Electrostatic Potential

The electrostatic potential at a point in an electric field is the ~~energy~~ work done in bringing a unit positive test charge from infinity to that point, against the field, without any acceleration of the charge.

$$\boxed{V = W / q_0}$$

<-work done per unit
<-positive charge

SI unit : J / C = 1 volt (V)

Dimensional formula : $[M L^2 T^{-3} A^{-1}]$

V is a scalar quantity (has no direction).

Potential Difference

Work done per unit positive test charge in carrying it from B to A against E :

$$\boxed{V_A - V_B = W_{BA} / q_0}$$

<-path-independent
<-in electrostatics

Note : V is defined only upto an additive constant ; only the difference of V between two points has a physical meaning.

Convention : V(infinity) = 0 , then V at any point is uniquely defined.

Potential due to a Point Charge

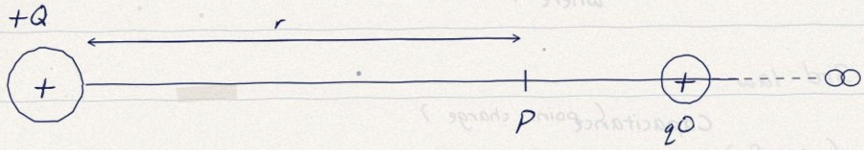


Fig. Test charge q_0 brought from infinity

Setup

Isolated point charge $+Q$ at O.

Point P at ~~origin~~ distance r from O.

Bring test charge q_0 from infinity to P.

Derivation

1. Force on q_0 at distance x from O :

$$F = \kappa Q q_0 / x^2 \quad \text{(radially outward)}$$

2. Work done ~~by~~ against F over dx towards O :

$$dW = -F\,dx = -\kappa Q q_0\,dx / x^2$$

3. Total work from infinity to P :

$$W = -\int_{inf}^{r} \kappa Q q_0\,dx / x^2 = \kappa Q q_0 / r$$

4. $V = W / q_0$ gives :

$$\boxed{V = \frac{1}{4\pi\varepsilon_0} Q/r = \kappa Q/r}$$

<- V scalar ;
<- V > 0 if +Q

System of Charges & E - V Relation

Superposition of Potentials

Potential is scalar. So total V at point P due to q1, q2, ... qn is algebraic sum :

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\left[q_1/r_1 + q_2/r_2 + \dots \right]}$$

<- signs of qi
<- are kept

ri is distance of qi from observation point.

No vector resolution needed - just add.

Uniformly Charged Shell (radius R)

Outside (r > R) :

$V = \kappa Q / r$

On surface (r = R) :

$V = \kappa Q / R$

Inside (r < R) :

$V = \kappa Q / R$ (constant inside)

<- E = 0 inside,
<- but V finite

Relation between E and V

For displacement dl : dV = - E . dl

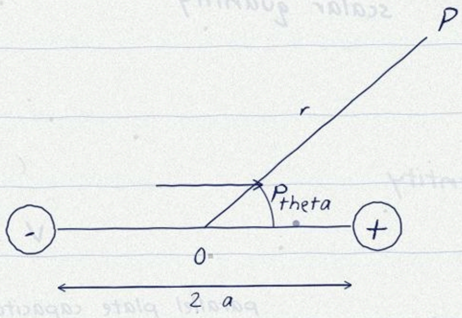
$$\boxed{E = - dV / dr}$$

<- E points along
<- steepest decrease

Unit of E : V/m = N/C (same thing).

If V is constant in a region, then E = 0.

Potential due to an Electric Dipole



Setup

Dipole : +q at A , -q at B , separation 2a.

P at distance r from centre 0 , at Θ afigden axis.

Dipole moment $p = q\ (2a)$, along (-q -> +q).

General Result (r >> a)

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\ p \cos\Theta / r^2}$$

<-p cos = p . r-hat

Special positions

Axial (theta = 0) :

$$V = \frac{1}{4\pi\varepsilon_0}\ p / r^2$$

Equatorial (theta = 90) :

$$V = 0 \quad (\cos 90 = 0)$$

Note : V of dipole falls as $1 / r^2$,

whereas of a point charge as 1 / r.

Faster fall : +q , -q nearly cancel far away.

Equipotential Surfaces

Surface on which V has the same value at every point. e.g. concentric spheres around a point charge ; parallel planes for uniform E.

Properties

(1) Work done to move a charge along an equipotential surface is zero ($dV = 0$).

(2) E is always perpendicular to the equipotential surface.
(if not , a tangential E would do work).

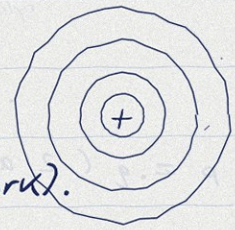


Fig. V = const around a point charge

(3) E points from higher V to lower V (along steepest decrease).

(4) Two equipotential *surfaces cannot intersect (else V would have two values at the same point).

(5) Closer the surfaces , stronger the field ($E = dV / dr$).

Uniform E : eq.surfaces = parallel planes.

Potential Energy of a System

Two Point Charges

Work done to assemble $q1$, $q2$ from infinity ; this work is stored as PE of the system.

$$\boxed{U = \frac{1}{4\pi\varepsilon_0} q_1 q_2 / r_{12}}$$

<- $U > 0$ like sign
<- $U < 0$ unlike sign

U is scalar . SI unit : J (or eV in atomic).

Three Point Charges

Take pairs one at a time :

$$\boxed{U = \frac{1}{4\pi\varepsilon_0} [q q_2/r_{12} + q_2 q_3/r_{23} + q q_3/r_{13}]}$$

n charges -> sum over all distinct pairs ; $n(n-1)/2$ terms in total.

U in terms of Potential

PE of charge q at a point where the existing potential (due to all other charges) is V :

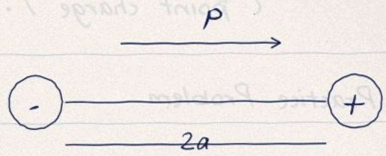
$$\boxed{U = q V}$$

<- useful for single
<- charge in external V

Definition of electron volt :

$1 \text{ eV} = 1.6 \times 10^{-19}$ J (KE gained by e in 1 V).

Dipole in a Uniform Field



Torque on the dipole

Each charge feels $+qE$ and $-qE$:

Net force = 0 (uniform E) ; but couple acts.

$$\boxed{T = p E \sin \theta}$$

<-(T = torque)
<-vector form : p x E

Stable equilibrium : theta = 0 (p E)

Unstable equilibrium : theta = 180 (p anti-E)

Potential Energy of the Dipole

Work done to rotate dipole from theta1 to theta2 :

$$W = p E (\cos \theta_1 - \cos \theta_2)$$

Take U(theta = 90) = 0 (perpendicular) :

$$\boxed{U = - p E \cos \theta = - p \cdot E}$$

<-min at theta=0
<-max at theta=180

U(0) = -pE (stable) ; U(180) = +pE.

Dipole tends to align along E to minimise U.

Electrostatics of Conductors

Inside a conductor there are free electrons. In static equilibrium, they redistribute so that the net force on each becomes zero.

Properties (memorise)

1. E inside a conductor is zero.

2. Net charge inside is zero; all excess charge resides on the surface.

3. E just outside is perpendicular to the surface; magnitude $E = \sigma / \varepsilon_0$

4. Potential is constant throughout the volume and on the surface.

5. Surface charge density is higher where the curvature is sharper. (* point -> large sigma -> large E)

6. Cavity: if there is no charge inside a hollow conductor, the field in the cavity is zero - electrostatic shielding.

Dielectrics & Polarisation

Dielectric : an insulator that develops an induced dipole moment in an external E. No free charges to move (unlike metal).

Two kinds of molecules

(a) Non - polar : no permanent dipole

(H_2 , O_2 , CO_2) . In E , the +ve and -ve centres shift -> induced dipole.

(b) Polar : permanent dipole already present (H_2O , HCl) . In E , dipoles rotate to align along E.

Polarisation P

Dipole moment per unit volume :

$$\boxed{P = \alpha E \quad \text{(linear dielectrics)}}$$

<- alpha = electric
<- susceptibility

Net field inside dielectric reduces :

$$\boxed{E_{net} = E_0 / K}$$

<- K = dielectric
<- constant (>= 1)

K(vacuum)=1 , K(air) 1 , K(water) 80.

Capacitor & Parallel Plate

Capacitor : two conductors of equal and opposite charge, separated by an insulator.

$$\boxed{C = Q / V}$$

<- SI unit : farad
<- 1 F = 1 C / V

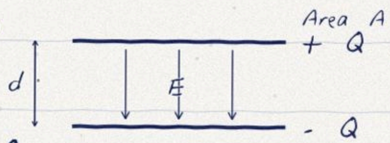
Parallel plate (vacuum)

Plates of area A, separation d, charge +- Q.

Surface density sigma = Q / A.

Field between plates :

$$E = \sigma / \varepsilon_0 = Q / \varepsilon_0 A$$



Potential difference :

$$V = E \cdot d = Q d / \varepsilon_0 A$$

$$\boxed{C = \varepsilon_0 A / d}$$

<- depends only on
<- geometry, not Q or V

Common observations

Increase A -> C increases linearly.

Increase d -> C decreases (inverse).

1 F is huge ; practical : pF, nF, uF.

Edge effects ignored when d << sqrt(A).

Battery sets V, isolated capacitor sets Q.

Both can't be fixed at the same time.

Dielectric in Capacitor & Combinations

With dielectric (full fill)

Insert a slab of constant K between plates :

E reduces E_0 / K

V reduces V_0 / K

C increases $K C_0$.

$$\boxed{C = K \varepsilon_0 A / d}$$

<-K > 1 always
<-so C grows

Series combination

Same charge Q on each ; voltages add.

$$\boxed{1/C_s = 1/C_1 + 1/C_2 + \dots}$$

<-C_s < smallest
<-in the group

Parallel combination

Same V across each ; charges add.

$$\boxed{C_p = C_1 + C_2 + \dots}$$

<-Cp > largest
<-in the group

Quick checks

Two equal C in series -> C/2 .

Two equal C in parallel -> 2 C .

For ~~~~ 3 caps of C in series : C/3.

Series -> divides V like resistors in parallel

(dual circuit relations).

Energy Stored in a Capacitor

Derivation

Charge a capacitor in tiny steps dq against the existing potential $v = q / C$:

$$dW = v\, dq = (q / C)\, dq$$

Total work to bring charge from 0 to Q :

$$W = \int_0^Q (q / C)\, dq = Q^2 / (2C)$$

This W is stored as electrostatic PE = U :

$$\boxed{U = Q^2 / 2C = 1/2\, C V^2 = 1/2\, Q V}$$

<- three equiva
<- forms (u

Energy density in the field

For parallel plate (vol = A d) :

$$u = U / (A d) = 1/2\, \varepsilon_0 E^2$$

$$\boxed{u = 1/2\, \varepsilon_0 E^2}$$

<- energy per unit
<- volume of the field

Common questions

When two charged caps are connected, charge redistributes -> some energy lost as heat / radiation. Common V = (C1V1 + C2V2)/(C1+C2).

Energy lost = U_initial - U_final > 0.