Electric Current

Rate of flow of charge across any cross-section of a conductor.

$$\boxed{I = dq/dt}$$ <-instantaneous <-current

Average current : $I_{avg} = q/t$

SI unit : ampere (A) = 1 C / s.

Dimensional formula : $[A] = [M^0 L^0 T^0 A]$

Direction Convention

Conventional current : direction of ~~electron~~ * , * motion of positive charge.

Electron current : opposite to it.

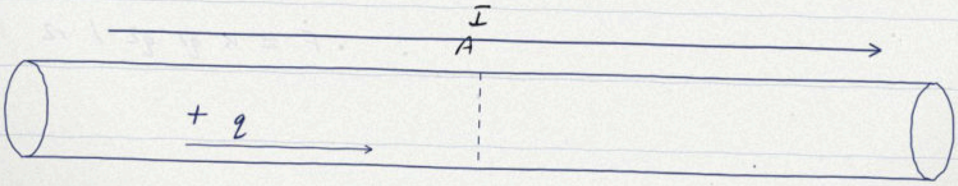

Fig. . charge crossing area A in time dt

Note : I is scalar (algebraic ; sign tells direction).

Current in a Conductor

A metal has a sea of free electrons. They move in random directions with thermal speeds 10^5 m/s, colliding with the lattice.
No external E -> no net current (vectors of v cancel out on average).

(a) No field - random (b) With field E -> drift

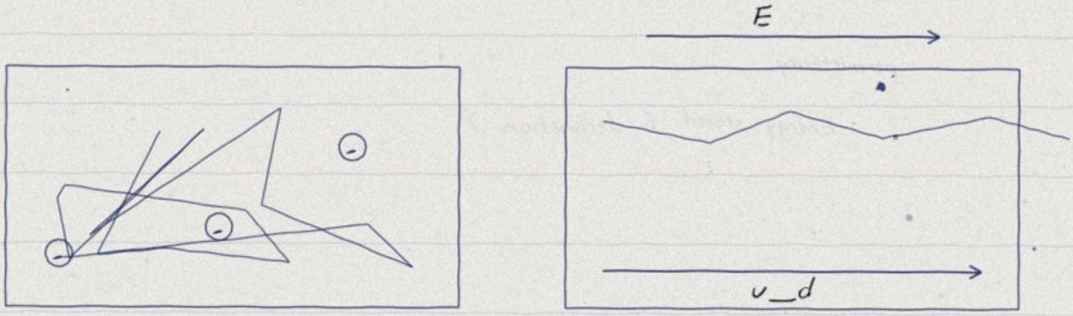

Fig. random motion vs. drift in an E field

Drift Velocity v_d

Average velocity acquired by free electrons in the direction opposite to E.

Typical magnitude : v_d a few mm/s (slow ?), but bulb glows instantly because field travels at c.

Drift Velocity - Derivation

Apply E along the wire. Force on each free electron : $F = -eE$

$$\boxed{a = -eE/m}$$

<- acceleration
<- during free flight

Relaxation Time

Average time between two successive collisions of an electron with ions : T

(T denotes relaxation time ; 10^{-14} s)

Drift Velocity Formula

If v_i = random velocity of i-th electron just after a collision , and t_i = time elapsed since then , then velocity now =

$$v_i = u_i + a t_i$$

Average over all electrons : $\langle u \rangle = 0$,
$\langle t \rangle = T$ (relaxation time)

$$\boxed{v_d = -eET/m}$$

<- v_d opposite
<- to E

Magnitude : $v_d = eET/m$

Higher E -> larger v_d ; higher T -> larger v_d.

Current in terms of Drift Velocity

Setup

Conductor of cross-section A, length L.

n = number of free electrons per unit volume.

v_d = drift velocity (all electrons same v_d).

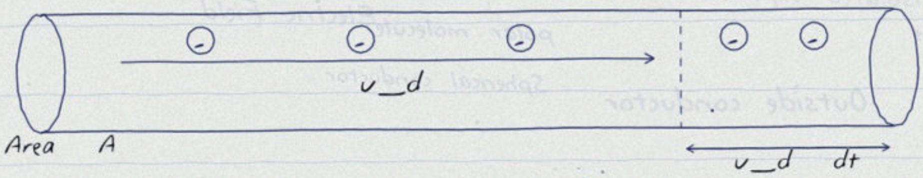

Fig. electrons cross A in time dt

Derivation

Volume of slice in time dt : $A\, v_d\, dt$

No. of electrons in slice : $n\, A\, v_d\, dt$

Charge crossing in dt : $e\, n\, A\, v_d\, dt$

So $I = dq / dt = n\, e\, A\, v_d$

$$\boxed{I = n\, e\, A\, v_d}$$

<- the famous
<- drift formula

Current density $J = I / A = n\, e\, v_d$

J is a vector : $J = n\, e\, v_d$ (along v_d).

SI unit of J : A / m^2.

Ohm's Law

At constant physical conditions, the current through a conductor is directly proportional to the potential difference across its ends.

$$\boxed{V = I R}$$

<- R = constant of
<- proportionality

Resistance R

Ratio of V to I for that conductor.

SI unit : ohm (Greek capital omega).

1 ohm = 1 V / 1 A. Dim. : $[M L^2 T^{-3} A^{-2}]$

Geometrical Formula

For a wire of length L and area A :

$$\boxed{R = \rho L / A}$$

<- rho = resistivity
<- material property

R depends on : material (rho), length L, area A, and temperature T.

Microscopic form of Ohm's law

$$\boxed{J = \sigma E}$$

<- sigma = conductivity
<- sigma = 1 / rho

(vector form ; valid at every point in conductor)

Origin of Resistivity

Combining $I = neA\, v\_d$ with $v\_d = eET/m$:

$$I = neA \cdot (eET/m) = ne^2AET/$$

$$J = I/A = (ne^2 T/m)E$$

$$\boxed{\sigma = ne^2 T/m}$$

<-conductivity from
<-free-electron model

$$\boxed{\rho = m/(ne^2 T)}$$

<-rho is material
<-property

Important Observations

1. rho 1 / n (more free electrons, lower resistivity). Metals : n 10 28 /m 3.

2. rho 1 / T ; T decreases with rise in temperature for metals -> rho rises.

3. Drift current is what carries I ; the random part of velocity gives no net flow.

Mobility : $\upsilon = v_d / E = eT/m$

(υ denotes mobility ; SI unit : m 2 V -1 s -1)

Limitations of Ohm's Law

$V = IR$ holds only for ohmic materials
(most metals at moderate I, const T).
Non-ohmic devices show 3 kinds of deviation :

(a) V not proportional to I

V-I curve is non-linear (e.g. GaAs).
Slope of V vs I is not constant ; $R = dV/dI$ depends on

(b) V depends on sign of I

Diode : large I when forward biased ,
tiny I when reverse biased.

(c) Multi-valued V for same I

Gas discharge tube , thyristor : same V can
give two different I values.

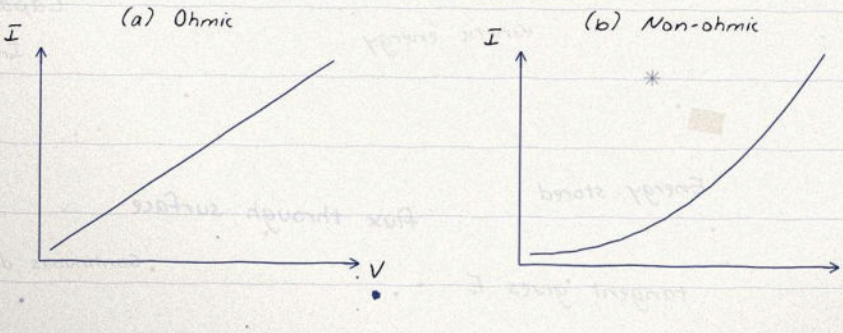

Fig. ohmic vs non-ohmic V-I behaviour

Resistivity of Materials

Materials are classified by rho :

(1) Conductors (metals)

Many free electrons , low resistivity.

rho 10^{-8} to 10^{-6} ohm . m

Examples : silver , copper , aluminium , gold , iron.

(2) Semiconductors

Intermediate : few charge carriers ,

rho 10^{-5} to 10^{0} ohm . m

Examples : silicon , germanium.

rho decreases with rising T (opposite of metals)

(3) Insulators

Very few free carriers , huge rho .

rho 10^{8} to 10^{16} ohm . m

Examples : glass , rubber , wood , teflon.

Quick comparison

$rho_{Cu} = 1.7 \times 10^{-8}$ ohm m

rho_{Si} 2.3×10^{3} ohm m (pure)

rho_{glass} 10^{11} ohm m

(span of 25 orders of magnitude !)

Temperature Dependence of rho

Over a small range :

$$\boxed{\rho = \rho_0 [1 + \alpha (T - T_0)]}$$

<- alpha = T - coeff
<- of resistivity (/K)

Sign of alpha

Metals : alpha > 0 (rho rises with T)

Reason : T decreases ; v_d falls -> rho rises.

Semiconductors / insulators : alpha < 0

Reason : more carriers excited at higher T.

Alloys (nichrome , manganin , constantan) have very small alpha -> used as std resistors.

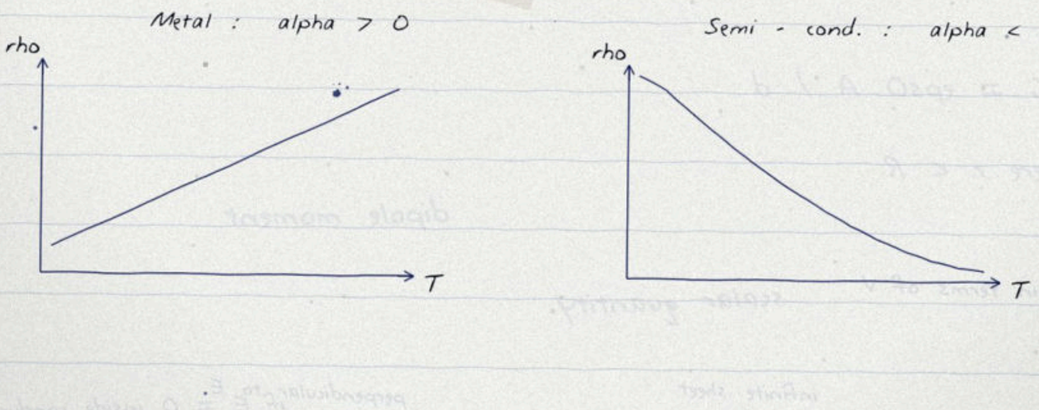

Fig. rho - T curves for metal & semiconductor

Resistors in Series

End-to-end ; the same current I flows through every resistor.

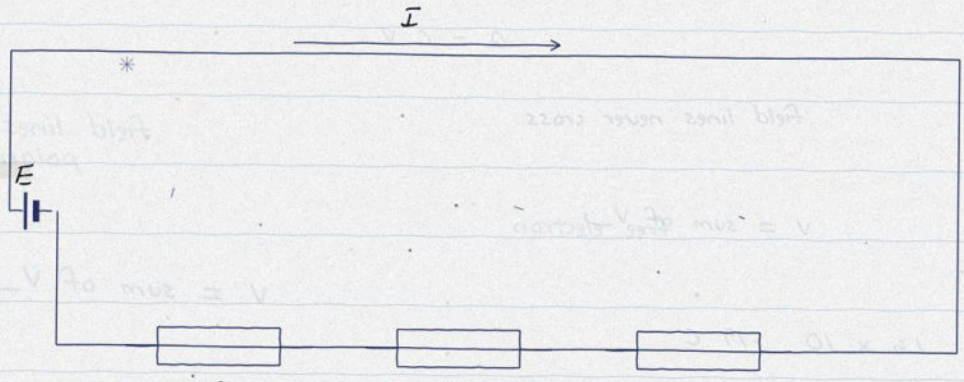

Fig. three resistors in series with cell E

Total potential drop

$$V = V_1 + V_2 + V_3$$
$$= I R_1 + I R_2 + I R_3$$
$$= I (R_1 + R_2 + R_3)$$

$$\boxed{R_s = R_1 + R_2 + R_3 + \dots}$$

<-R_s > any single
<-R in the chain

Two equal R in series -> 2 R .

Use : when we want a larger R from given ones, or to share V (potential ~~Drop~~ divider).

If one R burns / opens -> entire chain stops.

Resistors in Parallel

All resistors share the same two nodes ;
potential difference V is the same across each.

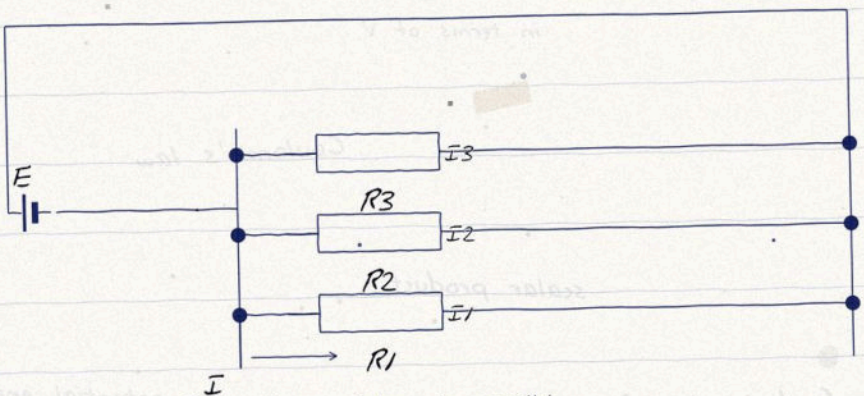

Fig. three resistors in parallel

Total current

$I = I_1 + I_2 + I_3$

$\quad = V/R_1 + V/R_2 + V/R_3$

$$\boxed{1 / R_p = 1 / R_1 + 1 / R_2 + 1 / R_3 + \dots}$$

<- R_p
<- R i

Two equal R in parallel -> R / 2 .

Use : when we want a smaller R , OR to share I (current divider).

If one branch opens -> others still work fine.

Electrical Energy & Power

When charge q moves through potential drop V, work done $= qV$. If steady current I flows for time t, then $q = I\,t$:

$$W = V\,I\,t$$

Power dissipated

$$\boxed{P = W / t = V\,I}$$

<- for any device
<- between its leads

Using $V = I\,R$, we get two more useful forms :

$$\boxed{P = I^2\,R}$$

<- Joule heating
<- in resistor

$$\boxed{P = V^2 / R}$$

<- useful when V
<- is fixed (mains)

Units of Energy

SI unit of energy : joule (J).

Commercial unit : kilowatt - hour (kWh).

$1\ \text{kWh} = 1000\ \text{W} \times 3600\ \text{s} = 3.6 \times 10^6\ \text{J}$

Power transmission

Send power as high V, low I to cut I 2 R loss in transmission lines. Step up at source.

EMF & Internal Resistance

EMF (E)

Work done by the source in driving unit positive charge around the complete circuit ;
equals open-circuit PD across cell terminals.

Internal Resistance r

Resistance of the electrolyte / cell material to its own current. Depends on size , ions, temperature.

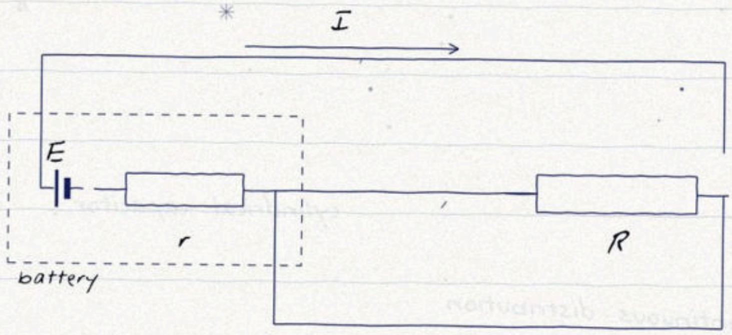

Fig. cell of EMF E , internal r , load R

Terminal Voltage V

$$\boxed{V = E - I r}$$

<- V < E when
<- I flows out

Also $I = E / (R + r)$ -> $V = E R / (R + r)$

Cells in Series

Two cells ($E1$, $r1$) and ($E2$, $r2$) connected so that + of one joins - of the other (aiding).

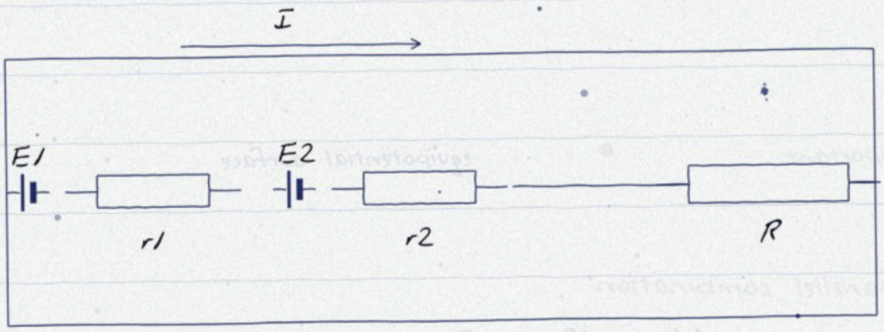

Fig. two cells aiding in series

Equivalent cell (aiding)

$$\boxed{E_{eq} = E_1 + E_2}$$ <-EMFs add

$$\boxed{r_{eq} = r_1 + r_2}$$ <-internal r add too

If one cell is reversed (opposing)

$$\boxed{E_{eq} = E_1 - E_2}$$

r_eq still = $r1 + r2$; use when higher EMF needed.

Cells in Parallel

•

Two cells (E1, r1) and (E2, r2) connected so that their + terminals are joined together and - to -.

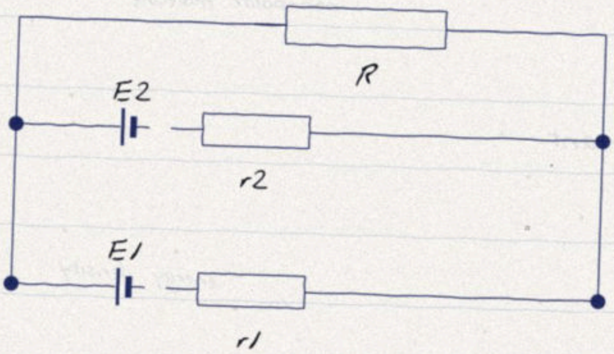

Fig. two cells in parallel with load R

Equivalent cell

$$\boxed{E_{eq} / r_{eq} = E_1 / r_1 + E_2 / r_2}$$

$$\boxed{1 / r_{eq} = 1 / r_1 + 1 / r_2}$$

<- internal r
<- add in parallel

Use parallel when : need large I , or backup in case one cell fails.

Equal cells : $E_{eq} = E$, $r_{eq} = r/n$ (n cells).

Kirchhoff's Junction Rule (KCL)

Statement

Algebraic sum of currents meeting at any junction (node) in a circuit is zero.

$$\boxed{\text{Sum } I_{in} = \text{Sum } I_{out}}$$

<- or Sum $I = 0$
<- with sign convention

Based on conservation of charge : no charge piles up at any junction in steady state.

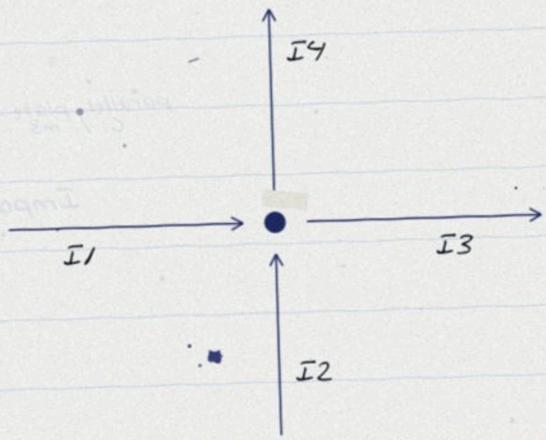

Fig. $I1 + I2 = I3 + I4$ at the junction

Sign convention

Currents entering the junction -> +

Currents leaving the junction -> -

Choose any direction for unknown I ; if the answer comes negative , true direction is opposite.

Kirchhoff's Loop Rule (KVL)

Statement

Algebraic sum of changes in potential around any closed loop in a network is zero.

$$\boxed{\text{Sum } E = \text{Sum } I R}$$

<- EMFs balance
<- IR drops

Based on conservation of energy : an electron returning to the same point has same PE.

Sign conventions

1. Choose a loop direction (CW or CCW).

2. Cross a resistor in direction of I : drop $-IR$. Against I : $+IR$.

3. Cross a cell from - to + : gain $+E$. From + to - : $-E$.

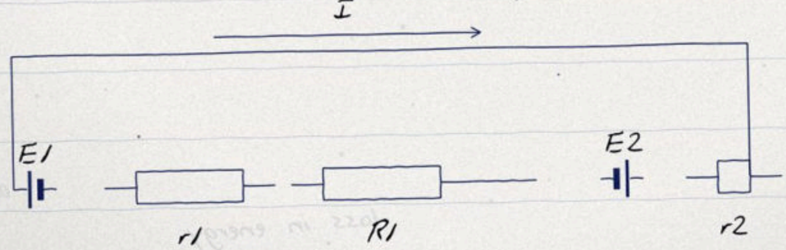

Fig. single-loop circuit ; apply Sum E = Sum I R

Wheatstone Bridge

Construction

Four resistors P , Q , R , S in a diamond.
Cell of EMF E between corners A and C ;
galvanometer G between corners B and D.

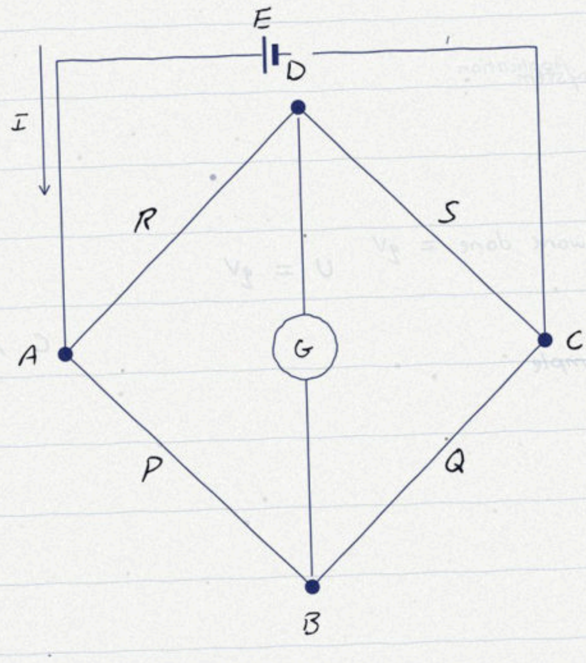

Fig. Wheatstone network ABCD with G across BD

Balance condition

$$\boxed{P / Q = R / S}$$

Wheatstone Bridge - Derivation

Balanced condition

At balance, galvanometer reads zero current.

-> $V_B = V_D$,

Let I1 flow through arm P (and Q),
I2 flow through arm R (and S).

Loop A B D (KVL)

$$I_1 P = I_2 R \quad \text{... (i)}$$

Loop B C D (KVL)

$$I_1 Q = I_2 S \quad \text{... (ii)}$$

Divide (i) by (ii)

$$\boxed{P / Q = R / S}$$

<- Wheatstone
<- balance condition

Use of the bridge

1. Measure an unknown resistance with three known ones (one variable).
2. Bridge is most sensitive when all four arms are of comparable resistance.

If P/Q != R/S, a current flows through G.

Worked Example : KCL & KVL

Problem : In the loop below, find current I.

Take $E1 = 10$ V, $E2 = 4$ V, $r1 = r2 = 1$ ohm, $R = 4$ ohm. Cells aid each other.

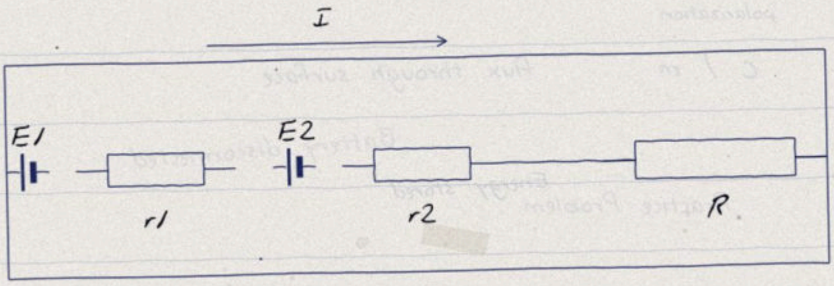

Fig. loop for the worked example

Solution (KVL clockwise)

Sum of EMFs - Sum $IR = 0$:

$$(E_1 + E_2) - I(r_1 + r_2 + R) = 0$$

$$(10 + 4) - I(1 + 1 + 4) = 0$$

$14 - 6I = 0 \rightarrow$ $\cancel{I = 4\ A}$ $I = 7/3$ A

Power dissipated in R :

$$P = I^2 R = (7/3)^2 \cdot 4 = 21.8\ W$$

Terminal V across $(E1 + r1)$: $V = 10 - (7/3)(1) = 7.67$ V

Worked Example : Wheatstone

Problem : In a balanced Wheatstone bridge, $P = 10$ ohm, $Q = 20$ ohm, $R = 15$ ohm. Find the unknown resistance S.

Solution

Balance condition : $P / Q = R / S$

$10 / 20 = 15 / S$

$1 / 2 = 15 / S$

$S = 30$ ohm.

Cross-check at junction

If I1 flows in P-Q arm, I2 in R-S arm :

$I_1 P = I_2 R \rightarrow I_1 (10) = I_2 (15)$

$I_1 / I_2 = 3 / 2$

Also $I_1 Q = I_2 S \rightarrow 20 I_1 = 30 I_2$

$I_1 / I_2 = 3 / 2$. Consistent !

Tip : how to recognise balance

Galvanometer shows zero deflection on closing K2 (galvanometer key) after K1 (cell key).

If not balanced, adjust the variable resistance until the galvanometer reads exactly zero.

Summary - Key Formulae

Definitions

$I = dq / dt$; $J = I / A$; $V = I R$

$v_d = e E T / m$ (drift velocity)

$I = n e A v_d$; $mu = v_d / E$

Material

$R = \rho L / A$; $\sigma = 1 / \rho$

$\sigma = n e^2 T / m$; $\rho = m / (n e^2 T)$

$\rho = \rho_0 [1 + \alpha (T - T_0)]$

Combinations

Series : $R_s = R_1 + R_2 + \dots$

Parallel : $1/R_p = 1/R_1 + 1/R_2 + \dots$

Power

$P = V I = I^2 R = V^2 / R$

Cells

Terminal V : $V = E - I r$

Series : $E_{eq} = \text{Sum } E$; $r_{eq} = \text{Sum } r$

Parallel : $E_{eq}/r_{eq} = \text{Sum } (E/r)$; $1/r_{eq} = \text{Sum } (1/r)$

Kirchhoff & Wheatstone

Sum $I = 0$ (KCL) ; Sum E = Sum $I R$ (KVL) ; $P/Q =$