NCERT SOLUTIONS

Class 12 Physics

Chapter 2: Electrostatic Potential and Capacitance

Detailed Step-by-Step Exercise Solutions

Q1 **Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.**

Solution

Understanding the Problem:
We have two point charges:

- $q_1 = 5 \times 10^{-8}$ C (positive charge)
- $q_2 = -3 \times 10^{-8}$ C (negative charge)
- Distance between them: $d = 16$ cm $= 0.16$ m

We need to find points on the line joining these charges where the net electric potential is zero. Since potential is a scalar quantity, the net potential at any point is the algebraic sum of potentials due to individual charges.

Step 1: Identify Possible Regions for Zero Potential
Let the two charges be placed on the x-axis with q_1 at origin ($x = 0$) and q_2 at $x = 0.16$ m. The electric potential at a distance r from a point charge q is given by:

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

Since the charges have opposite signs, their potentials will have opposite signs. For the net potential to be zero, the magnitudes of potentials due to both charges must be equal.
There are three regions to consider:

1. **Region I:** To the left of q_1 $(x < 0)$
2. **Region II:** Between the charges $(0 < x < 0.16)$
3. **Region III:** To the right of q_2 $(x > 0.16)$

Step 2: Analyze Region I (Left of q_1)
Let the point P be at a distance x from q_1 on the left side.
Distance from $q_1 = x$ Distance from $q_2 = x + 0.16$
Net potential at P:

$$V = \frac{kq_1}{x} + \frac{kq_2}{x + 0.16} = 0$$

where $k = \frac{1}{4\pi\varepsilon_0}$
Canceling k from both terms:

$$\frac{5 \times 10^{-8}}{x} + \frac{-3 \times 10^{-8}}{x + 0.16} = 0$$

$$\frac{5}{x} = \frac{3}{x + 0.16}$$

Cross multiplying:

$$5(x + 0.16) = 3x$$

$$5x + 0.80 = 3x$$

$$2x = -0.80$$

$$x = -0.40 \text{ m}$$

Since x is negative, this point is actually on the right side of q_1, not on the left. Therefore, **no solution exists in Region I**.

Step 3: Analyze Region II (Between the charges)
Let the point P be at a distance x from q_1.
Distance from $q_1 = x$ Distance from $q_2 = 0.16 - x$
Net potential at P:

$$\frac{k(5 \times 10^{-8})}{x} + \frac{k(-3 \times 10^{-8})}{0.16 - x} = 0$$

$$\frac{5}{x} = \frac{3}{0.16 - x}$$

Cross multiplying:

$$5(0.16 - x) = 3x$$

$$0.80 - 5x = 3x$$

$$0.80 = 8x$$

$$x = 0.10 \text{ m} = 10 \text{ cm}$$

This point lies between the charges (since $0 < 0.10 < 0.16$), so this is a valid solution.

Step 4: Analyze Region III (Right of q_2)
Let the point P be at a distance x from q_2 on the right side.
Distance from $q_1 = 0.16 + x$ Distance from $q_2 = x$
Net potential at P:

$$\frac{k(5 \times 10^{-8})}{0.16 + x} + \frac{k(-3 \times 10^{-8})}{x} = 0$$

$$\frac{5}{0.16 + x} = \frac{3}{x}$$

Cross multiplying:

$$5x = 3(0.16 + x)$$

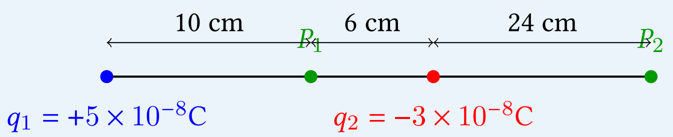
$$5x = 0.48 + 3x$$

$$2x = 0.48$$

$$x = 0.24 \text{ m} = 24 \text{ cm}$$

Distance from $q_1 = 0.16 + 0.24 = 0.40 \text{ m} = 40 \text{ cm}$

Visual Representation:



The electric potential is zero at two points on the line joining the charges:

1. **At a distance of 10 cm from the 5×10^{-8} C charge, between the two charges.**
2. **At a distance of 40 cm from the 5×10^{-8} C charge, on the outer side of the negative charge.**

Expert's Solution — Aarav Sharma, B.Tech CSE, IIT Bombay

Alternative Approach Using Ratio Method:
Instead of solving equations for each region separately, we can use the principle that for zero potential due to opposite charges:

$$\left|\frac{q_1}{r_1}\right| = \left|\frac{q_2}{r_2}\right|$$

$$\frac{r_1}{r_2} = \frac{|q_1|}{|q_2|} = \frac{5 \times 10^{-8}}{3 \times 10^{-8}} = \frac{5}{3}$$

Case 1: Point lies between the charges

$$r_1 + r_2 = 16 \text{ cm}$$

$$\frac{r_1}{r_2} = \frac{5}{3} \implies r_1 = \frac{5}{3}r_2$$

Substituting:

$$\frac{5}{3}r_2 + r_2 = 16$$

$$\frac{8}{3}r_2 = 16 \implies r_2 = 6 \text{ cm}$$

$$r_1 = 10 \text{ cm}$$

Case 2: Point lies outside the charges (beyond the smaller magnitude charge)

$$r_1 - r_2 = 16 \text{ cm}$$

$$\frac{r_1}{r_2} = \frac{5}{3} \implies r_1 = \frac{5}{3}r_2$$

Substituting:

$$\frac{5}{3}r_2 - r_2 = 16$$

$$\frac{2}{3}r_2 = 16 \implies r_2 = 24 \text{ cm}$$

$$r_1 = 40 \text{ cm}$$

Did You Know?

For two opposite charges, zero potential points always exist *between* them and *beyond the smaller magnitude charge*. For two like charges (both positive or both negative), zero potential points exist only *between* them, and only if the charges have equal magnitudes!

Q2 A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon.

Solution

Given Data:

- Side of hexagon = 10 cm = 0.10 m
- Charge at each vertex $q = 5\ \mu\text{C} = 5 \times 10^{-6}$ C
- Number of vertices $n = 6$
- $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2$

Step 1: Find distance from centre to each vertex

A regular hexagon can be divided into 6 equilateral triangles. In an equilateral triangle, all sides are equal.
Since the side of hexagon forms one side of this triangle, and the distance from centre to vertex forms another side, we have:

$$\text{Distance from centre to vertex } (r) = \text{Side of hexagon} = 0.10 \text{ m}$$

All 6 charges are at the same distance $r = 0.10$ m from the centre.

Step 2: Potential due to a single charge
The electric potential V at distance r from a point charge q is:

$$V_1 = \frac{kq}{r}$$

Substituting values:

$$V_1 = \frac{9 \times 10^9 \times 5 \times 10^{-6}}{0.10} = \frac{45 \times 10^3}{0.10} = 4.5 \times 10^5 \text{ V}$$

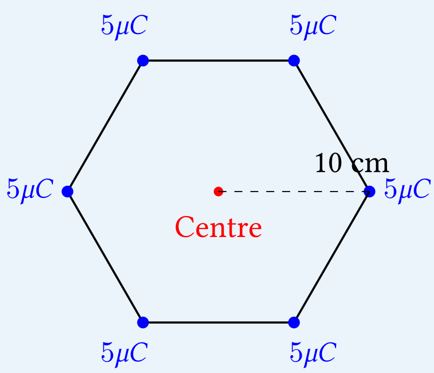
Step 3: Calculate total potential at centre
Electric potential is a **scalar quantity** (it has magnitude only, no direction). Therefore, the net potential is simply the algebraic sum of potentials due to all individual charges.
Since all 6 charges are:

- Equal in magnitude (5 μC each)
- Same sign (all positive)
- At equal distance from centre (0.10 m)

The total potential is:

$$V_{\text{total}} = 6 \times V_1 = 6 \times 4.5 \times 10^5 = 27 \times 10^5 = 2.7 \times 10^6 \text{ V}$$

Visual Representation:



2.7×10^6 **V** or 2.7 **MV**

Expert's Solution — Harshit Singh, B.Tech CSE, IIT Delhi

Quick Formula Approach:
For n identical charges placed symmetrically at equal distance r from a point:

$$V_{\text{total}} = n \times \frac{kq}{r}$$

Substitute directly:

$$V = 6 \times \frac{9 \times 10^9 \times 5 \times 10^{-6}}{0.10} = 6 \times 4.5 \times 10^5 = 2.7 \times 10^6 \text{ V}$$

Did You Know?

Key Points to Remember:

- In a regular hexagon, centre-to-vertex distance = side length
- Potential is a **scalar** — just add algebraically
- Electric field is a **vector** — would be zero at centre here
- Always convert units to SI before calculation (cm $\rightarrow$ m, μC $\rightarrow$ C)

Q3 Two charges 2 μC and -2 μC are placed at points A and B 6 cm apart.

(a) Identify an equipotential surface of the system.

(b) What is the direction of the electric field at every point on this surface?

Solution

Given:

- $q_1 = +2$ μC at point A
- $q_2 = -2$ μC at point B
- Distance AB = 6 cm

This is an **electric dipole** system with equal and opposite charges.

(a) Equipotential Surface:
An equipotential surface is a surface where the electric potential is constant at every point.

For a dipole (equal and opposite charges), the potential at any point P is:

$$V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = kq\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where r_1 and r_2 are distances from P to A and B respectively.
For $V = 0$, we need:

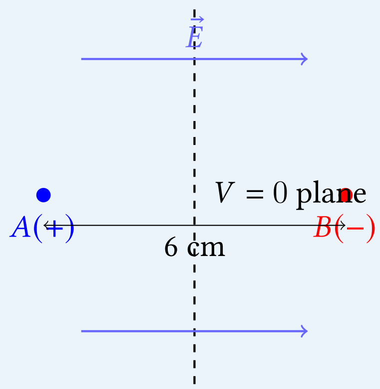
$$\frac{1}{r_1} - \frac{1}{r_2} = 0 \implies r_1 = r_2$$

This means the potential is zero at all points that are **equidistant** from both charges.
The locus of points equidistant from two fixed points is the **perpendicular bisector** of the line joining them.

(b) Direction of Electric Field:
The electric field $\vec{E}$ is always **perpendicular** to an equipotential surface.
For this dipole system:

- The equipotential surface (V = 0) is the perpendicular bisector plane
- At any point on this plane, the electric field vector is **parallel to the line AB** (the dipole axis)
- The direction is from the **positive charge towards the negative charge**

Visual Representation:



(a) The plane normal to the line AB and passing through its mid-point (the perpendicular bisector plane) is an equipotential surface with zero potential.

(b) The electric field is perpendicular to the equipotential surface at every point, directed parallel to the line joining the two charges, from the positive charge (A) towards the negative charge (B).

Expert's Solution — Rohan Verma, B.Tech CSE, IIT Delhi

Understanding the Concept:
For any two equal and opposite charges (dipole):

$$V = \frac{kq}{r_1} - \frac{kq}{r_2} = kq\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Key Observations:

- $V = 0$ when $r_1 = r_2 \rightarrow$ Perpendicular bisector plane
- $V > 0$ on the positive charge side ($r_1 < r_2$)
- $V < 0$ on the negative charge side ($r_1 > r_2$)

Direction of $\vec{E}$ on V = 0 surface:

- $\vec{E}$ is always $\perp$ to equipotential surface
- Direction: From higher potential to lower potential
- Since V = 0 surface lies between positive (V > 0) and negative (V < 0) regions, $\vec{E}$ points from positive to negative charge

Did You Know?

Remember:

- Equipotential surfaces are always $\perp$ to electric field lines
- For a dipole, the V = 0 surface is the perpendicular bisector plane
- Electric field on this surface is uniform in direction (parallel to dipole axis) but varies in magnitude
- Work done in moving a charge on an equipotential surface is zero ($W = q\Delta V = 0$)

Q4 A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7} C distributed uniformly on its surface. What is the electric field

(a) inside the sphere

(b) just outside the sphere

(c) at a point 18 cm from the centre of the sphere?

Solution

Given:

- Radius of spherical conductor, $R = 12$ cm $= 0.12$ m
- Charge on the sphere, $q = 1.6 \times 10^{-7}$ C
- Charge is uniformly distributed on the surface
- $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2$

(a) Electric field inside the sphere:
For a spherical conductor with charge distributed on its surface, we apply **Gauss's law**.
Key property of conductors in electrostatic equilibrium:

- All excess charge resides on the outer surface
- There is no charge inside the conductor

Consider a Gaussian surface (a sphere) of radius $r < R$ inside the conductor.
By Gauss's law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

Since there is no charge inside the conductor, $q_{\text{enclosed}} = 0$.
Therefore:

$$\oint \vec{E} \cdot d\vec{A} = 0 \implies E = 0$$

(b) Electric field just outside the sphere:
"Just outside" means at a distance $r = R$ from the centre (on the surface).
For a spherical charge distribution, the electric field outside behaves as if all charge is concentrated at the centre.
Using the formula for electric field due to a point charge:

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}$$

Substitute the values:

$$E = \frac{9 \times 10^9 \times 1.6 \times 10^{-7}}{(0.12)^2}$$

Calculate numerator:

$$9 \times 10^9 \times 1.6 \times 10^{-7} = 14.4 \times 10^2 = 1440$$

Calculate denominator:

$$(0.12)^2 = 0.0144$$

Now:

$$E = \frac{1440}{0.0144} = 100,000 = 1 \times 10^5\ \text{N/C}$$

(c) Electric field at a point 18 cm from the centre:
Here, distance from centre $r = 18$ cm $= 0.18$ m.

Since $r > R$ (0.18 m > 0.12 m), the point is outside the sphere.
Again, the sphere behaves like a point charge at the centre:

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

Substitute the values:

$$E = \frac{9 \times 10^9 \times 1.6 \times 10^{-7}}{(0.18)^2}$$

Numerator (same as before):

$$9 \times 10^9 \times 1.6 \times 10^{-7} = 1440$$

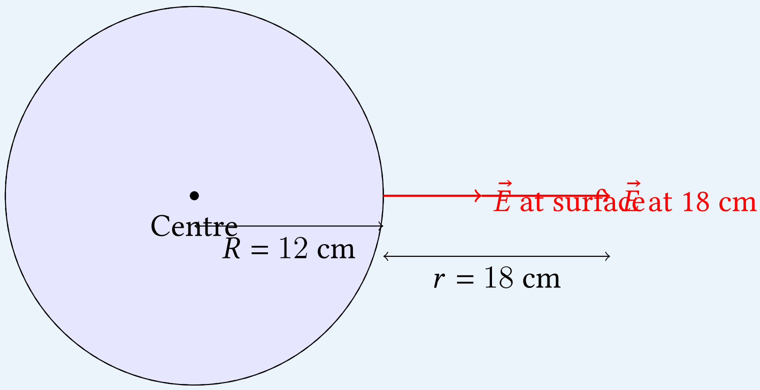
Denominator:

$$(0.18)^2 = 0.0324$$

Now:

$$E = \frac{1440}{0.0324} = 44,444.44 \text{ N/C} \approx 4.44 \times 10^4 \text{ N/C}$$

Visual Representation:



(a) 0

(b) 1×10^5 **N/C**

(c) 4.44×10^4 **N/C**

Expert's Solution — Satyam Yadav, B.Tech CSE, IIT Delhi

Quick Summary Using Gauss's Law:
For a charged spherical conductor of radius R with total charge q:

$$E(r) = \begin{cases} 0 & \text{for } r < R \text{ (inside)} \\ \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2} & \text{for } r = R \text{ (at surface)} \\ \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} & \text{for } r > R \text{ (outside)} \end{cases}$$

Alternative Calculation for (c):
Notice that $r = 18$ cm $= 1.5R$ (since $R = 12$ cm).
Since $E \propto \frac{1}{r^2}$, we can write:

$$\frac{E_c}{E_b} = \frac{R^2}{r^2} = \frac{1}{(1.5)^2} = \frac{1}{2.25}$$

Therefore:

$$E_c = \frac{E_b}{2.25} = \frac{1 \times 10^5}{2.25} = 4.44 \times 10^4 \text{ N/C}$$

Did You Know?

Key Points:

- **Inside conductor:** $E = 0$ always (electrostatic shielding)
- **Outside:** Behaves like a point charge at centre
- **At surface:** Maximum electric field for given charge
- Use proportionality to save time: $E_1 r_1^2 = E_2 r_2^2$

Q5 **A parallel plate capacitor with air between the plates has a capacitance of 8 pF (1pF = 10^{-12} F). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?**

Solution

Given:

- Initial capacitance, $C_0 = 8$ pF $= 8 \times 10^{-12}$ F
- Initial medium: Air (dielectric constant $K_0 \approx 1$)
- Initial plate separation: $d_0 = d$
- New plate separation: $d' = \frac{d}{2}$ (reduced by half)
- New medium: Dielectric constant $K = 6$

Step 1: Formula for capacitance of parallel plate capacitor
The capacitance of a parallel plate capacitor is given by:

$$C = \frac{K\varepsilon_0 A}{d}$$

where:

- K = dielectric constant of the medium
- ε_0 = permittivity of free space
- A = area of plates
- d = distance between plates

Step 2: Express initial capacitance
For air between plates ($K_0 \approx 1$):

$$C_0 = \frac{\varepsilon_0 A}{d} = 8 \text{ pF}$$

Step 3: Express new capacitance
With dielectric constant $K = 6$ and new distance $d' = \frac{d}{2}$:

$$C' = \frac{K\varepsilon_0 A}{d'} = \frac{6 \times \varepsilon_0 A}{d/2} = 6 \times 2 \times \frac{\varepsilon_0 A}{d}$$

Step 4: Relate new capacitance to initial capacitance
Since $\frac{\varepsilon_0 A}{d} = C_0$, we have:

$$C' = 6 \times 2 \times C_0 = 12 \times C_0 = 12 \times 8 = 96 \text{ pF}$$

96 **pF**

Expert's Solution — Vivek Jain, B.Tech Civil, IIT kanpur

Shortcut Method:
For a parallel plate capacitor:

$$C \propto \frac{K}{d}$$

When multiple changes occur, multiply the individual factors:

$$C_{\text{new}} = C_{\text{initial}} \times \frac{K_{\text{new}}}{K_{\text{initial}}} \times \frac{d_{\text{initial}}}{d_{\text{new}}}$$

Substitute values:

$$C_{\text{new}} = 8 \times \frac{6}{1} \times \frac{d}{d/2} = 8 \times 6 \times 2 = 96 \text{ pF}$$

Did You Know?

Key Points to Remember:

- $C \propto \frac{K}{d}$ (for fixed area)
- Reducing distance $\longrightarrow$ Increases capacitance
- Adding dielectric ($K > 1$) $\longrightarrow$ Increases capacitance
- Multiply factors to get overall change

Q6 Three capacitors each of capacitance 9 pF are connected in series.

(a) What is the total capacitance of the combination?

(b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

Solution

Given:

- Three capacitors: $C_1 = C_2 = C_3 = 9$ pF
- Connection: Series
- Supply voltage (for part b): $V = 120$ V

(a) Total capacitance of the combination:
For capacitors connected in **series**, the reciprocal of the equivalent capacitance is the sum of reciprocals of individual capacitances:

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Since all three capacitors are identical ($C_1 = C_2 = C_3 = C = 9$ pF):

$$\frac{1}{C_s} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$$

Therefore:

$$C_s = 3 \text{ pF}$$

(b) Potential difference across each capacitor:

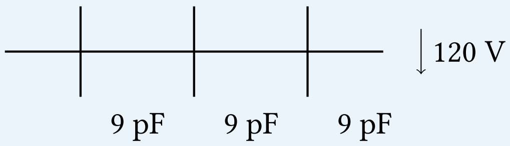
When the series combination is connected to a 120 V supply, the total voltage across the combination is $V = 120$ V.
In a series connection, **the charge on each capacitor is the same**.
The total charge Q on the equivalent capacitor is:

$$Q = C_s \times V = 3 \text{ pF} \times 120 \text{ V} = 360 \text{ pC}$$

For each capacitor ($C = 9$ pF):

$$V_1 = V_2 = V_3 = \frac{Q}{C} = \frac{360 \text{ pC}}{9 \text{ pF}} = 40 \text{ V}$$

Visual Representation:



(a) 3 **pF**

(b) 40 **V** each

Expert's Solution — Harshit Singh, B.Tech CSE, IIT Delhi

Shortcut Method for Series Capacitors:
For part (a): For n identical capacitors in series:

$$C_{eq} = \frac{C}{n} = \frac{9}{3} = 3 \text{ pF}$$

For part (b): For identical capacitors in series, the voltage divides **equally** across each capacitor:

$$V_{each} = \frac{V_{total}}{n} = \frac{120}{3} = 40 \text{ V}$$

Did You Know?

Key Points to Remember:

- **Series:** $C_{eq} = \frac{C}{n}$ (for identical capacitors)
- In series, voltage divides equally if capacitances are equal
- Charge on each capacitor in series is the same

Q7 **Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.**

(a) What is the total capacitance of the combination?

(b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.

Solution

Given:

- $C_1 = 2$ pF
- $C_2 = 3$ pF
- $C_3 = 4$ pF
- Connection: Parallel
- Supply voltage (for part b): $V = 100$ V

(a) Total capacitance of the combination:
For capacitors connected in **parallel**, the equivalent capacitance is the sum of individual capacitances:

$$C_p = C_1 + C_2 + C_3$$

Substitute the given values:

$$C_p = 2 + 3 + 4 = 9 \text{ pF}$$

(b) Charge on each capacitor:
In a parallel connection, **the potential difference across each capacitor is the same** and equals the supply voltage.
Therefore:

$$V_1 = V_2 = V_3 = V = 100 \text{ V}$$

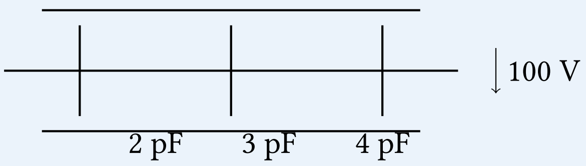
Using the formula $Q = CV$:

$$Q_1 = C_1 \times V = 2 \text{ pF} \times 100 \text{ V} = 200 \text{ pC}$$

$$Q_2 = C_2 \times V = 3 \text{ pF} \times 100 \text{ V} = 300 \text{ pC}$$

$$Q_3 = C_3 \times V = 4 \text{ pF} \times 100 \text{ V} = 400 \text{ pC}$$

Visual Representation:



(a) 9 **pF**

(b)

- **Charge on 2 pF capacitor** = 200 **pC**
- **Charge on 3 pF capacitor** = 300 **pC**
- **Charge on 4 pF capacitor** = 400 **pC**

Expert's Solution — vivek Sharma, B.Tech CSE, NIT Delhi

Key Formulas for Parallel Capacitors:
For part (a):

$$C_{eq} = C_1 + C_2 + C_3 = 2 + 3 + 4 = 9 \text{ pF}$$

For part (b): In parallel connection:

- Voltage across each capacitor = Supply voltage
- $Q = CV$ for each capacitor individually

Did You Know?

Key Points to Remember:

- **Parallel:** $C_p = C_1 + C_2 + C_3 + \ldots$
- In parallel, voltage is **same** across all capacitors
- Charge on each capacitor is proportional to its capacitance: $Q \propto C$

Q8 **In a parallel plate capacitor with air between the plates, each plate has an area of 6×10^{-3} m^2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?**

Solution

Given:

- Area of each plate, $A = 6 \times 10^{-3}\ \text{m}^2$
- Distance between plates, $d = 3\ \text{mm} = 3 \times 10^{-3}\ \text{m}$
- Medium between plates: Air (dielectric constant $K \approx 1$)
- Permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2$
- Supply voltage, $V = 100\ \text{V}$

Part 1: Calculate the capacitance
The capacitance of a parallel plate capacitor is given by:

$$C = \frac{\varepsilon_0 A}{d}$$

Substitute the given values:

$$C = \frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}}$$

Simplify:

$$C = \frac{8.85 \times 10^{-12} \times 6}{3} = 8.85 \times 10^{-12} \times 2 = 17.7 \times 10^{-12}\ \text{F}$$

Part 2: Calculate the charge on each plate
When connected to a 100 V supply, the charge on the capacitor is:

$$Q = C \times V$$

Substitute the values:

$$Q = 17.7 \times 10^{-12} \times 100 = 1.77 \times 10^{-9}\ \text{C}$$

Capacitance: 17.7 **pF**

Charge: 1.77×10^{-9} **C** or 1.77 **nC**

Expert's Solution — Bindu Singh, B.Tech CSE, IIT Bombay

Quick Calculation Method:
Capacitance:

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}} = 8.85 \times 10^{-12} \times 2 = 17.7 \times 10^{-12}\ \text{F} = 17.7\ \text{pF}$$

Charge:

$$Q = CV = 17.7 \times 10^{-12} \times 100 = 1.77 \times 10^{-9}\ \text{C}$$

Did You Know?

Key Points:

- Always convert all units to SI before calculation (mm → m)
- Capacitance depends only on geometry (A, d) and medium (ε)
- Charge $Q = CV$ (directly proportional to voltage)

Q9 **Explain what would happen if in the capacitor given in Exercise 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,**

(a) while the voltage supply remained connected.

(b) after the supply was disconnected.

Solution

Recall from Exercise 2.8:

- Area of plates, $A = 6 \times 10^{-3}$ m^2
- Plate separation, $d = 3$ mm $= 3 \times 10^{-3}$ m
- Initial capacitance with air, $C_0 = 17.7$ pF
- Supply voltage, $V = 100$ V
- Initial charge, $Q_0 = 1.77 \times 10^{-9}$ C
- Dielectric constant of mica, $K = 6$

When a dielectric is inserted, the capacitance increases by a factor of K:

$$C = KC_0 = 6 \times 17.7 = 106.2 \text{ pF}$$

(a) While the voltage supply remained connected:
When the voltage supply remains connected, the potential difference across the capacitor stays constant.
Given: $V = 100$ V (constant)
Effect on Capacitance:

$$C = KC_0 = 6 \times 17.7 = 106.2 \text{ pF}$$

The capacitance increases by a factor of 6.
Effect on Charge: Since V is constant and C increases:

$$Q = C \times V = 106.2 \times 10^{-12} \times 100 = 1.062 \times 10^{-8} \text{ C}$$

The charge increases by a factor of 6.
Effect on Electric Field:

$$E = \frac{V}{d} = \frac{100}{3 \times 10^{-3}} = 3.33 \times 10^{4} \text{ V/m}$$

The electric field remains unchanged.

(b) After the supply was disconnected:
When the supply is disconnected, the capacitor is isolated and the charge on the plates remains constant.
Given: $Q_0 = 1.77 \times 10^{-9}$ C (constant)
Effect on Capacitance:

$$C = KC_0 = 6 \times 17.7 = 106.2 \text{ pF}$$

Effect on Voltage: Since Q is constant and C increases:

$$V = \frac{Q_0}{C} = \frac{Q_0}{KC_0} = \frac{V_0}{K} = \frac{100}{6} = 16.67 \text{ V}$$

Effect on Electric Field:

$$E = \frac{V}{d} = \frac{V_0/K}{d} = \frac{E_0}{K} = \frac{3.33 \times 10^{4}}{6} = 5.56 \times 10^{3} \text{ V/m}$$

(a) With supply connected:

- **Capacitance:** Increases to 106.2 **pF**
- **Charge:** Increases to 1.062×10^{-8} **C**
- **Voltage:** Remains constant at 100 **V**
- **Electric Field:** Remains unchanged

(b) With supply disconnected:

- **Capacitance:** Increases to 106.2 **pF**
- **Charge:** Remains constant at 1.77×10^{-9} **C**
- **Voltage:** Decreases to 16.67 **V**
- **Electric Field:** Decreases by factor of 6

Expert's Solution — Arab Sharma, B.Tech CSE, NIT Bhubaneswar

Summary Comparison Table:

Quantity	Supply Connected	Supply Disconnected
Capacitance C	KC_0 (Increases)	KC_0 (Increases)
Charge Q	KQ_0 (Increases)	Q_0 (Constant)
Voltage V	V_0 (Constant)	V_0/K (Decreases)
Electric Field E	E_0 (Constant)	E_0/K (Decreases)

Did You Know?

Key Principles:

- **Constant V:** $Q = CV \propto C$ (Charge follows capacitance)
- **Constant Q:** $V = Q/C \propto 1/C$ (Voltage inversely follows capacitance)
- Dielectric always increases capacitance ($C = KC_0$)

Q10 A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor?

Solution

Given:

- Capacitance, $C = 12 \text{ pF} = 12 \times 10^{-12}$ F
- Voltage, $V = 50$ V

Step 1: Formula for electrostatic energy stored in a capacitor
The energy stored in a capacitor is given by:

$$U = \frac{1}{2}CV^2$$

Step 2: Substitute the values

$$U = \frac{1}{2} \times (12 \times 10^{-12}) \times (50)^2$$

Step 3: Calculate step by step
First, calculate V^2:

$$V^2 = 50^2 = 2500$$

Now multiply:

$$U = \frac{1}{2} \times 12 \times 10^{-12} \times 2500$$
$$U = 6 \times 10^{-12} \times 2500$$
$$U = 15000 \times 10^{-12}$$
$$U = 1.5 \times 10^{-8}\ \text{J}$$

1.5×10^{-8} **J** or 15 **nJ**

Expert's Solution — Harshit Singh, B.Tech CSE, IIT Delhi

Alternative Formulas for Energy:
The energy stored in a capacitor can be expressed in three equivalent forms:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$$

Quick Mental Calculation:

$$U = \frac{1}{2} \times 12 \times 10^{-12} \times 2500$$
$$= 6 \times 2500 \times 10^{-12}$$
$$= 15000 \times 10^{-12}$$
$$= 1.5 \times 10^{-8}\ \text{J}$$

Did You Know?

Key Points:

- Always convert pF to F before calculation: $1\ \text{pF} = 10^{-12}\ \text{F}$
- Choose the energy formula based on known quantities
- Common units: $1\ \text{nJ} = 10^{-9}\ \text{J}$, $1\ \mu\text{J} = 10^{-6}\ \text{J}$

Q11 **A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?**

Solution

Given:

- Capacitance of first capacitor, $C_1 = 600 \text{ pF} = 600 \times 10^{-12}$ F
- Capacitance of second capacitor, $C_2 = 600 \text{ pF} = 600 \times 10^{-12}$ F
- Initial voltage across C_1, $V_1 = 200$ V
- Initial voltage across C_2, $V_2 = 0$ V (uncharged)

Step 1: Calculate initial charge on the first capacitor

$$Q_0 = C_1 \times V_1 = 600 \times 10^{-12} \times 200 = 1.2 \times 10^{-7} \text{ C}$$

Step 2: Calculate initial energy stored in the system
Since only C_1 is charged initially:

$$U_i = \frac{1}{2}C_1V_1^2 = \frac{1}{2} \times (600 \times 10^{-12}) \times (200)^2 = 1.2 \times 10^{-5} \text{ J}$$

Step 3: Calculate final voltage after connecting both capacitors
When the two capacitors are connected in parallel, charge redistributes between them.
Total charge is conserved:

$$Q_{\text{total}} = Q_0 = 1.2 \times 10^{-7} \text{ C}$$

Equivalent capacitance in parallel:

$$C_{\text{eq}} = C_1 + C_2 = 600 + 600 = 1200 \text{ pF} = 1200 \times 10^{-12} \text{ F}$$

Final common voltage across both capacitors:

$$V_f = \frac{Q_{\text{total}}}{C_{\text{eq}}} = \frac{1.2 \times 10^{-7}}{1200 \times 10^{-12}} = 100 \text{ V}$$

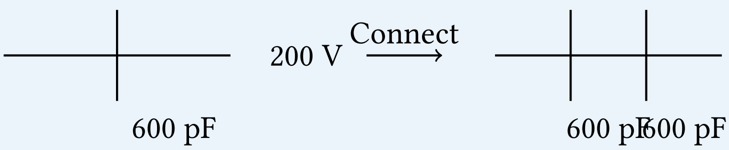
Step 4: Calculate final energy stored in the system

$$U_f = \frac{1}{2}C_{\text{eq}}V_f^2 = \frac{1}{2} \times (1200 \times 10^{-12}) \times (100)^2 = 0.6 \times 10^{-5} \text{ J}$$

Step 5: Calculate energy lost

$$\Delta U = U_i - U_f = 1.2 \times 10^{-5} - 0.6 \times 10^{-5} = 0.6 \times 10^{-5} \text{ J} = 6 \times 10^{-6} \text{ J}$$

Visual Representation:



6×10^{-6} **J or 6** μ**J**

Expert's Solution — Rahul Mehta, B.Tech ECE, IIT Roorkee

Shortcut Formula for Energy Loss:
When a charged capacitor (C_1) is connected to an uncharged capacitor (C_2), the energy loss is given by:

$$\Delta U = \frac{1}{2} \cdot \frac{C_1 C_2}{C_1 + C_2} \cdot (V_1 - V_2)^2$$

Since $V_2 = 0$ and $C_1 = C_2 = C$:

$$\Delta U = \frac{1}{2} \cdot \frac{C \cdot C}{C + C} \cdot V_1^2 = \frac{1}{2} \cdot \frac{C^2}{2C} \cdot V_1^2 = \frac{1}{4} C V_1^2 = \frac{U_i}{2}$$

Thus, **half of the initial energy is always lost** when two identical capacitors (one charged, one uncharged) are connected!
Verification:

$$U_i = 1.2 \times 10^{-5} \text{ J}$$

$$\Delta U = \frac{U_i}{2} = 0.6 \times 10^{-5} \text{ J} = 6 \times 10^{-6} \text{ J}$$

Did You Know?

Key Points:

- When capacitors are connected, **charge is conserved**, not energy
- For two identical capacitors (one charged, one uncharged):
  - Final voltage = Half of initial voltage
  - Final energy = Half of initial energy
  - Energy lost = Half of initial energy (50%)