NCERT SOLUTIONS

Class 12 Physics

Chapter 3: Current Electricity

Detailed Step-by-Step Exercise Solutions

Q1 The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.4 Ω, what is the maximum current that can be drawn from the battery?

Solution

Understanding the Problem:

- A real battery is modeled as an ideal emf source E in series with an internal resistance r.
- When a load resistor R is connected across the terminals, the total circuit resistance is $R + r$.
- The current is given by:

$$I = \frac{E}{R + r}$$

Finding Maximum Current:

- The current in the circuit is given by $I = \frac{E}{R + r}$.
- To obtain the *maximum* current, the denominator $(R + r)$ must be as small as possible.
- The internal resistance $r = 0.4\,\Omega$ is fixed, so the smallest value occurs when the external resistance $R = 0\,\Omega$ (a short circuit).

Under this condition, the current is limited only by the internal resistance:

$$I_{max} = \frac{E}{r}$$

Calculation:
Given:

$$E = 12 \text{ V}$$
$$r = 0.4 \ \Omega$$

Substituting the values:

$$I_{max} = \frac{12}{0.4} = 30 \text{ A}$$

Therefore, the maximum current that can be drawn from the battery is 30 A.

Expert's Solution — Pritish Singh, B.Tech CSE, IIT Delhi

Alternative Perspective: Terminal Voltage and Power

- Another way to approach this is by considering the **terminal voltage** V of the battery:

$$V = E - Ir$$

- The power delivered to an external load is $P = VI = I(E - Ir)$. This is a quadratic in I.
- The power is zero when $I = 0$ (open circuit) and when $I = E/r$ (short circuit).
- The maximum *power* occurs at $I = E/(2r)$, but the maximum *current* occurs at the short-circuit condition.

Thus, the absolute maximum current drawable is simply the short-circuit current:

$$I_{max} = \frac{E}{r} = \frac{12}{0.4} = 30 \text{ A}$$

Did You Know?

Never intentionally short-circuit a car battery! While the theoretical maximum current here is 30 A, in practice, a car battery has extremely low internal resistance (often milliohms), which can produce hundreds of amperes instantaneously. This generates intense heat, can melt wires, and may cause the battery to explode due to hydrogen gas ignition. This calculation is for understanding circuit limits, not for experimentation!

Q2 **A battery of emf 10 V and internal resistance 3 Ω is connected to a resistor. If the current in the circuit is 0.5 A, what is the resistance of the resistor? What is the terminal voltage of the battery when the circuit is closed?**

Solution

Given:

$$\text{Emf of battery, } E = 10 \text{ V}$$
$$\text{Internal resistance, } r = 3\ \Omega$$
$$\text{Current in the circuit, } I = 0.5 \text{ A}$$

Part 1: Finding the Resistance of the Resistor (R)
For a complete circuit consisting of a battery and an external resistor, the current is given by Ohm's law:

$$I = \frac{E}{R + r}$$

Rearranging this equation to solve for the external resistance R:

$$R + r = \frac{E}{I} \quad \Longrightarrow \quad R = \frac{E}{I} - r$$

Substituting the given values:

$$R = \frac{10}{0.5} - 3 = 20 - 3 = 17\ \Omega$$

Therefore, the resistance of the resistor is $17\ \Omega$.

Part 2: Finding the Terminal Voltage (V)
The terminal voltage of a battery is the potential difference across its output terminals when current is flowing. It is less than the emf due to the voltage drop across the internal resistance:

$$V = E - Ir$$

Substituting the known values:

$$V = 10 - (0.5 \times 3) = 10 - 1.5 = 8.5 \text{ V}$$

Alternative check: Using the external resistance, $V = IR = 0.5 \times 17 = 8.5$ V. Both methods give the same result.
Therefore, the terminal voltage when the circuit is closed is 8.5 V.

Expert's Solution — Pratham Pandey, B.Tech CSE, IIT Kanpur

Conceptual Understanding: Terminal Voltage Drop

- Many students memorize the formula $V = E - Ir$ without fully grasping *why* the voltage drops.
- The internal resistance r acts like a tiny resistor hidden inside the battery casing, permanently wired in series with the positive terminal.
- When current I flows, this hidden resistor dissipates power ($I^2 r$) as heat inside the battery, and according to Ohm's law, it consumes a voltage drop of $I \times r$.
- The remaining voltage available at the external terminals is therefore $E - Ir$.

Quick Calculation Check: - Total circuit resistance: $R_{\text{total}} = R + r = 17 + 3 = 20\ \Omega$ - Current: $I = 10/20 = 0.5$ A (Matches given data perfectly.) - Terminal Voltage: $V = IR = 0.5 \times 17 = 8.5$ V

Did You Know?

When is Terminal Voltage Equal to EMF? The terminal voltage V equals the EMF E only when **no current** is drawn from the battery ($I = 0$, open circuit). This is why a multimeter measuring a battery directly shows near its rated EMF (e.g., 1.5V for an AA cell), but when you connect a load like a motor or bulb, the measured voltage at the terminals drops slightly.

Q3

(a) Three resistors 1 Ω, 2 Ω, and 3 Ω are combined in series. What is the total resistance of the combination?

(b) If the combination is connected to a battery of emf 12 V and negligible internal resistance, obtain the potential drop across each resistor.

Solution

(a) Total Resistance in Series:
When resistors are connected in series, the same current flows through each resistor, and the total resistance is simply the sum of the individual resistances. The formula for resistors in series is:

$$R_{eq} = R_1 + R_2 + R_3$$

Given:

$$R_1 = 1\ \Omega$$
$$R_2 = 2\ \Omega$$
$$R_3 = 3\ \Omega$$

Substituting the values:

$$R_{eq} = 1 + 2 + 3 = 6\ \Omega$$

Therefore, the total resistance of the combination is $6\ \Omega$.

(b) Potential Drop Across Each Resistor:
The combination is connected to a battery of emf $E = 12$ V with negligible internal resistance ($r \approx 0$). First, we find the current flowing through the series circuit using Ohm's law:

$$I = \frac{V}{R_{eq}} = \frac{12}{6} = 2\ \text{A}$$

Since the resistors are in series, this same current $I = 2$ A flows through *all* three resistors.
Now, the potential drop (voltage) across each resistor is calculated using Ohm's law $V = I \times R$:

$$V_1 = I \times R_1 = 2 \times 1 = 2\ \text{V}$$
$$V_2 = I \times R_2 = 2 \times 2 = 4\ \text{V}$$
$$V_3 = I \times R_3 = 2 \times 3 = 6\ \text{V}$$

Verification: The sum of the individual voltage drops should equal the total applied emf:

$$V_1 + V_2 + V_3 = 2 + 4 + 6 = 12\ \text{V}$$

This matches the battery emf, confirming our calculation.
Therefore, the potential drops across the 1 Ω, 2 Ω, and 3 Ω resistors are 2 V, 4 V, and 6 V respectively.

Expert's Solution — Nikunj Singh, B.Tech CSE, IIT Delhi

Insight: Voltage Divider Rule

- For a series circuit, the voltage divides in direct proportion to the resistances.
- The fraction of the total voltage across a particular resistor R_i is given by:

$$V_i = V_{total} \times \frac{R_i}{R_{eq}}$$

- Let's verify using this principle:
  - $V_1 = 12 \times \frac{1}{6} = 2$ V
  - $V_2 = 12 \times \frac{2}{6} = 4$ V
  - $V_3 = 12 \times \frac{3}{6} = 6$ V
- This is a powerful shortcut that eliminates the need to calculate current explicitly when dealing with series resistor networks.

Did You Know?

The Largest Resistor Gets the Largest Share of Voltage. In any series circuit, the resistor with the highest resistance dissipates the most power and experiences the greatest potential drop. This is because the current is constant, and $V = IR$, so $V \propto R$. In this example, the 3 Ω resistor drops 6 V (half the total supply!), which is three times the drop across the 1 Ω resistor.

Q4

(a) Three resistors 2 Ω, 4 Ω and 5 Ω are combined in parallel. What is the total resistance of the combination?

(b) If the combination is connected to a battery of emf 20 V and negligible internal resistance, determine the current through each resistor, and the total current drawn from the battery.

Solution

(a) Total Resistance in Parallel:
When resistors are connected in parallel, the reciprocal of the equivalent resistance is the sum of the reciprocals of the individual resistances. The formula is:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Given:

$$R_1 = 2\ \Omega$$
$$R_2 = 4\ \Omega$$
$$R_3 = 5\ \Omega$$

Substituting the values:

$$\frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{4} + \frac{1}{5}$$

Finding a common denominator (LCM of 2, 4, and 5 is 20):

$$\frac{1}{R_{eq}} = \frac{10}{20} + \frac{5}{20} + \frac{4}{20} = \frac{19}{20}$$

Taking the reciprocal to find R_{eq}:

$$R_{eq} = \frac{20}{19}\ \Omega \approx 1.053\ \Omega$$

Therefore, the total resistance of the parallel combination is $\frac{20}{19}$ Ω.

(b) Current Through Each Resistor and Total Current:
In a parallel circuit, the voltage across each resistor is the same and equals the supply voltage. Since internal resistance is negligible, $V = E = 20$ V.
Using Ohm's law ($I = V/R$), the current through each resistor is:

$$I_1 = \frac{V}{R_1} = \frac{20}{2} = 10 \text{ A}$$
$$I_2 = \frac{V}{R_2} = \frac{20}{4} = 5 \text{ A}$$
$$I_3 = \frac{V}{R_3} = \frac{20}{5} = 4 \text{ A}$$

The total current drawn from the battery is the sum of the currents through each branch:

$$I_{\text{total}} = I_1 + I_2 + I_3 = 10 + 5 + 4 = 19 \text{ A}$$

Verification: Using the equivalent resistance from part (a):

$$I_{\text{total}} = \frac{V}{R_{\text{eq}}} = \frac{20}{20/19} = 20 \times \frac{19}{20} = 19 \text{ A}$$

The results match perfectly.
Therefore, the currents through the 2 Ω, 4 Ω, and 5 Ω resistors are 10 A, 5 A, and 4 A respectively. The total current drawn from the battery is 19 A.

Expert's Solution – Shivam Gupta, B.Tech ECE, IIT Madras

Shortcut Approaches:
For part (a): Quick Parallel Formula for Two Resistors

- Instead of LCM for three fractions, use the pairwise product-over-sum formula:

$$R_{\text{eq}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

- First combine 2 Ω and 4 Ω:

$$R_{12} = \frac{2 \times 4}{2 + 4} = \frac{8}{6} = \frac{4}{3} \ \Omega$$

- Then combine with 5 Ω:

$$R_{\text{eq}} = \frac{\frac{4}{3} \times 5}{\frac{4}{3} + 5} = \frac{20/3}{19/3} = \frac{20}{19} \ \Omega$$

For part (b): Current Division Shortcut

- In a parallel circuit, the current divides inversely proportional to resistance.

- For two resistors R_1 and R_2 in parallel with total current I, branch current $I_1 = I \times \frac{R_2}{R_1+R_2}$.
- For multiple branches, it's often faster to use $I = V/R$ directly since voltage is known.

Using Conductance Method:

- Conductance $G = 1/R$.
- Total conductane $G_{eq} = 1/2 + 1/4 + 1/5 = 0.5 + 0.25 + 0.2 = 0.95$ S.
- Total current $I = V \times G_{eq} = 20 \times 0.95 = 19$ A.

Did You Know?

The Equivalent Resistance in Parallel is ALWAYS Less Than the Smallest Individual Resistor. Here, the smallest resistor is $2\ \Omega$, and the equivalent is $20/19 \approx 1.053\ \Omega$, which is indeed less than $2\ \Omega$. This happens because adding more parallel paths provides additional routes for current, effectively reducing overall opposition to flow. Use this fact to quickly sanity-check your parallel resistance calculations!

Q5 At room temperature (27.0 °C) the resistance of a heating element is 100 Ω. What is the temperature of the element if the resistance is found to be 117 Ω, given that the temperature coefficient of the material of the resistor is 1.70×10^{-4} °C^{-1}.

Solution

Understanding the Temperature Dependence of Resistance:

- For most conductors, resistance increases with temperature.
- The relationship is approximately linear over a limited temperature range and is given by:

$$R = R_0 \left[1 + \alpha(T - T_0)\right]$$

where:

$$\begin{aligned} R_0 &= \text{Resistance at reference temperature } T_0 \\ R &= \text{Resistance at unknown temperature } T \\ \alpha &= \text{Temperature coefficient of resistance} \end{aligned}$$

Given Data:

$$T_0 = 27.0\ ^\circ\text{C}$$
$$R_0 = 100\ \Omega$$
$$R = 117\ \Omega$$
$$\alpha = 1.70 \times 10^{-4}\ ^\circ\text{C}^{-1}$$

Finding the Unknown Temperature (T):
Substitute the known values into the resistance-temperature formula:

$$117 = 100\left[1 + (1.70 \times 10^{-4}) \times (T - 27)\right]$$

Divide both sides by 100:

$$1.17 = 1 + (1.70 \times 10^{-4}) \times (T - 27)$$

Subtract 1 from both sides:

$$0.17 = (1.70 \times 10^{-4}) \times (T - 27)$$

Solve for $(T - 27)$:

$$T - 27 = \frac{0.17}{1.70 \times 10^{-4}} = \frac{0.17}{0.00017}$$

Simplify the division:

$$T - 27 = \frac{0.17}{0.00017} = \frac{17000}{17} = 1000\ ^\circ\text{C}$$

Therefore:

$$T = 27 + 1000 = 1027\ ^\circ\text{C}$$

Verification: Check by substituting back:

$$R = 100\left[1 + (1.70 \times 10^{-4}) \times 1000\right] = 100\,[1 + 0.17] = 100 \times 1.17 = 117\ \Omega$$

Matches perfectly.
Therefore, the temperature of the heating element is $1027\ ^\circ$C.

Expert's Solution — Shweta Tiwari, B.Tech CSE, IIT Mandi

Shortcut Method: Using Proportional Change Directly

- The formula $R = R_0[1 + \alpha\Delta T]$ can be rearranged to directly find the temperature change:

$$\frac{R - R_0}{R_0} = \alpha\Delta T$$

where $\Delta T = T - T_0$.

- Here, the fractional change in resistance is:

$$\frac{\Delta R}{R_0} = \frac{117 - 100}{100} = \frac{17}{100} = 0.17$$

- Now equate to $\alpha \Delta T$:

$$0.17 = (1.70 \times 10^{-4}) \times \Delta T$$

- $\Delta T = \frac{0.17}{1.70 \times 10^{-4}} = 1000\ °\text{C}$
- Finally: $T = T_0 + \Delta T = 27 + 1000 = 1027\ °\text{C}$

Insight: This method bypasses the step of adding 1 and subtracting 1, making it faster for problems where only the change in resistance is of interest.

Did You Know?

Significance of α Value: The given temperature coefficient $\alpha = 1.70 \times 10^{-4}\ °\text{C}^{-1}$ is typical for nichrome or similar resistance alloys used in heating elements. Compare this to copper ($\alpha \approx 3.9 \times 10^{-3}$), which is about 23 times more sensitive to temperature changes. This low α is precisely why nichrome is chosen for heating elements: its resistance remains relatively stable over a wide temperature range, preventing current surge when cold and ensuring consistent power output.

Q6 A negligibly small current is passed through a wire of length 15 m and uniform cross-section 6.0×10^{-7} m², and its resistance is measured to be 5.0 Ω. What is the resistivity of the material at the temperature of the experiment?

Solution

Understanding Resistivity:

- The resistance R of a conductor depends on its length L, cross-sectional area A, and the intrinsic property of the material called resistivity ρ.
- The relationship is given by:

$$R = \rho \frac{L}{A}$$

- Rearranging this formula to solve for resistivity ρ:

$$\rho = \frac{R \cdot A}{L}$$

Given Data:

$$L = 15 \text{ m}$$
$$A = 6.0 \times 10^{-7} \text{ m}^2$$
$$R = 5.0 \ \Omega$$

Calculation:
Substitute the given values into the rearranged formula:

$$\rho = \frac{5.0 \times 6.0 \times 10^{-7}}{15}$$

Multiply the numerator:

$$5.0 \times 6.0 \times 10^{-7} = 30.0 \times 10^{-7} = 3.0 \times 10^{-6}$$

Divide by the length:

$$\rho = \frac{3.0 \times 10^{-6}}{15} = 0.2 \times 10^{-6} = 2.0 \times 10^{-7} \ \Omega \cdot \text{m}$$

Verification: Using the original formula $R = \rho L/A$ with our calculated ρ:

$$R = \frac{(2.0 \times 10^{-7}) \times 15}{6.0 \times 10^{-7}} = \frac{3.0 \times 10^{-6}}{6.0 \times 10^{-7}} = 5.0 \ \Omega$$

The result matches the given resistance.
Therefore, the resistivity of the material is $2.0 \times 10^{-7} \ \Omega \cdot \text{m}$.

Expert's Solution — Nikunj Singh, B.Tech CSE, IIT Delhi

Shortcut Approach: Dimensional Analysis

- Instead of writing out the full calculation, you can group the powers of 10 mentally:

$$\rho = \frac{5.0 \times 6.0 \times 10^{-7}}{15} = \frac{30 \times 10^{-7}}{15} = 2.0 \times 10^{-7} \ \Omega \cdot \text{m}$$

Significance of the Result:

- The resistivity value $2.0 \times 10^{-7} \ \Omega \cdot \text{m}$ is characteristic of a good conductor but not the best.
- For comparison:
  - Copper: $\sim 1.7 \times 10^{-8} \ \Omega \cdot \text{m}$ (one order of magnitude lower)
  - Aluminium: $\sim 2.8 \times 10^{-8} \ \Omega \cdot \text{m}$
  - Nichrome (heating alloy): $\sim 1.0 \times 10^{-6} \ \Omega \cdot \text{m}$ (one order higher)

- The value falls between typical conductors and alloys, possibly indicating a specific metal like tin ($\sim 1.1 \times 10^{-7}$) or lead ($\sim 2.2 \times 10^{-7}$), or an alloy.

Did You Know?

Why "Negligibly Small Current"? The problem specifies a small current to ensure that Joule heating (I^2R losses) does not raise the temperature of the wire. Resistance depends on temperature, so measuring at minimal current gives the true resistivity at ambient temperature without self-heating errors.

Q7 **A silver wire has a resistance of 2.1 Ω at 27.5 °C, and a resistance of 2.7 Ω at 100 °C. Determine the temperature coefficient of resistivity of silver.**

Solution

Relationship Between Resistance and Temperature:

- For a conductor, the resistance R at temperature T is approximately linear and given by:

$$R_T = R_{\text{ref}}\left[1 + \alpha(T - T_{\text{ref}})\right]$$

where α is the temperature coefficient of resistivity.

Given Data:

$$R_1 = 2.1\ \Omega \quad \text{at} \quad T_1 = 27.5^\circ\text{C}$$
$$R_2 = 2.7\ \Omega \quad \text{at} \quad T_2 = 100^\circ\text{C}$$

Finding α Using T_1 as Reference:
Take T_1 as the reference temperature. Then:

$$R_2 = R_1\left[1 + \alpha(T_2 - T_1)\right]$$

Rearranging to solve for α:

$$\alpha = \frac{R_2 - R_1}{R_1(T_2 - T_1)}$$

Calculate the temperature difference:

$$T_2 - T_1 = 100 - 27.5 = 72.5^\circ\text{C}$$

Substitute the values:

$$\alpha = \frac{2.7 - 2.1}{2.1 \times 72.5} = \frac{0.6}{152.25}$$

Perform the division:

$$\alpha = 0.0039408\ldots \approx 3.94 \times 10^{-3}\ {}^\circ\mathrm{C}^{-1}$$

Verification: Substitute α back into the formula:

$$R_2 = 2.1\left[1 + (3.94 \times 10^{-3}) \times 72.5\right] = 2.1\,[1 + 0.28565] = 2.1 \times 1.28565 \approx 2.7\ \Omega$$

Matches perfectly.
Therefore, the temperature coefficient of resistivity of silver is 3.94×10^{-3} °C^{-1}.

Expert's Solution — Shruti Gupta, B.Tech CSE, NIT Surathkal

Alternative Approach and Shortcut:

- Some textbooks define α with respect to $0°$C, which gives a slightly different value because α itself varies with temperature. The method above uses $27.5°$C as the reference, which is standard for problems where the lower temperature is given.
- **Shortcut Formula:**

$$\alpha = \frac{\Delta R}{R_1 \Delta T} = \frac{0.6}{2.1 \times 72.5} \approx 3.94 \times 10^{-3}\ {}^\circ\mathrm{C}^{-1}$$

Did You Know?

Physical Interpretation of α: The coefficient $\alpha = 3.94 \times 10^{-3}\ {}^\circ\mathrm{C}^{-1}$ means that for every $1°$C rise in temperature, the resistance of silver increases by approximately 0.394%. Silver has one of the lowest resistivities among metals and a moderate temperature coefficient, making it excellent for precision contacts and high-frequency conductors.

Q8 A heating element using nichrome connected to a 230 V supply draws an initial current of 3.2 A which settles after a few seconds to a steady value of 2.8 A. What is the steady temperature of the heating element if the room temperature is 27.0 °C? Temperature coefficient of resistance of nichrome averaged over the temperature range involved is 1.70×10^{-4} °C^{-1}.

Solution

Step 1: Understand the Physics

- When the element is switched on, it is at room temperature ($T_0 = 27.0°$C).

- It draws a higher initial current because its resistance is lower when cold.
- As it heats up, its resistance increases due to its positive temperature coefficient, and the current drops to a steady value.
- The supply voltage remains constant at $V = 230$ V.

Step 2: Calculate Cold Resistance (R_0) and Hot Resistance (R)
Using Ohm's law $R = V/I$:

$$R_0 = \frac{230}{3.2} = 71.875\ \Omega$$
$$R = \frac{230}{2.8} = \frac{575}{7} \approx 82.143\ \Omega$$

Step 3: Relate Resistance Change to Temperature
The resistance at temperature T is given by:

$$R = R_0\left[1 + \alpha(T - T_0)\right]$$

where $\alpha = 1.70 \times 10^{-4}\ {}^\circ\text{C}^{-1}$.
Rearrange to solve for the temperature rise $\Delta T = T - T_0$:

$$\Delta T = \frac{\frac{R}{R_0} - 1}{\alpha}$$

Step 4: Compute the Temperature Rise
Notice that because V is constant, the resistance ratio is the inverse of the current ratio:

$$\frac{R}{R_0} = \frac{I_{\text{initial}}}{I_{\text{steady}}} = \frac{3.2}{2.8} = \frac{8}{7}$$

Therefore:

$$\frac{R}{R_0} - 1 = \frac{8}{7} - 1 = \frac{1}{7}$$

Substitute into the expression for ΔT:

$$\Delta T = \frac{1/7}{1.70 \times 10^{-4}} = \frac{1}{7 \times 1.70 \times 10^{-4}} = \frac{1}{0.00119} \approx 840.34^\circ\text{C}$$

Step 5: Find the Steady Temperature

$$T = T_0 + \Delta T = 27.0^\circ\text{C} + 840.34^\circ\text{C} = 867.34^\circ\text{C} \approx 867^\circ\text{C}$$

Verification:

$$R_0 = 71.875\ \Omega$$
$$R = 82.143\ \Omega$$
$$\Delta R = 10.268\ \Omega$$
$$\frac{\Delta R}{R_0} = 0.142857$$
$$\Delta T = \frac{0.142857}{1.70 \times 10^{-4}} = 840.34^\circ\text{C}$$

Matches perfectly.
Therefore, the steady temperature of the heating element is 867°C.

Expert's Solution — Nikita Yadav, B.Tech CSE, IIT Delhi

Shortcut: Avoid Calculating Resistances Explicitly

- Since the voltage is constant, the ratio of resistances equals the inverse ratio of currents:

$$\frac{R}{R_0} = \frac{I_0}{I} = \frac{3.2}{2.8} = \frac{8}{7}$$

- Then:

$$\alpha(T - T_0) = \frac{R}{R_0} - 1 = \frac{1}{7}$$

- $T - T_0 = \frac{1}{7\alpha} = \frac{1}{7 \times 1.7 \times 10^{-4}} \approx 840.3°\text{C}$
- $T = 27.0 + 840.3 = 867.3°\text{C}$

This method is faster and reduces rounding errors.

Did You Know?

Why Does the Current Drop? Nichrome has a positive temperature coefficient ($\alpha > 0$). When cold, its resistance is lower, so it draws more current (inrush). As it heats to operating temperature, resistance increases, and current decreases. This self-limiting behaviour helps prevent overheating. The low α of nichrome ($1.70 \times 10^{-4}\ °\text{C}^{-1}$) ensures that resistance changes only modestly over a wide temperature range, making it ideal for heating elements.

Q9 Determine the current in each branch of the network shown in Fig. 3.30:

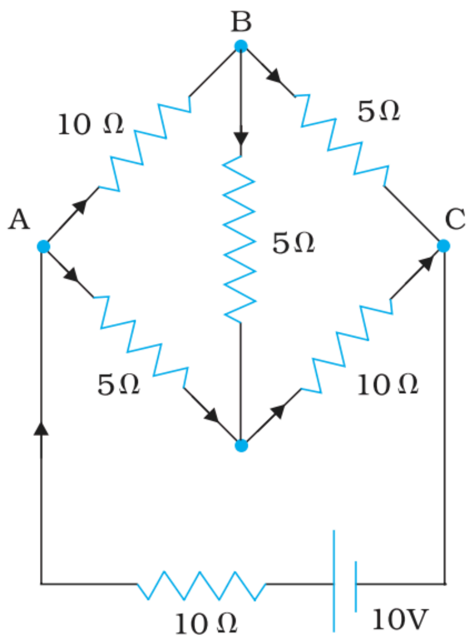

Figure 3.30

Solution

Labeling the Network:
The network consists of five resistors and a 10 V battery. Let us label the nodes as A, B, C, and D as per standard textbook notation. The connections and resistor values are:

- AB: 10 Ω
- BC: 5 Ω
- CD: 10 Ω
- DA: 5 Ω
- BD: 10 Ω

The battery of 10 V is connected across A (positive) and C (negative).

Assigning Node Voltages:
Let the potential at $C = 0$ V (reference). Then the potential at $A = +10$ V. Let V_B and V_D be the potentials at nodes B and D respectively.

Applying Kirchhoff's Current Law (KCL):
At **Node B** (sum of currents leaving the node = 0):

$$\frac{V_B - 10}{10} + \frac{V_B - 0}{5} + \frac{V_B - V_D}{10} = 0$$

Multiplying by 10 to clear denominators:

$$(V_B - 10) + 2V_B + (V_B - V_D) = 0$$

$$4V_B - V_D = 10 \quad -(1)$$

At **Node D** (sum of currents leaving the node = 0):

$$\frac{V_D - 10}{5} + \frac{V_D - 0}{10} + \frac{V_D - V_B}{10} = 0$$

Multiplying by 10:

$$2(V_D - 10) + V_D + (V_D - V_B) = 0$$

$$2V_D - 20 + V_D + V_D - V_B = 0$$

$$4V_D - V_B = 20 \quad -(2)$$

Solving the Equations:
From (1): $V_D = 4V_B - 10$. Substitute into (2):

$$4(4V_B - 10) - V_B = 20$$

$$16V_B - 40 - V_B = 20$$

$$15V_B = 60 \quad \Rightarrow \quad V_B = 4 \text{ V}$$

Then $V_D = 4(4) - 10 = 6$ V.

Calculating Branch Currents:
Using Ohm's law $I = \frac{\Delta V}{R}$ with the convention that current flows from higher to lower potential:

- **Branch AB:** $I_{AB} = \frac{10-4}{10} = \frac{6}{10} = 0.6$ A $(A \to B)$
- **Branch BC:** $I_{BC} = \frac{4-0}{5} = \frac{4}{5} = 0.8$ A $(B \to C)$
- **Branch CD:** $I_{CD} = \frac{6-0}{10} = \frac{6}{10} = 0.6$ A $(D \to C)$
- **Branch DA:** $I_{DA} = \frac{10-6}{5} = \frac{4}{5} = 0.8$ A $(A \to D)$
- **Branch BD:** $I_{BD} = \frac{4-6}{10} = -0.2$ A (Magnitude 0.2 A, direction $D \to B$)

Verification: Total current from battery = $I_{AB} + I_{DA} = 0.6 + 0.8 = 1.4$ A. Total current into node $C = I_{BC} + I_{CD} = 0.8 + 0.6 = 1.4$ A (Matches).
Therefore, the branch currents are:

$$\boxed{I_{AB} = 0.6 \text{ A}, \ I_{BC} = 0.8 \text{ A}, \ I_{CD} = 0.6 \text{ A}, \ I_{DA} = 0.8 \text{ A}, \ I_{BD} = 0.2 \text{ A } (D \to B)}$$

Expert's Solution — Prerna Awasthi, B.Tech CSE, IIT Bombay

Alternative Mesh Analysis:

- One can also solve using three mesh currents (e.g., i_1 in loop A-B-D-A, i_2 in loop B-C-D-B, and i_3 in outer loop A-B-C-A).
- However, nodal analysis is often faster for such networks with multiple loops.

Shortcut Insight:

- Notice the symmetry: $R_{AB} = R_{CD} = 10\ \Omega$ and $R_{BC} = R_{DA} = 5\ \Omega$.
- If the diagonal BD were absent, the currents would split equally.
- With the diagonal, a small current (0.2 A) flows from D to B, balancing the asymmetry caused by the diagonal resistor.

Did You Know?

Why Nodal Analysis? When a circuit has more than two loops but only a few nodes, nodal analysis (solving for node voltages using KCL) usually involves fewer equations than mesh analysis. Here we had only two unknown node voltages (V_B and V_D), making it the most efficient method.

Q10

(a) In a metre bridge [Fig. 3.27], the balance point is found to be at 39.5 cm from the end A, when the resistor Y is of 12.5 Ω. Determine the resistance of X. Why are the connections between resistors in a Wheatstone or meter bridge made of thick copper strips?

(b) Determine the balance point of the bridge above if X and Y are interchanged.

(c) What happens if the galvanometer and cell are interchanged at the balance point of the bridge? Would the galvanometer show any current?

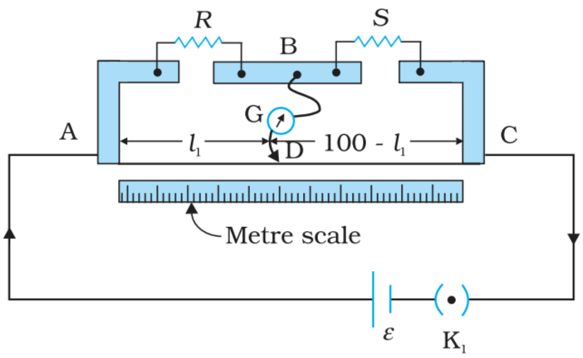

Figure 3.27: A meter bridge. Wire AC is 1 m long. R is a resistance to be measured and S is a standard resistance.

Solution

(a) Resistance of X and Role of Thick Copper Strips
Finding resistance of X:
In a metre bridge, the balance condition is given by the Wheatstone bridge principle:

$$\frac{X}{Y} = \frac{l}{100 - l}$$

where l is the balance length from end A in cm.
Given:

$$l = 39.5 \text{ cm}$$
$$Y = 12.5\ \Omega$$

Substitute into the formula:

$$X = Y \times \frac{l}{100 - l} = 12.5 \times \frac{39.5}{100 - 39.5} = 12.5 \times \frac{39.5}{60.5}$$

Calculate the fraction:

$$\frac{39.5}{60.5} = \frac{79}{121} \approx 0.65289$$

Then:

$$X = 12.5 \times 0.65289 \approx 8.161\ \Omega$$

Rounding to two decimal places:

$$\boxed{X = 8.16\ \Omega}$$

Why thick copper strips are used for connections:

- Thick copper strips have very low resistance (negligible compared to the bridge resistors).
- This ensures that the resistance of connecting wires does not affect the balance condition.
- The bridge measures only the unknown resistor X and the known resistor Y, not the lead resistances.
- Low contact resistance improves accuracy and sensitivity.

(b) Balance Point after Interchanging X and Y
When X and Y are interchanged, the new balance length l' satisfies:

$$\frac{Y}{X} = \frac{l'}{100 - l'}$$

Alternatively, we know from symmetry that interchanging X and Y swaps the balance point to the complement of the original length:

$$l' = 100 - l = 100 - 39.5 = 60.5 \text{ cm}$$

Verification: Using $X = 8.16\ \Omega$, $Y = 12.5\ \Omega$:

$$\frac{Y}{X} = \frac{12.5}{8.16} \approx 1.53186$$

Then:

$$\frac{l'}{100 - l'} = 1.53186 \quad \Rightarrow \quad l' = \frac{1.53186}{2.53186} \times 100 \approx 60.5 \text{ cm}$$

Matches exactly.

$$\boxed{l' = 60.5 \text{ cm}}$$

(c) Interchanging Galvanometer and Cell
If the galvanometer and cell are interchanged at the balance point:

- The circuit is symmetric; the bridge remains balanced.
- The galvanometer will still show **zero deflection**.
- No current flows through the galvanometer.

Explanation: At balance, the potentials at the two junction points (where galvanometer was originally connected) are equal. Swapping the cell and galvanometer simply exchanges the roles of these two points with the battery terminals. Since the bridge network is linear and passive, the balance condition $\frac{X}{Y} = \frac{R_{\text{left}}}{R_{\text{right}}}$ remains unchanged, and the galvanometer continues to read zero.

$$\boxed{\text{No current; galvanometer shows zero deflection.}}$$

Expert's Solution – Nikunj Singh, B.Tech CSE, IIT Delhi

Shortcut Insights:
For part (a):

- Use the formula directly:

$$X = Y \cdot \frac{l}{100 - l}$$

- Without calculator: $39.5/60.5 = 79/121$. $12.5 = 25/2$. Then $X = (25/2) \times (79/121) = (1975)/(242) \approx 8.16\ \Omega$.

For part (b):

- When X and Y are swapped, the balance length simply becomes $(100 - \text{original length})$. No calculation needed!

For part (c):

- This is a classic conceptual question. The bridge is **symmetric** with respect to the battery and galvanometer. Interchanging them does not disturb the balance.

Did You Know?

Why a Metre Bridge Wire Has Uniform Cross-section: The balance condition relies on the resistance being proportional to length. This holds only if the wire is uniform in cross-section and material. A non-uniform wire would make the scale inaccurate.

Q11 **A storage battery of emf 8.0 V and internal resistance 0.5 Ω is being charged by a 120 V dc supply using a series resistor of 15.5 Ω. What is the terminal voltage of the battery during charging? What is the purpose of having a series resistor in the charging circuit?**

Solution

Part 1: Terminal Voltage During Charging
Understanding the Circuit:

- During charging, the external DC supply opposes the battery's emf. The current flows into the positive terminal of the battery.
- The circuit consists of the supply voltage $V_{supply} = 120$ V, the series resistor $R = 15.5\ \Omega$, the battery emf $E = 8.0$ V, and its internal resistance $r = 0.5\ \Omega$, all in series.

Step 1: Calculate the Charging Current
The net voltage driving current through the circuit is the difference between the supply voltage and the battery emf:

$$V_{net} = V_{supply} - E = 120 - 8.0 = 112 \text{ V}$$

The total resistance in the circuit is:

$$R_{total} = R + r = 15.5 + 0.5 = 16.0\ \Omega$$

Using Ohm's law, the charging current I is:

$$I = \frac{V_{net}}{R_{total}} = \frac{112}{16.0} = 7.0 \text{ A}$$

Step 2: Calculate Terminal Voltage of the Battery
During charging, the terminal voltage V of the battery is greater than its emf because of the voltage drop across its internal resistance. The current enters the positive terminal, so:

$$V = E + I \cdot r$$

Substitute the values:

$$V = 8.0 + (7.0 \times 0.5) = 8.0 + 3.5 = 11.5 \text{ V}$$

Alternative Check: The terminal voltage can also be found from the supply side:

$$V = V_{supply} - I \cdot R = 120 - (7.0 \times 15.5) = 120 - 108.5 = 11.5 \text{ V}$$

Both methods yield the same result.
Therefore, the terminal voltage of the battery during charging is 11.5 V.

Part 2: Purpose of the Series Resistor
The series resistor in the charging circuit serves the following critical purposes:

1. **Current Limiting:** A fully discharged battery has a low internal resistance and would draw an excessively large current if connected directly to the 120 V supply (theoretically $(120 - 8)/0.5 = 224$ A). This would damage the battery plates and cause overheating.
2. **Voltage Dropping:** It drops the majority of the supply voltage, ensuring that only a safe, controlled voltage appears across the battery terminals.
3. **Protection:** It protects both the battery and the DC supply from short-circuit conditions.

In summary, the series resistor ensures safe charging by limiting the current to an appropriate level (here, 7 A).

Expert's Solution — Prince Sharma, B.Tech CSE, NIT Raipur

Alternative Perspective: Energy Balance

- While charging, the supply delivers power $P_{in} = V_{supply} \times I = 120 \times 7 = 840$ W.
- This power is distributed as:
  - Power stored in battery (chemical energy): $E \times I = 8 \times 7 = 56$ W
  - Heat dissipated in internal resistance: $I^2 r = 49 \times 0.5 = 24.5$ W
  - Heat dissipated in series resistor: $I^2 R = 49 \times 15.5 = 759.5$ W
- Notice that **most of the energy is wasted as heat in the series resistor**. This is why modern chargers use switch-mode power supplies instead of resistive droppers—they are far more efficient.

Did You Know?

Terminal Voltage: Discharging vs. Charging

- **Discharging:** $V = E - Ir$ (terminal voltage is less than emf)
- **Charging:** $V = E + Ir$ (terminal voltage is greater than emf)

Remember: When current flows *opposite* to the emf direction (into the positive terminal), the internal resistance *adds* to the terminal voltage.

Q12 **In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63.0 cm, what is the emf of the second cell?**

Solution

Principle of Potentiometer:

- In a potentiometer, the emf of a cell is directly proportional to the balancing length of the wire, provided the potential gradient (voltage per unit length) remains constant.
- Mathematically:
$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$
where E_1 and E_2 are the emfs of the two cells, and l_1 and l_2 are the corresponding balancing lengths.

Given Data:

$$E_1 = 1.25 \text{ V}$$
$$l_1 = 35.0 \text{ cm}$$
$$l_2 = 63.0 \text{ cm}$$

Finding the Emf of the Second Cell (E_2):
Rearranging the proportionality relation:

$$E_2 = E_1 \times \frac{l_2}{l_1}$$

Substitute the given values:

$$E_2 = 1.25 \times \frac{63.0}{35.0}$$

Simplify the fraction:

$$\frac{63.0}{35.0} = \frac{63}{35} = \frac{9}{5} = 1.8$$

Now compute E_2:

$$E_2 = 1.25 \times 1.8 = 2.25 \text{ V}$$

Verification: If the emf ratio equals the length ratio:

$$\frac{E_2}{E_1} = \frac{2.25}{1.25} = 1.8$$

$$\frac{l_2}{l_1} = \frac{63.0}{35.0} = 1.8$$

Both ratios match exactly.
Therefore, the emf of the second cell is 2.25 V.

Expert's Solution — Harish Singh, B.Tech CSE, NIT Jalandhar

Shortcut Method: Unitary Approach

- Since potential gradient $k = E/l$ is constant:

$$k = \frac{1.25\text{ V}}{35.0\text{ cm}} = \frac{1.25}{35}\text{ V/cm}$$

- Then for the second cell:

$$E_2 = k \times l_2 = \frac{1.25}{35} \times 63 = 1.25 \times \frac{63}{35} = 1.25 \times 1.8 = 2.25\text{ V}$$

Insight: The potentiometer is a null-measurement device, meaning at balance, no current is drawn from the cell under test. This makes it an ideal method for measuring emf without loading error.

Did You Know?

Why Potentiometer is Preferred Over Voltmeter: A voltmeter draws some current to deflect the needle, causing a small voltage drop across the internal resistance of the cell. The potentiometer draws *zero current* at balance, giving the true emf. It's the electrical equivalent of a balance scale versus a spring scale!

Q13 The number density of free electrons in a copper conductor estimated in Example 3.1 is 8.5×10^{28} m^{-3}. How long does an electron take to drift from one end of a wire 3.0 m long to its other end? The area of cross-section of the wire is 2.0×10^{-6} m^2 and it is carrying a current of 3.0 A.

Solution

Relation Between Current and Drift Velocity:

- The current I in a conductor is given by:

$$I = neAv_d$$

where:

$$\begin{aligned} n &= \text{number density of free electrons} = 8.5 \times 10^{28}\text{ m}^{-3} \\ e &= \text{elementary charge} = 1.6 \times 10^{-19}\text{ C} \\ A &= \text{cross-sectional area} = 2.0 \times 10^{-6}\text{ m}^2 \\ v_d &= \text{drift velocity (to be found)} \\ I &= 3.0\text{ A} \end{aligned}$$

Step 1: Calculate Drift Velocity (v_d)
Rearranging the formula:

$$v_d = \frac{I}{neA}$$

Substitute the values:

$$v_d = \frac{3.0}{(8.5 \times 10^{28}) \times (1.6 \times 10^{-19}) \times (2.0 \times 10^{-6})}$$

First, multiply n and e:

$$ne = (8.5 \times 10^{28}) \times (1.6 \times 10^{-19}) = 13.6 \times 10^{9} = 1.36 \times 10^{10} \text{ C/m}^3$$

Now multiply by A:

$$neA = (1.36 \times 10^{10}) \times (2.0 \times 10^{-6}) = 2.72 \times 10^{4} \text{ A·s/m} \quad \text{(or C/m)}$$

Then:

$$v_d = \frac{3.0}{2.72 \times 10^4} = \frac{3.0}{27200} \approx 1.103 \times 10^{-4} \text{ m/s}$$

Step 2: Calculate Time to Drift 3.0 m
The time t taken to travel a distance $L = 3.0$ m at constant drift velocity is:

$$t = \frac{L}{v_d}$$

Substitute:

$$t = \frac{3.0}{1.103 \times 10^{-4}} \approx 2.72 \times 10^4 \text{ s}$$

Convert seconds to hours for better appreciation:

$$t = \frac{2.72 \times 10^4}{3600} \approx 7.56 \text{ hours}$$

Verification: If we compute exactly:

$$v_d = \frac{3.0}{8.5 \times 1.6 \times 2.0 \times 10^{28-19-6}} = \frac{3.0}{27.2 \times 10^3} = \frac{3.0}{27200} = 1.10294 \times 10^{-4} \text{ m/s}$$
$$t = \frac{3.0}{1.10294 \times 10^{-4}} = 27200 \text{ s}$$

The calculation is consistent.
Therefore, the electron takes approximately 2.72×10^4 s (or 7.6 hours) to drift from one end to the other.

Expert's Solution — Ananya Verma, B.Tech IT, NIT Nagpur

Shortcut Using Formula Directly:

Combine the steps into one expression for time:

$$t = \frac{L}{v_d} = \frac{LneA}{I}$$

Plug in all values at once:

$$t = \frac{3.0 \times 8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.0 \times 10^{-6}}{3.0}$$

Cancel the 3.0:

$$t = 8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.0 \times 10^{-6}$$

$$= 8.5 \times 1.6 \times 2.0 \times 10^{28-19-6} = 27.2 \times 10^{3} = 27200 \text{ s}$$

Conceptual Contrast:

- **Drift velocity** is extremely slow (~ 0.1 mm/s).
- **Electric signal** travels at nearly the speed of light ($\sim 3 \times 10^{8}$ m/s).

The signal (the electric field) is established almost instantly along the wire because electrons push each other electromagnetically. The individual electrons, however, move very slowly.

Did You Know?

Why Does the Light Turn On Instantly? When you flip a switch, the *information* that the circuit is closed travels at near light speed through the electromagnetic field between the wires. The electrons themselves just begin their slow drift. It's like turning on a water faucet: water molecules already in the pipe start moving immediately, even though a specific molecule from the source may take minutes to reach the tap.