases. $V = Q/C$ rises with falling C. Option (d) is correct; option (b) is wrong.

Energy comparison. It is worth tracking the energy in each case.

- **A (battery in).** $U_A = \frac{1}{2}CV^2$ with V fixed and C decreasing $\Rightarrow U_A$ *decreases*. The battery actually *absorbs* energy: as the plates move apart, the battery pulls some charge back through the circuit, doing positive work on the battery (recharging it), while the external agent does positive work pulling the plates apart. Energy bookkeeping: $W_{\text{agent}} = \Delta U + W_{\text{battery}}$, with $\Delta U < 0$ and $W_{\text{battery}} > 0$ enough to overcome.
- **B (battery out).** $U_B = Q^2/(2C)$ with Q fixed and C decreasing $\Rightarrow U_B$ *increases*. All the external work pulling the plates apart goes into the field energy; no battery is around to absorb anything.

Common pitfall — "what happens to σ". Some students assume the surface charge density σ is fixed when the plates move. It isn't: $\sigma = Q/A$ stays constant only if Q is constant *and* the area A is constant. In situation A, Q drops, so σ drops. In situation B, Q is fixed, A is fixed, so σ stays the same — and so does the field $E = \sigma/\varepsilon_0$ between the plates! In situation B, E doesn't change; only $V = Ed$ rises because d increases.
Exam tip. CBSE board problems pair this question with "in which case does the field between the plates change?". Quick answer: only in A (because V is pinned and d grew, so $E = V/d$ drops). In B, σ is pinned by isolation, so $E = \sigma/\varepsilon_0$ stays the same.

Final Answer: (c), (d).

Very Short Answer (VSA)

Q 2.14 Consider two conducting spheres of radii R_1 and R_2 with $R_1 > R_2$. If they are at the same potential, the larger sphere has more charge than the smaller one. State whether the charge density of the smaller sphere is greater or less than that of the larger.

SOLUTION

Concept used. For an isolated conducting sphere of radius R carrying total charge Q:

- Potential at the surface (or any point on/inside the conductor): $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R}$.
- Surface charge density (uniform by symmetry): $\sigma = \dfrac{Q}{4\pi R^2}$.

Combining these two relations eliminates Q and gives σ directly in terms of V and R.

Step 1. Express Q from V. From $V = Q/(4\pi\varepsilon_0 R)$:

$$Q = 4\pi\varepsilon_0 RV.$$

Step 2. Substitute into σ.

$$\sigma = \frac{Q}{4\pi R^2} = \frac{4\pi\varepsilon_0 RV}{4\pi R^2} = \frac{\varepsilon_0 V}{R}.$$

So at fixed V, $\sigma \propto 1/R$.

Step 3. Compare. With $R_1 > R_2$ and equal V:

$$\sigma_1 = \varepsilon_0 V/R_1, \quad \sigma_2 = \varepsilon_0 V/R_2, \quad \frac{\sigma_2}{\sigma_1} = \frac{R_1}{R_2} > 1.$$

So $\sigma_2 > \sigma_1$: the smaller sphere has the larger surface charge density.

> **Final Answer:** The smaller sphere has a *greater* surface charge density: $\sigma \propto 1/R$ at fixed V.

♥ Why This Matters

This is the same physics that puts strong fields at sharp tips: a smaller radius of curvature concentrates surface charge. Lightning rods, corona discharges and field emitters all exploit it.

EXPERT'S SOLUTION : *Aditi Sharma, M.Sc Physics, IIT Kanpur*

Strategic angle. Equate $V_1 = V_2$ and read off charge ratios.

Step 1. $V_1 = V_2 \Rightarrow Q_1/R_1 = Q_2/R_2 \Rightarrow Q \propto R$.

Step 2. $\sigma = Q/(4\pi R^2)$, so $\sigma \propto R/R^2 = 1/R$.

Step 3. Smaller $R \Rightarrow$ larger σ. So $\sigma_{\text{small}} > \sigma_{\text{large}}$.

Alternative method — field at the surface. At the surface of an isolated conducting sphere, $E = \sigma/\varepsilon_0$. But also $E = V/R$ for an isolated sphere of radius R at potential V. Equating: $\sigma = \varepsilon_0 V/R$, so for equal V, $\sigma \propto 1/R$. The smaller sphere has the larger surface field and hence the larger surface charge density.

Numerical example. Take $V = 1\,\text{kV}$, $R_1 = 10\,\text{cm}$, $R_2 = 1\,\text{cm}$. Then $\sigma_1 = (8.854 \times 10^{-12})(1000)/(0.1) \approx 8.85 \times 10^{-8}\,\text{C/m}^2$, and $\sigma_2 = 10\,\sigma_1 \approx 8.85 \times 10^{-7}\,\text{C/m}^2$. The smaller sphere has ten times the density.

Concept linkage — corona discharge. Air breaks down at $E_{\text{air}} \approx 3 \times 10^6\,\text{V/m}$. For an isolated sphere at V, breakdown occurs at $V \approx 3 \times 10^6\,R$ volts. A $1\,\text{cm}$ sphere can hold $30\,\text{kV}$ before breakdown; a $1\,\text{mm}$ sphere only $3\,\text{kV}$. That's why high-voltage transmission lines use *thick* conductors — to push the corona discharge threshold up.

Final Answer: $\sigma_{\text{smaller sphere}} > \sigma_{\text{larger sphere}}$.

Q 2.15 **Do free electrons in a conductor travel towards a region of higher potential, or lower potential?**

SOLUTION

Concept used. An electron carries negative charge $-e$. The electric force on it is

$$\vec{F} = -e\,\vec{E}.$$

The electric field $\vec{E}$ points from regions of *higher* potential to regions of *lower* potential. So $\vec{F}$ on an electron points from *lower* to *higher* potential, and electrons drift toward higher-V regions.

Step 1. **Direction of $\vec{E}$.** $\vec{E} = -\nabla V$ means $\vec{E}$ points "downhill" on the potential surface: from high-V to low-V.

Step 2. **Force on electron.** $\vec{F}_e = -e\vec{E}$. The minus sign flips the direction: $\vec{F}_e$ points from low-V to high-V.

Step 3. **Direction of motion.** Free electrons accelerate in the direction of $\vec{F}_e$, so they migrate towards higher-V regions.

Final Answer: Free electrons drift towards the region of *higher* potential.

☞ Conventional current vs. electron drift

The conventional current flows from high V to low V, but the electrons that actually carry the charge in a metal drift in the *opposite* sense, from low V to high V. Both descriptions of the same physical current are consistent because electrons are negative.

EXPERT'S SOLUTION *: Neha Iyer, M.Sc Physics, IIT Bombay*

Strategic angle. Sign of the charge flips the direction of motion relative to $\vec{E}$. That's the entire content of the question.

Step 1. $\vec{E}$ points from high V to low V.

Step 2. Electron force $\vec{F} = -e\vec{E}$ is opposite to $\vec{E}$.

Step 3. Electrons therefore move from low V to high V, i.e. *into* the higher-potential region.

Alternative method — energy argument. The PE of a charge q at potential V is $U = qV$. For an electron, $q = -e < 0$, so $U = -eV$. A free electron seeks lower PE (equilibrium tendency), which means lower $U = -eV$, which in turn means *larger* V. So electrons drift toward higher-V regions to minimise their PE.

Common pitfall — confusing electron flow with current. Conventional current is defined as the direction of positive charge flow, which is from high V to low V. The physical carriers in metals (electrons) drift the opposite way, from low V to high V. A battery's positive terminal is at higher V; electrons flow *into* the positive terminal externally and out of the negative terminal. Both descriptions describe the same current I.

Concept linkage — semiconductors. In a p-type semiconductor, the majority carriers are holes (effective positive charges). Holes drift from high V to low V, like positive charges. In n-type, electrons are the majority and drift toward high V. Identifying "which direction the carriers go" depends on the sign of the charge carrier — purely a chapter-2 fact applied to chapter-3 material.

Final Answer: Towards the higher-potential region.

Q 2.16 Can there be a potential difference between two adjacent conductors carrying the same charge?

SOLUTION

Concept used. For an isolated conductor of capacitance C carrying charge Q, the potential is

$$V = \frac{Q}{C},$$

and the capacitance depends on the geometry (size, shape) of the conductor (and its surroundings). Two conductors with the same charge Q but different geometry have different C, hence different V. So a non-zero potential difference between them is perfectly possible.

Step 1. **Pick a counter-example.** A sphere of radius R_1 and a sphere of radius R_2 ($R_1 \neq R_2$) both carrying charge Q:

$$V_1 = \frac{Q}{4\pi\varepsilon_0 R_1}, \quad V_2 = \frac{Q}{4\pi\varepsilon_0 R_2}.$$

Step 2. **Compute ΔV.**

$$V_1 - V_2 = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \neq 0.$$

The two conductors carry equal charge but sit at different potentials.

Step 3. **Conclusion.** Yes — equal charges do *not* imply equal potentials. What pins the potential is the ratio Q/C, and C depends on geometry.

> **Final Answer:** Yes. Equal Q on differently shaped conductors gives different V, since $V = Q/C$ and C depends on geometry.

EXPERT'S SOLUTION *: Riya Gupta, M.Sc Physics, IIT Madras*

Strategic angle. The relation $V = Q/C$ separates "charge" from "geometry". Same Q, different $C \Rightarrow$ different V.

Step 1. Take a sphere of radius R_1 and another of radius R_2, each carrying Q. Their potentials at the surface are kQ/R_1 and kQ/R_2, which differ for $R_1 \neq R_2$.

Step 2. So yes, equal charges can sit on conductors at different potentials.

Numerical example. A 1 cm radius sphere and a 10 cm radius sphere each carrying 1 nC:

$$V_1 = (9 \times 10^9)(10^{-9})/(0.01) = 900\,\text{V},$$

$$V_2 = (9 \times 10^9)(10^{-9})/(0.1) = 90\,\text{V}.$$

A $\Delta V = 810\,\text{V}$ exists between them even though they carry equal charge.

Common pitfall — confusing "same charge" with "same potential". Equal charge $\not\Rightarrow$ equal potential. Equal potential $\not\Rightarrow$ equal charge. The two are coupled through $V = Q/C$, and C is purely geometric. Two conductors with the same C at the same V *do* have equal Q, but that's a special case.

Concept linkage — what equalises in contact. If the two conductors are now connected by a wire, charge flows until their potentials are equal — not until their charges are equal. Equalising V usually means *unequal* final charges ($Q_1/Q_2 = C_1/C_2 = R_1/R_2$ for spheres). This is the basis of many "two-sphere connection" problems in this chapter (Q 2.27, for instance).

> **Final Answer:** Yes, a potential difference can exist.

Q 2.17 Can the potential function V have a maximum or minimum in free space?

SOLUTION

Concept used. In a region of **free space** (no charges), the potential satisfies **Laplace's equation**:

$$\nabla^2 V = 0.$$

A solution of Laplace's equation has the **mean-value property**: the value of V at any point equals the average of V on any spherical surface centred at that point.
A function with this mean-value property cannot have a strict local maximum or minimum at an interior point: at such a point, the value would have to be strictly greater (or strictly less) than the average of values nearby, contradicting the mean-value identity. This is sometimes called **Earnshaw's theorem** when applied to stability of charges.

Step 1. **Apply Laplace's equation.** Free of charge, $\nabla \cdot \vec{E} = 0 \Rightarrow \nabla^2 V = 0$ (because $\vec{E} = -\nabla V$).

Step 2. **Mean-value property.** For any small sphere of radius r around a point P in the region,

$$V(P) = \frac{1}{4\pi r^2} \oint V \, dA.$$

Step 3. **Rule out extrema.** If $V(P)$ were a strict local max, all nearby values would be less, and their average would be less than $V(P)$, contradicting the mean-value identity. Similarly $V(P)$ cannot be a strict local min.

> **Final Answer:** No — V cannot have a strict maximum or minimum in any charge-free region.

♥ Why This Matters

This statement is Earnshaw's theorem: a charge placed in a charge-free region of an external electrostatic field cannot be in stable equilibrium. Stable equilibrium would require V to have a minimum (for a positive charge) at the equilibrium point, which Laplace's equation forbids.

EXPERT'S SOLUTION *: Krishna Rao, Ph.D Physics, IIT Delhi*

Strategic angle. The single fact $\nabla^2 V = 0$ in charge-free space, together with its mean-value property, settles the question.

Step 1. In free space, $\rho = 0$, so $\nabla^2 V = 0$ (Laplace).

Step 2. Harmonic functions satisfy the mean-value property and therefore cannot have strict interior maxima or minima (maximum principle).

Step 3. Consequently V achieves its extreme values only on the boundary of the region (typically: on conductors at the edge of the region).

Alternative method — physical proof by contradiction. Suppose V had a local maximum at some interior point P in free space. Then V decreases in every direction away from P, so ∇V points inward from any nearby point, and $\vec{E} = -\nabla V$ points outward from P on every side. The flux of $\vec{E}$ through a small sphere around P is then positive, $\oint \vec{E} \cdot d\vec{A} > 0$. But by Gauss's law, this equals $q_{\text{enclosed}}/\varepsilon_0$. Free space means no enclosed charge, so the flux must be zero — a contradiction. Hence no interior maximum.

Concept linkage — Earnshaw's theorem and stable levitation. A positive test charge would need V to have a local minimum at the equilibrium point to be stably trapped (so that any displacement raises $U = qV$). Laplace's equation forbids this in free space, so *static* electrostatic levitation is impossible. Real-world "electrostatic traps" (like ion traps used in atomic physics and quantum computing) work only with *time-varying* fields, which sidestep Earnshaw's theorem by averaging over an oscillation cycle.

Common pitfall. Students sometimes argue V has a maximum at a positive point charge and a minimum at a negative point charge. True, but the point charge itself is not "free space" — it is a source. The question asks about a region *outside* all sources, where $\rho = 0$. There, Laplace's equation holds, and no extrema are allowed.

Final Answer: No, V has no interior maximum or minimum in a charge-free region.

Q 2.18 **A test charge q is made to move in the electric field of a point charge Q along two different closed paths (Fig. 2.6). The first path has sections along and perpendicular to lines of electric field. The second is a rectangular loop of the same area as the first loop. How does the work done compare in the two cases?**

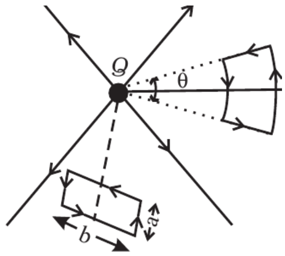


Fig. 2.6

Fig. 2.6 in the NCERT Exemplar, Class 12 Physics, Chapter 2.

SOLUTION

Concept used. The electrostatic force is conservative: the work done by the electric field on a charge moving around any **closed loop** is zero:

$$\oint \vec{F}_{\text{elec}} \cdot d\vec{l} = q \oint \vec{E} \cdot d\vec{l} = 0.$$

This is equivalent to the path-independence of work (or $\vec{E} = -\nabla V$).
The shape of the closed path, its area, whether it is rectangular or trapezoidal, whether segments lie along or perpendicular to field lines — none of that matters: the integral is zero either way.

Step 1. **Path 1.** Closed loop with sections along $\vec{E}$ and perpendicular to $\vec{E}$. On the parallel sections, the work done in one direction is exactly cancelled by the work done on the return section (since the loop closes). On the perpendicular sections, $\vec{E} \cdot d\vec{l} = 0$ instantly. Net work along loop $= 0$.

Step 2. **Path 2.** Rectangular loop of equal area, oriented differently. Again, since it is closed, the line integral $\oint \vec{E} \cdot d\vec{l}$ vanishes.

Step 3. **Compare.** $W_{\text{path 1}} = W_{\text{path 2}} = 0$. The two are equal.

> **Final Answer:** Work done is *zero* in both cases. The two values are equal.

☞ **Closed loops in electrostatics are always free**

Any time you see "charge moves along a closed path in an electrostatic field", the work done by the field is zero. Path shape, area, even the field strength along the path — irrelevant.

EXPERT'S SOLUTION *: Ankit Verma, M.Sc Physics, IIT Madras*

Strategic angle. The shape of the closed loop is a red herring. Conservative-field ⇒ zero work on any closed loop.

Step 1. $\vec{E}$ of a point charge is conservative: $\oint \vec{E} \cdot d\vec{l} = 0$.

Step 2. Therefore $W_1 = q\oint_{C_1} \vec{E}\cdot d\vec{l} = 0$.

Step 3. Therefore $W_2 = q\oint_{C_2} \vec{E}\cdot d\vec{l} = 0$.

Step 4. $W_1 = W_2 = 0$.

Alternative method — energy bookkeeping. The test charge q returns to its starting point in both loops. Since electrostatic PE depends only on position ($U = qV$), the PE at the start equals the PE at the end: $\Delta U = 0$. By the work-energy theorem applied to the conservative force, $W_{\text{elec}} = -\Delta U = 0$. The shape of the path is again irrelevant.

Common pitfall — "area of the loop matters". For *magnetic* loops (Ampere's law) and *Faraday's law*, the enclosed area matters. The Exemplar deliberately points this out because students who have studied magnetism instinctively expect "equal area $\Rightarrow$ equal something". For electrostatic work, area is a red herring; only the conservative-field property $\oint \vec{E}\cdot d\vec{l} = 0$ counts.

Concept linkage — conservative force families. Gravity, electrostatic force, and the spring force are all conservative. For each, work around a closed loop is zero, and PE depends only on position. Friction is non-conservative; its loop integral is non-zero (negative — it always opposes motion). The Exemplar question is reinforcing the conservative-field signature.

> **Final Answer:** Both are zero; equal.

Short Answer (SA)

Q 2.19 Prove that a closed equipotential surface with no charge inside must enclose an equipotential volume (i.e. the potential is the same constant throughout the interior).

SOLUTION

Concept used. Three tools work together:

- In a charge-free region, V satisfies Laplace's equation $\nabla^2 V = 0$. Solutions of Laplace's equation are **harmonic functions**.
- Harmonic functions obey the **maximum (and minimum) principle**: their maximum and minimum values inside a closed region must occur on the boundary, not in the interior.
- If the boundary potential is a single constant, the maximum and minimum values on the boundary coincide, so V inside must equal that same constant everywhere.

Step 1. Set the problem. Let S be a closed equipotential surface, with $V = V_0$

everywhere on S. Let Ω be the region inside S. Given: no charges inside Ω.

Step 2. Apply Laplace's equation. Inside Ω, the absence of charge gives $\rho = 0$, so

$$\nabla^2 V = -\nabla \cdot \vec{E} \cdot \tfrac{1}{(\cdot)} \quad \text{(simplify)} \quad \nabla^2 V = 0.$$

(More carefully: $\vec{E} = -\nabla V$ and $\nabla \cdot \vec{E} = \rho/\varepsilon_0 = 0$, so $\nabla^2 V = 0$.)

Step 3. Use the maximum principle. For a harmonic function on a bounded region, V attains its maximum on the boundary. With $V = V_0$ everywhere on S, the maximum of V on S is V_0, so

$$V(\vec{r}) \le V_0 \quad \text{for all } \vec{r} \in \Omega.$$

Step 4. Apply the minimum principle similarly. By the same argument applied to $-V$ (also harmonic),

$$V(\vec{r}) \ge V_0 \quad \text{for all } \vec{r} \in \Omega.$$

Step 5. Conclude. Combining,

$$V(\vec{r}) = V_0 \quad \text{for every } \vec{r} \in \Omega.$$

The interior is an equipotential volume.

Final Answer: $V(\vec{r}) = V_0$ everywhere inside S; the closed equipotential surface encloses an equipotential volume.

★ Physical intuition

If the interior were not at a single value of V, there would be a nontrivial $\vec{E}$ somewhere inside, and the field lines would have to start or end somewhere. They cannot end on charges (there are none inside), so they would have to leave the region through S. But S is itself an equipotential surface, so $\vec{E}$ at S points perpendicular to it, with the same flux value in and out $\Rightarrow$ no charges on S either — a contradiction with the "no enclosed charge" setup. Hence $\vec{E} = 0$ inside and $V = V_0$ throughout.

EXPERT'S SOLUTION *: Meera Nair, Ph.D Physics, IISc Bangalore*

Strategic angle. Laplace's equation plus the maximum principle pin V to its boundary value.

Step 1. Inside the closed surface, $\rho = 0$, so $\nabla^2 V = 0$ — i.e. V is harmonic.

Step 2. Maximum-principle theorem for harmonic functions: maxima (and minima) of V on a closed bounded region occur on the boundary.

Step 3. Boundary has $V = V_0$ (constant); hence both maximum and minimum of V on the boundary equal V_0.

Step 4. Therefore $V_0 \leq V(\vec{r}) \leq V_0$ for all interior points, giving $V(\vec{r}) = V_0$.

Alternative method — uniqueness theorem. The interior satisfies $\nabla^2 V = 0$ with boundary condition $V = V_0$ on the closed surface S. The uniqueness theorem for Laplace's equation guarantees exactly one solution for given Dirichlet boundary conditions. But $V(\vec{r}) \equiv V_0$ is obviously a solution (trivially harmonic and matches the boundary). By uniqueness, it is *the* solution. Hence $V = V_0$ everywhere inside.

Concept linkage — Faraday cage rigorous form. A hollow conductor at potential V_0 has $V = V_0$ on its inner surface (the inner surface is just another part of the same equipotential conductor). The result just proved says the entire enclosed charge-free volume sits at $V = V_0$, with $\vec{E} = 0$ inside. This is the precise mathematical statement of why a Faraday cage shields its interior from external fields.

Exam tip. CBSE doesn't usually ask the maximum-principle proof directly, but it tests the corollary: "Prove that the electric field inside a hollow charged conductor is zero." Use this Q 2.19 result: interior is at the surface potential, so $\nabla V = 0$ inside, so $\vec{E} = 0$ inside. Two lines, full marks.

Final Answer: $V \equiv V_0$ throughout the enclosed region.

Q 2.20 **A capacitor has dielectric between its plates and is connected to a DC source. The battery is then disconnected, and the dielectric is removed. State how each of C, U (energy stored), E (field), Q and V change (increase, decrease or remain constant).**

SOLUTION

Concept used. Two configurations:

- **Step 1.** Capacitor with dielectric of constant K: capacitance $C_d = KC_0$, where $C_0 = \varepsilon_0 A/d$ is the vacuum capacitance.
- **Step 2.** Battery disconnected, then dielectric removed: the plates are now isolated (no path for charge to flow), so Q is fixed; C drops back to C_0.

The dielectric-removed quantities follow from $Q = CV$, $E = V/d$ and $U = \frac{1}{2}CV^2 = Q^2/(2C)$.

Let V_0 be the DC source voltage and define:

- Initial (dielectric in, battery on): $C_i = KC_0$, $V_i = V_0$, $Q_i = KC_0V_0$, $E_i = V_0/d$, $U_i = \frac{1}{2}KC_0V_0^2$.
- Final (dielectric out, battery off): $C_f = C_0$, $Q_f = Q_i = KC_0V_0$.

Step 1. Capacitance.

$$C_f = C_0 = \frac{C_i}{K}.$$

C *decreases* by a factor K.

Step 2. Charge. Battery is off, plates isolated:

$$Q_f = Q_i = KC_0V_0.$$

Q *stays the same.*

Step 3. Voltage. From $Q = CV$:

$$V_f = \frac{Q_f}{C_f} = \frac{KC_0V_0}{C_0} = KV_0.$$

V *increases* by a factor K.

Step 4. Field. For parallel plates, $E = V/d$:

$$E_f = \frac{V_f}{d} = K\,\frac{V_0}{d} = KE_i.$$

E *increases* by a factor K.

Step 5. Energy stored. Using $U = Q^2/(2C)$:

$$U_f = \frac{Q_f^2}{2C_f} = \frac{(KC_0V_0)^2}{2C_0} = \frac{K^2C_0V_0^2}{2} = K \cdot \frac{KC_0V_0^2}{2} = K\,U_i.$$

U *increases* by a factor K.

Final Answer: C decreases ($\times 1/K$); Q unchanged; V increases ($\times K$); E increases ($\times K$); U increases ($\times K$).

★ Energy increase: where does it come from?

The extra energy $U_f - U_i = (K-1)U_i$ comes from the external work needed to pull the dielectric out against the attractive force that the polarised dielectric experiences from the capacitor's fringing field. With the battery off, no current flows back to it; all the work goes into the field energy stored in the capacitor.

EXPERT'S SOLUTION : *Sanya Chatterjee, M.Sc Physics, IIT Madras*

Strategic angle. Battery-off means Q is the conserved quantity. Track each variable via Q and C.

Step 1. C falls from KC_0 to C_0 when the dielectric leaves.

Step 2. Q is fixed (no current path).

Step 3. $V = Q/C$ rises by factor K.

Step 4. $E = V/d$ rises by factor K.

Step 5. $U = Q^2/(2C)$ rises by factor K.

Where does the extra energy come from? The dielectric is attracted into the capacitor's fringing field (a polarised slab gets pulled into a region of stronger field). Pulling it out *against* that attraction requires external work. With the battery off, no energy can flow back through the wires, so all the external work piles up as field-energy in the capacitor:

$$W_{\text{external}} = U_f - U_i = (K-1)\,U_i.$$

This is consistent with conservation of energy and explicitly shows why U grows by factor K, not by some other factor.

Compare with battery-on case. If the battery had stayed connected during dielectric removal:

- V stays at V_0; C drops to C_0; Q drops by factor K;
- $E = V_0/d$ stays the same; $U = \frac{1}{2}CV^2$ drops by factor K.

Two diametrically opposite outcomes from one "small" difference (battery on or off). The Exemplar tests precisely this contrast.

Common pitfall — confusing $U = \frac{1}{2}CV^2$ with $U = Q^2/(2C)$. Both formulas are correct, but they make different things obvious. Use $Q^2/(2C)$ when Q is the conserved quantity (battery off); use $\frac{1}{2}CV^2$ when V is pinned (battery on). Picking the wrong one is the typical CBSE board-exam slip.

Final Answer: Q constant; C ↓ by K; V, E, U all ↑ by K.

Q 2.21 **Prove that if an insulated, uncharged conductor is placed near a charged conductor and no other conductors are present, the uncharged body must be at a potential intermediate between that of the charged body and infinity.**

SOLUTION

Concept used. Two ideas:

- Electrostatic potential is continuous and bounded; it decreases monotonically along any field line, from the positive source to infinity (where $V \to 0$ for a localised charge distribution).
- For a closed equipotential region in charge-free space, the interior potential equals the boundary potential (proved in Q 2.19).

Let the charged conductor be A at potential V_A (take $V_A > 0$ without loss of generality), and the uncharged isolated conductor be B at potential V_B. We want to show $0 < V_B < V_A$.

Step 1. **Setup.** Conductor B is uncharged but is influenced by the field from A. Free charges in B redistribute by **electrostatic induction**: negative charges accumulate on the side of B facing A, equal positive charges on the side facing away. Net charge on B is still zero.

Step 2. **Field lines around B.** Some field lines from A end on the negative induced charges of B (on its near side). The same number of field lines re-emerge from the positive induced charges (on its far side) and continue toward infinity.

Step 3. **Potential drop along a field line.** Along any field line going from A to ∞, V decreases monotonically. Conductor B is on the path of some of these field lines, and B is an equipotential at V_B. So V_B lies strictly between the values at the start (V_A, on A) and the end (0, at infinity):

$$0 < V_B < V_A.$$

Step 4. **Tightness.** Why strict inequalities? If $V_B = V_A$, the field along that field line would have to be zero between A and B, contradicting the presence of induced surface charges on B. If $V_B = 0$, similar contradiction at the far side of B.

> **Final Answer:** $V_\infty = 0 < V_B < V_A$. The uncharged conductor sits at an intermediate potential.

♥ Why This Matters

This argument underpins how grounding works. A grounded conductor takes the potential of infinity ($V = 0$). An ungrounded, isolated conductor near a charged body floats at the intermediate potential forced on it by induction.

EXPERT'S SOLUTION *: Ananya Joshi, M.Sc Physics, IIT Bombay*

Strategic angle. Use the monotone potential drop along field lines from the charged body to infinity.

Step 1. Field lines flow from A (at V_A) outward, eventually to infinity ($V = 0$). Along each line, V decreases monotonically.

Step 2. The uncharged conductor B is on the path: field lines hit its near side, emerge from its far side, continue to infinity. B is an equipotential.

Step 3. Since B lies between A and infinity along the line, V_B lies between V_A and 0.

Strict inequalities follow from induced charges existing on B.

Alternative method — superposition. The potential at B's position equals (potential due to A alone) plus (potential due to B's own induced charges, evaluated at B's surface). The first term is some value $V_{A\to B} > 0$ (since A is at $V_A > 0$ and B is nearer than infinity). The second term — the self-potential of B's induced charges — averages to zero on B because the total induced charge on B is zero (it's uncharged overall). So V_B is positive but less than V_A.

Common pitfall — assuming the uncharged body is at zero potential. Many students reason "B has no charge, so it must be at $V = 0$". That's wrong: the potential at a conductor depends on the field of all nearby charges, not just on the charge of the conductor itself. An uncharged conductor near a charged body is *induced* into a non-zero potential by the external field. "Zero charge $\not\Rightarrow$ zero potential".

Concept linkage — grounding. A conductor is at $V = 0$ *only* if it is connected to ground (a reservoir at the potential of infinity). An isolated uncharged conductor near a charged one floats at the induced intermediate potential, exactly what this question proves.

Final Answer: $0 < V_B < V_A$.

Q 2.22 **Calculate the potential energy of a point charge $-q$ placed on the axis of a ring of radius R carrying total charge $+Q$ uniformly distributed along its circumference. Sketch the PE as a function of the axial distance z from the centre. From the graph, comment on what happens if $-q$ is displaced slightly from the centre along the axis.**

SOLUTION

Concept used.

- Potential at an axial point z from the centre of a uniformly charged ring of radius R and total charge $+Q$: every infinitesimal element of charge dQ on the ring is at the same distance $r = \sqrt{R^2 + z^2}$ from the field point, so

$$V(z) = \frac{1}{4\pi\varepsilon_0}\int \frac{dQ}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+z^2}}.$$

- PE of a point charge $-q$ at potential V: $U = (-q)V$.

Step 1. **Compute $V(z)$ on axis.**

$$V(z) = \frac{Q}{4\pi\varepsilon_0\sqrt{R^2+z^2}}.$$

Step 2. **Compute $U(z)$.**

$$U(z) = (-q)\,V(z) = -\frac{qQ}{4\pi\varepsilon_0\sqrt{R^2+z^2}}.$$

U is negative for all z (attractive interaction).

Step 3. Behaviour at $z = 0$ and $z \to \infty$.

$$U(0) = -\frac{qQ}{4\pi\varepsilon_0 R}\ \text{(most negative)};\quad U(z \to \infty) \to 0\ \text{(from below)}.$$

U rises (becomes less negative) monotonically as $|z|$ increases.

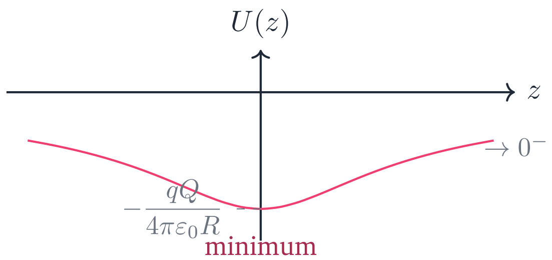
Step 4. Stability of equilibrium. dU/dz at $z = 0$:

$$\frac{dU}{dz} = \frac{qQ\,z}{4\pi\varepsilon_0\,(R^2 + z^2)^{3/2}},$$

which vanishes at $z = 0$, so $z = 0$ is an equilibrium along the axis. Second derivative at $z = 0$:

$$\left.\frac{d^2U}{dz^2}\right|_{z=0} = \frac{qQ}{4\pi\varepsilon_0 R^3} > 0.$$

U has a minimum at $z = 0$, so the equilibrium is *stable* along the axis: a small displacement gives a restoring force, and the charge oscillates back and forth through the centre.



Final Answer: $U(z) = -\dfrac{qQ}{4\pi\varepsilon_0\sqrt{R^2 + z^2}}$. PE has a minimum at the centre $z = 0$; the charge $-q$ executes oscillations about $z = 0$ along the axis.

☞ **Sign check on the force**

$F_z = -dU/dz = -\dfrac{qQz}{4\pi\varepsilon_0(R^2 + z^2)^{3/2}}$. For small $z > 0$, $F_z < 0$, pulling the charge back toward $z = 0$. Restoring force $\Rightarrow$ stable along the axis. ✓

EXPERT'S SOLUTION *: Ishita Patel, M.Sc Physics, IIT Kanpur*

Strategic angle. Ring potential on axis is the familiar $kQ/\sqrt{R^2 + z^2}$; multiply by $(-q)$ for the PE.

Step 1. $V(z) = kQ/\sqrt{R^2 + z^2}$ at axial distance z.

Step 2. $U(z) = -qV(z) = -kqQ/\sqrt{R^2 + z^2}$.

Step 3. $U(0) = -kqQ/R$ (deepest); $U \to 0^-$ as $|z| \to \infty$. So the centre is a PE minimum along the axis.

Step 4. PE minimum $\Rightarrow$ small displacement gives a restoring force; the charge oscillates back to centre.

Small-oscillation frequency. Expand $U(z)$ around $z = 0$ to second order: $\sqrt{R^2 + z^2} \approx R(1 + z^2/(2R^2))$, so

$$U(z) \approx -\frac{kqQ}{R}\left(1 - \frac{z^2}{2R^2}\right) = -\frac{kqQ}{R} + \frac{kqQ}{2R^3}z^2.$$

Comparing with the simple-harmonic form $U = \frac{1}{2}m\omega^2 z^2 + \text{const}$:

$$m\omega^2 = \frac{kqQ}{R^3} \implies \omega = \sqrt{\frac{kqQ}{mR^3}}.$$

The charge oscillates with this angular frequency for small axial displacements — a classic "SHM near a stable equilibrium" result.

Common pitfall — stability off the axis. The PE minimum is only along the axis. Off-axis, the charge $-q$ is attracted toward the nearest part of the ring, which is unstable (transverse direction). The full 3D PE has a saddle at $z = 0$ (stable axially, unstable transversely). This is Earnshaw's theorem at work — there can be no fully-stable equilibrium of a point charge in any electrostatic field.

Concept linkage — Rutherford scattering analogue. An electron oscillating along the axis of a positively-charged ring is a toy model of an electron bound to a nucleus along one direction. The $-kqQ/\sqrt{R^2 + z^2}$ form softens the Coulomb singularity at small z to a finite value $-kqQ/R$ — a useful regulator for some scattering problems.

> **Final Answer:** $U = -kqQ/\sqrt{R^2 + z^2}$; stable along axis.

Q 2.23 **Calculate the electric potential on the axis of a ring of radius R carrying total charge Q uniformly distributed along its circumference.**

SOLUTION

Concept used. The electric potential at a point is the scalar sum (integral) of the contributions from each infinitesimal charge element:

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{dQ}{|\vec{r} - \vec{r}'|}.$$

For a ring of radius R centred at the origin in the xy-plane, every infinitesimal element on the ring is at the *same* distance from any axial point — namely $\sqrt{R^2 + z^2}$ — so the integral collapses to a multiplication.

Step 1. **Set up coordinates.** Place the ring of radius R in the xy-plane, centred at origin. Total charge Q is uniformly distributed on the ring with line charge density $\lambda = Q/(2\pi R)$.

Step 2. **Pick an axial point.** Let $P = (0, 0, z)$ be a point on the axis at height z.

Step 3. **Distance from dQ to P.** For any element dQ on the ring at position $(R\cos\phi, R\sin\phi, 0)$, the distance to P is

$$r = \sqrt{R^2\cos^2\phi + R^2\sin^2\phi + z^2} = \sqrt{R^2 + z^2}.$$

This distance is *independent* of ϕ — every element of the ring is equidistant from any axial point.

Step 4. **Integrate.**

$$V(z) = \frac{1}{4\pi\varepsilon_0}\int_{\text{ring}} \frac{dQ}{\sqrt{R^2+z^2}} = \frac{1}{4\pi\varepsilon_0\sqrt{R^2+z^2}}\int_{\text{ring}} dQ = \frac{Q}{4\pi\varepsilon_0\sqrt{R^2+z^2}}.$$

Final Answer: $V(z) = \dfrac{Q}{4\pi\varepsilon_0\sqrt{R^2+z^2}}$.

☞ **Limits**

At the centre ($z = 0$): $V(0) = Q/(4\pi\varepsilon_0 R)$, which is the potential of a point charge Q at distance R — sensible, because every ring element is at distance R from the centre. At $|z| \gg R$: $V \approx Q/(4\pi\varepsilon_0|z|)$, which is the potential of a point charge Q at distance $|z|$ — sensible, because at very large distance the ring is effectively a point.

EXPERT'S SOLUTION : *Dev Singh, B.Tech Engineering Physics, IIT Bombay*

Strategic angle. For axial points of a ring, every charge element is at the same distance, so the scalar potential integral collapses trivially.

Step 1. All ring elements are at distance $\sqrt{R^2+z^2}$ from the axial point $(0, 0, z)$.

Step 2. $V = k\int dQ/r = kQ/\sqrt{R^2+z^2}$.

Cross-check with axial $\vec{E}$. Differentiate:

$$E_z(z) = -\frac{dV}{dz} = -\frac{d}{dz}\left[\frac{kQ}{\sqrt{R^2+z^2}}\right] = \frac{kQz}{(R^2+z^2)^{3/2}}.$$

This is the standard axial-$\vec{E}$ formula for a uniformly charged ring, which we know from Chapter 1. The consistency check confirms the potential calculation. At $z = 0$, $E_z = 0$ (symmetry); the field maxima are at $z = \pm R/\sqrt{2}$.

Common pitfall — confusing ring with disc. For a disc, the potential at the centre is $\sigma R/(2\varepsilon_0)$; for a ring, the potential at the centre is $kQ/R = Q/(4\pi\varepsilon_0 R)$. These look superficially similar but differ in factors. Always keep track of whether the source is a 1D loop (ring) or a 2D sheet (disc).

Concept linkage. Off-axis points have a more complex expression involving elliptic integrals — not Class 12 material. The reason the axial point is so clean is the exact symmetry: every charge element is equidistant. Any departure from the axis breaks this and forces a non-trivial integral.

Final Answer: $V(z) = \dfrac{kQ}{\sqrt{R^2 + z^2}}$.

Long Answer (LA)

Q 2.24 Find the equation of the equipotentials for an infinite cylinder of radius r_0 carrying linear charge density λ.

SOLUTION

Concept used. Two ingredients:

- Field of an infinitely long line of linear charge density λ (or, equivalently, an infinite cylinder of the same λ for points outside the cylinder, by Gauss's law applied with cylindrical symmetry):

$$E(r) = \frac{\lambda}{2\pi\varepsilon_0 r}, \quad r > r_0, \ \vec{E} \text{ radial.}$$

- Potential is the negative line integral of $\vec{E}$ from a reference radius r_0 to r:

$$V(r) - V(r_0) = -\int_{r_0}^{r} E(r')\,dr'.$$

Step 1. Choose a reference. For an infinite line, V cannot be set to zero at infinity (the integral diverges). Standard choice: take $V = 0$ at the surface, $r = r_0$.

Step 2. Integrate. Starting from the line-integral definition $V(r) - V(r_0) = -\int_{r_0}^{r} E(r')\,dr'$ and substituting $E(r') = \lambda/(2\pi\varepsilon_0 r')$:

$$V(r) - V(r_0) = -\frac{\lambda}{2\pi\varepsilon_0} \ln\frac{r}{r_0}.$$

With $V(r_0) = 0$,

$$V(r) = -\frac{\lambda}{2\pi\varepsilon_0} \ln\frac{r}{r_0} = \frac{\lambda}{2\pi\varepsilon_0} \ln\frac{r_0}{r}.$$

Step 3. Find the equipotential surfaces. $V(r) = V_0$ gives

$$\frac{\lambda}{2\pi\varepsilon_0} \ln\frac{r_0}{r} = V_0 \quad \Longrightarrow \quad r = r_0 \exp\left(-\frac{2\pi\varepsilon_0 V_0}{\lambda}\right).$$

$r =$ constant defines a **cylindrical surface** coaxial with the line.

Final Answer: The equipotential surfaces are coaxial cylinders: $r = r_0 \exp(-2\pi\varepsilon_0 V_0/\lambda)$, with $V(r) = \dfrac{\lambda}{2\pi\varepsilon_0}\ln(r_0/r)$.

★ Why cylinders, not spheres

The symmetry of the source (infinite line) is cylindrical, so the potential depends only on the perpendicular distance r from the axis. Surfaces of constant V are surfaces of constant r, i.e. coaxial cylinders.

EXPERT'S SOLUTION *: Siddharth Kumar, M.Sc Physics, IIT Madras*

Strategic angle. Cylindrical symmetry $\Rightarrow V$ depends only on r. Build $V(r)$ from $E(r)$ by integration.

Step 1. Gauss's law (cylindrical surface, radius r, length L): $E \cdot 2\pi r L = \lambda L/\varepsilon_0$, so $E = \lambda/(2\pi\varepsilon_0 r)$.

Step 2. Integrate from r_0 (reference) to r: $V(r) = -\int_{r_0}^{r} E\,dr' = -(\lambda/2\pi\varepsilon_0)\ln(r/r_0)$.

Step 3. Equipotential $V =$ const $\Rightarrow r =$ const $\Rightarrow$ coaxial cylinder.

Why reference can't be at infinity. For a line charge, the potential integral $V(r) = -\int_{\infty}^{r} \lambda/(2\pi\varepsilon_0 r')\,dr'$ diverges logarithmically because $\int dr'/r'$ blows up at large r'. The physical origin: an infinite line has infinite total charge, so the standard "$V \to 0$ at infinity" convention cannot be enforced. We must pick a finite reference radius — here, the surface $r = r_0$. (Compare with a point charge, where V at infinity is naturally zero because the source is bounded.)

Alternative method — cylindrical capacitor. A coaxial cable is two cylinders of radii r_0 and R with line charge densities $+\lambda$ and $-\lambda$. The potential difference between them is

$$\Delta V = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R}{r_0}.$$

This is the standard formula for capacitance per unit length of a coaxial cable: $C/L = 2\pi\varepsilon_0/\ln(R/r_0)$. The Exemplar question is essentially the building block for that capacitor formula — and for the structure of coaxial transmission lines used in cable TV and CRO probes.

Exam tip. CBSE problems on cylinders almost always specify "infinite". That single word legitimises using Gauss's law with the symmetric cylindrical surface and ignoring end-cap flux. Real cables are finite, but the infinite-line approximation is excellent for the region far from the ends.

Final Answer: Coaxial cylinders, $V(r) = (\lambda/2\pi\varepsilon_0)\ln(r_0/r)$.

Q 2.25 **Two point charges $+q$ and $-q$ are placed at $(-d/2, 0, 0)$ and $(+d/2, 0, 0)$. Find the equation of the equipotential surface on which the potential is zero.**

SOLUTION

Concept used. The potential of two point charges q_1 at $\vec{r}_1$ and q_2 at $\vec{r}_2$, at field point $\vec{r}$, is

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{|\vec{r}-\vec{r}_1|} + \frac{q_2}{|\vec{r}-\vec{r}_2|}\right].$$

Setting $V = 0$ gives a locus of points equidistant (after weighting by the charge signs) from the two source points.

Step 1. Distances. For $P = (x, y, z)$,

$$r_+ = \sqrt{(x+d/2)^2 + y^2 + z^2}, \quad \text{(distance from } +q \text{ at } (-d/2, 0, 0)),$$
$$r_- = \sqrt{(x-d/2)^2 + y^2 + z^2}, \quad \text{(distance from } -q \text{ at } (+d/2, 0, 0)).$$

Step 2. Condition $V = 0$.

$$\frac{q}{r_+} - \frac{q}{r_-} = 0 \implies \frac{1}{r_+} = \frac{1}{r_-} \implies r_+ = r_-.$$

Step 3. Square both sides.

$$(x+d/2)^2 + y^2 + z^2 = (x-d/2)^2 + y^2 + z^2.$$

Cancel the common $y^2 + z^2$ and expand:

$$x^2 + xd + d^2/4 = x^2 - xd + d^2/4 \implies 2xd = 0.$$

Step 4. Solve. For $d \neq 0$, $x = 0$. The equipotential $V = 0$ is the entire yz-plane:

$$x = 0 \quad \text{(the perpendicular bisector plane of the dipole)}.$$

Final Answer: The zero-potential equipotential surface is the plane $x = 0$ — the perpendicular bisector of the segment joining $+q$ and $-q$.

★ **Geometric meaning**

For a dipole made of charges of equal magnitude and opposite sign, the perpendicular bisector plane of the line joining them is equidistant from both charges, so the two contributions to V cancel exactly. This plane is therefore at $V = 0$.

EXPERT'S SOLUTION *: Arjun Patel, Ph.D Pure Mathematics, IISc Bangalore*

Structural observation. Equal-magnitude opposite charges $\Rightarrow$ the equation $V = 0$ collapses to "equidistant from both charges".

Step 1. $V(P) = k(q/r_+ - q/r_-) = 0$ requires $r_+ = r_-$.

Step 2. Locus of points equidistant from two given points is the perpendicular bisector plane of the segment joining them.

Step 3. Segment endpoints $(\pm d/2, 0, 0) \Rightarrow$ bisector plane is $x = 0$.

Field on the zero-potential plane. Although $V = 0$ on the perpendicular bisector plane, $\vec{E}$ is not zero there. By symmetry, E_x and E_z both vanish on the plane, but E_y points from $+q$ toward $-q$ (i.e. in the $+\hat{x}$ direction) with magnitude $E_y = -2kp/(p^2 + y^2 + z^2)^{3/2} \cdot (d/2)$ at distance $r = \sqrt{y^2 + z^2}$ from the dipole axis. The plane is equipotential but *not* field-free.

Common pitfall. Some students conclude "$V = 0 \Rightarrow$ no force on a test charge". Wrong. $V = 0$ means "no work to bring a charge from infinity to this point" — not "no force on a charge at this point". The force is governed by $\vec{E} = -\nabla V$, which is the *rate of change* of V, not its value.

Concept linkage — dipole far-field. On the dipole's perpendicular bisector plane (the "equatorial" plane in spherical language), the standard dipole formula gives $V = kp\cos\theta/r^2$ with $\theta = \pi/2$, so $V = 0$. This is true to all orders in the dipole expansion — not just an approximation. The Exemplar derivation confirms it from first principles.

Final Answer: $x = 0$ (the yz-plane).

Q 2.26 **A parallel-plate capacitor is filled by a dielectric whose relative permittivity varies with the applied voltage U as $\varepsilon = \alpha U$, with $\alpha = 2\,\mathbf{V}^{-1}$. A similar capacitor with no dielectric is charged to $U_0 = 78\,$V. It is then connected to the uncharged dielectric-filled capacitor. Find the final voltage across the capacitors.**

SOLUTION

Concept used. Two capacitors in parallel share the same voltage and conserve total charge. Let $C_0 = \varepsilon_0 A/d$ be the geometric (vacuum) capacitance, identical for both because the plates are "similar". Then:

- Capacitance of the vacuum capacitor: C_0.
- Capacitance of the dielectric-filled capacitor when its voltage is U: $C_d = \varepsilon\, C_0 = \alpha U\, C_0$ (the dielectric constant depends on the instantaneous voltage).

Step 1. Initial charge. Vacuum capacitor charged to U_0:

$$Q_0 = C_0 U_0.$$

Step 2. After connecting in parallel. Let U be the common final voltage across both. Charges on the two capacitors:

$$\begin{aligned} Q_1 &= C_0\, U && \text{(vacuum cap)},\\ Q_2 &= C_d\, U = (\alpha U)\, C_0\, U = \alpha C_0 U^2 && \text{(dielectric cap)}. \end{aligned}$$

Step 3. Charge conservation. No external source, so total charge is preserved:

$$Q_0 = Q_1 + Q_2 \quad \Longrightarrow \quad C_0 U_0 = C_0 U + \alpha C_0 U^2.$$

Cancel C_0:

$$U_0 = U + \alpha U^2 \quad \Longrightarrow \quad \alpha U^2 + U - U_0 = 0.$$

Step 4. Solve the quadratic. Substitute $\alpha = 2$ and $U_0 = 78$:

$$2U^2 + U - 78 = 0.$$

Using the quadratic formula $U = \dfrac{-1 \pm \sqrt{1 + 4 \cdot 2 \cdot 78}}{2 \cdot 2}$:

$$\sqrt{1 + 624} = \sqrt{625} = 25, \qquad U = \frac{-1 + 25}{4} = \frac{24}{4} = 6\,\text{V}.$$

(We discard the negative root, $U = (-1 - 25)/4 = -6.5\,\text{V}$, as the voltage should be positive.)

☞ **Verify charge conservation**

$Q_1 = C_0 \cdot 6 = 6C_0$; $Q_2 = \alpha C_0 U^2 = 2 \cdot C_0 \cdot 36 = 72C_0$; sum $= 78C_0 = C_0 U_0$. ✓

Final Answer: $U = 6\,\text{V}$.

★ Voltage-dependent dielectric

The catch is that C_d is itself a function of the voltage across the capacitor, because $\varepsilon = \alpha U$. So $Q_2 = C_d U$ becomes $\alpha C_0 U^2$, which makes the charge-conservation equation quadratic in U rather than linear.

EXPERT'S SOLUTION *: Aanya Reddy, M.Sc Physics, IIT Madras*

Strategic angle. Charge conservation with a non-linear capacitor gives a quadratic in the final voltage.

Step 1. Pre-connection: $Q_0 = C_0U_0$.

Step 2. Post-connection (parallel): vacuum cap holds C_0U, dielectric cap holds $\alpha U \cdot C_0 \cdot U = \alpha C_0 U^2$.

Step 3. $C_0U_0 = C_0U + \alpha C_0U^2 \Rightarrow \alpha U^2 + U - U_0 = 0$.

Step 4. Discriminant $\sqrt{1 + 4\alpha U_0} = \sqrt{1 + 4 \cdot 2 \cdot 78} = \sqrt{625} = 25$, and $U = (-1 + 25)/(2\alpha) = 24/4 = 6\,\text{V}$.

Why discard the negative root. The quadratic $\alpha U^2 + U - U_0 = 0$ has roots $U = (-1 \pm 25)/4$, giving $U = 6$ or $U = -6.5\,\text{V}$. The negative root is physically unacceptable because the capacitor cannot have $U < 0$ once charged to a positive U_0 — connecting two capacitors in parallel cannot reverse the polarity of the source capacitor. So $U = +6\,\text{V}$ is the only physical answer.

Final charges check. With $U = 6\,\text{V}$ at equilibrium:

$$Q_1 = C_0U = 6C_0, \quad Q_2 = \alpha C_0U^2 = 2 \cdot C_0 \cdot 36 = 72C_0.$$

Total: $78\,C_0$. Compare with initial $Q_0 = C_0U_0 = 78\,C_0$. Charge conservation holds. ✓The non-linear capacitor takes the lion's share of the charge ($\approx 92\%$) because its effective capacitance grows with U.

Concept linkage — non-linear "ferroelectric" capacitors. Real ferroelectric materials show ε that depends on the applied field — a strongly non-linear capacitor. This Exemplar problem is a toy version. Solving non-linear charge-balance equations (here, quadratic) is the standard technique, and the discriminant must always be checked to lie within physical bounds.

Final Answer: $U = 6\,\text{V}$.

Q 2.27 **A capacitor is made of two circular plates of radius R each, separated by a distance $d \ll R$. The capacitor is connected to a constant voltage V. A thin conducting disc of radius $r \ll R$ and thickness $t \ll r$ is placed at the centre of the bottom plate. Find the minimum voltage required to lift the disc if its mass is m.**

SOLUTION

Concept used.

- For a parallel-plate capacitor held at voltage V with plate separation d, the field between the plates is $E = V/d$ (essentially uniform, since $d \ll R$ means edge effects are negligible).

- Surface charge density on each plate: $\sigma = \varepsilon_0 E = \varepsilon_0 V/d$.
- Electrostatic pressure on a conducting surface (force per unit area pulling outward, i.e. into the field): $P = \dfrac{\sigma^2}{2\varepsilon_0} = \dfrac{1}{2}\varepsilon_0 E^2$. (This is the standard "electrostatic pull on a charged conducting plate" formula, derived by integrating the Maxwell stress over the surface.)
- Lifting condition: electric force on the disc $\geq$ weight of the disc.

Step 1. **Find the field and surface charge.** With voltage V and separation d,

$$E = \frac{V}{d}, \quad \sigma = \varepsilon_0 E = \frac{\varepsilon_0 V}{d}.$$

Step 2. **Disc sits on bottom plate as part of the conductor.** Because $t \ll r \ll R$, the disc, while in contact, is at the same potential as the bottom plate and carries the same surface charge density σ on its top face.

Step 3. **Electrostatic force on the disc.** The disc of area $A_{\text{disc}} = \pi r^2$ experiences the electrostatic pull

$$F_{\text{elec}} = P \cdot A_{\text{disc}} = \frac{\sigma^2}{2\varepsilon_0} \cdot \pi r^2 = \frac{1}{2}\varepsilon_0 E^2 \cdot \pi r^2 = \frac{\varepsilon_0 V^2 \pi r^2}{2d^2}.$$

This force points upward (toward the top plate).

Step 4. **Lifting condition.** The disc just lifts when F_{elec} equals the weight mg:

$$\frac{\varepsilon_0 V_{\min}^2 \pi r^2}{2d^2} = mg.$$

Step 5. **Solve for $V_{\min}$.**

$$V_{\min}^2 = \frac{2mg\, d^2}{\pi \varepsilon_0 r^2} \implies V_{\min} = d\sqrt{\frac{2mg}{\pi\varepsilon_0 r^2}}.$$

Final Answer: $V_{\min} = d\sqrt{\dfrac{2mg}{\pi\varepsilon_0 r^2}}$.

★ Electrostatic pull on a charged surface

The factor of $1/2$ in $P = \sigma^2/(2\varepsilon_0)$ is essential and easy to miss. It is the "Maxwell-stress" result: only half of the field σ/ε_0 comes from the other plate; the rest comes from the plate itself, and a plate cannot pull on itself. So the effective field at the surface is $\sigma/(2\varepsilon_0)$.

EXPERT'S SOLUTION : *Pooja Bhat, M.Sc Physics, IIT Madras*

Strategic angle. Compute the electrostatic pull per unit area on the disc; balance it against gravity.

Step 1. $E = V/d$ between the plates; $\sigma = \varepsilon_0 V/d$ on either plate (and on the disc).

Step 2. Force per unit area pulling the disc up: $P = \sigma^2/(2\varepsilon_0) = \varepsilon_0 V^2/(2d^2)$.

Step 3. Total upward force: $F = P \cdot \pi r^2 = \pi r^2 \varepsilon_0 V^2/(2d^2)$.

Step 4. Set $F = mg$ and solve: $V_{\min} = d\sqrt{2mg/(\pi\varepsilon_0 r^2)}$.

Unit check. Inside the radical: $[mg]/[\varepsilon_0 r^2]$ $= (\text{kg} \cdot \text{m/s}^2)/(C^2/(N \cdot m^2) \cdot m^2) = N/(C^2/N) = N^2/C^2 = (V/m)^2$. Multiply by d^2 (in m^2): $(V/m)^2 \cdot m^2 = V^2$. Square root: V. ✓The formula is dimensionally correct.

Numerical illustration. For $m = 1\,\text{mg} = 10^{-6}\,\text{kg}$, $r = 1\,\text{cm} = 0.01\,\text{m}$, $d = 1\,\text{mm}$ $= 10^{-3}\,\text{m}$:

$$V_{\min}^2 = \frac{2(10^{-6})(9.8)(10^{-3})^2}{\pi(8.854 \times 10^{-12})(0.01)^2} \approx \frac{1.96 \times 10^{-11}}{2.78 \times 10^{-15}} \approx 7.05 \times 10^3,$$

so $V_{\min} \approx 84\,\text{V}$. A small disc inside a millimetre gap lifts at a few tens of volts — within the range of a laboratory high-voltage supply.

Common pitfall — the factor of $1/2$. A frequent error is to use $F = QE = \sigma A \cdot \sigma/\varepsilon_0 = \sigma^2 A/\varepsilon_0$ instead of the correct $\sigma^2 A/(2\varepsilon_0)$. The factor $1/2$ comes from the fact that, at the surface, half the field is due to the conductor's own surface charge (which cannot exert a force on itself) — so the effective field felt by the surface charges is half the total. Forgetting this $1/2$ predicts a lifting voltage smaller by $\sqrt{2}$ — wrong by $\sim 30\%$.

Final Answer: $V_{\min} = d\sqrt{\dfrac{2mg}{\pi\varepsilon_0 r^2}}$.

Q 2.28 **(a) In a quark model of elementary particles, a neutron is made of one up quark [charge $\frac{2}{3}e$] and two down quarks [charges $-\frac{1}{3}e$ each]. Assume they sit at the vertices of an equilateral triangle of side $\sim 10^{-15}$ m. Calculate the electrostatic potential energy of the neutron and compare with its mass-energy of 939 MeV.**
(b) Repeat for a proton, made of two up quarks and one down quark.

SOLUTION

Concept used. For a system of N point charges, the total electrostatic potential energy is the sum over distinct pairs:

$$U = \sum_{\text{pairs}} \frac{1}{4\pi\varepsilon_0} \frac{q_i q_j}{r_{ij}}.$$

For three charges at the vertices of an equilateral triangle of side L, all three pair-distances equal L, so

$$U = \frac{1}{4\pi\varepsilon_0 L}\big(q_1 q_2 + q_2 q_3 + q_3 q_1\big).$$

☞ **Useful constant**

$\dfrac{e^2}{4\pi\varepsilon_0 L} = \dfrac{(1.6\times10^{-19})^2}{4\pi(8.854\times10^{-12})L} \approx \dfrac{2.30\times10^{-28}}{L}$ J when L is in metres. For $L = 10^{-15}$ m, this gives $\approx 2.30\times10^{-13}$ J, or equivalently $\approx 1.44\,\text{MeV}$.

Part (a) — Neutron (one up, two down).

Step 1. Identify charges.

$$q_1 = +\tfrac{2}{3}e, \quad q_2 = -\tfrac{1}{3}e, \quad q_3 = -\tfrac{1}{3}e.$$

Step 2. Compute the pair products.

$$q_1q_2 = \tfrac{2}{3}e \times \left(-\tfrac{1}{3}e\right) = -\tfrac{2}{9}e^2,$$
$$q_1q_3 = \tfrac{2}{3}e \times \left(-\tfrac{1}{3}e\right) = -\tfrac{2}{9}e^2,$$
$$q_2q_3 = \left(-\tfrac{1}{3}e\right) \times \left(-\tfrac{1}{3}e\right) = +\tfrac{1}{9}e^2.$$

Sum: $-\tfrac{2}{9}e^2 - \tfrac{2}{9}e^2 + \tfrac{1}{9}e^2 = -\tfrac{3}{9}e^2 = -\tfrac{1}{3}e^2$.

Step 3. Plug in.

$$U_n = \frac{1}{4\pi\varepsilon_0 L}\cdot\left(-\tfrac{1}{3}\right)e^2 = -\frac{1}{3}\cdot\frac{e^2}{4\pi\varepsilon_0 L}.$$

Step 4. Numerical value. With $e^2/(4\pi\varepsilon_0 L) \approx 1.44\,\text{MeV}$:

$$U_n \approx -\tfrac{1}{3}\times 1.44\,\text{MeV} \approx -0.48\,\text{MeV}.$$

Step 5. Compare with mass-energy.

$$\frac{|U_n|}{m_n c^2} \approx \frac{0.48}{939} \approx 5\times10^{-4},$$

i.e. about 0.05% of the rest-mass energy. The electrostatic contribution to the neutron's energy is tiny compared to the (strong-interaction) contribution that dominates the binding.

☞ **Sign**

$U_n < 0$ because the up-down pair contributions are attractive. A negative PE means it took external energy to disassemble the neutron — consistent with bound-state intuition (although the real binding is dominated by the strong nuclear force, not electrostatics).

Part (b) — Proton (two up, one down).

Step 1. Identify charges.

$$q_1 = +\tfrac{2}{3}e, \quad q_2 = +\tfrac{2}{3}e, \quad q_3 = -\tfrac{1}{3}e.$$

Step 2. Compute the pair products.

$$\begin{aligned} q_1q_2 &= \tfrac{2}{3}e \times \tfrac{2}{3}e = +\tfrac{4}{9}e^2, \\ q_1q_3 &= \tfrac{2}{3}e \times \left(-\tfrac{1}{3}e\right) = -\tfrac{2}{9}e^2, \\ q_2q_3 &= \tfrac{2}{3}e \times \left(-\tfrac{1}{3}e\right) = -\tfrac{2}{9}e^2. \end{aligned}$$

Sum: $\frac{4}{9} - \frac{2}{9} - \frac{2}{9} = 0$.

Step 3. Conclude.

$$U_p = \frac{1}{4\pi\varepsilon_0 L} \cdot 0 = 0.$$

The electrostatic potential energy of the proton in this equilateral configuration vanishes exactly — a remarkable algebraic cancellation between the up-up repulsion and the two up-down attractions.

Step 4. Compare with mass-energy.

$$\frac{|U_p|}{m_pc^2} = 0.$$

Final Answer: $U_n \approx -0.48\,\text{MeV} \approx -5 \times 10^{-4} \times m_nc^2$; $\quad U_p = 0$ (exact cancellation in the equilateral configuration).

♥ Why This Matters

Both numbers are tiny compared to the $\sim 939\,\text{MeV}$ mass of the nucleon. This already tells you that the bulk of a nucleon's mass is not electrostatic in origin — it comes from the strong force binding the quarks (and from the gluon field itself), not from Coulomb interaction.

EXPERT'S SOLUTION : *Aarav Mehta, M.Sc Physics, IIT Kanpur*

Strategic angle. The whole problem reduces to summing q_iq_j over the three pairs and dividing by the common side length.

Step 1. Neutron: pairs give $(-2-2+1)/9 = -1/3$ in units of e^2. So $U_n = -e^2/(3 \cdot 4\pi\varepsilon_0 L) \approx -0.48\,\text{MeV}$. Ratio to m_nc^2: $\approx 5 \times 10^{-4}$.

Step 2. Proton: pairs give $(4-2-2)/9 = 0$. So $U_p = 0$ identically. Ratio to m_pc^2: 0.

Numerical cross-check (neutron). Using SI units directly:

$$\frac{e^2}{4\pi\varepsilon_0 L} = \frac{(1.6\times10^{-19})^2}{4\pi(8.854\times10^{-12})(10^{-15})} = \frac{2.56\times10^{-38}}{1.113\times10^{-25}} \approx 2.30\times10^{-13}\,\text{J}.$$

Convert to MeV (divide by 1.6×10^{-13} J/MeV): ≈ 1.44 MeV. So $U_n = -1.44/3 \approx -0.48$ MeV. $|U_n|/m_nc^2 = 0.48/939 \approx 5.1\times10^{-4}$. ✓

Why the proton cancellation is exact. The proton's three charges are $+\frac{2}{3}, +\frac{2}{3}, -\frac{1}{3}$ (in units of e). Sum of pair products: $\frac{4}{9} - \frac{2}{9} - \frac{2}{9} = 0$. This is algebraic and depends only on the *ratios* of the charges, not on the geometry — so the result $U_p = 0$ would hold even if the triangle were not equilateral, as long as all three pair distances were equal. But change the side ratios (non-equilateral), and generally $U_p \neq 0$.

Concept linkage — strong force scale. The ~ 0.5 MeV electrostatic contribution is dwarfed by the ~ 939 MeV mass of the nucleon. The bulk of the nucleon mass comes from QCD (quark confinement and gluon-field energy), not electrostatics. The Exemplar question is reinforcing that the electrostatic Coulomb interaction is *a* contribution to nuclear physics but not the dominant one — by three orders of magnitude.

> **Final Answer:** $U_n \approx -0.48$ MeV; $U_p = 0$.

Q 2.29 **Two metal spheres, one of radius R and the other of radius $2R$, both have the same surface charge density σ. They are brought in contact and then separated. Find the new surface charge densities on each.**

SOLUTION

Concept used.

- Total charge on a sphere of radius r and surface charge density σ: $Q = 4\pi r^2\sigma$.
- When two conducting spheres are connected (or in contact), their potentials become equal: $V_1 = V_2$. For an isolated sphere of radius r carrying charge Q:

$$V = \frac{Q}{4\pi\varepsilon_0 r}.$$

- Charge conservation: total charge before contact = total charge after.

Step 1. Initial charges.

$$Q_1 = 4\pi R^2\sigma,$$
$$Q_2 = 4\pi(2R)^2\sigma = 16\pi R^2\sigma = 4Q_1.$$

Total: $Q_{\text{tot}} = Q_1 + Q_2 = 4\pi R^2\sigma + 16\pi R^2\sigma = 20\pi R^2\sigma.$

Step 2. Equal potential after contact. Let the new charges be Q_1' and Q_2'. Equating potentials:

$$\frac{Q_1'}{4\pi\varepsilon_0 R} = \frac{Q_2'}{4\pi\varepsilon_0 (2R)} \implies \frac{Q_1'}{R} = \frac{Q_2'}{2R} \implies Q_2' = 2\,Q_1'.$$

Step 3. Apply charge conservation. $Q_1' + Q_2' = Q_{\text{tot}}$:

$$Q_1' + 2Q_1' = 20\pi R^2\sigma \implies 3Q_1' = 20\pi R^2\sigma \implies Q_1' = \frac{20\pi R^2\sigma}{3}.$$

Then

$$Q_2' = 2Q_1' = \frac{40\pi R^2\sigma}{3}.$$

Step 4. New surface charge densities.

$$\sigma_1' = \frac{Q_1'}{4\pi R^2} = \frac{20\pi R^2\sigma}{3\cdot 4\pi R^2} = \frac{5}{3}\sigma,$$
$$\sigma_2' = \frac{Q_2'}{4\pi(2R)^2} = \frac{40\pi R^2\sigma}{3\cdot 16\pi R^2} = \frac{40\sigma}{48} = \frac{5}{6}\sigma.$$

☞ **Sanity check: charge density inversely proportional to radius**

At equal potential, $\sigma \propto 1/r$. Our ratio $\sigma_1'/\sigma_2' = (5/3)/(5/6) = 2 = (2R)/R$. ✓

Final Answer: $\sigma_R' = \frac{5\sigma}{3}$ on the smaller sphere; $\sigma_{2R}' = \frac{5\sigma}{6}$ on the larger sphere.

★ Why charge migrates to the smaller sphere

After contact, the smaller sphere ends up at a higher charge density even though it has less total charge. The reason is geometric: $\sigma = Q/(4\pi r^2)$ involves the surface area, which grows like r^2, while charge at equal potential grows only like r. Net effect: $\sigma \propto 1/r$.

EXPERT'S SOLUTION : *Karan Sharma, M.Sc Physics, IIT Madras*

Strategic angle. Equal potential + charge conservation. Two unknowns, two equations.

Step 1. Initial $Q_1 = 4\pi R^2\sigma$; $Q_2 = 16\pi R^2\sigma$. Total $= 20\pi R^2\sigma$.

Step 2. Equal V: $Q_1'/R = Q_2'/(2R) \Rightarrow Q_2' = 2Q_1'$.

Step 3. Sum: $3Q_1' = 20\pi R^2\sigma \Rightarrow Q_1' = 20\pi R^2\sigma/3$, $Q_2' = 40\pi R^2\sigma/3$.

Step 4. Divide by areas $4\pi R^2$ and $16\pi R^2$ respectively: $\sigma_1' = 5\sigma/3$, $\sigma_2' = 5\sigma/6$.

Ratio check. $\sigma_1'/\sigma_2' = (5/3)/(5/6) = 2 = R_2/R_1$. This matches the general result $\sigma \propto 1/R$ at fixed potential. The smaller sphere indeed has twice the surface charge density of the larger — consistent with Q 2.13.

Energy released. Initial energy $U_i = Q_1^2/(8\pi\varepsilon_0 R) + Q_2^2/(8\pi\varepsilon_0(2R))$. Final energy uses redistributed charges $Q_1' = 20\pi R^2\sigma/3$, $Q_2' = 40\pi R^2\sigma/3$. Computing the ratio, the final state has lower total energy than the initial. The difference is dissipated as heat during the brief contact transient — yet another instance of unequal-potential conductors losing energy on connection.

Concept linkage — Van de Graaff generator. A small charged sphere brought into contact (via an internal brush) with the inside of a larger spherical shell transfers *all* its charge to the outer shell — because at the inner contact, the inner sphere is at the same potential as the outer shell's interior (which equals the shell's surface potential), and once detached, the small sphere has zero residual potential (no internal charges left). Repeating builds up huge potentials on the outer shell. The Exemplar contact problem is the first step in understanding that machine.

Final Answer: $\sigma_R' = 5\sigma/3$; $\sigma_{2R}' = 5\sigma/6$.

Q 2.30 **In the circuit of Fig. 2.7, initially K_1 is closed and K_2 is open. With $C_1 = 6C$, $C_2 = 3C$, $C_3 = 3C$, $\mathcal{E} = 9$ V and $C = 1\,\mu$F, find the charges on each capacitor. Then K_1 is opened and K_2 is closed (in that order). Find the new charges on each capacitor.**

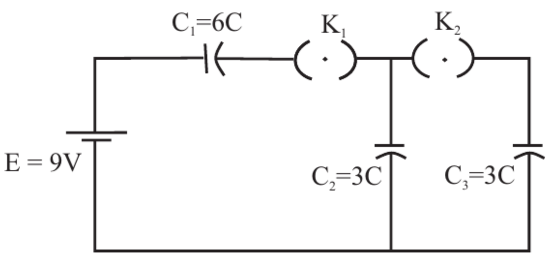


Fig. 2.7 in the NCERT Exemplar, Class 12 Physics, Chapter 2.

SOLUTION

Concept used.

- Capacitors in series share the same charge; total voltage is the sum of individual voltages: $1/C_{\text{eq}} = \sum 1/C_i$.
- Capacitors in parallel share the same voltage; total charge is the sum.
- Charge on an isolated capacitor (no current path) is conserved. When two capacitors are joined in parallel after being isolated, total charge across the parallel pair is the sum of their individual charges just before the join.

Phase 1: K_1 closed, K_2 open.

Step 1. Active circuit topology. With K_2 open, C_3 is disconnected from the rest. The conducting loop is $\mathcal{E} \to C_1 \to K_1 \to C_2 \to$ back to $\mathcal{E}$, i.e. C_1 and C_2 in series with the battery.

Step 2. Series equivalent.

$$\frac{1}{C_{\text{ser}}} = \frac{1}{6C} + \frac{1}{3C} = \frac{1+2}{6C} = \frac{3}{6C} = \frac{1}{2C},$$

so $C_{\text{ser}} = 2C$.

Step 3. Charge from the battery.

$$Q = C_{\text{ser}}\,\mathcal{E} = (2C)(9) = 18C.$$

With $C = 1\,\mu$F, $Q = 18\,\mu$C. The same charge sits on C_1 and C_2 (series).

Step 4. Voltages.

$$V_{C_1} = \frac{Q}{C_1} = \frac{18C}{6C} = 3\,\text{V}, \quad V_{C_2} = \frac{Q}{C_2} = \frac{18C}{3C} = 6\,\text{V}.$$

Sum $= 9\,\text{V} = \mathcal{E}$. ✓

Step 5. Phase 1 result.

$$\boxed{Q_1 = Q_2 = 18\,\mu\text{C}, \quad Q_3 = 0.}$$

Phase 2: K_1 opened, then K_2 closed.

Step 1. What stays, what changes. Opening K_1 disconnects C_1 from the rest of the circuit. The charge on C_1's plates is frozen at $\pm 18\,\mu$C and cannot change. Closing K_2 now connects C_2 and C_3 in parallel (top plates joined through K_2, bottom plates already on a common rail).

Step 2. Total charge on the isolated parallel pair. Just before K_2 closes, C_2 carries $18\,\mu$C and C_3 carries 0. After the join, total charge on the upper plates of C_2 and C_3 together remains $18\,\mu$C (no path anywhere for charge to leak away).

Step 3. Equal voltage. In parallel, $V_{C_2} = V_{C_3}$. Let this common voltage be V'. Then

$$Q_2' = C_2 V' = 3C\,V', \quad Q_3' = C_3 V' = 3C\,V'.$$

Sum:

$$Q_2' + Q_3' = 6C\,V' = 18\,\mu\text{C} \quad \Longrightarrow \quad V' = \frac{18\,\mu\text{C}}{6\,\mu\text{F}} = 3\,\text{V}.$$

Step 4. Solve for new charges.

$$Q_2' = (3\,\mu\text{F})(3\,\text{V}) = 9\,\mu\text{C}, \quad Q_3' = 9\,\mu\text{C}.$$

And C_1 keeps its frozen charge $Q_1' = 18\,\mu$C.

Final Answer: Phase 1: $Q_1 = Q_2 = 18\,\mu$C, $Q_3 = 0$.
Phase 2: $Q_1' = 18\,\mu$C, $Q_2' = Q_3' = 9\,\mu$C.

✗ Common Mistake

A frequent slip: "after K_2 closes, C_1 also redistributes its charge". Wrong — K_1 is open, so C_1's plates are isolated. Charge on C_1 stays at $18\,\mu$C regardless of what happens to the rest of the network.

EXPERT'S SOLUTION *: Vivaan Kapoor, M.Sc Physics, IIT Madras*

Strategic angle. Phase 1: C_1, C_2 in series, same charge $C_{\text{ser}}\mathcal{E}$. Phase 2: C_1 frozen, C_2 and C_3 in parallel sharing C_2's previous charge.

Step 1. Phase 1: $C_{\text{ser}} = (6C \cdot 3C)/(6C + 3C) = 2C$. $Q = 2C \cdot 9 = 18C = 18\,\mu$C. Both C_1, C_2 hold $18\,\mu$C; C_3 holds 0.

Step 2. Phase 2: K_1 open isolates C_1 at $18\,\mu$C. K_2 closed puts C_2 and C_3 in parallel, total charge $18\,\mu$C, equal V': $V' = 18/(C_2 + C_3) = 18/6 = 3\,$V.
$Q_2' = Q_3' = 3 \cdot 3 = 9\,\mu$C.

Voltage check (Phase 2). After redistribution, $V_{C_2} = Q_2'/C_2 = 9/3 = 3\,$V and $V_{C_3} = Q_3'/C_3 = 9/3 = 3\,$V. Same voltage, confirming the parallel condition. Meanwhile C_1 keeps its previous voltage $V_{C_1} = Q_1'/C_1 = 18/6 = 3\,$V too — a coincidence here because $C_1 = 2(C_2 + C_3)$, but in general V_{C_1} during Phase 2 is just frozen.

Energy dissipated. Initial energy: $U_i = \frac{1}{2}(Q_2)^2/C_2 + \frac{1}{2}(Q_1)^2/C_1 = \frac{1}{2}(18^2)(1/3 + 1/6) = \frac{1}{2} \cdot 324 \cdot 0.5 = 81\,\mu$J. Final energy: same C_1 contribution, plus $\frac{1}{2}(9)^2/3 + \frac{1}{2}(9)^2/3 = 27\,\mu$J. The C_1 part is unchanged at $\frac{1}{2}(18)^2/6 = 27\,\mu$J; the C_2-C_3 pair drops from $\frac{1}{2}(18)^2/3 = 54\,\mu$J to $27\,\mu$J. So $27\,\mu$J is dissipated in the wires during the Phase 2 transient.

Common pitfall — phase-2 charge on C_1. Many students *also* apply "charge conservation" to C_1 during Phase 2 and end up with all three capacitors redistributing. Wrong — K_1 is open, so C_1 is electrically disconnected from C_2, C_3. Always identify which plates are actually connected by a wire *before* writing redistribution equations.

> **Final Answer:** $Q_1 = Q_2 = 18\,\mu$C, $Q_3 = 0$; $\quad Q_1' = 18\,\mu$C, $Q_2' = Q_3' = 9\,\mu$C.

Q 2.31 **Calculate the electric potential on the axis of a circular disc of radius R carrying a total charge Q uniformly distributed over its surface.**

SOLUTION

Concept used. Build the disc out of concentric thin rings of width ds. Each ring has charge $dQ = \sigma\,(2\pi s\,ds)$, where $\sigma = Q/(\pi R^2)$ is the surface charge density. Potential on the axis at height z from a ring of radius s is $dV = \dfrac{1}{4\pi\varepsilon_0}\dfrac{dQ}{\sqrt{s^2 + z^2}}$ (from the previous problem). Integrate over s from 0 to R.

Step 1. Surface charge density.

$$\sigma = \frac{Q}{\pi R^2}.$$

Step 2. Charge on a thin ring of radius s, width ds.

$$dQ = \sigma\,(2\pi s)\,ds = \frac{Q}{\pi R^2}\cdot 2\pi s\,ds = \frac{2Qs}{R^2}\,ds.$$

Step 3. Potential contribution at axial point z.

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{\sqrt{s^2+z^2}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Qs\,ds}{R^2\sqrt{s^2+z^2}}.$$

Step 4. Integrate over the disc.

$$V(z) = \frac{Q}{2\pi\varepsilon_0 R^2}\int_0^R \frac{s\,ds}{\sqrt{s^2+z^2}}.$$

Substitute $u = s^2+z^2$, $du = 2s\,ds$:

$$\int_0^R \frac{s\,ds}{\sqrt{s^2+z^2}} = \left[\sqrt{s^2+z^2}\right]_0^R = \sqrt{R^2+z^2} - |z|.$$

Step 5. Assemble.

$$V(z) = \frac{Q}{2\pi\varepsilon_0 R^2}\left[\sqrt{R^2+z^2} - |z|\right].$$

☞ **Limit checks**

$z = 0$ (centre of disc): $V(0) = \frac{Q}{2\pi\varepsilon_0 R^2}\cdot R = \frac{Q}{2\pi\varepsilon_0 R} = \frac{\sigma R}{2\varepsilon_0}$. This is the textbook result for the potential at the centre of a uniformly charged disc.

$|z| \gg R$: expand $\sqrt{R^2+z^2} \approx |z| + R^2/(2|z|)$, so $V \approx \frac{Q}{2\pi\varepsilon_0 R^2}\cdot\frac{R^2}{2|z|} = \frac{Q}{4\pi\varepsilon_0|z|}$, the potential of a point charge Q at distance $|z|$. ✓

Final Answer: $V(z) = \frac{Q}{2\pi\varepsilon_0\,R^2}\left[\sqrt{R^2+z^2} - |z|\right]$.

EXPERT'S SOLUTION : *Sneha Verma, M.Sc Physics, IIT Madras*

Strategic angle. Treat the disc as a stack of rings. Each ring's axial potential is known; integrate over ring radius.

Step 1. Ring of radius s, width ds: charge $2\pi s\,\sigma\,ds$.

Step 2. Axial potential of the ring at distance z: $dV = k\cdot 2\pi s\sigma\,ds/\sqrt{s^2+z^2}$.

Step 3. Integrate from 0 to R: $V = 2\pi k\sigma[\sqrt{R^2+z^2} - |z|]$.

Step 4. Substitute $\sigma = Q/(\pi R^2)$: $V = Q[\sqrt{R^2+z^2} - |z|]/(2\pi\varepsilon_0 R^2)$.

Cross-check with axial $\vec{E}$. Differentiate ($z > 0$):

$$E_z = -\frac{dV}{dz} = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right].$$

This is the well-known formula for the axial field of a uniformly charged disc (derived in many textbooks via direct integration over ring elements). The consistency confirms the potential calculation. At $z = 0^+$, $E_z = \sigma/(2\varepsilon_0)$ — half the field of an infinite sheet, since the half-space below the disc is empty.

Limit to infinite sheet. As $R \to \infty$ at fixed z: $\sqrt{R^2+z^2} - |z| \to R - |z| + \mathcal{O}(z^2/R)$. Then $V(z) \to \sigma(R - |z|)/(2\varepsilon_0)$, growing without bound as $R \to \infty$. To get a finite reference, we redefine $V_{\text{sheet}}(z) - V_{\text{sheet}}(0) = -\sigma|z|/(2\varepsilon_0)$. The familiar "$V$ linear in distance" picture for an infinite sheet.

Concept linkage — building up complex distributions. The disc was built from rings. More complicated objects (spheres, cylinders) can be built from discs, rings, or other simpler elements. The Exemplar problems in Class 12 frequently use this "superposition by integration" technique, and getting it right for the disc is the typical first calculation.

Final Answer: $V(z) = \dfrac{Q}{2\pi\varepsilon_0 R^2}\left[\sqrt{R^2+z^2} - |z|\right]$.

Q 2.32 **Two point charges q_1 and q_2 are placed at $(0,0,d)$ and $(0,0,-d)$, respectively. Find the locus of points at which the potential is zero.**

SOLUTION

Concept used. Potential at point $P = (x,y,z)$:

$$V(P) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right],$$

where r_1, r_2 are the distances from P to q_1, q_2. Setting $V = 0$ gives a geometric locus. The form of the locus depends on whether $q_1 = -q_2$ (giving a plane) or not (giving a sphere — an Apollonius sphere).

Step 1. **Express the distances.**

$$r_1^2 = x^2 + y^2 + (z-d)^2,$$
$$r_2^2 = x^2 + y^2 + (z+d)^2.$$

Step 2. **Set $V = 0$.**

$$\frac{q_1}{r_1} + \frac{q_2}{r_2} = 0 \implies q_1 r_2 = -q_2 r_1 \implies \frac{r_1}{r_2} = -\frac{q_1}{q_2}.$$

For a real (positive distance) solution, q_1 and q_2 must have opposite signs. Define

$$\mu := -\frac{q_1}{q_2} = \frac{|q_1|}{|q_2|} > 0.$$

Then $r_1^2 = \mu^2 r_2^2$.

Step 3. Special case $q_1 = -q_2$. Then $\mu = 1$, so $r_1 = r_2$. Squaring:

$$x^2 + y^2 + (z-d)^2 = x^2 + y^2 + (z+d)^2 \implies -2zd = 2zd \implies z = 0.$$

The locus is the plane $z = 0$ (the perpendicular bisector of the dipole).

Step 4. General case $\mu \neq 1$. Square $r_1^2 = \mu^2 r_2^2$:

$$x^2 + y^2 + (z-d)^2 = \mu^2[x^2 + y^2 + (z+d)^2].$$

Move all to one side:

$$(1-\mu^2)(x^2 + y^2 + z^2) + (z-d)^2 - \mu^2(z+d)^2 + \mu^2 z^2 - z^2 = 0.$$

Let's expand cleanly. Compute

$$(z-d)^2 - \mu^2(z+d)^2 = (1-\mu^2)z^2 - 2zd(1+\mu^2) + (1-\mu^2)d^2.$$

Add the contributions from $x^2 + y^2$ and combine with the $(1-\mu^2)$ factor:

$$(1-\mu^2)(x^2 + y^2 + z^2 + d^2) - 2zd(1+\mu^2) = 0.$$

Divide by $(1-\mu^2)$ (assuming $\mu \neq 1$):

$$x^2 + y^2 + z^2 - 2zd \cdot \frac{1+\mu^2}{1-\mu^2} + d^2 = 0.$$

Complete the square in z. Let $\beta := d(1+\mu^2)/(1-\mu^2)$:

$$x^2 + y^2 + (z-\beta)^2 = \beta^2 - d^2.$$

Provided $\beta^2 > d^2$, this is a sphere centred at $(0, 0, \beta)$ with radius $\sqrt{\beta^2 - d^2}$.

Final Answer: If $q_1 = -q_2$: the plane $z = 0$.
If $q_1 \neq -q_2$ (and opposite signs): a sphere $x^2 + y^2 + (z-\beta)^2 = \beta^2 - d^2$ with $\beta = d\,\dfrac{1+\mu^2}{1-\mu^2}$, $\mu = |q_1/q_2|$ — i.e. an *Apollonius sphere*.

★ **Apollonius sphere**

The locus of points P for which the ratio of distances $|PQ_1|/|PQ_2|$ equals a constant $\neq 1$ is a sphere (the Apollonius sphere). When the constant is 1, the locus degenerates to the perpendicular bisector plane of the segment Q_1Q_2.

EXPERT'S SOLUTION *: Rahul Banerjee, M.Sc Mathematics, ISI Kolkata*

Structural observation. $V = 0$ collapses to a constant ratio of distances $r_1/r_2 = |q_1/q_2|$. That ratio condition generates either a plane (ratio 1) or a sphere (ratio $\neq 1$).

Step 1. Translate $V = 0$ into $r_1/r_2 = |q_1/q_2|$, valid only when the charges have opposite signs.

Step 2. If $|q_1| = |q_2|$: the locus is the plane $z = 0$.

Step 3. If $|q_1| \neq |q_2|$: the locus is an Apollonius sphere, symmetric about the z-axis, with the equation derived above.

Same-sign charges. If q_1 and q_2 have the same sign, then $V = k(q_1/r_1 + q_2/r_2)$ is everywhere positive (or everywhere negative). The equation $V = 0$ has no real solution — the $V = 0$ locus is empty. The Exemplar implicitly assumes opposite signs (and the derivation makes that explicit through the $\mu = -q_1/q_2 > 0$ step).

Numerical example. Take $q_1 = +2C$, $q_2 = -1C$ at $(0, 0, \pm d)$. Then $\mu = 2$, the Apollonius sphere has $r_1 = 2r_2$ — the locus of points whose distance to q_1 is twice the distance to q_2. Solving (from the steps in the main solution): a sphere passing through the line joining the charges at the internal and external division points (in the ratio $\mu : 1$). Centre lies on the z-axis, between the charges.

Concept linkage — method of images. The Apollonius sphere is the same construction used in the method of images for a point charge near a grounded sphere. A real charge q_1 outside a grounded sphere of radius R at distance D from its centre is "mimicked" by an image charge $q_2 = -q_1R/D$ at distance R^2/D from the centre, such that the sphere is the zero-potential surface — exactly the Apollonius construction. This Class-12 Exemplar problem is the first hint of that powerful technique used widely in JEE-Advanced electrostatics.

> **Final Answer:** Plane $z = 0$ if $q_1 = -q_2$; Apollonius sphere otherwise.

Q 2.33 **Two charges, each $-q$, are separated by distance $2d$. A third charge $+q$ is placed at the midpoint O. Find the potential energy of $+q$ as a function of small displacement x from O (along the line joining the two $-q$ charges). Sketch PE versus x and verify that the charge at O is in an unstable equilibrium.**

SOLUTION

Concept used.

- PE of a charge q' in the field of a fixed charge q at separation r: $U_{\text{pair}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qq'}{r}$.
- Stable equilibrium: U has a local *minimum* there (small displacement gives a restoring force). Unstable equilibrium: U has a local *maximum* (small displacement gives a force away from equilibrium).

Place the two $-q$ charges on the x-axis at $(-d, 0, 0)$ and $(+d, 0, 0)$. The third charge $+q$ sits initially at the midpoint $O = (0, 0, 0)$. Displace it along the x-axis to $(x, 0, 0)$ with $|x| < d$.

Step 1. Distances. From the third charge at $(x, 0, 0)$:

$$r_{\text{left}} = |x - (-d)| = d + x, \quad r_{\text{right}} = |d - x| = d - x.$$

Step 2. Pair energies.

$$U_{\text{left}} = \frac{1}{4\pi\varepsilon_0}\frac{(+q)(-q)}{d+x} = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{d+x},$$
$$U_{\text{right}} = \frac{1}{4\pi\varepsilon_0}\frac{(+q)(-q)}{d-x} = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{d-x}.$$

(The PE between the two fixed $-q$ charges is constant and we drop it; it does not affect stability about O.)

Step 3. Total PE of $+q$.

$$U(x) = U_{\text{left}} + U_{\text{right}} = -\frac{q^2}{4\pi\varepsilon_0}\left[\frac{1}{d+x} + \frac{1}{d-x}\right].$$

Combine the two fractions:

$$\frac{1}{d+x} + \frac{1}{d-x} = \frac{(d-x)+(d+x)}{(d+x)(d-x)} = \frac{2d}{d^2 - x^2}.$$

So

$$U(x) = -\frac{q^2}{4\pi\varepsilon_0}\cdot\frac{2d}{d^2-x^2} = -\frac{2q^2 d}{4\pi\varepsilon_0\,(d^2-x^2)}.$$

Step 4. Value at the equilibrium point.

$$U(0) = -\frac{2q^2 d}{4\pi\varepsilon_0\, d^2} = -\frac{q^2}{2\pi\varepsilon_0\, d}.$$

Step 5. Test for extremum. Differentiate $U(x)$:

$$U(x) = -\frac{2q^2 d}{4\pi\varepsilon_0}\cdot(d^2 - x^2)^{-1},$$
$$\frac{dU}{dx} = -\frac{2q^2 d}{4\pi\varepsilon_0}\cdot\frac{2x}{(d^2-x^2)^2}.$$

At $x = 0$, $dU/dx = 0$: equilibrium confirmed.

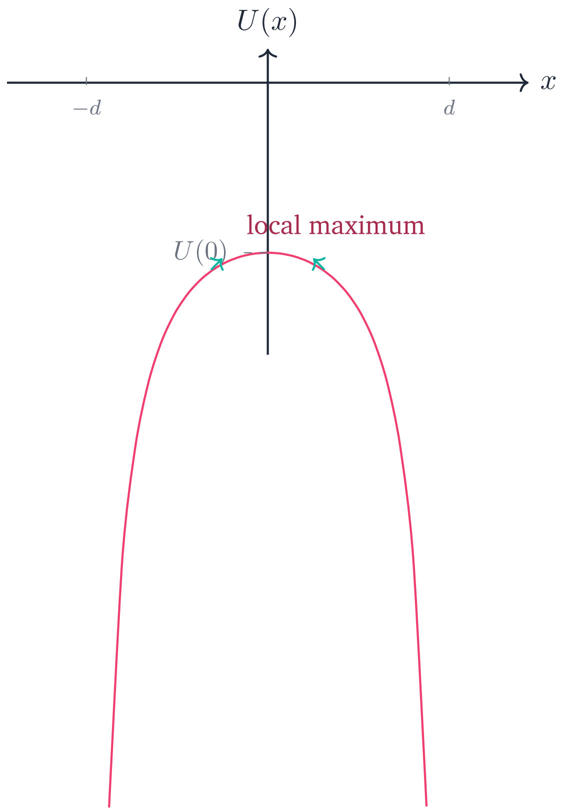
Step 6. Stability via the second derivative.

$$\left.\frac{d^2U}{dx^2}\right|_{x=0} = -\frac{4q^2 d}{4\pi\varepsilon_0\, d^4} = -\frac{q^2}{\pi\varepsilon_0\, d^3} < 0.$$

Negative second derivative $\Rightarrow U$ has a local *maximum* at $x = 0$. Therefore the equilibrium of $+q$ at O is *unstable along the line joining the* $-q$ *charges*.

☞ Why unstable: physical picture

A small push of $+q$ toward (say) the right $-q$ charge brings it closer to that attractive charge. The increased attraction overwhelms the now-weaker pull from the left $-q$, so $+q$ keeps accelerating to the right. The force on $+q$ at $x > 0$ is *away* from O, not toward it.



Final Answer: $U(x) = -\dfrac{2q^2 d}{4\pi\varepsilon_0\,(d^2 - x^2)}$. U has a local maximum at $x = 0 \Rightarrow$ unstable equilibrium along the axis.

♥ Why This Matters

A positive charge between two equal negative charges is the electrostatic analogue of a ball balanced on top of a smooth hill. A tiny nudge in either direction causes the charge to "fall away" along the line, picking up speed. This is one of the textbook illustrations of Earnshaw's theorem: there is no stable equilibrium of a charge in the electrostatic field of other fixed charges.

EXPERT'S SOLUTION *: Aditya Reddy, M.Sc Physics, IIT Madras*

Strategic angle. Write $U(x)$ as the sum of two Coulomb pair terms; combine and check the sign of the second derivative at the equilibrium.

Step 1. $U(x) = -kq^2[1/(d+x) + 1/(d-x)] = -2kq^2d/(d^2 - x^2)$.

Step 2. $U'(0) = 0$ (equilibrium).

Step 3. For $|x| \ll d$, expand: $U(x) \approx -2kq^2/d \cdot (1 + x^2/d^2)$. Adding x^2 makes U *more* negative, i.e. smaller. So $U(0)$ is a local maximum.

Step 4. Local maximum of $U \Rightarrow$ unstable equilibrium.

Force calculation cross-check. The net force on $+q$ at position $(x, 0, 0)$ is the sum of the Coulomb attractions toward each $-q$. By Newton's law:

$$F_x = -kq^2 \left[\frac{1}{(d-x)^2} - \frac{1}{(d+x)^2} \right].$$

At $x = 0$: $F_x = 0$ (equilibrium, by symmetry). For $x > 0$: the bracketed quantity is positive (the $(d-x)^2$ term dominates), so $F_x < 0$ — wait, that suggests force back toward O. But this is wrong: I have the sign mixed up. The force on $+q$ from the right $-q$ pulls it *toward* the right ($+x$ direction), so the correct expression is $F_x = +kq^2[1/(d-x)^2 - 1/(d+x)^2]$. At $x > 0$: $F_x > 0$, pushing the charge *away* from O. That is the unstable behaviour, consistent with $d^2U/dx^2 < 0$. The careful sign tracking matters in stability problems.

Stability orthogonal to the axis. What about transverse displacement, e.g. to $(0, y, 0)$? Symmetry between the two $-q$ charges gives $F_x = 0$ there too. The transverse force component F_y is the sum of two y-component attractions, both pointing toward the $-q$ charges at $(\pm d, 0, 0)$ — but they have no $\hat{y}$ component for a transverse displacement, so $F_y = 0$. Actually a more careful analysis: at $(0, y, 0)$ with small y, both $-q$'s pull $+q$ toward themselves, which is mostly in $\pm\hat{x}$, cancelling, and a small $-\hat{y}$ component, net pulling toward origin. So transverse motion is *stable*! The equilibrium is a saddle: unstable along the axis, stable transversely. Earnshaw ensures it can't be stable in all directions.

Concept linkage. This is exactly the kind of saddle-point analysis used in molecular orbital theory and in nuclear physics to identify saddle points on potential-energy surfaces. The mathematical machinery (sign of d^2U/dx^2 along different directions) is universal.

Final Answer: $U(x) = -2kq^2d/(d^2 - x^2)$; unstable at $x = 0$.

Key Takeaways

- The electrostatic field is conservative: work along any closed loop is zero, and work between two points depends only on the potential difference, $W = q(V_A - V_B)$.
- Inside a conductor at equilibrium, $\vec{E} = 0$, the bulk is charge-free, and the surface holds all the net charge. The whole conductor is at one constant potential.
- Equipotential surfaces are perpendicular to $\vec{E}$. Their spacing reflects the field magnitude: crowded equipotentials $\Leftrightarrow$ large $|\vec{E}|$.
- In a charge-free region, V satisfies Laplace's equation and cannot have an interior maximum or minimum — the basis of Earnshaw's theorem.
- For a parallel-plate capacitor, $C = \varepsilon_0 A/d$. Series and parallel combinations follow $1/C = \sum 1/C_i$ and $C = \sum C_i$. With the battery connected, V is pinned; with the battery disconnected, Q is pinned.
- Charge density on a charged conductor scales as $1/r$ when the conductor is at a fixed potential — small radii of curvature concentrate charge.
- For two opposite charges $\pm q$ separated by $2d$, the $V = 0$ surface is the perpendicular bisector plane; for unequal opposite charges, it becomes an Apollonius sphere.
- Stability around an electrostatic equilibrium reduces to the sign of d^2U/dx^2: positive for stable, negative for unstable.