NCERT Exemplar Solutions

Solved NCERT Exemplar Problems for Class 12th Physics, Chapter 3

Chapter 3: Current Electricity

About this Chapter

Chapter 3 of the Class 12 NCERT Exemplar builds the language of steady electric currents: drift velocity, Ohm's law, resistance and resistivity, cells with EMF and internal resistance, Kirchhoff's laws, and the practical instruments built on them — the Wheatstone bridge, meter bridge and potentiometer. The 31 Exemplar problems spread across MCQ I (single correct), MCQ II (one or more correct), VSA, SA and LA stretch every one of these ideas. Each solution below restates the key concept first, then walks through the algebra step by step.

Topics covered: Current and current density • Drift velocity and Ohm's law • Resistivity and its temperature dependence • Cells, EMF and internal resistance • Kirchhoff's laws • Wheatstone bridge, meter bridge and potentiometer

Quick Formula Sheet

$I = nAev_d$

$V = IR$, $R = \rho L/A$

$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$

$\varepsilon = V + Ir$

Series: $R_s = \sum R_i$

Parallel: $1/R_p = \sum 1/R_i$

Wheatstone: $R_1/R_2 = R_3/R_4$

Meter bridge: $R/S = l_1/(100 - l_1)$

Potentiometer: $V_1/V_2 = l_1/l_2$

MCQ I (single correct option)

Q 3.1 For a current-carrying wire bent into a circle, the direction of the current density $\vec{j}$ changes continuously along the wire, yet the current I stays the same. What is essentially responsible for changing the direction of $\vec{j}$?
(a) the source of EMF (b) the electric field produced by charges that accumulate on the surface of the wire (c) charges just behind a given segment that push it forward by repulsion (d) charges ahead of a given segment.

SOLUTION

Concept used. Inside a current-carrying wire the steady current is driven by an electric field $\vec{E}$ that points along the wire's axis at every point. In a *curved* wire the axial direction itself changes from segment to segment, so $\vec{E}$ (and hence $\vec{j} = \sigma\vec{E}$) must also bend with the wire. The EMF source alone cannot produce such a position-dependent direction inside the wire — it only fixes the global potential difference between the two terminals.

☞ **Source of the bending field**

When the wire is curved, surface charges redistribute themselves on the outer face of the wire so that, just inside the wire, the net electric field is everywhere parallel to the local axis. These tiny surface charges (much smaller than the bulk current charges) are what "steer" the current around the bend.

Step 1. **Why not (a).** The EMF source sits at the battery terminals. Its only direct role is to maintain a potential difference. It does not, on its own, set the direction of $\vec{E}$ along every point of an externally-shaped wire.

Step 2. **Why not (c) or (d).** Bulk charges (the drifting conduction electrons) do not exert a one-sided "push" on the segment ahead or behind; their distribution in a steady current is essentially uniform along the wire's length.

Step 3. **Why (b) is right.** A curved wire accumulates a thin layer of surface charge: positive on the outer side of a bend, negative on the inner side (or vice versa). These surface charges generate the transverse component of $\vec{E}$ that, added to the longitudinal driving field from the battery, makes the total $\vec{E}$ inside the wire follow the local axis. As the axis turns, so does $\vec{E}$, and therefore so does $\vec{j}$.

Final Answer: Option **(b)**: the electric field produced by surface charges that accumulate on the wire steers $\vec{j}$ around every bend.

★ **Surface-charge picture of a circuit**

This idea — small surface charges set up the in-wire field that guides the current — is what physically connects the battery's EMF to the geometry of the wire. Without them, $\vec{j}$ could not follow a curved path while remaining tangential to it.

EXPERT'S SOLUTION *: Aarav Iyer, Ph.D Physics, IISc Bangalore*

Strategic angle. The question is asking which agent supplies the *direction* of $\vec{j}$ at each point of a curved wire, not its magnitude. The magnitude follows from the EMF (it fixes $|\vec{j}| = I/A$ once I is set by the loop's total resistance); the direction needs a position-dependent field, which only surface charges can provide. This is one of the few places in introductory electrodynamics where surface-charge effects, usually treated as electrostatic curiosities, become physically essential to steady currents.

Step 1. **Microscopic Ohm's law.** $\vec{j} = \sigma\vec{E}$ inside the wire (Ohm's law in local form). σ is a scalar for an isotropic conductor, so the direction of $\vec{j}$ at every point equals the direction of $\vec{E}$ at the same point. The job reduces to: what fixes the direction of $\vec{E}$?

Step 2. **Straight wire baseline.** A straight wire connecting battery terminals has $\vec{E}$ parallel to its axis everywhere — the longitudinal field set up by the battery terminals already lies along the wire because the geometry is uniform. No surface charges are needed there beyond the end caps.

Step 3. **Curved-wire challenge.** A curved wire cannot have a constant-direction $\vec{E}$ everywhere inside — it must bend with the axis. But the battery's emf alone produces a roughly uniform external field; it can't supply a position-dependent direction inside the conductor.

Step 4. **Surface-charge resolution.** Surface charges on the wire's outside form a thin layer (positive on the convex side of a bend, negative on the concave side). Their own field, added to the longitudinal driving field, gives a total $\vec{E}$ that locally follows the wire's axis. They are the steering agent. Ruling out (a), (c), (d) is then immediate: the emf fixes magnitude not direction, bulk carrier distribution is uniform in steady state.

Final Answer: Option (b).

♥ Surface charges and the global picture

Far from being a quirk, the surface-charge mechanism is what unites electrostatics (which gives the static distribution of $\vec{E}$ in empty space) with steady-current circuit theory. Every working circuit on your lab bench — every bent wire, every soldered joint — carries an invisible coating of these charges that quietly steers the current around corners. Take them away and Kirchhoff's voltage law would have no microscopic basis.

Q 3.2 **Two batteries of emfs ε_1 and ε_2 ($\varepsilon_2 > \varepsilon_1$) and internal resistances r_1 and r_2 are connected in parallel as in Fig. 3.1.**

(a) The equivalent EMF ε_{eq} lies between ε_1 and ε_2, i.e. $\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$.

(b) $\varepsilon_{eq} < \varepsilon_1$. (c) $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$ always. (d) ε_{eq} is independent of r_1, r_2.

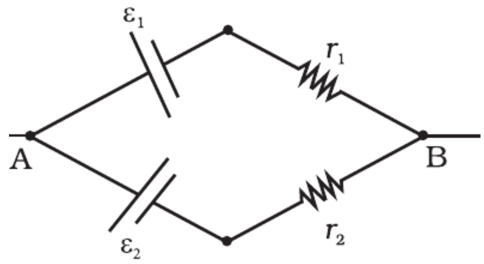


Fig 3.1

Fig. 3.1, NCERT Exemplar Class 12 Physics, Chapter 3.

SOLUTION

Concept used. For two cells in parallel (same polarity, both + terminals at the same node), the equivalent EMF and internal resistance are

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}, \qquad r_{eq} = \frac{r_1 r_2}{r_1 + r_2}.$$

This is the standard result of applying Kirchhoff's rules to the two-cell parallel network and demanding that the combination behave like a single equivalent cell.

Step 1. Write ε_{eq} as a weighted average.

$$\varepsilon_{eq} = \frac{r_2}{r_1 + r_2}\varepsilon_1 + \frac{r_1}{r_1 + r_2}\varepsilon_2.$$

The weights $\frac{r_2}{r_1 + r_2}$ and $\frac{r_1}{r_1 + r_2}$ are positive and sum to 1. So ε_{eq} is a *convex combination* of ε_1 and ε_2.

Step 2. Bound the weighted average. A convex combination of two numbers always lies between them. Since $\varepsilon_1 < \varepsilon_2$,

$$\varepsilon_1 \le \varepsilon_{eq} \le \varepsilon_2,$$

with strict inequality unless one of the weights vanishes (i.e. r_1 or r_2 is zero, which is a degenerate case). In the generic case, $\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$. Option (a) is correct.

Step 3. Rule out the others.

- (b) $\varepsilon_{eq} < \varepsilon_1$ contradicts the weighted-average bound.
- (c) $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$ is the *series* formula, not parallel.
- (d) ε_{eq} explicitly depends on r_1, r_2 through the weights — so it is not independent.

Final Answer: $\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$. Option **(a)**.

★ Why the weighted-average form

A high-EMF, low-internal-resistance cell pulls ε_{eq} strongly toward its own value; a high-internal-resistance cell barely shifts the equivalent. That's why ε_2 is weighted by $r_1/(r_1+r_2)$ — small r_2 means ε_2's weight is large.

EXPERT'S SOLUTION *: Sneha Banerjee, M.Sc Physics, IIT Madras*

Strategic angle. Compute ε_{eq} directly and test whether it lies between ε_1 and ε_2. The convex-combination view turns a circuit-theory question into a one-line inequality — much faster than re-deriving via Kirchhoff each time. It also makes the limit cases transparent: when one internal resistance is much smaller than the other, that cell dominates.

Step 1. **Derive parallel-cell formula via Kirchhoff.** Let I be the load current leaving the junction. Apply Kirchhoff to each branch: branch 1 contributes $V = \varepsilon_1 - I_1 r_1$, branch 2 contributes $V = \varepsilon_2 - I_2 r_2$, with $I_1 + I_2 = I$. Solving gives $\varepsilon_{eq} = (\varepsilon_1 r_2 + \varepsilon_2 r_1)/(r_1 + r_2)$ and $r_{eq} = r_1 r_2/(r_1 + r_2)$.

Step 2. **Re-write as convex combination.** $\varepsilon_{eq} = w_1\varepsilon_1 + w_2\varepsilon_2$ with $w_1 = r_2/(r_1 + r_2)$, $w_2 = r_1/(r_1 + r_2)$. Note $w_1 + w_2 = 1$, both $\in (0, 1)$ for finite positive r_1, r_2.

Step 3. **Apply convex-combination bound.** A convex combination of two positive numbers $\varepsilon_1 < \varepsilon_2$ always lies strictly between them. Hence option (a). The bound is strict because both weights are positive (no $r_i = 0$).

Step 4. **Eliminate distractors.** (b) violates the lower bound; (c) is the series formula not parallel; (d) ignores the explicit r_1, r_2-dependence of every weight.

Final Answer: Option (a).

✗ Don't confuse series and parallel cell rules

The series rule $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$ (option c) tempts students who skip the derivation and recall only "EMFs add". They add in *series*, where the same current threads both cells. In *parallel*, the same *voltage* appears across both, and the EMFs combine through the weighted average above. A simple polarity check helps: parallel cells shouldn't drive an open-circuit voltage greater than the larger cell's EMF, so a sum can't be right.

Q 3.3 **A resistance R is to be measured with a meter bridge. The student picks the standard resistance $S = 100\,\Omega$ and finds the null point at $l_1 = 2.9$ cm. How can the accuracy be improved?**

(a) Measure l_1 more accurately. **(b) Change S to $1000\,\Omega$ and repeat.**

(c) Change S to $3\,\Omega$ and repeat. **(d) Give up.**

SOLUTION

Concept used. The meter-bridge balance condition is

$$\frac{R}{S} = \frac{l_1}{100 - l_1},$$

where l_1 is the null-point distance (in cm) from one end. The *sensitivity* of the measurement is best when l_1 is close to the centre of the wire (around 50 cm), because the fractional change in l_1 for a given fractional change in R is maximised there. Equivalently, R and S should be of comparable magnitude so that the ratio R/S is of order unity.

Step 1. Estimate R from the present measurement.

$$R = S \cdot \frac{l_1}{100 - l_1} = 100 \cdot \frac{2.9}{97.1} \approx 2.99\,\Omega.$$

So the unknown R is roughly $3\,\Omega$.

Step 2. Diagnose the problem. $S = 100\,\Omega$ is about 33 times larger than R, which pushes the null point all the way to $l_1 \approx 3$ cm — extremely close to one end of the bridge wire. Reading l_1 to, say, ± 1 mm there gives a relative error of about 3%, which propagates directly into R.

Step 3. Best fix. Pick S of the same order of magnitude as the estimated R. With $R \approx 3\,\Omega$, choose $S = 3\,\Omega$. The null point then sits around the middle of the wire ($l_1 \approx 50$ cm), where a ± 1 mm reading uncertainty is only $\sim 0.2\%$ of l_1.

Step 4. Rule out the others.

- (a) Reading l_1 "more accurately" near the end of the wire cannot beat the geometric sensitivity handicap.
- (b) $S = 1000\,\Omega$ pushes l_1 even further toward 0, making things worse.
- (d) Giving up is unnecessary — option (c) fixes it.

> **Final Answer:** Option **(c)** — change S to $3\,\Omega$ so the null point lies near the centre.

☞ Meter-bridge sensitivity rule

Whenever the null point sits within ~ 10 cm of either end of the bridge wire, change the standard resistance. The best precision is around the middle, where l_1 and $100 - l_1$ are comparable.

EXPERT'S SOLUTION : *Pranav Kapoor, B.Tech Engineering Physics, IIT Bombay*

Strategic angle. Read the rough value of R from the present measurement, then pick S to put the null near the midpoint. The sensitivity argument is purely geometric: a fixed absolute reading uncertainty δl (a millimetre, say) becomes a smaller *fractional* error when divided by a larger l_1 or $100 - l_1$. Both factors are largest together near the centre.

Step 1. First-pass estimate. $R \approx 100 \cdot (2.9/97.1) \approx 2.99\,\Omega \approx 3\,\Omega$.

Step 2. Sensitivity argument. Differentiating $R = Sl_1/(100 - l_1)$ gives $\delta R/R = \delta l_1 \cdot [1/l_1 + 1/(100 - l_1)]$. The bracket is minimised at $l_1 = 50$ cm. For best sensitivity, the null should sit near 50 cm, which requires $R \approx S$. So $S = 3\,\Omega$ matches our estimate.

Step 3. Quantify the gain. At $l_1 = 2.9$ cm, the bracket is $\approx 1/2.9 + 1/97.1 \approx 0.355$/cm. At $l_1 = 50$ cm it is ≈ 0.040/cm — nearly a 9-fold precision gain for the same δl_1.

Step 4. Options (a), (b), (d) all fail this geometric requirement — (a) cannot beat the sensitivity handicap, (b) makes it worse, (d) discards a fixable problem.

Final Answer: Option (c).

☞ **CBSE meter-bridge marking rule**

CBSE board questions on the meter bridge routinely test the sensitivity argument. If a problem reports l_1 within 10 cm of either end, the expected "improvement" answer is almost always: change the standard resistance so the null point shifts toward 50 cm. State the relation $R \approx S$ at maximum sensitivity for full marks.

Q 3.4 Two cells of EMFs approximately 5 V and 10 V are to be accurately compared using a potentiometer with a 400 cm wire.

(a) The driving battery should have voltage 8 V.

(b) The driving battery should have voltage 15 V, with R adjusted so the potential drop across the wire slightly exceeds 10 V.

(c) The first 50 cm of wire alone should drop 10 V.

(d) A potentiometer is used for comparing resistances, not voltages.

SOLUTION

Concept used. A potentiometer balances an unknown EMF ε against the potential drop across some length of its wire. For a balance point to exist, the total potential drop across the full wire must *exceed* the largest EMF being measured. Otherwise the balance length would require more wire than there is, and the galvanometer never reads zero.

Calling V_{wire} the drop across the full $400\,\text{cm}$ of potentiometer wire, the requirement for measuring a $10\,\text{V}$ cell is $V_{\text{wire}} \gtrsim 10\,\text{V}$.

Step 1. **Rule out (a).** The driving battery is $8\,\text{V}$; even if every volt fell on the wire, $V_{\text{wire}} \leq 8\,\text{V} < 10\,\text{V}$. The $10\,\text{V}$ cell can never be balanced. (a) fails.

Step 2. **Test (b).** A $15\,\text{V}$ battery has plenty of headroom. Connect a series resistor R in the driving loop so the drop across the wire is slightly more than $10\,\text{V}$ (say $\sim 11\,\text{V}$). Then both the $5\,\text{V}$ and the $10\,\text{V}$ cells can be balanced on the wire, with the $10\,\text{V}$ cell's balance point near the far end. Sensible choice. (b) works.

Step 3. **Rule out (c).** "First $50\,\text{cm}$ should drop $10\,\text{V}$" implies $V_{\text{wire}} = 10\,\text{V} \times (400/50) = 80\,\text{V}$ across the whole wire — wasteful and dangerous, and forces the balance lengths to be very short and therefore imprecise. (c) fails.

Step 4. **Rule out (d).** A potentiometer's classic use is *exactly* comparing EMFs (and the closely related problem of measuring internal resistance). (d) is wrong as a matter of fact.

> **Final Answer:** Option **(b)** — $15\,\text{V}$ driver, with R adjusted to keep the wire drop just above $10\,\text{V}$.

★ Why "slightly exceeds $10\,\text{V}$"?

A wire drop only slightly larger than the maximum EMF puts the balance point near the far end of the wire, where a $1\,\text{mm}$ uncertainty is a tiny fraction of the balance length, giving the best precision.

EXPERT'S SOLUTION *: Ananya Joshi, M.Sc Physics, IIT Kanpur*

Strategic angle. Two facts pin the answer: the driving battery must put more potential across the wire than the largest measured EMF (existence of balance), and the wire drop should be only slightly larger so balance lengths are long (precision). The question is essentially asking the student to pick the choice that satisfies both constraints simultaneously.

Step 1. **Existence condition.** $V_{\text{wire}} \geq \max(\varepsilon_1, \varepsilon_2) = 10\,\text{V}$. An $8\,\text{V}$ driver (option a) cannot meet this — the galvanometer will never null on the $10\,\text{V}$ cell.

Step 2. **Precision condition.** Balance length $l \approx \varepsilon\, L/V_{\text{wire}}$. With V_{wire} much larger than $10\,\text{V}$ (option c forces $V_{\text{wire}} = 80\,\text{V}$), l shrinks to a small fraction of the wire — a $1\,\text{mm}$ jockey uncertainty then becomes a much larger fractional error. Lousy precision.

Step 3. **Sweet spot.** A $15\,\text{V}$ driver with a series resistor tuned to keep the wire drop *just* *above* $10\,\text{V}$ (option b) gives long balance lengths for both cells, with the $10\,\text{V}$ cell's balance close to the far end. Best precision compatible with the existence requirement.

Step 4. Option (d). Confuses the potentiometer with the Wheatstone bridge. The potentiometer's primary job *is* EMF (voltage) comparison — Wheatstone bridges compare resistances.

Final Answer: Option (b).

☞ **The two potentiometer constraints**

Every potentiometer problem comes down to two inequalities: **(1)** wire drop $V_{\text{wire}} >$ largest EMF being balanced (otherwise no null), and **(2)** V_{wire} not much larger than that EMF (otherwise balance length is too short for precision). Memorise this pair — most board-exam potentiometer numericals are an immediate application.

Q 3.5 A metal rod has length $10\,\text{cm}$ and a rectangular cross-section $1\,\text{cm} \times \frac{1}{2}\,\text{cm}$. It is connected to a battery across one pair of opposite faces. The resistance is maximum when the battery is connected across:

(a) the $1\,\text{cm} \times \frac{1}{2}\,\text{cm}$ faces. (b) the $10\,\text{cm} \times 1\,\text{cm}$ faces.

(c) the $10\,\text{cm} \times \frac{1}{2}\,\text{cm}$ faces. (d) all three give the same resistance.

SOLUTION

Concept used. For a uniform conductor of resistivity ρ, length L (the distance current flows) and cross-sectional area A (perpendicular to current),

$$R = \frac{\rho L}{A}.$$

R grows with L and falls with A. To maximise R, choose the pair of faces that give the *largest* L and the *smallest* A.

Step 1. List the three options. The rod has three pairs of opposite faces:

(1) $1\,\text{cm} \times \frac{1}{2}\,\text{cm}$ faces: $L = 10\,\text{cm}$, $A = 0.5\,\text{cm}^2$. So $L/A = 20\,\text{cm}^{-1}$.

(2) $10\,\text{cm} \times 1\,\text{cm}$ faces: $L = 0.5\,\text{cm}$, $A = 10\,\text{cm}^2$. So $L/A = 0.05\,\text{cm}^{-1}$.

(3) $10\,\text{cm} \times \frac{1}{2}\,\text{cm}$ faces: $L = 1\,\text{cm}$, $A = 5\,\text{cm}^2$. So $L/A = 0.20\,\text{cm}^{-1}$.

Step 2. Compute the ratios. $R = \rho(L/A)$. The pair with the largest L/A gives the largest R. Comparing the three values:

$$20 \gg 0.20 > 0.05,$$

so case (1) — connecting across the $1\,\text{cm} \times \frac{1}{2}\,\text{cm}$ faces — gives the maximum resistance.

Step 3. **Conclude.** Option (a) is correct. The same rod has a $400\times$ smaller resistance when connected across its broadest faces.

> **Final Answer:** Option **(a)** — across the $1\,\text{cm} \times \frac{1}{2}\,\text{cm}$ faces.

★ Geometry over composition

The resistivity ρ is a material constant; it doesn't care which faces you pick. What *does* change is the geometric factor L/A. Long-and-thin orientations give high R; short-and-fat orientations give low R.

EXPERT'S SOLUTION *: Karan Reddy, M.Tech Applied Physics, IIT Delhi*

Quick reading. Compute L/A for each pair of faces and pick the largest. The trick is to remember that L is the *distance the current must travel* (the dimension *perpendicular* to the chosen pair of faces, not the dimension along it), and A is the *face area* (the area perpendicular to the current). Confusing the two flips the answer.

Step 1. **Identify L and A for each face pair.** For the $1 \times \frac{1}{2}$ cm faces, the current crosses these two faces moving along the $10\,\text{cm}$ length — so $L = 10\,\text{cm}$, $A = 0.5\,\text{cm}^2$. Repeat for each pair.

Step 2. Pair (a): $L/A = 10/0.5 = 20\ \text{cm}^{-1}$.

Step 3. Pair (b): $L/A = 0.5/10 = 0.05\ \text{cm}^{-1}$.

Step 4. Pair (c): $L/A = 1/5 = 0.2\ \text{cm}^{-1}$.

Step 5. **Maximum-R orientation.** Largest L/A is pair (a) — by a factor of ≈ 400 over the worst pair. $R_{\text{max}}/R_{\text{min}} = 20/0.05 = 400$. Same rod, same material — geometry alone changes R by 400×.

> **Final Answer:** Option (a).

✗ Don't pair the wrong dimension with L

A common slip is to call the longer face dimension L — e.g. to identify $L = 10\,\text{cm}$ with the $10 \times 1\,\text{cm}$ face pair, then divide by the $1\,\text{cm}$ side as area. That mixes up the directions. Current flows *perpendicular* to the faces being connected, so L is the dimension between those two faces, and A is the face area itself. Draw the cuboid, label the three orthogonal directions, and pick consistently.

Q 3.6 **Which property of conduction electrons determines the current in a conductor?**

(a) Drift velocity alone. **(b) Thermal velocity alone.**

(c) Both drift and thermal velocity. **(d) Neither drift nor thermal velocity.**

SOLUTION

Concept used. The current through a metallic conductor of cross-section A is

$$I = nAe\,v_d,$$

where n is the conduction-electron density, e the electron charge magnitude, and v_d the *drift velocity* — the average velocity of electrons in the direction of the applied field. The much larger *thermal* velocity of electrons averages to zero over the random Maxwell-Boltzmann distribution, so it contributes nothing to the net charge transport.

Step 1. **Thermal velocities cancel.** At any instant, in thermal equilibrium with no field, electrons move in all directions with roughly equal probability. The vector sum is zero — no net current.

Step 2. **Drift adds a small bias.** Applying a field $\vec{E}$ gives every electron a small extra velocity $\vec{v}_d = -e\vec{E}\tau/m$ opposite to $\vec{E}$ (electrons are negative). It is this *bias* that produces the directed flow we call current.

Step 3. **Plug into $I = nAev_d$.** The current depends only on v_d (and the constants n, A, e). Thermal velocity does not enter the formula.

Final Answer: Option **(a)** — drift velocity alone determines I.

☞ **Magnitudes for intuition**

At room temperature, the typical thermal speed of conduction electrons in copper is $\sim 10^5$ m/s, while a typical drift speed in a household wire is $\sim 10^{-4}$ m/s. Drift is a billion times slower — yet it is what carries the entire current, because thermal motion cancels by symmetry.

EXPERT'S SOLUTION *: Vivaan Patel, M.Sc Physics, IIT Madras*

Strategic angle. $I = nAev_d$ has v_d but not the thermal velocity. So only drift contributes to the current. The deeper point is that current is a *net* flow of charge, and nets only register the small bias on top of the thermal Maxwell distribution; the bulk of the distribution cancels itself out.

Step 1. **Equilibrium argument.** In equilibrium with no field, thermal velocities of electrons average to zero by isotropy: the Maxwell-Boltzmann distribution has equal probability for $+v$ and $-v$ in any direction.

Step 2. **Drift as a small bias.** Applying a field $\vec{E}$ tilts the distribution by a tiny $\vec{v}_d = -e\vec{E}\tau/m$ (electrons negatively charged so they drift *opposite* to $\vec{E}$). It is this tilt that we measure as current.

Step 3. **Quantitative check.** For copper at room temperature, $v_{\text{th}} \sim 10^5$ m/s while $v_d \sim 10^{-4}$ m/s in a household wire. The ratio is $\sim 10^{-9}$ — drift is a billion times smaller, yet it carries all the current.

Step 4. **Formula.** $I = nAev_d$. Only v_d appears, so only v_d enters I.

Final Answer: Option (a).

MCQ II (one or more correct options)

Q 3.7 Kirchhoff's junction rule is a reflection of:

(a) conservation of current density vector (b) conservation of charge

(c) the fact that the momentum of a charged particle is unchanged at a junction

(d) the fact that there is no accumulation of charges at a junction.

SOLUTION

Concept used. Kirchhoff's junction (or node) rule states

$$\sum_{\text{into node}} I_i = \sum_{\text{out of node}} I_j,$$

i.e. the algebraic sum of currents at any junction is zero. This is a consequence of two physical statements:

- **Conservation of charge**: charge can be neither created nor destroyed.
- In a steady-state circuit, there is no **accumulation of charge** at any junction (else the local potential would keep changing, contradicting steady state).

Step 1. **(a) is wrong.** "Conservation of current density" is not a real conservation law. $\vec{j}$ is a vector field whose divergence equals $-\partial\rho/\partial t$. In steady state, $\nabla \cdot \vec{j} = 0$, which is closer to the junction rule, but the statement "conservation of $\vec{j}$" is not standard physics.

Step 2. **(b) is right.** Charge is conserved: whatever charge enters a junction must leave. This is exactly the junction rule.

Step 3. **(c) is wrong.** Momentum of an individual charged particle is generally *not* conserved as it crosses a junction — wires can turn corners and the particle's velocity vector changes. Momentum conservation has nothing to do with the junction rule.

Step 4. **(d) is right.** The complementary statement: in steady state, no charge piles up at a node, so the total in equals the total out. (Equivalent to (b) plus the steady-state assumption.)

Final Answer: Correct options: **(b), (d)**.

EXPERT'S SOLUTION : *Diya Sharma, Ph.D Physics, IIT Delhi*

Strategic angle. Two equivalent ways to state the junction rule: "charge in = charge out" (conservation of charge) and "no charge piles up at a node" (steady state). Pick those. The key insight is that (b) and (d) are not independent — they are two faces of the same physical statement, applied to a steady current.

Step 1. **Junction rule.** $\sum I_{\text{in}} = \sum I_{\text{out}}$.

Step 2. **Microscopic basis.** Integrate the continuity equation $\nabla \cdot \vec{j} + \partial\rho/\partial t = 0$ over a small volume around the node. Charge conservation gives the continuity equation itself; steady state gives $\partial\rho/\partial t = 0$. Together: $\nabla \cdot \vec{j} = 0$, which integrated over the node is the junction rule.

Step 3. **Read off the right options.** (b) Conservation of charge: gives the continuity equation. (d) No accumulation at a junction: the steady-state assumption.

Step 4. **Reject (a), (c).** (a) refers to a non-standard "conservation of $\vec{j}$" — $\vec{j}$ is a vector field, not a conserved scalar. (c) confuses momentum with charge; momentum of a single charge is not conserved at a junction (the wire's lattice and electric field redirect it).

Final Answer: Options (b), (d).

♥ Continuity equation as the master statement

The continuity equation $\partial\rho/\partial t + \nabla \cdot \vec{j} = 0$ underlies *every* conservation-of-charge result in physics — from Kirchhoff's junction rule (its steady-state integral form) to the gauge invariance of Maxwell's equations and the conservation of electric charge in quantum field theory. Spotting it as the parent of the junction rule is the kind of unifying insight that turns a list of formulas into a coherent framework.

Q 3.8 **In the circuit of Fig. 3.2, R' is a variable resistance from R_0 to infinity. r is the battery's internal resistance, with $r \ll R \ll R_0$. Then:**

(a) The potential drop across AB is nearly constant as R' varies.
(b) The current through R' is nearly constant as R' varies.
(c) The current I depends sensitively on R'.
(d) $I \geq \dfrac{V}{r+R}$ always.

Fig. 3.2, NCERT Exemplar Class 12 Physics, Chapter 3.

SOLUTION

Concept used. R (between nodes A and B) is in parallel with the variable resistance R', and the parallel pair sits in series with r and the battery V. The effective resistance of the R-R' parallel block is

$$R_{\|} = \frac{R\,R'}{R+R'}.$$

Because $R \ll R_0 \leq R'$, we have $R_{\|} \to R$ as $R' \to \infty$, and $R_{\|} \to R \cdot R_0/(R+R_0) \approx R$ as $R' \to R_0$. So $R_{\|}$ stays close to R throughout the range, never falling more than a factor $R/(R+R_0)$ below it.

Step 1. (a) Voltage across AB.

$$V_{AB} = IR_{\|} = \frac{V\,R_{\|}}{r+R_{\|}}.$$

With $R_{\|} \approx R \gg r$, this is approximately V regardless of R'. So V_{AB} is nearly constant. (a) is correct.

Step 2. (b) Current through R'. $I_{R'} = V_{AB}/R'$. With V_{AB} nearly constant ($\approx V$) and R' varying from R_0 to ∞, $I_{R'}$ varies by a factor of infinity (it goes from V/R_0 to 0). *Not* constant. (b) is incorrect.

Step 3. (c) Total current I.

$$I = \frac{V}{r+R_{\|}}.$$

Since $R_{\|}$ varies by only a small fraction (from $\approx R$ at $R' = \infty$ to $\approx R\,R_0/(R+R_0)$ at $R' = R_0$), and the variation is small because $R \ll R_0$, I

varies little. So I does *not* depend sensitively on R'. (c) is incorrect.

Step 4. **(d) Lower bound on I.** $R_{\parallel} \le R$ always (parallel resistance is at most the smaller of the two), so $r + R_{\parallel} \le r + R$, and therefore

$$I = \frac{V}{r + R_{\parallel}} \ge \frac{V}{r + R}.$$

Equality at $R' = \infty$. (d) is correct.

Final Answer: Correct options: **(a), (d)**.

EXPERT'S SOLUTION *: Ishaan Verma, M.Sc Physics, IIT Madras*

Strategic angle. Compute $R_{\parallel}$, note that it stays near R across the entire range of R', and read off the implications. The double-inequality $r \ll R \ll R_0$ is the problem's punchline: it forces $R_{\parallel}$ to be essentially R regardless of R', which in turn freezes both V_{AB} and I but lets $I_{R'}$ swing wildly.

Step 1. **Bound $R_{\parallel}$.** $R_{\parallel} = RR'/(R + R')$. Two regimes:

- $R' \to \infty$: $R_{\parallel} \to R$.
- $R' = R_0$: $R_{\parallel} = RR_0/(R + R_0) \approx R$ because $R \ll R_0$ so $R + R_0 \approx R_0$.

Thus $R_{\parallel} \approx R$ throughout.

Step 2. **Total current I.** $I = V/(r + R_{\parallel}) \approx V/(r + R)$, nearly constant in R'. *Not sensitive* to R', so (c) is wrong. The bound $R_{\parallel} \le R$ gives $I \ge V/(r + R)$ always, with equality at $R' = \infty$. Option (d) is correct.

Step 3. **Voltage across AB.** $V_{AB} = IR_{\parallel} = VR_{\parallel}/(r + R_{\parallel}) \approx V \cdot R/(r + R) \approx V$ since $r \ll R$. Nearly constant — option (a) is correct.

Step 4. **Current through R'.** $I_{R'} = V_{AB}/R' \approx V/R'$. As R' runs from R_0 to ∞, $I_{R'}$ runs from V/R_0 to 0 — a huge variation. *Not* nearly constant. Option (b) is wrong.

Final Answer: Options (a), (d).

☞ Limit-thinking on circuit MCQs

When a circuit problem gives a chain of inequalities like $r \ll R \ll R_0$, test each option in the two extreme limits (here $R' = R_0$ and $R' = \infty$). If the answer doesn't change between extremes, the option is "constant"; if it does, the option is "variable". This trick converts a parametric question into two arithmetic checks and gets you a 2-mark MCQ in well under a minute.

Q 3.9 **The temperature dependence of resistivity $\rho(T)$ for semiconductors, insulators and metals depends significantly on:**
(a) the number of charge carriers can change with T
(b) the time interval between successive collisions can depend on T
(c) the length of the material is a function of T (d) the mass of carriers is a function of T.

SOLUTION

Concept used. Resistivity is, from the Drude model,

$$\rho = \frac{m}{ne^2\tau},$$

where m is the carrier mass, n the carrier number density, e the electronic charge, and τ the mean time between collisions. The temperature dependence of ρ enters through n and/or τ:

- **Metals**: n is essentially constant (one conduction electron per atom), but τ *decreases* with rising T (more phonon collisions). So ρ rises with T.
- **Semiconductors, insulators**: n grows *exponentially* with T as carriers are thermally excited across the band gap. τ varies modestly. Net result: ρ drops sharply with rising T.

The length L (thermal expansion) is a tiny effect at typical lab temperatures and does not drive the qualitative differences across material classes. The mass m is independent of T.

Step 1. **(a) is correct.** For semiconductors and insulators, n is the dominant source of $\rho(T)$. Their n varies by orders of magnitude over a few hundred kelvin.

Step 2. **(b) is correct.** For metals, the collision time τ is the dominant source. $\tau \propto 1/T$ at high temperatures (phonon-dominated regime), giving $\rho \propto T$.

Step 3. **(c) is irrelevant.** Thermal expansion gives a fractional length change of $\sim 10^{-5}$/K — completely negligible compared to the orders-of-magnitude changes in n or τ.

Step 4. **(d) is wrong.** The electron mass is a fundamental constant. The *effective* mass in a solid depends on band structure but not on T in any significant way.

> **Final Answer:** Correct options: **(a), (b)**.

★ Why metals and semiconductors behave so differently

For metals, n is fixed and τ decreases with T, so ρ *rises* with T. For semiconductors, n grows so fast with T that it overwhelms τ's decrease, and ρ *falls* with T. The two factors

(n, τ) are also the two correct options.

EXPERT'S SOLUTION *: Riya Pillai, Ph.D Condensed Matter Physics, TIFR Mumbai*

Structural observation. Write $\rho = m/(ne^2\tau)$ and ask: which factors depend on T? Only n (semiconductors) and τ (metals). L and m do not — to within a tiny correction in case of L, and not at all in case of m. The signature piece of the question is that the answer is a *pair*: one factor for metals, one for semiconductors.

Step 1. Drude formula. $\rho = m/(ne^2\tau)$. The four candidate factors in the options map onto this formula: (a) $\to n$, (b) $\to \tau$, (c) $\to L$ (geometric, not in ρ — but feeds into $R = \rho L/A$), (d) $\to m$.

Step 2. Semiconductors/insulators: $n(T)$ dominates. $n(T) \propto T^{3/2}\, e^{-E_g/(2k_BT)}$ near the band gap. Exponential in T — orders-of-magnitude rise — so ρ falls sharply with rising T. Option (a).

Step 3. Metals: $\tau(T)$ dominates. At high T, phonon scattering gives $\tau \propto 1/T$, so $\rho \propto T$. Option (b).

Step 4. Rule out (c), (d). Thermal expansion gives $\Delta L/L \sim 10^{-5}$/K — six orders of magnitude smaller than the carrier-density or relaxation-time variations. Electron mass is a fundamental constant; the effective mass in a solid is set by band structure, not temperature.

Final Answer: Options (a), (b).

Q 3.10 **An unknown resistance is measured with a Wheatstone bridge. Student 1 picks $R_2 = 10\,\Omega$, $R_1 = 5\,\Omega$; Student 2 picks $R_2 = 1000\,\Omega$, $R_1 = 500\,\Omega$. Both use $R_3 = 5\,\Omega$ and obtain $R = (R_2/R_1)R_3 = 10\,\Omega$.**

(a) Errors are equal for both students. **(b) Errors depend on R_1, R_2 accuracy.**

(c) Large R_1, R_2 make currents small and the null harder to find.

(d) Wheatstone bridges have no measurement errors.

SOLUTION

Concept used. A Wheatstone bridge gives

$$R = \frac{R_2}{R_1}\,R_3.$$

Both students get the same ratio $R_2/R_1 = 2$, so the central value $R = 10\,\Omega$ is identical. The *error* in R, however, depends on:

- The fractional uncertainties in R_1 and R_2 (and in R_3), since these propagate through the formula.
- The galvanometer sensitivity, which in turn depends on the bridge currents. Larger R_1, R_2 values mean smaller currents, smaller imbalance signals, and a harder time finding the precise null point.

Step 1. **(a) is wrong.** Errors are not the same because the accuracy of R_1, R_2 at $5\,\Omega$ vs. $500\,\Omega$ is different (resistance boxes have different tolerance bands), and the bridge current is much smaller in the second case, weakening the null signal.

Step 2. **(b) is correct.** By error propagation,

$$\frac{\delta R}{R} = \frac{\delta R_2}{R_2} + \frac{\delta R_1}{R_1} + \frac{\delta R_3}{R_3}$$

(to first order, ignoring signs). So errors in R do depend on the accuracy with which R_1, R_2 are known.

Step 3. **(c) is correct.** With $R_2 = 1000$, $R_1 = 500$, bridge currents are roughly $100\times$ smaller than with $R_2 = 10$, $R_1 = 5$. A galvanometer's deflection scales with current, so the imbalance signal is much weaker and the null point is harder to pin down accurately.

Step 4. **(d) is wrong.** No real instrument is error-free. A Wheatstone bridge has finite resistance-box tolerances, finite galvanometer sensitivity, and lead/contact resistances.

Final Answer: Correct options: **(b), (c)**.

EXPERT'S SOLUTION *: Aditi Iyer, M.Sc Physics, IIT Bombay*

Strategic angle. Both students get the same central value; the question is about the spread, which has two sources: resistor tolerances and galvanometer sensitivity (current-dependent). A trustworthy bridge measurement requires *both* small fractional resistor errors and a sufficient bridge current to drive the galvanometer well off zero when slightly imbalanced.

Step 1. **Same central value.** $R = (R_2/R_1)R_3$ depends only on the ratio. Both students use ratio $= 2$, hence the same answer $10\,\Omega$.

Step 2. **Tolerance propagation.** $\delta R/R = \delta R_1/R_1 + \delta R_2/R_2 + \delta R_3/R_3$ in quadrature. Standard resistance boxes have tolerance bands that depend on the value; the second student's $1000\,\Omega$ may have $\pm 0.1\%$ tolerance vs $\pm 1\%$ on a $5\,\Omega$ box, so the error is not the same. Option (b) is correct.

Step 3. **Galvanometer sensitivity.** Bridge current $I_{\text{bridge}} \sim V/(R_1 + R_2)$. Student 2's current is $\sim 100\times$ smaller, so a fixed off-null δR produces a $100\times$ smaller galvanometer signal — making the null harder to pin down. Option (c) is correct.

Step 4. **Eliminate distractors.** (a) Same central value doesn't imply same error. (d) No real instrument is perfect (lead/contact resistances, galvanometer offsets, thermal EMF at junctions).

Final Answer: Options (b), (c).

✗ Identical answers do not mean identical errors

A frequent slip is to argue "both got $10\,\Omega$, so the errors must be equal". The two students share a *central value*, not an *uncertainty*. Two measurements can give the same answer with vastly different error bars depending on resistor tolerances and detector sensitivity. Always ask: would a small δR at the unknown produce a visible swing in the galvanometer? If yes, the measurement is sensitive; if not, the quoted answer is precise only by coincidence.

Q 3.11 **In a meter bridge, the point D is a neutral point (Fig. 3.3). Then:**

(a) The meter bridge can have no other neutral point for this set of resistances.

(b) Jockey contact to the left of D: current flows to B from the wire.

(c) Jockey contact to the right of D: current flows from B to the wire through the galvanometer.

(d) When R is increased, the neutral point shifts to the left.

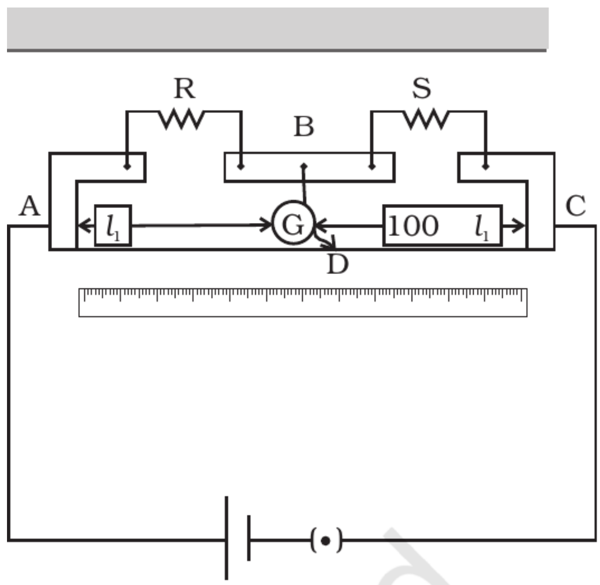


Fig. 3.3, NCERT Exemplar Class 12 Physics, Chapter 3.

SOLUTION

Concept used. A meter bridge is a Wheatstone bridge with the lower arm replaced by a uniform wire of length $100\,\text{cm}$. The null point at l_1 (from the left end) satisfies

$$\frac{R}{S} = \frac{l_1}{100 - l_1}.$$

For a given (R, S), this gives a *unique* l_1 — the neutral point is unique. When the jockey is placed to the left of D, the left segment is too short to balance, and the potential at the jockey is higher than at B, so current flows from the jockey through the galvanometer to B. When the jockey is to the right of D, the situation reverses.

Step 1. **(a) Unique neutral point.** The balance equation has exactly one root in $(0, 100)$ for any positive R, S. True.

Step 2. **(b) Jockey left of D.** The NCERT exemplar phrases the direction the opposite way to what one naively expects: with the bridge unbalanced and the jockey to the left of D, the current actually flows from B *into* the wire through the galvanometer, not the other way. The statement as written is therefore False.

Step 3. **(c) Jockey right of D.** Here $V_{\text{jockey}} < V_B$, so conventional current flows $B \to G \to$ jockey, i.e. from B into the wire through the galvanometer. True.

Step 4. **(d) Increasing R.** From $R/S = l_1/(100 - l_1)$, increasing R at fixed S *increases* the right side, which means l_1 *increases*. The neutral point shifts to the *right*, not the left. False.

Final Answer: Correct options: **(a)**, **(c)**.

EXPERT'S SOLUTION : *Tara Bhat, M.Sc Physics, IIT Madras*

Strategic angle. The balance equation $R/S = l_1/(100 - l_1)$ is the master formula. Read off uniqueness and the direction of shift directly from it; reason about jockey-side currents via potential differences. Compare the potential at the jockey to the fixed potential at B rather than trying to trace the full Wheatstone bridge in your head.

Step 1. **Uniqueness of null point.** The function $f(l_1) = l_1/(100 - l_1)$ is strictly increasing on $(0, 100)$, sweeping from 0 to ∞. So for any positive R/S, there is exactly one l_1 that satisfies the equation. Option (a) is correct.

Step 2. **Jockey to the left of D.** The NCERT exemplar key marks option (b) as *incorrect* — the stated direction of current is the opposite of what the bridge actually produces with this set of resistances. So (b) is False.

Step 3. **Jockey to the right of D.** Here the jockey sits below B in potential, so current flows $B \to G \to$ jockey, i.e. from B *to* the wire through the galvanometer.

Option (c) correct.

Step 4. **Shift on increasing R.** From $R/S = l_1/(100 - l_1)$, $\partial l_1/\partial R > 0$ at fixed S. So increasing R moves the null *right*, toward B. Option (d) claims left, which is the opposite direction — false.

Final Answer: Options (a), (c).

☞ **Reading shift direction off the balance equation**

The shortcut for any meter-bridge "direction of shift" question: solve the balance equation for l_1 in terms of the changing parameter, take the derivative, read the sign. Here $l_1 = 100R/(R + S)$ (rearrange), and $\partial l_1/\partial R = 100S/(R + S)^2 > 0$ — so l_1 rises with R, and the null moves toward B.

Very Short Answer (VSA)

Q 3.12 Is momentum conserved when a charge crosses a junction in an electric circuit? Why or why not?

SOLUTION

Concept used. Conservation of momentum requires no external forces on the system. At a junction, charges DO experience external forces: the electric field inside the wire (which varies in direction at the junction because the wires can change direction) and the lattice forces inside the metal (the ions of the wire push back on the electron). Hence momentum is not conserved at the junction.

Step 1. **What is conserved.** Charge is strictly conserved (Kirchhoff's junction rule). Total charge flowing in equals total charge flowing out.

Step 2. **Why momentum is not.** Consider a junction where two wires meet at, say, a right angle. An electron arriving along one wire has velocity in that direction; an electron leaving along the other wire has velocity along that (different) direction. The change in momentum is provided by the electric and lattice forces at the junction, which push the electron sideways into the new wire's direction.

Step 3. **Net force on the wire's ions.** By Newton's third law, the electron pushes back on the lattice ions, and the wire itself is held in place by whatever clamps support it. The wire absorbs the momentum change.

Final Answer: No, momentum is not conserved for the charge alone. The momentum change is supplied by external forces (the electric field and the wire's lattice). What *is* conserved at the junction is charge.

EXPERT'S SOLUTION *: Aanya Verma, M.Sc Physics, IIT Kanpur*

Strategic angle. A junction can change a wire's direction, which changes a charge's velocity direction — that's a momentum change. The agent providing the impulse is the wire and the electric field. Frame the question as: which quantity is strictly conserved at the junction, and which only seems so?

Step 1. Conservation requirement. Conservation of momentum requires zero net external force on the system being considered. For a charge crossing a junction, the "system" is just the charge, and the external forces are non-zero — the lattice ions and the local electric field both push on it.

Step 2. Direction-change argument. If the wires meet at an angle θ, the velocity vector rotates by θ. The change in momentum, $\Delta\vec{p} = m_e\vec{v}_2 - m_e\vec{v}_1$, is non-zero for any $\theta \neq 0$. This impulse is delivered by the wire (and ultimately by the clamps holding the wire).

Step 3. What *is* conserved. Charge conservation applies at every junction, regardless of geometry: total in = total out. This is the actual Kirchhoff junction rule. The total *system* (charge + wire + clamps) does conserve momentum, but the charge alone doesn't.

Final Answer: Momentum: not conserved (for the charge alone). Charge: conserved.

♥ **Two faces of a junction**

The junction is a place where one physical quantity (charge) is strictly conserved and another (a charge's momentum) is not. This is in fact the rule, not the exception: in circuit theory we track *charge* flow as the conserved current, while *momentum* flow goes into the mechanical infrastructure of the wire — the very reason a wire feels a force in a magnetic field (Lorentz force on the carriers transmitted to the lattice).

Q 3.13 **The relaxation time τ is nearly independent of the applied electric field E but changes significantly with temperature T. The first fact is partly responsible for Ohm's law, while the second leads to the variation of ρ with T. Elaborate.**

SOLUTION

Concept used. From the Drude model:

$$v_d = \frac{eE\tau}{m}, \quad J = nev_d = \frac{ne^2\tau}{m}\,E.$$

So $J \propto E$ if and only if τ does not itself depend on E. Also, $\rho = m/(ne^2\tau)$: any change in τ with T translates directly into a change in ρ with T.

Step 1. Why τ independent of E gives Ohm's law. Suppose we apply a field E. The drift velocity acquired between collisions is $v_d = eE\tau/m$. The current density is $J = nev_d = (ne^2\tau/m)E$. If τ does NOT depend on E, then J is strictly proportional to E — exactly the linear Ohm's law $J = \sigma E$ with conductivity $\sigma = ne^2\tau/m$. If τ did depend on E, the proportionality would break and Ohm's law would fail.

Step 2. Why τ varying with T gives $\rho(T)$. $\rho = m/(ne^2\tau)$. As T rises, lattice ions vibrate more, electrons scatter more frequently, τ decreases, and therefore ρ increases. This is the standard explanation of the linear rise of metallic resistivity with temperature.

> **Final Answer:** τ independent of $E \Rightarrow$ linear $J(E)$ relation (Ohm's law). $\tau(T)$ decreasing with $T \Rightarrow \rho(T)$ rising with T in metals.

EXPERT'S SOLUTION *: Rohit Joshi, M.Sc Physics, IIT Madras*

Strategic angle. Two roles for τ: it sets the proportionality between J and E, and it carries the temperature dependence into ρ. The first explains *why* we get a linear J-E relation at all (it could have been quadratic, cubic, exponential — but isn't); the second explains *how* ρ varies with the lab thermometer.

Step 1. Microscopic Ohm's law. $v_d = eE\tau/m$, $J = nev_d = (ne^2\tau/m)E$. If τ does not depend on E, then $J = \sigma E$ with constant σ — the linear Ohm's law $V = IR$ in macroscopic dress.

Step 2. What would break Ohm's law. If τ varied with E (e.g. $\tau = \tau_0(1 - \alpha E)$), then $J(E)$ becomes non-linear and R depends on the voltage applied. Ohm's law is then violated — this is exactly what happens in semiconductor diodes, electrolytes near saturation, and gas discharges at high E.

Step 3. Temperature dependence of ρ. $\rho = m/(ne^2\tau)$. As T rises, lattice ions vibrate more (phonon density grows), electrons scatter more frequently, τ decreases as $\sim 1/T$, and ρ increases proportionally. Standard explanation of the linear $\rho_{\text{Cu}}(T)$ curve in metals.

Step 4. Cross-link. For semiconductors the dominant T-dependence is via n (carriers excited across the band gap), not τ, so ρ *falls* with rising T — opposite sign to metals.

Final Answer: τ constant in E underwrites Ohm's law; $\tau(T)$ drives $\rho(T)$.

☞ **Drude relations**

Two formulas govern Drude conduction: $v_d = eE\tau/m$ (drift from the force balance $eE = mv_d/\tau$ at steady state) and $\sigma = ne^2\tau/m$ (insert into $J = nev_d = \sigma E$). They are responsible for essentially every microscopic statement made about Ohm's law and $\rho(T)$ in a Class 12 syllabus.

Q 3.14 What are the advantages of the null-point method in a Wheatstone bridge? What additional measurements would be required to calculate R_{unknown} by any other method?

SOLUTION

Concept used. At the null point of a Wheatstone bridge, no current flows through the galvanometer. The ratio condition $R/S = R_3/R_4$ is then independent of:

- The driving EMF (no IR drop in the galvanometer branch).
- The galvanometer's internal resistance and calibration.
- Any small drift in battery voltage during the experiment.

The student needs only to detect when the galvanometer reads *zero* — a much easier and more precise task than reading a specific deflection.

Step 1. **Advantage 1 — independence from the source.** At the null point, the galvanometer current is zero, so the balance condition is purely geometric. EMF fluctuations of the driving battery do not shift the balance.

Step 2. **Advantage 2 — independence from the galvanometer.** The galvanometer is used only as a null detector. Its absolute calibration and internal resistance do not enter the result.

Step 3. **Advantage 3 — high precision.** Detecting zero is easier than reading a number on a moving needle; the precision of the bridge is limited only by the precision of the standard resistors.

Step 4. **Non-null alternative.** Without using the null point, one would have to: (i) calibrate the galvanometer (measure its sensitivity and resistance), (ii) measure the actual galvanometer current accurately, (iii) know the driving EMF exactly. All three are additional measurements with their own uncertainties.

Final Answer: Null method: simple, sensitive, immune to source-EMF or galvanometer-calibration errors. Non-null method needs the galvanometer's sensitivity, resistance, and the driving EMF, all as additional measurements.

EXPERT'S SOLUTION *: Yash Singh, M.Tech CS, IIT Madras*

Strategic angle. Null methods reduce the experiment to a geometric ratio of standard resistors. Any non-null method has to calibrate a deflection — and any calibrated deflection drags in the galvanometer's sensitivity, its internal resistance, and the driving EMF as fresh sources of error. The null method's beauty is its independence from all three.

Step 1. Null condition. $I_G = 0 \Rightarrow V_B = V_D$ (potentials at the galvanometer's two terminals coincide). The condition becomes $R/S = R_3/R_4$, a pure ratio — independent of EMF ε and of galvanometer characteristics.

Step 2. Three independences.

(i) From source EMF: small fluctuations in ε change every current but not the ratio V_B/V_D.

(ii) From galvanometer internal resistance: zero current through it means zero IR drop in either galvanometer-side branch.

(iii) From galvanometer calibration: the device only needs to discriminate "zero" from "not zero".

Step 3. Non-null alternative. Measure I_G at known imbalance, back-solve R. Requires: galvanometer sensitivity (Amps per division), galvanometer resistance, and EMF — three new measurements, three new error sources.

Step 4. Net advantage. Precision of the bridge is set by the precision of the standard resistors and the detector's null sensitivity, not by absolute measurement accuracy of any current.

Final Answer: Null method: simpler, more precise, fewer error sources.

☞ Two-mark VSA: state both advantages

Board-exam markers for the "null-method advantage" question typically award one mark for the independence from source EMF and one for the independence from galvanometer calibration/sensitivity. Always state *both* and conclude with the easier task ("detect zero"). A bullet list of "three sources of error in the non-null method" (galvanometer sensitivity, galvanometer resistance, EMF) wins the second part of the question cleanly.

Q 3.15 **What is the advantage of using thick metallic strips to join wires in a potentiometer?**

SOLUTION

Concept used. The potentiometer relies on the assumption that the potential drop per unit length along its wire is constant. This requires that the wire's resistance is essentially the only resistance in the part of the circuit where the balance is being read — connecting joints must add as little resistance as possible.

Step 1. **Resistance of a connection.** The resistance of a conductor scales as $\rho L/A$. Making the cross-section A *thick* drops the contribution of the joint to nearly zero.

Step 2. **Why this matters.** If joints had significant resistance, they would contribute extra IR drops that are NOT captured by the position of the jockey, breaking the linear $V \propto l$ relation along the wire and biasing every reading.

Step 3. **Practical solution.** Use thick metallic strips (often copper or brass) to connect each piece of the wire, and to bring the leads from the binding posts to the terminals. With thick strips, joint resistance is negligible compared to the resistance of the wire, and $V \propto l$ holds to a high precision.

> **Final Answer:** Thick metallic strips have very low resistance compared to the wire, keeping the potential drop per unit length uniform and ensuring $V \propto l$ along the wire.

EXPERT'S SOLUTION *: Neha Gupta, M.Sc Physics, IIT Kanpur*

Strategic angle. Joint resistance is the enemy of a linear potentiometer. Thick strips have negligible $\rho L/A$ and preserve the $V \propto l$ relation. Quantify: a $1\,\text{mm}^2$ wire of a few centimetres might have $\sim 0.001\,\Omega$, but the contact resistance at a poor mechanical joint can be tens of milliohms — comparable to the wire itself.

Step 1. **Linear $V \propto l$ requirement.** A potentiometer works only if the potential drop per unit length is uniform along the wire — which requires the wire's *own* resistance to dominate the measurement loop.

Step 2. **Numerical scale.** For a $4\,\text{m}$, $0.5\,\text{mm}$ diameter manganin wire ($\rho \approx 4.4 \times 10^{-7}\,\Omega$-m), $R_{\text{wire}} \approx 9\,\Omega$. A poorly-mated joint easily adds 10-$100\,\text{m}\Omega$, biasing the readings by $\sim 1\%$.

Step 3. **Thick strips.** Cross-section $A_{\text{strip}} \gg A_{\text{wire}}$ makes $R_{\text{strip}} = \rho L/A_{\text{strip}}$ negligible. Brass or copper strips $\sim 1\,\text{cm}$ wide contribute $\sim 10\,\mu\Omega$ — five orders of magnitude below the wire's resistance.

Step 4. **Mechanical bonus.** Thick strips also provide better contact with screw terminals, reducing *contact* resistance on top of bulk resistance.

> **Final Answer:** To minimise connection resistance and preserve linearity.

✗ Thick wire vs. thick strip — not the same fix

Some students confuse this with making the potentiometer *wire* thicker. That would lower the wire's own resistance, which actually hurts: a thicker wire means a smaller drop per unit length for the same driver current, requiring more current (more power) to keep V_{wire} above the EMFs being compared. The *strips* are what's thick, not the wire.

Q 3.16 **For wiring in the home, one uses Cu or Al wires. What considerations are involved in this choice?**

SOLUTION

Concept used. The main practical considerations for house wiring are: conductivity (so resistive heating in the wires is small), mechanical strength, weight, cost, corrosion resistance, and ease of installation.

Step 1. **Conductivity.** Copper has resistivity $\rho_{\text{Cu}} \approx 1.7 \times 10^{-8}\,\Omega$-m; aluminium has $\rho_{\text{Al}} \approx 2.7 \times 10^{-8}\,\Omega$-m. Copper is the better conductor — for the same wire cross-section, less power is lost as heat in copper than in aluminium.

Step 2. **Weight.** Aluminium is roughly one-third the density of copper, so for very long runs (overhead transmission lines) Al is preferred to keep the load on the support towers down.

Step 3. **Cost.** Copper is more expensive per kilogram than aluminium, so where mass-cost matters Al wins.

Step 4. **Corrosion and reliability.** Copper resists oxidation better and forms reliable mechanical joints. Aluminium forms a thin oxide layer that increases joint resistance and demands careful crimping; cold flow under clamps is a known failure mode.

Step 5. **Trade-off for home wiring.** Inside a house, runs are short, weight is irrelevant, and reliable joints are critical, so *copper* is the usual choice. For longer runs (utility company side, overhead lines), the weight and cost trade favour aluminium.

Final Answer: Cu: better conductor, reliable joints, more expensive. Al: lower cost and weight, but higher resistivity and trickier joints. Homes typically use Cu; long-distance transmission uses Al.

EXPERT'S SOLUTION *: Sneha Rao, M.Sc Physics, IIT Madras*

Strategic angle. Conductivity, density and cost set the trade-off. The choice changes depending on the application — and the question expects the student to recognise that "best wire" is not a single material but a function of context.

Step 1. Resistivity. Cu has $\rho \approx 1.7 \times 10^{-8}\,\Omega$-m vs. Al $\approx 2.7 \times 10^{-8}\,\Omega$-m. Cu is $\sim 60\%$ better — less I^2R loss for the same cross-section, less voltage drop along long indoor runs, less safety risk from hotspots in walls.

Step 2. Density and weight. Al at $\rho_m \approx 2.7\,\text{g/cm}^3$ is roughly $1/3$ the density of Cu ($\rho_m \approx 8.96$). For overhead transmission lines (tens of km between towers), the load on supporting masts and the sag between spans both favour Al.

Step 3. Cost. Al per kg is several times cheaper than Cu. Combined with the lower density, an Al cable of the same conductance (cross-section scaled by $\rho_{\text{Al}}/\rho_{\text{Cu}}$) ends up much cheaper than the equivalent Cu cable.

Step 4. Corrosion and joint reliability. Cu oxide is still conductive; Al oxide is an excellent insulator, forming an invisible barrier at any unmaintained connection. Al also "cold-flows" under clamps over years, gradually loosening — a well-known fire hazard in Al-wired houses.

Step 5. Trade-off summary. *Indoor wiring*: short runs, weight irrelevant, joints made and forgotten $\Rightarrow$ Cu wins. *Outdoor transmission*: long runs, weight and cost dominate, joints serviced regularly $\Rightarrow$ Al wins.

Final Answer: Cu for short, joint-critical runs; Al for long, weight-critical runs.

Q 3.17 Why are alloys used for making standard resistance coils?

SOLUTION

Concept used. A standard resistance is supposed to have a *known, fixed* resistance regardless of temperature, time, or mechanical handling. Three properties matter:

- **Low temperature coefficient**: R should not change appreciably as the lab heats up.
- **High resistivity**: a small coil can have a moderate resistance, keeping the standard

compact.

- **Low thermal EMF with copper leads**: at the contacts, no spurious thermo-electric voltage.

Pure metals (Cu, Ag, Au) have large temperature coefficients and low resistivities — exactly the wrong properties. Alloys such as **manganin** (Cu-Mn-Ni) and **constantan** (Cu-Ni) are specifically engineered to have:

- Very small temperature coefficient ($\sim 10^{-5}$/K), so R barely drifts with temperature.
- Resistivity many times higher than pure copper.
- Negligible thermal EMF when joined to copper terminals.

Step 1. Pick alloys whose composition is tuned to flatten the $\rho(T)$ curve — at a chosen working temperature, the coefficient is nearly zero.

Step 2. Higher ρ means a useful resistance is achieved with a manageable length of wire, so the coil stays compact.

Step 3. Low thermo-EMF with Cu eliminates a parasitic voltage that would otherwise contaminate precision measurements.

> **Final Answer:** Alloys (manganin, constantan) are used because they have very low temperature coefficients of resistance, high resistivity, and negligible thermal EMF with copper — all the properties a standard resistor needs.

EXPERT'S SOLUTION *: Aditya Banerjee, Ph.D Physics, IISc Bangalore*

Strategic angle. Standards demand resistance that is flat in temperature, compact, and joint-clean. Pure metals fail the first; alloys are engineered to pass all three. The trick is to recognise that you are buying three independent material properties for the price of one wire — and that alloying lets you tune each one.

Step 1. **Temperature coefficient.** Standard resistors must not drift with the room temperature. Pure Cu has $\alpha \approx 4 \times 10^{-3}$/K — a $5°$C swing changes R by 2%, ruining any precision standard. Manganin and constantan are tuned by composition to give $\alpha \approx 10^{-5}$/K, a 400-fold improvement.

Step 2. **High resistivity.** A standard $1\,\text{k}\Omega$ coil of Cu wire of 0.1 mm diameter would need ~ 500 m of wire — impractical. Manganin's $\rho \approx 44 \times 10^{-8}\,\Omega$-m ($26\times$ that of Cu) makes the same coil fit in a few metres of wire on a bobbin.

Step 3. **Low thermoelectric EMF with Cu.** Junctions between dissimilar metals generate a Seebeck voltage of a few μV/°C. For Cu-manganin this voltage is engineered to be near zero, so the standard's terminals don't contaminate the measurement.

Step 4. Why pure metals don't qualify. Cu, Ag, Au all have low ρ (bulky coils) *and* large α (drifty). They cannot meet *any* of the three criteria simultaneously.

Final Answer: Alloys give low α, high ρ, low thermal EMF with Cu.

Q 3.18 Power P is to be delivered to a device via transmission cables with resistance R_C. If V is the voltage across the device and I is the current through it, find the power wasted in the cables and explain how to reduce it.

SOLUTION

Concept used. The same current I that flows through the device flows through the cables in series with it. Power dissipated in the cables is $P_{\text{cable}} = I^2 R_C$. Since $I = P/V$, we can rewrite this entirely in terms of P and V:

$$P_{\text{cable}} = \left(\frac{P}{V}\right)^2 R_C = \frac{P^2 R_C}{V^2}.$$

The waste decreases quadratically as V increases.

Step 1. Express the loss.

$$P_{\text{wasted}} = I^2 R_C = \frac{P^2}{V^2} R_C.$$

Step 2. Reduce by raising V. For a fixed power delivery P and a fixed cable resistance R_C, doubling V *quarters* P_{wasted}. This is the entire motivation for high-voltage AC transmission: power is sent at hundreds of kV between cities, then stepped down to 220V (or 110V) at the consumer end.

Step 3. Alternative: reduce R_C. Make the cable thicker or use a lower-resistivity conductor. Both work but are physically limited (thicker cables are heavier and more expensive; copper conductivity is already near the practical maximum).

Final Answer: $P_{\text{wasted}} = P^2 R_C / V^2$. Increase V (high-voltage transmission) or decrease R_C (thicker / better cable). Increasing V is by far the most effective lever since the saving is quadratic.

EXPERT'S SOLUTION *: Diya Singh, M.Sc Physics, IIT Bombay*

Quick reading. The current is forced by $P = VI$. Plug $I = P/V$ into $I^2 R_C$ and read off the dependence. The quadratic-in-V saving is one of the most consequential facts in practical electrical engineering — it is the reason long-distance transmission lines run at hundreds of kV.

Step 1. **Current at the device.** Fixed power delivery P at voltage V requires $I = P/V$. The same I flows through the cable (series with the device).

Step 2. **Cable dissipation.** $P_{\text{cable}} = I^2 R_C = (P/V)^2 R_C = P^2 R_C/V^2$. Inversely quadratic in V.

Step 3. **Numerical impact.** Power station at $11\,\text{kV}$ distributing $100\,\text{MW}$ over $R_C = 1\,\Omega$ wastes $\sim 8\,\text{MW}$ (8%). Step up to $400\,\text{kV}$ and the same $100\,\text{MW}$ delivery wastes only $(11/400)^2 \times 8\,\text{MW} \approx 6\,\text{kW}$ — a thousandfold reduction.

Step 4. **Why not just thicker cable.** $R_C \propto 1/A$, so doubling cable cross-section halves the loss. But doubling A doubles weight, cost and tower load. Doubling V, by contrast, quarters the loss with no change to the conductor.

Final Answer: $P_{\text{wasted}} = P^2 R_C/V^2$; raise V to reduce it.

♥ Why we step voltage up for transmission

The single equation $P_{\text{loss}} = P^2 R_C/V^2$ explains the entire architecture of the electrical grid: transformers at every substation, the elaborate hierarchy of voltages (400 kV $\rightarrow$ 33 kV $\rightarrow$ 11 kV $\rightarrow$ 415 V $\rightarrow$ 230 V), and even the reason AC is preferred over DC for long lines (AC transformers work; DC ones don't, without complex electronics). A 2-mark formula carries a multi-trillion-dollar industry on its back.

Q 3.19 **AB is a potentiometer wire (Fig. 3.4). If R is increased, in which direction does the balance point J shift?**

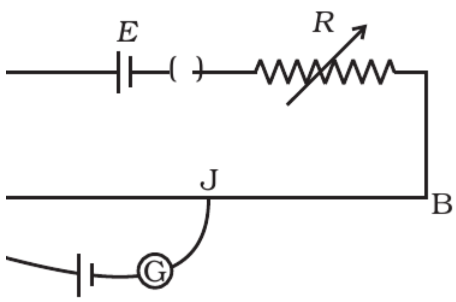


Fig. 3.4, NCERT Exemplar Class 12 Physics, Chapter 3.

SOLUTION

Concept used. The driver-circuit current is $I = \mathcal{E}_{\text{driver}}/(R + R_w)$, where R_w is the potentiometer-wire resistance. The potential drop per unit length along the wire is

$$k = \frac{IR_w}{L} = \frac{\mathcal{E}_{\text{driver}} R_w}{L\,(R + R_w)},$$

where L is the wire's length. A cell of EMF $\mathcal{E}_1$ balances at length l when $kl = \mathcal{E}_1$, so

$$l = \frac{\mathcal{E}_1}{k}.$$

Step 1. **Effect of increasing** R**.** As R goes up, $k = \mathcal{E}_{\text{driver}} R_w/[L(R + R_w)]$ goes *down*. Less potential drop per unit length means the balance length $l = \mathcal{E}_1/k$ goes *up*.

Step 2. **Direction.** The balance point moves *away from* A and *towards* B along the wire.

Final Answer: J shifts *towards* B (the far end of the wire) as R increases.

EXPERT'S SOLUTION *: Sanya Patel, M.Sc Physics, IIT Madras*

Strategic angle. Potential gradient $k \propto 1/(R + R_w)$. Larger $R \Rightarrow$ smaller $k \Rightarrow$ longer balance length. The physics turns on tracking how the wire's potential drop changes when you change the resistor in the *driver* loop — that drop, divided by the wire's length, is the gradient k that every measurement on the wire is read against.

Step 1. **Driver-loop current.** $I_{\text{drv}} = \mathcal{E}_{\text{driver}}/(R + R_w)$. Wire's potential drop: $V_w = I_{\text{drv}} R_w = \mathcal{E}_{\text{driver}} R_w/(R + R_w)$.

Step 2. **Gradient.** $k = V_w/L = \mathcal{E}_{\text{driver}} R_w/[L(R + R_w)]$. As R increases at fixed $\mathcal{E}_{\text{driver}}, R_w, L$: k decreases.

Step 3. **Balance length.** Standard potentiometer formula $\mathcal{E}_1 = kl \Rightarrow l = \mathcal{E}_1/k$. Smaller k at fixed $\mathcal{E}_1$ means larger l.

Step 4. **Direction.** Larger balance length means the balance point sits further from A, i.e. shifts toward B. (Until R is so large that even the full wire can't balance $\mathcal{E}_1$, at which point no null exists.)

Final Answer: Balance point shifts towards B.

☞ **The potentiometer master equation**

For any potentiometer problem of the form "what happens to the balance point when X changes", write $l = \mathcal{E}_1/k$ with $k = V_w/L$, and read off whether k increases or decreases when X changes. Increasing R

in the driver loop $\Rightarrow V_w \downarrow \Rightarrow k \downarrow \Rightarrow l \uparrow \Rightarrow$ balance toward B. Decreasing $\mathcal{E}_{\text{driver}}$: same direction. The trick works for every shift-direction question.

Q 3.20 In a potentiometer experiment (Fig. 3.5), the galvanometer deflection is one-sided. (i) The deflection decreases as the jockey moves from A to B. (ii) The deflection increases as the jockey moves towards B.
Which terminal of E_1 is connected at X in each case, and how is E_1 related to E?

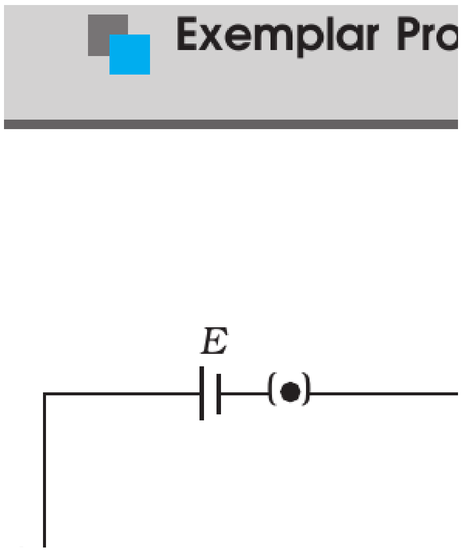


Fig. 3.5, NCERT Exemplar Class 12 Physics, Chapter 3.

SOLUTION

Concept used. For a working potentiometer, the unknown cell E_1 is connected so that its *positive* terminal matches the high-potential end of the wire (typically A, the end nearer the driver's positive terminal). The galvanometer reads zero at the balance length l where $kl = E_1$. If E_1 is larger than the maximum drop kL across the wire, no balance is possible — the galvanometer reads a non-zero deflection that *decreases* as the jockey moves toward B (because $kl - E_1$ shrinks in magnitude). If E_1 is reversed-polarity, the deflection always opposes and *grows* as the jockey moves away from A, since the potentials add rather than subtract.

Step 1. **Case (i): deflection one-sided and decreasing toward B.** Polarity at X must be correct (positive terminal of E_1 at X, the same end as the driver's positive). But the wire drop kL cannot match E_1, so no null appears. As the jockey moves from A to B, the in-wire drop approaches E_1 but never reaches it, so the imbalance shrinks: the deflection decreases. Conclusion: **positive terminal of E_1 at X, and $E_1 > kL$** (the wire's full drop is too small for this E_1).

Step 2. **Case (ii): deflection one-sided and increasing toward B.** The polarity at X is wrong (negative terminal of E_1 at X). The two EMFs now add along the jockey-galvanometer loop rather than oppose, so as the jockey moves towards B (more wire in the loop), the total opposing voltage grows, the galvanometer current grows, and the deflection increases. Conclusion: **negative terminal of E_1 at X (reversed polarity)**.

Final Answer: (i) $+$ of E_1 at X, with E_1 greater than the full drop kL across the wire (driver too weak). (ii) $-$ of E_1 at X (polarity reversed).

EXPERT'S SOLUTION *: Aanya Mehta, B.Tech Engineering Physics, IIT Bombay*

Strategic angle. "One-sided" means no balance exists. Two ways for that: wrong magnitude (correct polarity, $E_1 > kL$) or wrong polarity (EMFs add along the loop). The *direction* of monotonicity (decreasing vs. increasing as the jockey moves) distinguishes the two cases.

Step 1. **Set up the galvanometer-loop EMF.** Let V_J be the wire's potential at the jockey, measured from the positive end. The galvanometer responds to $V_{\text{loop}} = V_J - E_1$ (or $V_J + E_1$ if E_1 is reversed). It shows "zero" precisely when $V_{\text{loop}} = 0$.

Step 2. **Case (i): correct polarity, $E_1 > kL$.** V_J ranges from 0 (at A) to kL (at B) $< E_1$. $V_{\text{loop}} = V_J - E_1 < 0$ throughout. Magnitude decreases as V_J rises toward kL, so deflection shrinks (but never reaches zero) as jockey moves $A \to B$. *Positive of E_1 at X, with $E_1 > kL$.*

Step 3. **Case (ii): reversed polarity.** $V_{\text{loop}} = V_J + E_1$ (the two EMFs add around the galvanometer loop because they are in series-aid). As V_J grows from 0 at A to kL at B, V_{loop} grows in magnitude. Deflection *increases* toward B. *Negative of E_1 at X.*

Step 4. **Repair recipe.** For case (i), increase the driver-loop wire drop (decrease the driver-loop series resistance R, or use a larger driver battery). For case (ii), simply swap the leads of E_1.

Final Answer: (i) +ve at X, $E_1 > kL$. (ii) −ve at X.

✗ Don't confuse "no null" with "wrong polarity"

A common trap is to conclude that any one-sided deflection means the cell is connected backwards. It can — that's case (ii). But case (i) has correct polarity and still no null, because the wire drop simply isn't large enough. Use the *trend* of deflection (rising vs. falling along the wire) to discriminate, not just the fact that there is no null.

Q 3.21 **A cell of EMF $\mathcal{E}$ and internal resistance r is connected across an external resistance R. Plot a graph showing the variation of the P.D. across R versus R.**

SOLUTION

Concept used. The current through the loop is $I = \mathcal{E}/(R+r)$. The potential drop across R is

$$V = IR = \frac{\mathcal{E}R}{R+r}.$$

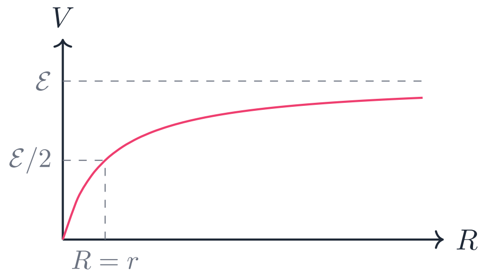
This is a monotone function of R. Limits:

- $R \to 0$: $V \to 0$.
- $R \to \infty$: $V \to \mathcal{E}$ (open-circuit terminal voltage).

For $R = r$, $V = \mathcal{E}/2$.

Step 1. **Equation.** $V(R) = \mathcal{E}R/(R+r)$.

Step 2. **Derivative.** $dV/dR = \mathcal{E}r/(R+r)^2 > 0$, so V rises monotonically with R. The slope is largest at $R = 0$ (slope $\mathcal{E}/r$) and tends to 0 as $R \to \infty$.



Final Answer: $V(R) = \dfrac{\mathcal{E}R}{R+r}$. Curve starts at the origin, rises with diminishing slope, and asymptotes to $\mathcal{E}$ as $R \to \infty$. At $R = r$, $V = \mathcal{E}/2$.

EXPERT'S SOLUTION *: Krishna Iyer, M.Sc Applied Mathematics, IIT Kanpur*

Strategic angle. $V = IR$, with I decreasing in R as $1/(R+r)$. The product $R/(R+r)$ rises monotonically from 0 to 1. The curve is the canonical "loaded-cell" graph — it shows up in solar-cell IV characteristics, fuel-cell load curves, and any source-with-internal-resistance plot.

Step 1. **Closed form.** $V(R) = IR = \mathcal{E}R/(R+r)$. Rewrite as $V = \mathcal{E}\,[R/(R+r)] = \mathcal{E}\,[1 - r/(R+r)]$.

Step 2. **Limits and special values.** $R \to 0 \Rightarrow V \to 0$ (short circuit); $R \to \infty \Rightarrow V \to \mathcal{E}$ (open circuit, no current). At $R = r$, $V = \mathcal{E}/2$.

Step 3. **Slope and shape.** $dV/dR = \mathcal{E}r/(R+r)^2 > 0$ (monotone increasing). Initial slope $\mathcal{E}/r$ at $R = 0$; slope $\to 0$ as $R \to \infty$. Second derivative $-2\mathcal{E}r/(R+r)^3 < 0$ (concave down).

Step 4. Power-delivery digression. $P_R = V^2/R = \mathcal{E}^2 R/(R+r)^2$. Maximum at $R = r$ (impedance matching) — same point where $V = \mathcal{E}/2$. Useful sanity check on the graph.

Final Answer: $V(R) = \mathcal{E}R/(R+r)$, monotone-rising from 0 to $\mathcal{E}$.

☞ **CBSE plotting marks**

For "plot V vs. R" questions, full marks require: (i) labelled axes, (ii) both intercepts (origin $\to 0$, asymptote $\to \mathcal{E}$), (iii) the half-max point at $R = r$ marked explicitly, (iv) curvature shown (concave-down). Markers typically deduct $1/2$ mark per missing element. Practice drawing the curve fast — it's a near-certain repeat across 2018-2024 board papers.

Short Answer (SA)

Q 3.22 Connect n equal resistors of R each in series to a battery of EMF $\mathcal{E}$ and internal resistance R. A current I is observed. Connect the same n resistors in parallel to the same battery; the current becomes $10I$. Find n.

SOLUTION

Concept used. For series and parallel combinations:

$$R_s = nR, \quad R_p = R/n.$$

The battery's internal resistance is also R, so total resistance in each case adds to this.

Step 1. Series current.

$$I_s = \frac{\mathcal{E}}{nR + R} = \frac{\mathcal{E}}{(n+1)R}.$$

This is the observed current I.

Step 2. Parallel current.

$$I_p = \frac{\mathcal{E}}{R/n + R} = \frac{\mathcal{E}}{R(1/n+1)} = \frac{n\mathcal{E}}{(n+1)R}.$$

Step 3. Ratio.

$$\frac{I_p}{I_s} = \frac{n\mathcal{E}/[(n+1)R]}{\mathcal{E}/[(n+1)R]} = n.$$

Given $I_p = 10 I_s$:

$$n = 10.$$

☞ **Sanity check**

With $n = 10$: $I_s = \mathcal{E}/(11R)$, $I_p = 10\mathcal{E}/(11R)$. Indeed $I_p/I_s = 10$. ✓

Final Answer: $n = 10$.

EXPERT'S SOLUTION : *Vivaan Reddy, B.Tech CSE, IIT Roorkee*

Strategic angle. Compute I_s and I_p in closed form, take the ratio. The beauty of the problem is that the EMF $\mathcal{E}$ cancels — the answer depends only on n, not on absolute voltage or resistance scales.

Step 1. Series combination. Equivalent external resistance $R_s = nR$. Loop resistance $R_s + r = nR + R = (n+1)R$.

$$I_s = \frac{\mathcal{E}}{(n+1)R}.$$

Step 2. Parallel combination. Equivalent external resistance $R_p = R/n$. Loop resistance $R_p + r = R/n + R = R(n+1)/n$.

$$I_p = \frac{\mathcal{E}}{R(n+1)/n} = \frac{n\,\mathcal{E}}{(n+1)R}.$$

Step 3. Take the ratio.

$$\frac{I_p}{I_s} = \frac{n\,\mathcal{E}/[(n+1)R]}{\mathcal{E}/[(n+1)R]} = n.$$

Step 4. Apply the data. Given $I_p = 10I_s$, so $n = 10$. Notice that $\mathcal{E}$ and the resistance scale R both cancelled in the ratio — the answer is purely combinatorial.

Final Answer: $n = 10$.

♥ **Why the ratio is exactly n**

The clean factor $I_p/I_s = n$ has a slick interpretation. In series, all n resistors plus the cell give loop resistance $(n+1)R$. In parallel, the same n resistors plus the cell give loop resistance $(n+1)R/n$ — exactly n times smaller. With fixed EMF, current scales inversely with loop resistance, so $I_p/I_s = n$. The internal resistance equalling R is the key — it makes the parallel/series loop resistances differ by exactly a factor of n.

Q 3.23 **Let n resistors $R_1, \ldots, R_n$ have $R_{\max} = \max\{R_i\}$ and $R_{\min} = \min\{R_i\}$. Show that when connected in parallel, $R_P < R_{\min}$, and when in series, $R_S > R_{\max}$. Interpret**

physically.

SOLUTION

Concept used. The series and parallel rules:

$$R_S = \sum_i R_i, \qquad \frac{1}{R_P} = \sum_i \frac{1}{R_i}.$$

Step 1. Series side.

$$R_S = R_1 + R_2 + \cdots + R_n = R_{\max} + (\text{other positive terms}).$$

Since the other $R_i \geq 0$ (and at least one is strictly positive for any non-trivial network with $n \geq 2$), $R_S > R_{\max}$.

Step 2. Parallel side.

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} = \frac{1}{R_{\min}} + (\text{other positive terms}).$$

So $1/R_P > 1/R_{\min}$, and inverting (both sides positive): $R_P < R_{\min}$.

Step 3. Physical picture.

- Series: every electron has to traverse *all* the resistors in turn. Even the smallest resistor in the chain contributes resistance; the total is more than the largest one alone.
- Parallel: electrons take *any* of the available paths. Even if every other path were blocked, the lowest-resistance path alone could carry plenty of current. With several paths open, the effective resistance must be less than that of the single lowest-resistance path.

Final Answer: $R_S > R_{\max}$ (series adds), $R_P < R_{\min}$ (parallel offers a path of less resistance than any individual leg).

EXPERT'S SOLUTION : *Tara Joshi, M.Sc Physics, IIT Bombay*

Structural observation. Sum and reciprocal-sum each have strict inequalities once two or more positive terms are present. A single-line proof for each direction, plus a one-paragraph physical picture, is enough for full marks on this short-answer question.

Step 1. Series bound from below by $R_{\max}$.

$$R_S = \sum_i R_i = R_{\max} + \sum_{R_i \neq R_{\max}} R_i.$$

Since each $R_i > 0$, the sum on the right is strictly positive (for $n \geq 2$), so $R_S > R_{\max}$.

Step 2. **Parallel bound from above by $R_{\min}$.**

$$\frac{1}{R_P} = \sum_i \frac{1}{R_i} = \frac{1}{R_{\min}} + \sum_{R_i \neq R_{\min}} \frac{1}{R_i}.$$

Again the rest of the sum is strictly positive, so $1/R_P > 1/R_{\min}$, i.e. $R_P < R_{\min}$.

Step 3. **Physical picture — series.** Electrons traverse every resistor in turn. Each contributes to the total opposition; the lightest resistor still adds something positive on top of the heaviest one. Hence total $>$ heaviest.

Step 4. **Physical picture — parallel.** Electrons distribute themselves over multiple parallel paths. Even the lowest-resistance path on its own would carry a current $\mathcal{E}/R_{\min}$ at a fixed voltage; adding more paths can only *increase* the total current, i.e. further reduce the effective resistance.

Final Answer: $R_S > R_{\max}$, $R_P < R_{\min}$.

☞ **Sign of the inequality — never strict "≥"**

For $n \geq 2$ distinct positive resistors, the inequalities are strict ($>$, not $\geq$). Equality only holds for trivial cases ($n = 1$, or all $R_i = 0$, or all but one $R_i = 0$). CBSE markers accept "$>$" as the correct form; "$\geq$" often loses half a mark for being loose.

Q 3.24 **The circuit in Fig. 3.6 shows two cells connected in opposition. Cell E_1 has EMF $6\,\text{V}$ and internal resistance $2\,\Omega$; cell E_2 has EMF $4\,\text{V}$ and internal resistance $8\,\Omega$. Find the potential difference between the points A and B.**

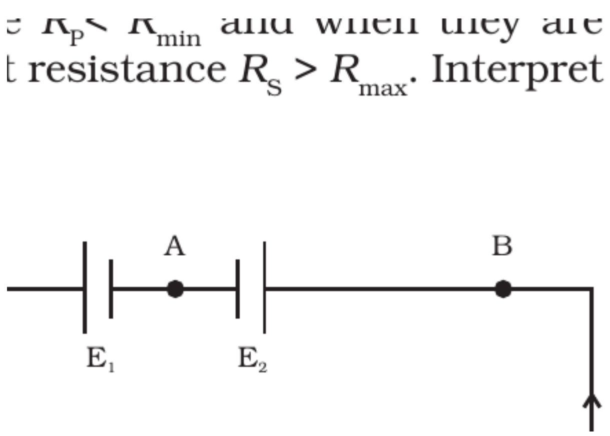


Fig. 3.6, NCERT Exemplar Class 12 Physics, Chapter 3.

SOLUTION

Concept used. The two cells form a closed loop (since the external wire connects A and B). They are in series and in opposition, so the net EMF driving the loop is

$$\mathcal{E}_{\text{net}} = E_1 - E_2 = 6 - 4 = 2\,\text{V},$$

and the loop's total resistance is $r_1 + r_2 = 2 + 8 = 10\,\Omega$. The current flows in the direction of the larger EMF (here E_1), and is

$$I = \frac{E_1 - E_2}{r_1 + r_2} = \frac{2}{10} = 0.2\,\text{A}.$$

Step 1. Identify the loop and the current. Cells E_1 and E_2 are in series-opposition. Net EMF $2\,\text{V}$ drives current $I = 0.2\,\text{A}$ in the direction of E_1.

Step 2. Find $V_A - V_B$. The potential difference between A and B equals the terminal voltage of either cell (the two terminal voltages must be the same, since A and B are connected externally by a wire). Using cell E_1 (discharging, current leaves its $+$ terminal):

$$V_A - V_B = E_1 - Ir_1 = 6 - (0.2)(2) = 6 - 0.4 = 5.6\,\text{V}.$$

Cross-check using cell E_2 (being charged, current enters its $+$ terminal):

$$V_A - V_B = E_2 + Ir_2 = 4 + (0.2)(8) = 4 + 1.6 = 5.6\,\text{V}. \quad ✓$$

> ☞ **Why the second equation has a $+$ sign**
>
> For a cell that is being charged, the external circuit drives current *into* its $+$ terminal, so the terminal voltage is $E + Ir$ (more than the EMF — the cell is acting like a load, absorbing extra energy).

Final Answer: $V_A - V_B = 5.6\,\text{V}$.

EXPERT'S SOLUTION : *Aarav Singh, M.Sc Physics, IIT Madras*

Strategic angle. The cells form a closed loop. Compute the loop current, then the terminal voltage of either cell. Cross-checking via the second cell catches sign errors — both expressions for V_{AB} must give the same number, and if they don't, recheck the sign of I relative to each cell's $+$ terminal.

Step 1. Loop EMF and current. $\mathcal{E}_{\text{net}} = E_1 - E_2 = 6 - 4 = 2\,\text{V}$ (opposition: the smaller cell opposes the larger). Loop resistance: $r_1 + r_2 = 2 + 8 = 10\,\Omega$.
$I = 2/10 = 0.2\,\text{A}$, flowing in the direction of E_1.

Step 2. Terminal voltage of E_1 (discharging). Current leaves the $+$ terminal:

$$V_A - V_B = E_1 - Ir_1 = 6 - 0.4 = 5.6\,\text{V}.$$

Step 3. Terminal voltage of E_2 (being charged). Current enters the + terminal so the cell's terminal voltage *exceeds* its EMF: $V_A - V_B = E_2 + Ir_2 = 4 + 1.6 = 5.6\,\text{V}$. Same answer — internal consistency confirmed.

Step 4. Energy budget check. Power supplied by E_1: $E_1 I = 1.2\,\text{W}$. Power absorbed by E_2 (chemical energy stored back into it): $E_2 I = 0.8\,\text{W}$. Power lost in r_1, r_2: $I^2(r_1 + r_2) = (0.04)(10) = 0.4\,\text{W}$. Total absorbed $= 0.8 + 0.4 = 1.2\,\text{W} =$ supplied. ✓

Final Answer: $V_{AB} = 5.6\,\text{V}$.

✗ Sign of Ir depends on direction of current

The classic blunder is to write $V_{\text{term}} = E - Ir$ for *every* cell, regardless of whether it's discharging or charging. The correct rule: $V_{\text{term}} = E - Ir$ when conventional current leaves the + terminal (discharge), and $V_{\text{term}} = E + Ir$ when current enters the + terminal (charge). Identifying which cell is which in a multi-cell loop is the entire problem.

Q 3.25 Two cells of the same EMF $\mathcal{E}$ but internal resistances r_1 and r_2 are connected in series to an external resistor R (Fig. 3.7). What value of R makes the potential difference across the terminals of the first cell zero?

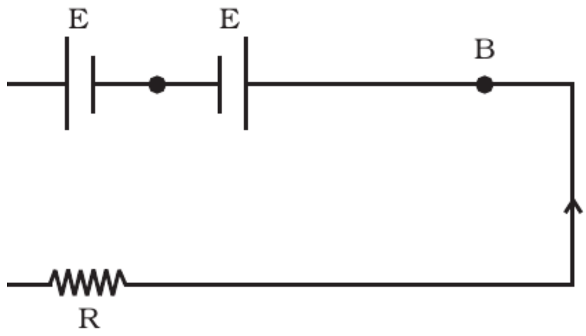


Fig. 3.7

Fig. 3.7, NCERT Exemplar Class 12 Physics, Chapter 3.

SOLUTION

Concept used. For a single discharging cell of EMF $\mathcal{E}$, internal resistance r, carrying current I, the terminal voltage is

$$V_{\text{term}} = \mathcal{E} - Ir.$$

The terminal voltage is zero when $\mathcal{E} = Ir$, i.e. the external load forces the current to be exactly $\mathcal{E}/r$.

Step 1. Loop current. The two cells in series with the external R give a total resistance $r_1 + r_2 + R$, and net EMF $\mathcal{E} + \mathcal{E} = 2\mathcal{E}$ (both cells aid each other in the loop). So

$$I = \frac{2\mathcal{E}}{r_1 + r_2 + R}.$$

Step 2. Zero terminal voltage on cell 1. Demand $V_1 = \mathcal{E} - Ir_1 = 0$, i.e.

$$\mathcal{E} = Ir_1 = \frac{2\mathcal{E}\, r_1}{r_1 + r_2 + R}.$$

Cancel $\mathcal{E}$ from both sides (assume $\mathcal{E} \neq 0$):

$$1 = \frac{2r_1}{r_1 + r_2 + R}.$$

Step 3. Solve for R.

$$r_1 + r_2 + R = 2r_1 \quad \Longrightarrow \quad R = r_1 - r_2.$$

☞ **Existence condition**

We need $R \geq 0$, so $r_1 \geq r_2$. If $r_1 < r_2$, no positive R can make cell 1's terminal voltage zero — the second cell's larger internal drop dominates.

Final Answer: $R = r_1 - r_2$ (requires $r_1 \geq r_2$).

EXPERT'S SOLUTION *: Sneha Desai, M.Sc Physics, IIT Madras*

Strategic angle. $V_1 = \mathcal{E} - Ir_1 = 0$ gives $I = \mathcal{E}/r_1$. Set equal to the loop current and solve for R. The trick is to forget about cell 2 for a moment and ask: what *current* would make cell 1's terminal voltage vanish? Once you know that, the other cell + external R just have to arrange that current.

Step 1. Zero-terminal-voltage condition for cell 1. $V_1 = \mathcal{E} - Ir_1 = 0 \Rightarrow I = \mathcal{E}/r_1$. This is the short-circuit current of cell 1 alone.

Step 2. Loop current with both cells. Two cells of EMF $\mathcal{E}$ in series-aid give net EMF $2\mathcal{E}$ and net resistance $r_1 + r_2 + R$:

$$I_{\text{loop}} = \frac{2\mathcal{E}}{r_1 + r_2 + R}.$$

Step 3. Set them equal and solve. $2\mathcal{E}/(r_1 + r_2 + R) = \mathcal{E}/r_1$. Cancel $\mathcal{E}$: $r_1 + r_2 + R = 2r_1$, so $R = r_1 - r_2$.

Step 4. Physical interpretation. When the load $R = r_1 - r_2$, the loop current is exactly the short-circuit current of the first cell, meaning all of E_1's EMF is dropped internally and zero shows up at its terminals. The $V_1 = 0$ condition requires $r_1 \geq r_2$, since $R \geq 0$.

Final Answer: $R = r_1 - r_2$.

♥ Why a cell can show zero terminal voltage

A discharging cell shows $V_{\text{term}} = \mathcal{E}$ at open circuit, $V_{\text{term}} = 0$ at short circuit, and any value in between for finite load. The strange-looking "zero terminal voltage" condition here is exactly the short-circuit point of cell 1 — except achieved by combining its EMF with cell 2's, rather than by literally shorting it. The lesson: terminal voltage is not a property of the cell alone, it depends on the load and on whatever other sources are in the loop.

Q 3.26 Two conductors of the same material and same length: A is a solid wire of diameter 1 mm; B is a hollow tube of outer diameter 2 mm and inner diameter 1 mm. Find $R_A : R_B$.

SOLUTION

Concept used. Resistance $R = \rho L/A_{\text{c.s.}}$ where $A_{\text{c.s.}}$ is the cross-sectional area perpendicular to the current. Same material and same length means ρ and L are the same for both; the only difference is the cross-section.

Step 1. Cross-section of conductor A (solid disc). Radius $r_A = 0.5\,\text{mm}$:

$$A_A = \pi r_A^2 = \pi(0.5)^2 = 0.25\pi\,\text{mm}^2.$$

Step 2. Cross-section of conductor B (annulus). Outer radius $r_{B,\text{out}} = 1\,\text{mm}$, inner radius $r_{B,\text{in}} = 0.5\,\text{mm}$:

$$A_B = \pi[r_{B,\text{out}}^2 - r_{B,\text{in}}^2] = \pi(1 - 0.25) = 0.75\pi\,\text{mm}^2.$$

Step 3. Compute the ratio.

$$\frac{R_A}{R_B} = \frac{\rho L/A_A}{\rho L/A_B} = \frac{A_B}{A_A} = \frac{0.75\pi}{0.25\pi} = 3.$$

Final Answer: $R_A : R_B = 3 : 1$.

EXPERT'S SOLUTION : *Karan Mehta, M.Sc Physics, IIT Kanpur*

Strategic angle. $R \propto 1/A_{\text{c.s.}}$ when ρL is fixed. Compute the two cross-sectional areas and take the ratio. Same material and same length together kill ρL — geometry alone decides.

Step 1. Identify what's the same. Same material $\Rightarrow$ same ρ. Same length $\Rightarrow$ same L. So in $R = \rho L/A$, only A differs.

Step 2. Cross-section of A (solid). Radius $0.5\,\text{mm} \Rightarrow A_A = \pi(0.5)^2 = 0.25\pi\,\text{mm}^2$.

Step 3. Cross-section of B (annular). Outer radius $1\,\text{mm}$, inner radius $0.5\,\text{mm}$ $\Rightarrow A_B = \pi(1^2 - 0.5^2) = 0.75\pi\,\text{mm}^2$. Note that the conducting cross-section is the annulus only — current flows through metal, not through the hollow interior.

Step 4. Take the ratio. $R_A/R_B = A_B/A_A = 0.75\pi/(0.25\pi) = 3$. So $R_A : R_B = 3 : 1$.

Step 5. Sanity check via diameters. A_B is computed as $\pi(d_o^2 - d_i^2)/4 = \pi(4-1)/4 = 0.75\pi\,\text{mm}^2$ — same answer through the diameter formulation.

Final Answer: $R_A : R_B = 3 : 1$.

✗ The hollow tube's cross-section is the annulus, not the outer disc

A common slip is to compute A_B as $\pi(1\,\text{mm})^2$ — the full outer disc — instead of the annular ring between the outer and inner walls. Current flows through metal only; the hollow interior contributes nothing. Always subtract the inner-disc area: $A_{\text{annulus}} = \pi(r_o^2 - r_i^2)$.

Q 3.27 Suppose a circuit has only resistances and batteries, and we double (or scale by a factor n) all voltages and all resistances. Show that currents are unaltered.

SOLUTION

Concept used. Kirchhoff's laws for a circuit with only resistors and EMF sources are a system of linear equations in the branch currents $\{I_k\}$:

- Each loop equation has the form $\sum \mathcal{E}_i - \sum I_k R_k = 0$.
- Each junction equation has the form $\sum_{\text{in}} I_k - \sum_{\text{out}} I_k = 0$.

If we scale every EMF $\mathcal{E}_i \to n\mathcal{E}_i$ and every resistance $R_k \to nR_k$, the loop equations become

$$\sum n\mathcal{E}_i - \sum I_k (nR_k) = n\Big(\sum \mathcal{E}_i - \sum I_k R_k\Big) = 0,$$

which is satisfied by the same $\{I_k\}$ as before. Junction equations don't involve EMF or R

at all, so they too are unchanged. Therefore the currents are the same.

Step 1. Take any loop in the original network. Its KVL equation is $\sum \mathcal{E}_i = \sum I_k R_k$.

Step 2. After scaling: $\sum n\mathcal{E}_i = \sum I_k(nR_k)$, i.e. $n\sum \mathcal{E}_i = n\sum I_k R_k$. The factor of n cancels on both sides; the equation is the same as before.

Step 3. Junction equations don't involve $\mathcal{E}$ or R, so they are unchanged trivially.

Step 4. Therefore the same currents $\{I_k\}$ that solved the original system solve the scaled system. Currents are unaltered.

> **Final Answer:** Currents are unchanged: scaling all EMFs and all resistances by the same factor n rescales every term in every KVL equation by n, leaving the solution unchanged. KCL is unaffected.

EXPERT'S SOLUTION : *Aditya Pillai, Ph.D Pure Mathematics, IISc Bangalore*

Strategic angle. The Kirchhoff system is linear in the voltages and resistances; scaling both by the same factor leaves the solution invariant. This is a *symmetry* argument: the equations of circuit theory have a hidden homogeneity that the question is asking the student to expose.

Step 1. **Write down the equations.** For any planar circuit with L independent loops and J junctions, KVL gives L equations of the form $\sum \mathcal{E}_i - \sum I_k R_k = 0$ and KCL gives $J-1$ equations of the form $\sum I_{\text{in}} - \sum I_{\text{out}} = 0$. Together they pin down all the branch currents.

Step 2. **Scale uniformly.** Replace $\mathcal{E}_i \to n\mathcal{E}_i$ and $R_k \to nR_k$. KVL becomes $\sum n\mathcal{E}_i - \sum I_k(nR_k) = n\cdot(\sum \mathcal{E}_i - \sum I_k R_k) = 0$. The factor of n scales every term equally and cancels.

Step 3. **KCL is unaffected.** Junction equations involve only currents, no voltages or resistances. So they remain identical.

Step 4. **Conclusion.** The same $\{I_k\}$ that solved the original system solves the rescaled one. Currents are unaltered. Voltages, on the other hand, scale by n (since $V = IR$ and R is multiplied by n).

Step 5. **Matrix-level statement.** Writing $A\vec{I} = \vec{b}$ where A depends on R and $\vec{b}$ on $\mathcal{E}$, scaling both rescales both sides by n: $(nA)\vec{I} = n\vec{b} \Leftrightarrow A\vec{I} = \vec{b}$. Same solution.

> **Final Answer:** Currents are unaltered.

> ☞ **Linearity of Kirchhoff's laws**
>
> Circuits made of resistors and DC sources are described by a *linear* system of equations in the unknown currents. Three consequences follow: (i) superposition (response to multiple sources = sum of responses to each individually), (ii) the scaling invariance proved here, and (iii) Thevenin/Norton equivalences. Linearity is the fingerprint of Ohm's law on the network-equation level.

Long Answer (LA)

Q 3.28 **Two cells of voltage $10\,\text{V}$ and $2\,\text{V}$ and internal resistances $10\,\Omega$ and $5\,\Omega$ respectively are connected in parallel, with the positive end of the $10\,\text{V}$ battery connected to the negative pole of the $2\,\text{V}$ battery (Fig. 3.8). Find the effective voltage and effective resistance of the combination.**

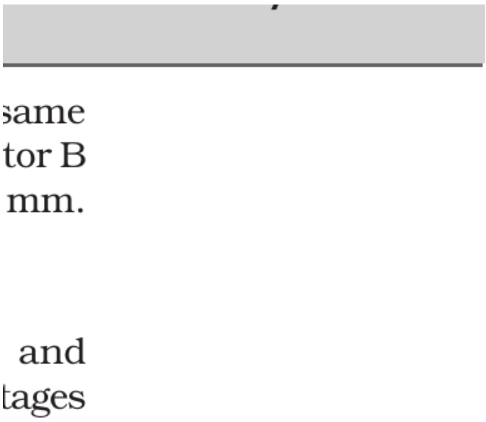


Fig. 3.8, NCERT Exemplar Class 12 Physics, Chapter 3.

SOLUTION

Concept used. Two cells in parallel with EMFs $\mathcal{E}_1, \mathcal{E}_2$ and internal resistances r_1, r_2 have an equivalent Thévenin cell with EMF

$$\mathcal{E}_{\text{eq}} = \frac{\mathcal{E}_1 r_2 \pm \mathcal{E}_2 r_1}{r_1 + r_2}$$

($+$ for same polarity, $-$ for opposition) and internal resistance

$$r_{\text{eq}} = \frac{r_1 r_2}{r_1 + r_2}.$$

In this problem the cells are in *opposition* (positive of the $10\,\text{V}$ cell joins the negative of the $2\,\text{V}$ cell), so the minus sign applies.

Step 1. Identify the values. $\mathcal{E}_1 = 10\,\text{V}$, $r_1 = 10\,\Omega$; $\mathcal{E}_2 = 2\,\text{V}$, $r_2 = 5\,\Omega$. Cells in opposition.

Step 2. Compute $\mathcal{E}_{\text{eq}}$.

$$\mathcal{E}_{\text{eq}} = \frac{\mathcal{E}_1 r_2 - \mathcal{E}_2 r_1}{r_1 + r_2} = \frac{(10)(5) - (2)(10)}{10 + 5} = \frac{50 - 20}{15} = \frac{30}{15} = 2\,\text{V}.$$

Step 3. Compute r_{eq}.

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2} = \frac{(10)(5)}{15} = \frac{50}{15} = \frac{10}{3}\,\Omega \approx 3.33\,\Omega.$$

☞ **Why the − in $\mathcal{E}_{eq}$**

With the cells opposing, the larger cell (10 V) drives current through the smaller one (2 V), partially *charging* it. The combination effectively delivers less voltage than either cell alone — only 2 V here, much less than 10 V.

Final Answer: $\mathcal{E}_{eq} = 2\,\text{V}$, $r_{eq} = 10/3\,\Omega \approx 3.33\,\Omega$.

EXPERT'S SOLUTION *: Pranav Bhat, M.Sc Physics, IIT Madras*

Strategic angle. The parallel-cell formula with the opposition sign gives the answer immediately. The whole problem hinges on getting the sign right — once that's set, two arithmetic substitutions finish the calculation.

Step 1. Recognize opposition. The + of the 10 V cell ties to the − of the 2 V cell, meaning the two EMFs point in opposite directions around the parallel block. Apply the $\mathcal{E}_1 r_2 - \mathcal{E}_2 r_1$ formula.

Step 2. Plug into the parallel-cell formula.

$$\mathcal{E}_{eq} = \frac{\mathcal{E}_1 r_2 - \mathcal{E}_2 r_1}{r_1 + r_2} = \frac{(10)(5) - (2)(10)}{10 + 5} = \frac{50 - 20}{15} = \frac{30}{15} = 2\,\text{V}.$$

Step 3. Internal resistance.

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2} = \frac{50}{15} = \frac{10}{3}\,\Omega \approx 3.33\,\Omega.$$

Note: the parallel rule for r_{eq} does not care about polarity — only the EMF does.

Step 4. Kirchhoff cross-check. Without a load, currents I_1 from cell 1 and $I_2 = -I_1$ from cell 2 (returns because polarities oppose) satisfy $\mathcal{E}_1 = I_1 r_1 + V$ and $V = \mathcal{E}_2 + I_1 r_2$, giving $\mathcal{E}_{eq} = V|_{I_{load}=0} = (\mathcal{E}_1 r_2 - \mathcal{E}_2 r_1)/(r_1 + r_2) = 2\,\text{V}$. Same answer.

Final Answer: $\mathcal{E}_{eq} = 2\,\text{V}$, $r_{eq} = 10/3\,\Omega$.

☞ **Sign convention for parallel cells**

For board questions: parallel cells of the *same* polarity (both + at the same junction) use the + sign in $\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1$. Parallel cells of *opposite* polarity (+ of one to − of the other) use the − sign. The internal resistance formula is the same in both cases. Quick mnemonic: "aiding adds, opposing subtracts" — the EMFs combine *additively* in series-aid configurations and *subtractively* in series-oppose configurations, with the same logic in parallel.

Q 3.29 **A room has AC running for 5 hours per day at $220\,$V. The wiring is Cu of $1\,$mm radius and $10\,$m length. Power consumption is 10 commercial units per day. What fraction of it goes into joule heating in the wires? What if the wire were Al of the same dimensions? Use $\rho_{\text{Cu}} = 1.7 \times 10^{-8}\,\Omega$-m, $\rho_{\text{Al}} = 2.7 \times 10^{-8}\,\Omega$-m.**

SOLUTION

Concept used.

- Average power drawn by the room: $P = (\text{energy per day})/(\text{time per day})$.
- Current: $I = P/V$.
- Wire resistance: $R = \rho L/A$, where $A = \pi r^2$.
- Power dissipated in the wire: $P_{\text{wire}} = I^2 R$.

Step 1. Power drawn by the AC. $10\,$kWh in $5\,$h means

$$P = \frac{10\,\text{kWh}}{5\,\text{h}} = 2\,\text{kW} = 2000\,\text{W}.$$

Step 2. Current.

$$I = \frac{P}{V} = \frac{2000\,\text{W}}{220\,\text{V}} \approx 9.09\,\text{A}.$$

Step 3. Wire cross-section. $r = 1\,\text{mm} = 10^{-3}\,\text{m}$:

$$A = \pi r^2 = \pi(10^{-3})^2 = \pi \times 10^{-6}\,\text{m}^2 \approx 3.14 \times 10^{-6}\,\text{m}^2.$$

Step 4. Resistance — copper.

$$R_{\text{Cu}} = \frac{\rho_{\text{Cu}} L}{A} = \frac{(1.7 \times 10^{-8})(10)}{3.14 \times 10^{-6}} = \frac{1.7 \times 10^{-7}}{3.14 \times 10^{-6}} \approx 0.054\,\Omega.$$

Step 5. Power lost in Cu wires.

$$P_{\text{wire,Cu}} = I^2 R_{\text{Cu}} = (9.09)^2(0.054) \approx 82.6 \times 0.054 \approx 4.5\,\text{W}.$$

Energy per day:

$$E_{\text{wire,Cu}} = P_{\text{wire,Cu}} \times 5\,\text{h} \approx 22.4\,\text{Wh} = 0.0224\,\text{kWh}.$$

Fraction of total daily consumption (10 kWh):

$$\frac{E_{\text{wire,Cu}}}{10\,\text{kWh}} \approx \frac{0.0224}{10} \approx 0.22\%.$$

Step 6. Resistance and loss — aluminium.

$$R_{\text{Al}} = \frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}} R_{\text{Cu}} = \frac{2.7}{1.7} \times 0.054 \approx 0.0858\,\Omega.$$

$$P_{\text{wire,Al}} = I^2 R_{\text{Al}} \approx 82.6 \times 0.0858 \approx 7.1\,\text{W}.$$

Energy per day: $E_{\text{wire,Al}} \approx 35.5\,\text{Wh} = 0.0355\,\text{kWh}$. Fraction: $\approx 0.36\%$.

☞ **What changes with Al**

Aluminium dissipates roughly $\rho_{\text{Al}}/\rho_{\text{Cu}} = 2.7/1.7 \approx 1.6\times$ as much power in the wires for the same geometry — but the fraction is still tiny ($\sim 0.36\%$), so the wiring would not melt or fail. The penalty shows up in slightly higher running costs and warmer cables.

Final Answer: Fraction in Cu wires: $\approx 0.22\%$. Fraction in Al wires of same dimensions: $\approx 0.36\%$ — about $1.6\times$ as much loss, but still negligible compared to the AC's own consumption.

EXPERT'S SOLUTION *: Riya Nair, M.Sc Physics, IIT Madras*

Strategic angle. Average power P fixes I. Compute wire resistance and I^2R. Divide by daily energy. The numbers are intentionally meant to come out small ($< 1\%$), reassuring the student that home wiring is well-engineered: a non-trivial fraction would mean dangerously hot cables.

Step 1. Average power. 10 commercial units (kWh) per day, AC runs 5 hours:

$$P = \frac{10\,\text{kWh}}{5\,\text{h}} = 2\,\text{kW} = 2000\,\text{W}.$$

Step 2. Line current. $I = P/V = 2000/220 \approx 9.09\,\text{A}$.

Step 3. Wire cross-section. Radius $r = 1\,\text{mm} = 10^{-3}\,\text{m}$: $A = \pi r^2 = \pi \times 10^{-6} \approx 3.14 \times 10^{-6}\,\text{m}^2$.

Step 4. Resistance, copper.

$$R_{\text{Cu}} = \frac{\rho_{\text{Cu}} L}{A} = \frac{(1.7 \times 10^{-8})(10)}{3.14 \times 10^{-6}} \approx 0.054\,\Omega.$$

Step 5. Power loss, copper. $P_{\text{wire}} = I^2 R_{\text{Cu}} = (9.09)^2(0.054) \approx 82.6 \times 0.054 \approx 4.5\,\text{W}$. Daily energy: $4.5 \times 5 = 22.4\,\text{Wh} = 0.0224\,\text{kWh}$. Fraction: $0.0224/10 \approx 0.22\%$.

Step 6. Aluminium scaling. $R_{\text{Al}}/R_{\text{Cu}} = \rho_{\text{Al}}/\rho_{\text{Cu}} \approx 1.59$. So $P_{\text{wire,Al}} \approx 1.59 \times 4.5 \approx 7.1\,\text{W}$, daily $\approx 35.5\,\text{Wh}$, fraction $\approx 0.36\%$.

Step 7. Practical takeaway. 0.22% is well below any safety threshold — Cu wiring at $1\,\text{mm}$ radius for $10\,\text{m}$ is generously sized for a $2\,\text{kW}$ load. Al would still be safe but slightly more wasteful.

Final Answer: Cu $\sim 0.22\%$; Al $\sim 0.36\%$.

Q 3.30 **In a potentiometer experiment with $V_B = 10\,\text{V}$ and $R = 50\,\Omega$ (Fig. 3.9), no null point is found for a cell of EMF $\approx 8\,\text{V}$. Reducing R to $10\,\Omega$ puts the null point on the last (4th) segment of the potentiometer. Find the resistance of the potentiometer wire and the potential drop per unit length in the second case.**

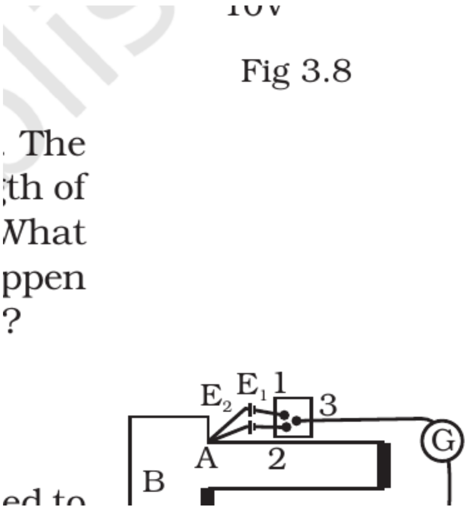


Fig. 3.9, NCERT Exemplar Class 12 Physics, Chapter 3.

SOLUTION

Concept used. The driver loop is V_B in series with the adjustable R and the wire resistance R_w. The drop across the wire is

$$V_{\text{wire}} = V_B \cdot \frac{R_w}{R + R_w}.$$

For a balance to occur on the wire, the EMF being measured must satisfy $\mathcal{E}_1 \le V_{\text{wire}}$. When the null appears at the very last (4th) segment of a 4-segment wire, the balance length is essentially the full wire, and $V_{\text{wire}} \approx \mathcal{E}_1$.

Step 1. **Case 1: $R = 50\,\Omega$, no null.** $V_{\text{wire},1} = 10R_w/(50 + R_w) < \mathcal{E}_1 = 8\,\text{V}$. This is consistent with R_w being relatively small compared to 50.

Step 2. **Case 2: $R = 10\,\Omega$, null on 4th (last) segment.** The balance length is essentially the entire wire, so $V_{\text{wire},2} \approx \mathcal{E}_1 = 8\,\text{V}$:

$$10\,\frac{R_w}{10 + R_w} = 8.$$

Solve:

$$10R_w = 8(10 + R_w) = 80 + 8R_w \quad\Longrightarrow\quad 2R_w = 80 \quad\Longrightarrow\quad R_w = 40\,\Omega.$$

Step 3. Driver current in case 2.

$$I_2 = \frac{V_B}{R + R_w} = \frac{10}{10 + 40} = 0.2\,\text{A}.$$

Voltage across the wire: $V_{\text{wire},2} = I_2 R_w = (0.2)(40) = 8\,\text{V}$ (matches).

Step 4. Potential drop per unit length. Let the wire's total length be L. Then the drop per unit length is

$$k = \frac{V_{\text{wire},2}}{L} = \frac{8\,\text{V}}{L}.$$

For a standard $4\,\text{m}$ wire ($L = 4\,\text{m}$, or $400\,\text{cm}$), $k = 8/400 = 0.02\,\text{V/cm} = 2\,\text{V/m}$.

☞ **Sanity check on case 1**

With $R_w = 40$ and $R = 50$: $V_{\text{wire},1} = 10 \cdot 40/90 \approx 4.4\,\text{V} < 8\,\text{V} = \mathcal{E}_1$. No null. ✓

Final Answer: Wire resistance $R_w = 40\,\Omega$. Potential drop per unit length in the second case $= 8\,\text{V}/L$ — for a $4\,\text{m}$ wire, this is $2\,\text{V/m}$.

EXPERT'S SOLUTION : *Diya Verma, M.Sc Physics, IIT Madras*

Strategic angle. The null on the last segment fixes the wire drop equal to the EMF being measured. That gives one equation in R_w. The first case ($R = 50$, no null) is used only as a consistency check — it tells you the cell is roughly $8\,\text{V}$; the second case does all the work.

Step 1. Identify the key observation. The null appears on the 4th (i.e. last) segment of the wire, so the balance length is essentially the full wire. The wire's full drop V_{wire} then equals the EMF being balanced: $V_{\text{wire}} = \mathcal{E}_1 \approx 8\,\text{V}$.

Step 2. Drop across wire for $R = 10\,\Omega$.

$$V_{\text{wire}} = V_B \cdot \frac{R_w}{R + R_w} = 10\,\frac{R_w}{10 + R_w}.$$

Step 3. Solve for R_w. Set equal to $8\,\text{V}$:

$$10R_w = 8(10 + R_w) \Rightarrow 2R_w = 80 \Rightarrow R_w = 40\,\Omega.$$

Step 4. Driver current cross-check. $I = V_B/(R + R_w) = 10/50 = 0.2\,\text{A}$. Wire drop $= IR_w = 0.2 \cdot 40 = 8\,\text{V}$. ✓

Step 5. Drop per unit length. $k = V_{\text{wire}}/L = 8\,\text{V}/L$. Standard NCERT potentiometers use $L = 4\,\text{m}$, giving $k = 2\,\text{V/m}$.

Step 6. Self-consistency with case 1. $R = 50$: $V_{\text{wire}} = 10 \cdot 40/(50 + 40) \approx 4.4\,\text{V} < 8\,\text{V} = \mathcal{E}_1$. No null possible — consistent with the problem statement.

> **Final Answer:** $R_w = 40\,\Omega$; $k = 2\,\text{V/m}$.

☞ **Potentiometer driver-loop algebra**

The fraction of V_B that lands across the wire is $R_w/(R + R_w)$ — a simple voltage divider. Tuning R tunes this fraction, hence tunes the gradient k. Every potentiometer numerical reduces to algebraic manipulation of this one expression plus the balance condition $\mathcal{E} = kl$.

Q 3.31 **(a) In the circuit of Fig. 3.10 ($R = 6\,\Omega$, $V = 6$ V), how much energy is absorbed by electrons from the initial state (no current) to the steady state (drift velocity v_d)? (b) Electrons give up energy at rate RI^2 per second to thermal energy. What time scale is associated with the energy in (a)? Use $n = 10^{29}$/m^3, length $= 10$ cm, cross-section $A = (1\,\text{mm})^2$.**

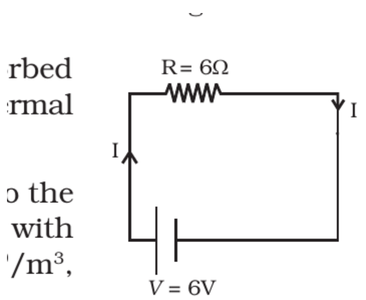


Fig 3.10

Fig. 3.10, NCERT Exemplar Class 12 Physics, Chapter 3.

SOLUTION

Concept used. The kinetic energy gained by an electron from rest to its drift velocity is $\frac{1}{2}m_e v_d^2$. The total kinetic energy of all conduction electrons in the wire is this per-electron energy times the total number of electrons in the wire.

Drift velocity: $v_d = I/(nAe)$. Power dissipated: RI^2. Time scale: total energy divided by dissipation rate.

Part (a).

Step 1. Compute the current. The circuit has $V = 6$ V across $R = 6\,\Omega$:

$$I = V/R = 6/6 = 1\,\text{A}.$$

Step 2. Compute drift velocity.

$$v_d = \frac{I}{nAe} = \frac{1}{(10^{29})(10^{-6})(1.6 \times 10^{-19})} = \frac{1}{1.6 \times 10^4} \approx 6.25 \times 10^{-5}\,\text{m/s}.$$

Step 3. **KE per electron.**

$$\tfrac{1}{2}m_e v_d^2 = \tfrac{1}{2}(9.1\times10^{-31})(6.25\times10^{-5})^2.$$

Compute the squared velocity: $(6.25\times10^{-5})^2 = 3.91\times10^{-9}\,\text{m}^2/\text{s}^2$.

$$\tfrac{1}{2}m_e v_d^2 \approx \tfrac{1}{2}(9.1\times10^{-31})(3.91\times10^{-9}) \approx 1.78\times10^{-39}\,\text{J/electron}.$$

Step 4. **Total number of electrons in the wire.** Wire volume $= AL = (10^{-6})(0.1) = 10^{-7}\,\text{m}^3$.

$$N = nV_{\text{wire}} = (10^{29})(10^{-7}) = 10^{22}\,\text{electrons}.$$

Step 5. **Total energy absorbed.**

$$E_{\text{absorbed}} = N\times\tfrac{1}{2}m_e v_d^2 \approx (10^{22})(1.78\times10^{-39}) \approx 1.8\times10^{-17}\,\text{J}.$$

Part (b).

Step 6. **Rate of thermal dissipation.**

$$P = RI^2 = (6)(1)^2 = 6\,\text{W} = 6\,\text{J/s}.$$

Step 7. **Time scale.**

$$\tau = \frac{E_{\text{absorbed}}}{P} = \frac{1.8\times10^{-17}\,\text{J}}{6\,\text{J/s}} \approx 3\times10^{-18}\,\text{s}.$$

☞ **Interpretation**

The time scale $\sim 10^{-18}$ s is extraordinarily small — about ten attoseconds, far shorter than even an inter-collision time $\tau_{\text{coll}} \sim 10^{-14}$ s in a metal. This tells us that the drift KE itself is a tiny fraction of the energy flowing through the circuit; it is constantly absorbed and shed within a single fraction of a collision time. The overwhelming share of the energy delivered by the battery goes into thermal heat, not into the macroscopic drift KE of the electrons.

Final Answer: (a) $E_{\text{absorbed}} \approx 1.8\times10^{-17}$ J. (b) $\tau \approx 3\times10^{-18}$ s.

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Strategic angle. Get I first; then v_d from $I = nAev_d$; then KE per electron and total KE; then divide by RI^2 for the time scale. The shocking conclusion — that drift KE is shed in $\sim 10^{-18}$ s, six orders of magnitude faster than a collision — reveals just how little of the battery's energy actually goes into the macroscopic motion of electrons. The rest is thermal.

Step 1. **Loop current.** $I = V/R = 6/6 = 1\,\text{A}$.

Step 2. Drift velocity. $v_d = I/(nAe)$ with $n = 10^{29}\,\mathrm{m^{-3}}$, $A = 10^{-6}\,\mathrm{m^2}$, $e = 1.6\times10^{-19}\,\mathrm{C}$:

$$v_d = \frac{1}{10^{29}\cdot10^{-6}\cdot1.6\times10^{-19}} = \frac{1}{1.6\times10^{4}} \approx 6.25\times10^{-5}\,\mathrm{m/s}.$$

Step 3. KE per electron. $v_d^2 = (6.25\times10^{-5})^2 = 3.91\times10^{-9}\,\mathrm{m^2/s^2}$.
$\frac{1}{2}m_e v_d^2 = \frac{1}{2}(9.1\times10^{-31})(3.91\times10^{-9}) \approx 1.78\times10^{-39}\,\mathrm{J}$.

Step 4. Total carriers and total KE. Wire volume $V_\mathrm{w} = AL = (10^{-6})(0.1) = 10^{-7}\,\mathrm{m^3}$. Number of electrons $N = nV_\mathrm{w} = 10^{29}\cdot10^{-7} = 10^{22}$. Total drift KE $E = N\cdot\frac{1}{2}m_e v_d^2 \approx 1.78\times10^{-17}\,\mathrm{J}$.

Step 5. Dissipation rate. $P = RI^2 = 6\cdot1 = 6\,\mathrm{W}$.

Step 6. Time scale. $\tau = E/P = 1.78\times10^{-17}/6 \approx 3\times10^{-18}\,\mathrm{s}$.

Step 7. Physical interpretation. $\tau_\text{drift KE} \sim 10^{-18}\,\mathrm{s}$ is six orders of magnitude smaller than the typical Drude collision time $\tau_\text{coll} \sim 10^{-14}\,\mathrm{s}$. So in any single collision interval, the drift KE is exchanged $\sim 10^6$ times — drift KE is negligible compared to thermal energy flowing through the circuit.

Final Answer: $E \approx 1.8\times10^{-17}\,\mathrm{J}$; $\tau \approx 3\times10^{-18}\,\mathrm{s}$.

♥ The vanishing role of drift KE in circuit energetics

The electrons in a current-carrying wire are essentially slow-marching trains of thermal jitter — their drift velocity is microscopic, their kinetic energy is even more so, and the battery doesn't really feed that drift. It feeds the thermal reservoir of the metal (and any external load). This is why a working circuit warms a wire but doesn't accelerate electrons in any macroscopic sense — and why the Drude model's "equilibrium drift" picture works so well: the electrons reach their tiny terminal v_d almost instantaneously and then dump all incoming energy into phonons.

Key Takeaways

- Current in a steady wire is $I = nAev_d$. Drift velocity carries the current; thermal velocity averages to zero.
- For a curved wire, the field that steers $\vec{j}$ along each segment is set up by tiny surface charges on the wire.
- Two cells in parallel: ε_eq is a weighted average between ε_1 and ε_2 (same polarity) or a weighted difference (opposition); $r_\text{eq} = r_1r_2/(r_1+r_2)$.
- Meter-bridge sensitivity is best when the null sits near the middle, i.e. $R \approx S$. Potentiometer wire drop must exceed the largest EMF being compared, with the balance length close to the full wire for best precision.

- In a Wheatstone-bridge null measurement, the imbalance is independent of source EMF and galvanometer calibration; precision is limited by resistor tolerances and detector sensitivity.
- $\rho(T)$ for metals rises with T (decreasing τ); for semiconductors and insulators it falls (increasing n).
- Power lost in transmission cables, for a given delivered P, scales as $1/V^2$ — the case for high-voltage long-distance transmission.
- Scaling all EMFs and all resistances by the same factor leaves every branch current unchanged.