NCERT Exemplar Solutions

Solved NCERT Exemplar Problems for Class 12th Physics, Chapter 4

Chapter 4: Moving Charges and Magnetism

About this Chapter

This chapter studies the magnetic effects of currents and the forces on currents in magnetic fields. We use **Biot–Savart's law** and **Ampère's circuital law** to compute fields of straight wires, loops and solenoids; the Lorentz force law $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ to predict circular and helical motion in cyclotrons; and the torque $\vec{\tau} = \vec{M} \times \vec{B}$ on current loops to analyse moving-coil galvanometers and their conversion into ammeters and voltmeters.

Topics covered: Lorentz force • Biot–Savart law • Ampère's law • Cyclotron • Magnetic dipole and torque • Galvanometer to ammeter / voltmeter

Quick Formula Sheet

Lorentz force:
$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Biot–Savart law:
$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$

Long straight wire:
$B = \frac{\mu_0 I}{2\pi r}$

Circular loop centre:
$B = \frac{\mu_0 I}{2R}$

Cyclotron frequency:
$\omega = \frac{qB}{m}$

Torque on loop:
$\vec{\tau} = \vec{M} \times \vec{B},\ M = NIA$

NCERT Exemplar Problems

MCQ I (single correct option)

Q 4.1 **Two charged particles traverse identical helical paths in a completely opposite sense in a uniform magnetic field $\vec{B} = B_0\,\hat{k}$.**

(A) They have equal z-components of momenta.

(B) They must have equal charges.

(C) They necessarily represent a particle–antiparticle pair.

(D) The charge to mass ratios satisfy $\left(\frac{e}{m}\right)_1 + \left(\frac{e}{m}\right)_2 = 0$.

SOLUTION

Correct option: (D) $\left(\frac{e}{m}\right)_1 + \left(\frac{e}{m}\right)_2 = 0.$

Concept used. A charged particle moving in a uniform magnetic field $\vec{B}$ describes a helix whose axis is along $\vec{B}$. The radius is $r = mv_\perp/(qB)$ and the pitch is $p = 2\pi m v_\parallel/(qB)$. Two helices are **identical** when their radii *and* pitches match, and they are traversed in *opposite sense* when the rotation senses about $\vec{B}$ are opposite. The rotation sense reverses when the sign of q flips (for the same $\vec{v}$).

Step 1. Equal radii: $\frac{m_1 v_{\perp,1}}{|q_1|B} = \frac{m_2 v_{\perp,2}}{|q_2|B}$. Equal pitches: $\frac{m_1 v_{\parallel,1}}{|q_1|B} = \frac{m_2 v_{\parallel,2}}{|q_2|B}$. Both give $\frac{m_1}{|q_1|} = \frac{m_2}{|q_2|}$ (taking equal speeds), i.e. the magnitudes of e/m are equal.

Step 2. Opposite sense of rotation requires q_1 and q_2 to have opposite signs.

Step 3. Combining the two conditions:

$$\left(\frac{e}{m}\right)_1 = -\left(\frac{e}{m}\right)_2 \implies \left(\frac{e}{m}\right)_1 + \left(\frac{e}{m}\right)_2 = 0.$$

Step 4. Why (A), (B), (C) fail: z-momenta could differ if masses differ and pitch is fixed by m/q; charges need not be *equal*, only *opposite*; the particles need not be a true particle–antiparticle pair (e.g. a proton and a negative muon have opposite signs but are not antiparticles).

Final Answer: Option **(D)**: $(e/m)_1 + (e/m)_2 = 0.$

☞ Recall

Helical motion of a charge in $\vec{B}$: radius $r = mv_\perp/(|q|B)$, pitch $p = 2\pi m v_\parallel/(|q|B)$, angular frequency $\omega_c = qB/m$ (the sign of ω_c encodes the sense of rotation about $\vec{B}$).

✗ Common Mistake

Identical helical paths do *not* force the charges to be equal in magnitude — only $|q/m|$ must match. Two particles with charges $+2e, +4m$ and $-e, -2m$ have the same $|q/m|$, so they share the helix shape; their charges differ by a factor of two. Always reduce the condition to q/m, not to q or m alone.

EXPERT'S SOLUTION *: Aarav Sharma, M.Sc Physics, IIT Madras*

Strategic angle. Rather than equating radius and pitch separately, use the gyration frequency $\omega_c = qB/m$ which alone fixes both shape and sense. This single scalar contains all the kinematic information about the helix and saves you two equations.

Step 1. Identical helices $\Rightarrow$ equal $|\omega_c|$, hence equal $|q/m|$. (The magnitude of ω_c sets both the radius via $v_\perp/\omega_c$ and the pitch via $2\pi v_\parallel/\omega_c$.)

Step 2. Opposite sense $\Rightarrow \omega_c$ values are opposite in sign, hence q/m values are opposite in sign. The sense of rotation about $\vec{B}$ is the sign of ω_c, full stop.

Step 3. Combining: $(q/m)_1 = -(q/m)_2$, i.e. $(q/m)_1 + (q/m)_2 = 0$, which is option (D).

Step 4. *Alternative angle — physical interpretation.* A positive charge spirals one way and a negative charge of the same $|q/m|$ spirals the other way at the same rate. The particle and "mirror" particle trace out identical helices in opposite senses; this is exactly the scenario in the problem.

Step 5. *Cross-check with options.* (A) Equal z-momenta would require equal $mv_\parallel$, but m can differ between the two while $m/|q|$ stays fixed. (B) Equal charges contradicts the opposite-sense requirement. (C) Particle–antiparticle pairs do satisfy (D), but the converse fails: an e^- and μ^+ also satisfy $(e/m)_1 + (e/m)_2 \neq 0$ in general but a proton and an electron with carefully chosen kinetic energies do not form a particle–antiparticle pair.

Why this matters. Helical motion is fully determined by q/m and the velocity; mass and charge separately are not needed. This is why mass spectrometers measure m/q, not m.

Final Answer: Option **(D)**.

Q 4.2 Biot–Savart law indicates that the moving electrons (velocity $\vec{v}$) produce a magnetic field $\vec{B}$ such that

(A) $\vec{B} \perp \vec{v}$.

(B) $\vec{B} \parallel \vec{v}$.

(C) it obeys inverse cube law.

(D) it is along the line joining the electron and point of observation.

SOLUTION

Correct option: (A) $\vec{B} \perp \vec{v}$.

Concept used. Biot–Savart's law gives the magnetic field $d\vec{B}$ produced by a small current element $I\,d\vec{\ell}$ at a point with position vector $\vec{r}$ from the element:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\ell} \times \hat{r}}{r^2}.$$

For a single moving charge $-e$ with velocity $\vec{v}$, $I\,d\vec{\ell}$ is replaced by $-e\vec{v}$, so $\vec{B} \propto \vec{v} \times \hat{r}$.

Step 1. By definition of the cross product, $\vec{v} \times \hat{r}$ is perpendicular to both $\vec{v}$ and $\hat{r}$. Hence $\vec{B} \perp \vec{v}$. Option (A) is correct.

Step 2. (B) fails for the same reason: $\vec{B}$ cannot be parallel to a vector to which it is perpendicular.

Step 3. (C) fails: the magnitude varies as $1/r^2$, not $1/r^3$.

Step 4. (D) fails: $\vec{B}$ is perpendicular to $\hat{r}$, not along it.

Final Answer: Option **(A)**: $\vec{B} \perp \vec{v}$.

☞ Exam Tip

In CBSE board MCQs on Biot–Savart, the field of a moving point charge $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}$ is a 1-mark recall expected at full speed. Memorise the structure "cross product over r^2"; the cross product immediately fixes the direction $\perp \vec{v}$ and $\perp \hat{r}$, while the $1/r^2$ rules out any inverse-cube option.

EXPERT'S SOLUTION *: Priya Iyer, Ph.D Physics, IISc Bangalore*

Quick reading. The Biot–Savart formula contains a *cross product*; cross products always yield a vector perpendicular to both factors. That single structural fact answers the entire question without any geometry.

Step 1. $\vec{B} \propto \vec{v} \times \hat{r}$ is the entire content of the law for a point charge.

Step 2. Perpendicularity is a property of the cross product, not of any special geometry. So (A) holds in every configuration.

Step 3. Magnitude scales with $|\vec{v}|\sin\theta/r^2$, ruling out (C) (inverse cube) and (D) (along $\hat{r}$).

Step 4. *Alternative method — direct vector computation.* Pick an electron moving along $+\hat{x}$ with $\hat{r}$ along $+\hat{y}$; then $\vec{v} \times \hat{r} \propto \hat{z}$. The field at that observation point is along $\pm\hat{z}$, which is perpendicular to both $\vec{v}$ ($\hat{x}$) and $\hat{r}$ ($\hat{y}$). The construction generalises: any other choice of $\hat{r}$ still gives a field perpendicular to $\vec{v}$.

Step 5. *Unit/structure check.* $[\mu_0] = \mathrm{T\,m/A}$, $[qv/r^2] = \mathrm{A\,m^{-1}}$; product gives Tesla. Correct.

Why this matters. The perpendicularity of $\vec{B}$ to $\vec{v}$ is what causes magnetic field lines to form closed loops around a current; if $\vec{B}$ were along $\vec{v}$ they would diverge from the charge like an electric field. This is why no magnetic monopoles are observed — the very algebra of Biot–Savart forbids a radial $\vec{B}$.

Final Answer: Option **(A)**.

♥ Why This Matters

The Biot–Savart law is the magnetic analogue of Coulomb's law in electrostatics, but the cross-product gives it a fundamentally different geometry: electric field lines start and end on charges, while magnetic field lines form closed loops. This single mathematical fact ($\nabla \cdot \vec{B} = 0$) underlies the absence of isolated magnetic monopoles in classical electromagnetism.

Q 4.3 A current carrying circular loop of radius R is placed in the x-y plane with centre at the origin. Half of the loop with $x > 0$ is now bent so that it now lies in the y-z plane.

(A) The magnitude of magnetic moment now diminishes.

(B) The magnetic moment does not change.

(C) The magnitude of B at $(0, 0, z)$, $z \gg R$ increases.

(D) The magnitude of B at $(0, 0, z)$, $z \gg R$ is unchanged.

SOLUTION

Correct option: (A) The magnitude of magnetic moment now diminishes.

Concept used. The magnetic moment of a planar current loop is $\vec{M} = I\vec{A}$, where $\vec{A}$ is the area vector (normal to the plane of the loop, magnitude equal to the area enclosed). When the loop is no longer planar, the moments of the two halves add as vectors: $\vec{M}_{\text{net}} = \vec{M}_1 + \vec{M}_2$.

Step 1. Before bending: full circle of radius R in the x-y plane carrying current I. Magnetic moment magnitude:

$$M_0 = I\,(\pi R^2).$$

The direction is along $\hat{z}$.

Step 2. After bending: half-loop in the x-y plane has area $\pi R^2/2$, moment $\vec{M}_1 = I(\pi R^2/2)\,\hat{z}$. The other half-loop in the y-z plane has area $\pi R^2/2$, moment $\vec{M}_2 = I(\pi R^2/2)\,\hat{x}$.

Step 3. The net moment magnitude is

$$|\vec{M}| = \sqrt{M_1^2 + M_2^2} = \sqrt{2}\,\frac{I\pi R^2}{2} = \frac{I\pi R^2}{\sqrt{2}} \approx 0.707\,M_0.$$

Since $0.707\,M_0 < M_0$, the magnitude diminishes. (A) holds.

Step 4. (C), (D) fail because B at large z along the axis is no longer purely a dipole field of strength M_0; the dipole moment along $\hat{z}$ alone is $M_1 = M_0/2$, so the on-axis field decreases.

Final Answer: Option **(A)**: magnetic moment diminishes to $M_0/\sqrt{2}$.

✗ Common Mistake

A frequent trap: students assume that because the total enclosed current (I) and the total wire length are unchanged, $|\vec{M}|$ must also be unchanged. But $\vec{M}$ is a *vector area integral*, not a scalar built from current alone. Bending the loop out of plane rotates the area-vector contributions, and orthogonal vectors add as $\sqrt{M_1^2 + M_2^2}$, not as $M_1 + M_2$.

EXPERT'S SOLUTION *: Vivaan Gupta, M.Sc Physics, IIT Bombay*

Picture-first. Two perpendicular half-discs of equal area behave like the legs of a right triangle: the vector sum is $\sqrt{2}$ times one leg, not 2 times. The bend has split a single $\hat{z}$-aligned moment of magnitude M_0 into two halved moments along $\hat{z}$ and $\hat{x}$, and their orthogonal addition yields $M_0/\sqrt{2}$.

Step 1. Each half-disc carries half the original area. Treating each as a half-loop with the wire current I, the magnitude of each moment is $|\vec{M}_1| = |\vec{M}_2| = I\pi R^2/2 = M_0/2$.

Step 2. These two moments are perpendicular ($\hat{z}$ and $\hat{x}$). Their resultant has magnitude $\sqrt{(M_0/2)^2 + (M_0/2)^2} = M_0/\sqrt{2} \approx 0.707M_0$.

Step 3. $M_0/\sqrt{2} \approx 0.71\, M_0 < M_0$. So the moment shrinks.

Step 4. *Alternative method — continuous deformation.* Imagine slowly bending the right half through angles $0, \pi/6, \pi/3, \pi/2$. The z-component of the right half's moment falls as $\cos\phi$ while a new x-component grows as $\sin\phi$. At $\phi = \pi/2$ the z-component of the right half is 0, the x-component is $M_0/2$, and combined with the unchanged left half ($M_0/2$ along $\hat{z}$) the resultant magnitude is $M_0/\sqrt{2}$. Same answer, by tracking the geometry.

Step 5. *Numerical sanity check.* $M_0/\sqrt{2} = 0.707\, M_0$, so the on-axis dipole field at large z falls by the same factor $1/\sqrt{2} \approx 0.71$ (since dipole field $\propto M$). The field does decrease, confirming options (C), (D) are wrong.

Why this matters. For a vector quantity, geometry matters: splitting along orthogonal directions does not preserve the magnitude. This is the same effect that makes the magnetic moment of a folded coil less than that of the flat one — a fact exploited in magnetic-shielding design.

Final Answer: Option **(A)**.

Q 4.4 **An electron is projected with uniform velocity along the axis of a current carrying long solenoid. Which of the following is true?**
(A) The electron will be accelerated along the axis.
(B) The electron path will be circular about the axis.
(C) The electron will experience a force at 45° to the axis and hence execute a helical path.
(D) The electron will continue to move with uniform velocity along the axis of the solenoid.

SOLUTION

Correct option: (D) continues with uniform velocity along the axis.
Concept used. Inside a long solenoid the magnetic field is uniform and directed along the axis: $\vec{B} = B\,\hat{n}$ where $\hat{n}$ is the axial unit vector. The magnetic force on a charge q moving with velocity $\vec{v}$ is $\vec{F} = q\vec{v} \times \vec{B}$.

Step 1. Here $\vec{v} = v_0\,\hat{n}$ (electron moves along the axis) and $\vec{B} = B\,\hat{n}$.

Step 2. Compute the magnetic force:

$$\vec{F} = -e\,(v_0\hat{n}) \times (B\hat{n}) = -ev_0B\,(\hat{n} \times \hat{n}) = \vec{0}.$$

(Any vector crossed with itself gives zero.)

Step 3. With zero net force, Newton's first law says the electron keeps moving with the same velocity along the axis. Option (D) holds.

Step 4. (A), (B), (C) all require a non-zero force, which does not exist here.

> **Final Answer:** Option **(D)**: electron moves with uniform velocity along the axis.

☞ **Recall**
Cross-product zeros: $\hat{n} \times \hat{n} = 0$ always. Hence $\vec{v} \parallel \vec{B} \Rightarrow \vec{v} \times \vec{B} = 0 \Rightarrow \vec{F}_{\text{mag}} = 0$. The magnetic force is *non-zero only when $\vec{v}$ has a component perpendicular to $\vec{B}$.*

EXPERT'S SOLUTION *: Aanya Mehta, B.Tech Engineering Physics, IIT Bombay*

Strategic angle. The cross product vanishes when the two vectors are parallel. Spot this and the answer is immediate; no calculation required.

Step 1. Velocity is along axis; field is along axis. Parallel.

Step 2. $\vec{v} \times \vec{B} = 0$, so $\vec{F}_{\text{mag}} = 0$.

Step 3. No force, no acceleration. Uniform motion continues.

Step 4. *Alternative method — decompose the velocity.* Write $\vec{v} = v_{\parallel}\hat{n} + \vec{v}_{\perp}$. The general motion in a solenoid would be helical: the parallel piece slides along $\hat{n}$ unaffected, and the perpendicular piece circles at $\omega_c = eB/m$. Here $\vec{v}_\perp = 0$, so the circle collapses to a point and only the parallel slide remains — straight-line uniform motion.

Step 5. *Concept linkage.* This is exactly the principle of magnetic confinement in fusion devices: charged particles spiral tightly around field lines but slide freely along them, just like the electron here.

Why this matters. A solenoid acts as a velocity filter: charges moving along its axis are unaffected, while charges with a transverse component spiral. The same physics underlies charged-particle beams in mass spectrometers and electron-microscope columns.

> **Final Answer:** Option **(D)**.

Q 4.5 **In a cyclotron, a charged particle**

(A) undergoes acceleration all the time.

(B) speeds up between the dees because of the magnetic field.

(C) speeds up in a dee.

(D) slows down within a dee and speeds up between dees.

SOLUTION

Correct option: (A) undergoes acceleration all the time.

Concept used. A cyclotron has two semicircular metallic hollow chambers (**dees**). Inside the dees the magnetic field keeps the charge on a circular arc; in the gap between the dees an oscillating electric field speeds the charge up. Centripetal acceleration is non-zero whenever the path curves.

Step 1. Inside a dee: $|\vec{v}|$ is constant but direction keeps changing along a circular arc. Acceleration magnitude $a = v^2/r \neq 0$ (centripetal).

Step 2. In the gap between the dees: the electric field along the gap exerts force $\vec{F} = q\vec{E}$ on the charge, accelerating it tangentially (speeding it up).

Step 3. In both regions acceleration is non-zero. Hence (A): the particle is accelerated all the time.

Step 4. (B) fails because the speed increase happens in the electric field (gap), not the magnetic field. (C) is wrong: speed stays constant inside the dee. (D) is wrong: speed does not decrease inside the dee.

Final Answer: Option **(A)**: the cyclotron particle is accelerated continuously.

☞ **Useful aside**

Distinguish "speeding up" (change in $|\vec{v}|$) from "acceleration" (change in $\vec{v}$). The first is one component of the second; the second is non-zero whenever the velocity vector turns, even at fixed speed.

EXPERT'S SOLUTION : *Arjun Kapoor, M.Sc Physics, IIT Madras*

Strategic angle. Acceleration includes change in direction, not just change in speed. Once that is clear, (A) is the only choice; the magnetic-field dee region and the gap each provide their own non-zero $\vec{a}$.

Step 1. Centripetal acceleration in the dees: speed fixed, direction rotates, $\vec{a}_c = v^2/r$ pointed toward the centre of the arc, $\vec{a} \neq 0$.

Step 2. Tangential acceleration in the gap: speed grows under $\vec{F} = q\vec{E}$, magnitude $a_t = qE/m$, $\vec{a} \neq 0$.

Step 3. Net: acceleration is non-zero throughout the orbit, so (A).

Step 4. *Quantitative check.* For protons in a 1.5 T cyclotron at radius $r = 0.5$ m: $v = qBr/m = (1.6 \times 10^{-19})(1.5)(0.5)/(1.67 \times 10^{-27}) \approx 7.2 \times 10^7$ m/s, so $a_c = v^2/r \approx 1.0 \times 10^{16}$ m/s^2. Even inside the dee the acceleration is enormous — option (A) is unambiguously right.

Step 5. *Alternative angle — energy ledger.* In the dee, the magnetic force does no work ($\vec{F} \perp \vec{v}$), so kinetic energy stays constant. In the gap, the electric force does positive work each time, and the kinetic energy grows. Both regions feature non-zero $\vec{a}$, even though the energy story is different in each.

Why this matters. The cyclotron is the textbook demonstration that constant-magnitude circular motion is still accelerated motion — a fact that students often miss when they conflate "speed" with "velocity". Real cyclotrons exploit this duality: the dees provide the geometric (centripetal) turning, the gap provides the energy injection.

Final Answer: Option **(A)**.

Q 4.6 A circular current loop of magnetic moment M is in an arbitrary orientation in an external magnetic field $\vec{B}$. The work done to rotate the loop by 30° about an axis perpendicular to its plane is

(A) MB.

(B) $\dfrac{\sqrt{3}\,MB}{2}$.

(C) $\frac{MB}{2}$**.**
(D) zero.

SOLUTION

Correct option: (D) zero.
Concept used. The potential energy of a magnetic dipole in an external field is $U = -\vec{M} \cdot \vec{B} = -MB\cos\theta$, where θ is the angle between $\vec{M}$ and $\vec{B}$. The magnetic moment vector $\vec{M}$ of a flat current loop is *perpendicular* to the plane of the loop. Rotating the loop about an axis *perpendicular to its plane* keeps $\vec{M}$ along the same direction.

Step 1. Initial orientation: $\vec{M}$ makes some angle θ with $\vec{B}$. So $U_i = -MB\cos\theta$.

Step 2. Rotation axis is perpendicular to the plane of the loop, i.e. along $\vec{M}$ itself. Rotating about $\vec{M}$ does *not* change $\vec{M}$'s direction; only the loop spins about that direction.

Step 3. Final orientation: $\vec{M}$ still makes angle θ with $\vec{B}$, so $U_f = -MB\cos\theta = U_i$.

Step 4. Work done by external agent = $\Delta U = U_f - U_i = 0$. Option (D).

Step 5. (A), (B), (C) all assume the rotation tilts $\vec{M}$ away from $\vec{B}$, which a perpendicular-axis spin does not do.

Final Answer: Option **(D)**: $W = 0$.

✗ Common Mistake

Students often jump to $W = MB(1 - \cos 30°)$ without reading the axis of rotation. Always identify the rotation axis first — if it is along $\vec{M}$ (i.e. perpendicular to the loop plane), $\vec{M}$ does not rotate at all and $W = 0$, regardless of the angle. The angle $30°$ is a distractor.

EXPERT'S SOLUTION *: Aditya Nair, Ph.D Physics, IISc Bangalore*

Strategic angle. Identify the rotation axis first; the behaviour of $\vec{M}$ follows. The axis-direction is doing all the work in this problem.

Step 1. Loop plane $\perp$ $\vec{M}$, so the rotation axis (perpendicular to plane) is parallel to $\vec{M}$.

Step 2. A vector rotated about its own direction is unchanged: the rotation matrix about $\hat{M}$ fixes $\vec{M}$.

Step 3. Energy depends only on $\vec{M} \cdot \vec{B}$. With $\vec{M}$ unchanged, U unchanged, $W = 0$.

Step 4. *Alternative method — torque integral.* Work done by an external agent rotating the loop by $d\theta$ at angle θ against the field torque is $dW_{\text{ext}} = -\vec{\tau}_{\text{ext}} \cdot d\vec{\theta} = MB\sin\theta\, d\theta$, but here $d\vec{\theta}$ is along $\vec{M}$ and $\vec{\tau} = \vec{M} \times \vec{B}$ is perpendicular to $\vec{M}$; their dot product is zero for every $d\theta$. So $\int dW = 0$ irrespective of the $30°$. Same answer.

Step 5. *Concept linkage.* A free coil in a magnetic field precesses about its own axis (like a top about gravity) — the precession does no work against the field for exactly this reason. Galvanometer suspension wires exploit this to allow rotation without energy loss into the magnetic background.

Why this matters. It explains why a freely-rotating coil about its own axis in a uniform field does not exchange energy with the field; only the alignment with $\vec{B}$ matters.

Final Answer: Option **(D)**.

♥ Why This Matters

The energy of a magnetic dipole, $U = -\vec{M} \cdot \vec{B}$, is the classical seed of nuclear magnetic resonance (NMR) and magnetic resonance imaging (MRI). When the rotation axis is *not* along $\vec{M}$, the energy changes as $-MB\,d(\cos\theta)$ and quantum- mechanically the change is restricted to discrete photon energies $\hbar\omega$. The current problem isolates the boring case where nothing happens; the interesting cases (where $W \neq 0$) drive every imaging machine in a hospital.

MCQ II (one or more correct options)

Q 4.7 The gyro-magnetic ratio of an electron in an H-atom, according to Bohr model, is

(A) independent of which orbit it is in.

(B) negative.

(C) positive.

(D) increases with the quantum number n.

SOLUTION

Correct options: (A) and (B).

Concept used. The gyro-magnetic ratio of a charged particle in orbital motion is the ratio of its magnetic moment to its orbital angular momentum, $\gamma = \frac{M_\ell}{L} = \frac{q}{2m}$. This expression is independent of the orbit's radius and the speed. For an electron, $q = -e < 0$, so γ is negative.

Step 1. For a charge q in a circular orbit of radius r with speed v:

$$M_\ell = I \cdot \pi r^2 = \frac{q}{T}\pi r^2, \qquad T = \frac{2\pi r}{v}.$$

So $M_\ell = \frac{qvr}{2}$.

Step 2. Angular momentum: $L = mvr$. Hence

$$\gamma = \frac{M_\ell}{L} = \frac{qvr/2}{mvr} = \frac{q}{2m}.$$

This depends only on q/m, not on r, v or n. So (A) is correct, and (D) is wrong.

Step 3. For an electron, $q = -e$, so $\gamma = -e/(2m_e) < 0$. Option (B) is correct; (C) is wrong.

> **Final Answer:** Options **(A)** and **(B)**.

☞ Recall

Gyro-magnetic ratio: $\gamma = M_\ell/L = q/(2m)$. The r and v cancel because both M_ℓ and L scale linearly with each. The sign of γ tracks the sign of q.

EXPERT'S SOLUTION *: Riya Singh, M.Sc Physics, IIT Madras*

Structural observation. The cancellation of r and v in the ratio M_ℓ/L is the key. Once that is seen, the orbit dependence disappears and the answer follows from the sign of q alone.

Step 1. Magnetic moment of orbit: $M_\ell = qvr/2$ (treat orbit as current loop with effective current $I = q/T = qv/(2\pi r)$, area πr^2).

Step 2. Orbital angular momentum: $L = mvr$.

Step 3. Ratio: $\gamma = q/(2m)$, no r or v or n.

Step 4. Sign of γ = sign of q. Electron: $q = -e$, so $\gamma = -e/(2m_e) < 0$.

Step 5. *Alternative method — Bohr-radius formulation.* In Bohr's model $r_n = n^2 a_0$, $v_n = v_1/n$. So $M_\ell^{(n)} = qv_n r_n/2 = (q/2)(v_1/n)(n^2 a_0) = (nqv_1 a_0)/2$ and $L_n = nmv_1 a_0$. Their ratio $M_\ell^{(n)}/L_n = q/(2m)$, with all the n-dependence cancelling. Same answer.

Step 6. *Concept linkage.* The Bohr magneton $\mu_B = e\hbar/(2m_e)$ pops out from this when $L = \hbar$ is substituted. So γ is the constant of proportionality between angular momentum and the magnetic dipole moment — a recurring theme in atomic and nuclear physics.

Why this matters. The same logic gives the electron's spin g-factor close to -1 for orbital motion; spin requires the relativistic correction $g \approx -2$. The orbital gyro-magnetic ratio is the simplest manifestation of a deep symmetry between angular momentum and magnetic moment.

Final Answer: Options **(A)**, **(B)**.

Q 4.8 Consider a wire carrying a steady current, I placed in a uniform magnetic field $\vec{B}$ perpendicular to its length. Consider the charges inside the wire. It is known that magnetic forces do no work. This implies that,
(A) motion of charges inside the conductor is unaffected by $\vec{B}$ since they do not absorb energy.
(B) some charges inside the wire move to the surface as a result of $\vec{B}$.
(C) if the wire moves under the influence of $\vec{B}$, no work is done by the force.
(D) if the wire moves under the influence of $\vec{B}$, no work is done by the magnetic force on the ions, assumed fixed within the wire.

SOLUTION

Correct options: (B) and (D).
Concept used. The magnetic force on a moving charge is $\vec{F} = q\vec{v} \times \vec{B}$, always perpendicular to $\vec{v}$, so the instantaneous power $\vec{F} \cdot \vec{v} = 0$: magnetic forces do no work on the charge. However, the *lateral push* on the drift electrons is real; in equilibrium it must be balanced by an electric field set up by surface charges (the **Hall effect**).

Step 1. Inside the wire, drift electrons feel a transverse magnetic force. They drift sideways until they pile up on one face of the wire (a Hall-type charge separation). So (B) is correct and (A) is wrong (the motion *is* affected even though no energy is absorbed).

Step 2. If the wire as a whole moves with velocity $\vec{u}$ under $\vec{B}$, the ions (lattice) move with velocity $\vec{u}$. The magnetic force on each ion is $q\vec{u} \times \vec{B}$, which is $\perp \vec{u}$, so it does no work *on the ions*. Hence (D) is correct.

Step 3. (C) is too strong: the wire *can* do mechanical work (e.g. rotating a coil in a motor) because the Hall-balancing electric field, originally set up by $\vec{B}$, transmits force to the lattice. The first law still applies: it is the *source maintaining the current* that supplies the energy.

Final Answer: Options **(B)** and **(D)**.

✗ Common Mistake

"Magnetic forces do no work" is true only for forces *on individual charges*. It is *not* true that a magnetic field "cannot do work on a wire" — a current-carrying wire in a magnetic field experiences a net macroscopic force ($\vec{F} = I\vec{\ell} \times \vec{B}$) that can deliver mechanical power. The energy comes from the battery maintaining the current, transmitted through the internal Hall field.

EXPERT'S SOLUTION *: Karan Bhat, Ph.D Condensed Matter Physics, TIFR Mumbai*

Strategic angle. Separate force on charges (drift electrons) from force on the bulk (ions). Magnetic forces do no work *on the carrier they push*, but the carriers can transfer momentum via internal fields.

Step 1. Drift electrons feel transverse $\vec{F}$; they pile up on the wire surface. (B) follows.

Step 2. For a wire moving under $\vec{B}$: ions move with the wire. $\vec{F}$ on ions is $\perp \vec{u}$, so no work on ions. (D) follows.

Step 3. (A) and (C) are too sweeping: (A) ignores Hall accumulation; (C) confuses "no work on the charges" with "no work on the wire".

Why this matters. This is the principle behind every electric motor: magnetic forces do no work on individual carriers, yet a motor delivers mechanical power because the carriers transmit the force to the lattice. The energy ledger ties back to the battery maintaining the current, not to the magnetic field as a source.

Alternative angle — microscopic check. The instantaneous power on a single drift electron is $\vec{F} \cdot \vec{v} = q(\vec{v} \times \vec{B}) \cdot \vec{v} = 0$ identically. Sum over $\sim 10^{28}\,\mathrm{m}^{-3}$ electrons and the integrated power is still zero per electron. The macroscopic power comes from the Hall electric field, which is a real, work-doing $\vec{E}$ even though it originated from a magnetic field.

Final Answer: Options **(B)**, **(D)**.

☞ Exam Tip

In CBSE 3-mark conceptual questions, "magnetic force does no work" is a standard quotation worth memorising verbatim from NCERT. The follow-up trap is always the wire-moves-under-$\vec{B}$ case — be ready with the answer that the magnetic force on the ions ($\perp \vec{u}$) does no work, but the wire as a whole gains energy via the source that drives the current.

Q 4.9 **Two identical current carrying coaxial loops, carry current I in an opposite sense. A simple amperian loop passes through both of them once. Calling the loop as C,**

(A) $\oint_C \vec{B} \cdot d\vec{\ell} = 2\mu_0 I$.

(B) the value of $\oint_C \vec{B} \cdot d\vec{\ell}$ **is independent of sense of** C.

(C) there may be a point on C **where** $\vec{B}$ **and** $d\vec{\ell}$ **are perpendicular.**

(D) $\vec{B}$ **vanishes everywhere on** C.

SOLUTION

Correct options: (B) and (C).

Concept used. Ampère's circuital law states $\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$, where I_{enc} is the algebraic sum of currents threading the Amperian loop C (sign by right-hand rule).

Step 1. The two loops carry currents $+I$ and $-I$ through the same Amperian loop C. Net enclosed current:

$$I_{\text{enc}} = +I + (-I) = 0.$$

Hence $\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0(0) = 0$. So (A) is wrong.

Step 2. Since the integral is 0, it is unchanged whether C is traversed clockwise or anticlockwise; the value 0 is independent of sense. (B) is correct.

Step 3. $\vec{B}$ is non-zero away from the wires (each loop produces its own field). On C there *may* be points where the local field is perpendicular to $d\vec{\ell}$, contributing zero to the integrand locally. (C) is correct.

Step 4. (D) is too strong: $\vec{B}$ is generally non-zero on C; only the line integral is zero.

Final Answer: Options **(B)** and **(C)**.

☞ **Useful aside**

Whenever Ampère's law gives a vanishing integral, do not jump to $\vec{B} = 0$. Only the *sum* $\oint \vec{B} \cdot d\vec{\ell} = 0$ is fixed; the local field can be (and usually is) non-zero. The integral-vs-integrand distinction is also crucial for Gauss's law in electrostatics.

EXPERT'S SOLUTION *: Ananya Joshi, M.Sc Astrophysics, IIT Kanpur*

Strategic angle. Distinguish "integral = 0" from "integrand = 0". The first holds because currents cancel; the second does not. The line integral is a global property; the integrand is a local one.

Step 1. $I_{\text{enc}} = I - I = 0 \Rightarrow$ integral $= 0$.

Step 2. A scalar that equals 0 has the same value under reversal of the integration sense. (B) follows.

Step 3. Local $\vec{B} \perp d\vec{\ell}$ on parts of C is consistent with non-zero $\vec{B}$. (C) follows.

Step 4. Field-cancellation everywhere on C (option D) would require $\vec{B} = 0$ globally, contradicted by Biot–Savart for each loop (each loop alone produces a non-zero field everywhere except on its own axis).

Step 5. *Alternative method — superposition.* Apply Ampère's law to each loop separately. For loop 1 alone: $\oint_C \vec{B}_1 \cdot d\vec{\ell} = +\mu_0 I$. For loop 2 alone: $\oint_C \vec{B}_2 \cdot d\vec{\ell} = -\mu_0 I$ (opposite sense). Their sum is zero. The local field $\vec{B} = \vec{B}_1 + \vec{B}_2$ is non-zero at any generic point.

Step 6. *Concept linkage.* This is the principle behind anti-Helmholtz coils used in magneto-optical traps: opposing currents give zero net field at the centre but a strong *gradient* that confines cold atoms.

Why this matters. Helmholtz coils and anti-Helmholtz coils exploit exactly this difference between net enclosed current and local field. Modern ion-trap experiments depend on getting this distinction right.

Final Answer: Options **(B)**, **(C)**.

Q 4.10 A cubical region of space is filled with some uniform electric and magnetic fields. An electron enters the cube across one of its faces with velocity $\vec{v}$ and a positron enters via opposite face with velocity $-\vec{v}$. At this instant,

(A) the electric forces on both the particles cause identical accelerations.

(B) the magnetic forces on both the particles cause equal accelerations.

(C) both particles gain or loose energy at the same rate.

(D) the motion of the centre of mass (CM) is determined by $\vec{B}$ alone.

SOLUTION

Correct options: (B), (C) and (D).

Concept used. For an electron ($-e$, mass m) and a positron ($+e$, mass m) in the same $\vec{E}, \vec{B}$ field: the electric force $\vec{F}_E = q\vec{E}$ has opposite sign for the two; the magnetic force $\vec{F}_B = q\vec{v} \times \vec{B}$ involves *both* q and $\vec{v}$, and both flip when going from electron to positron, so the net magnetic force is the same.

Step 1. Electron: $\vec{F}_E^{(e^-)} = -e\vec{E}$. Positron: $\vec{F}_E^{(e^+)} = +e\vec{E}$. Opposite directions $\Rightarrow$ accelerations ($\vec{a} = \vec{F}/m$) are opposite, *not identical*. So (A) is wrong.

Step 2. Electron: $\vec{F}_B^{(e^-)} = -e(\vec{v} \times \vec{B})$. Positron with velocity $-\vec{v}$: $\vec{F}_B^{(e^+)} = +e\left((-\vec{v}) \times \vec{B}\right) = -e(\vec{v} \times \vec{B})$. *Equal* forces, equal accelerations. (B) is correct.

Step 3. Power delivered by $\vec{E}$ on a charge is $\vec{F}_E \cdot \vec{v}$. Electron: $(-e\vec{E}) \cdot \vec{v} = -e\,\vec{E} \cdot \vec{v}$.

Positron: $(+e\vec{E})\cdot(-\vec{v}) = -e\,\vec{E}\cdot\vec{v}$. *Same rate* of energy change. (C) is correct.

Step 4. The centre-of-mass momentum is $\vec{p}_{\text{CM}} = m\vec{v} + m(-\vec{v}) = 0$. Net electric force on the system is $-e\vec{E} + (+e\vec{E}) = 0$, so $\vec{E}$ contributes nothing to CM motion. The two magnetic forces (computed in step 2) are equal, summing to a non-zero $2\vec{F}_B^{(e^-)}$, which alone drives the CM. (D) is correct.

Final Answer: Options **(B)**, **(C)**, **(D)**.

✗ Common Mistake

A frequent error: equating "equal magnitude" with "identical". The electric forces on the electron and positron *do* have equal magnitudes, but they point in opposite directions — their accelerations are not identical, so option (A) fails. Always check the direction alongside the magnitude.

☞ Recall

Symmetry rule: under charge conjugation ($q \to -q$) and velocity reversal ($\vec{v} \to -\vec{v}$) together, the magnetic force $q\vec{v}\times\vec{B}$ is unchanged. This is the deep reason particle–antiparticle pairs trace the same magnetic trajectory.

EXPERT'S SOLUTION : *Krishna Reddy, M.Tech Applied Physics, IIT Delhi*

Strategic angle. Track how each force depends on the sign of charge and the sign of velocity. Electric: only on q. Magnetic: on $q\,\vec{v}$, so on the *product*. This separation immediately sorts the options.

Step 1. Electric force flips between e^- and e^+ (different q). Accelerations are equal in magnitude but opposite. Rule out (A).

Step 2. Magnetic force is the same on both because both q and $\vec{v}$ flip and their product is unchanged. (B), and consequently (D) for the CM.

Step 3. Power $\vec{F}_E\cdot\vec{v}$ flips twice (in q and in $\vec{v}$), so it is the same number. (C).

Step 4. *Alternative method — CM coordinates.* The CM of the electron–positron system moves at $(m\vec{v} + m(-\vec{v}))/(2m) = \vec{0}$ at $t = 0$. The net external electric force is $-e\vec{E} + (+e\vec{E}) = 0$, so $\vec{E}$ never accelerates the CM. The net magnetic force is $2\vec{F}_B^{(e^-)} \neq 0$ in general, so it alone drives the CM. (D) follows from Newton's-second-law for the CM.

Step 5. *Concept linkage.* Annihilation experiments (such as positron-emission tomography, PET) rely on the symmetric kinematics of an e^+e^- pair: the two photons emitted in opposite directions can be tracked precisely because the magnetic environment treats both particles identically.

Why this matters. This is exactly the symmetry that makes e^+e^- pairs travel along the same trajectory inside a uniform $\vec{B}$ but opposite to each other inside a uniform $\vec{E}$. The electric–magnetic split is central to particle-physics detector design.

> **Final Answer:** Options **(B)**, **(C)**, **(D)**.

Q 4.11 A charged particle would continue to move with a constant velocity in a region wherein,

(A) $\vec{E} = 0,\ \vec{B} \neq 0$.
(B) $\vec{E} \neq 0,\ \vec{B} \neq 0$.
(C) $\vec{E} \neq 0,\ \vec{B} = 0$.
(D) $\vec{E} = 0,\ \vec{B} = 0$.

SOLUTION

Correct options: (A), (B) and (D).
Concept used. Constant velocity requires zero net force. The Lorentz force is $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$. The particle keeps a constant velocity if and only if $\vec{E} + \vec{v} \times \vec{B} = \vec{0}$.

Step 1. (A): $\vec{E} = 0$, $\vec{B} \neq 0$. If $\vec{v} \parallel \vec{B}$, then $\vec{v} \times \vec{B} = 0$ and the net force is zero. Constant velocity possible. (A) is correct.

Step 2. (B): $\vec{E} \neq 0,\ \vec{B} \neq 0$. Choose $\vec{E} = -\vec{v} \times \vec{B}$ (a **velocity selector** configuration). Then $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = 0$. Constant velocity possible. (B) is correct.

Step 3. (D): $\vec{E} = 0,\ \vec{B} = 0$. Free particle, no force. Trivially constant velocity. (D) is correct.

Step 4. (C): $\vec{E} \neq 0,\ \vec{B} = 0$. Then $\vec{F} = q\vec{E} \neq 0$. The particle accelerates, so its velocity changes. (C) fails.

> **Final Answer:** Options **(A)**, **(B)**, **(D)**.

☞ Exam Tip

For NCERT MCQ-II conceptual questions, list *every* condition under which the relevant force vanishes; tick each option that lies in that list. Treat the four options as four checkboxes against the condition $\vec{F}_{\text{net}} = 0$, not as alternatives.

EXPERT'S SOLUTION : *Yash Verma, B.Tech Engineering Physics, IIT Bombay*

Strategic angle. List all the ways to make the Lorentz force vanish; that gives the answer set immediately. The Lorentz force has exactly two pieces (electric, magnetic), and each can be killed in a specific way.

Step 1. No fields: no force. Always works (D).

Step 2. Magnetic only: force vanishes when $\vec{v} \parallel \vec{B}$ (so $\vec{v} \times \vec{B} = 0$). (A) works.

Step 3. Both fields: balance them out, $\vec{E} + \vec{v} \times \vec{B} = 0$. Equivalently $\vec{E} = -\vec{v} \times \vec{B}$, a velocity-selector geometry. (B) works.

Step 4. Electric only: $\vec{F} = q\vec{E} \neq 0$ always (for $q \neq 0$). Nothing can cancel an electric force on a stationary or moving charge in pure $\vec{E}$. (C) fails.

Step 5. *Alternative method — think about it kinematically.* Constant velocity $\Leftrightarrow$ $\vec{a} = 0 \Leftrightarrow \vec{F} = 0$. In each option, find conditions on $\vec{v}$ that make $q(\vec{E} + \vec{v} \times \vec{B}) = 0$. Only (C) admits no such $\vec{v}$ for a non-zero charge.

Step 6. *Numerical sanity for (B).* In a velocity selector with $E = 10^4$ V/m and $B = 10^{-2}$ T (crossed), the selected speed is $v = E/B = 10^6$ m/s — a typical thermionic-electron speed. The configuration is physically accessible, not pathological.

Why this matters. The crossed-fields configuration of (B) is the J. J. Thomson velocity selector that isolated a specific v from the cathode beam. It is also the working principle of the Wien filter used to purify ion beams in modern mass spectrometers.

Final Answer: Options **(A)**, **(B)**, **(D)**.

☞ **Useful aside**

A useful mnemonic: "E does work always; B does work never." This captures why (C) cannot give constant velocity (electric work changes energy and hence speed), while (A), (B), (D) can.

Very Short Answer (VSA)

Q 4.12 **Verify that the cyclotron frequency $\omega = eB/m$ has the correct dimensions of $[\mathrm{T}]^{-1}$.**

SOLUTION

Concept used. A formula is dimensionally correct when the dimensions on both sides match. Here we need $[\omega] = [\mathrm{T}]^{-1}$ (inverse seconds).

Step 1. Lorentz force gives $F = qvB$, so $B = F/(qv)$. Dimensions:

$$[B] = \frac{[F]}{[q][v]} = \frac{\text{MLT}^{-2}}{\text{AT} \cdot \text{LT}^{-1}} = \text{MT}^{-2}\text{A}^{-1}.$$

Step 2. Now compute $[eB/m]$. With $[e] = \text{AT}$, $[B] = \text{MT}^{-2}\text{A}^{-1}$, $[m] = \text{M}$:

$$\left[\frac{eB}{m}\right] = \frac{\text{AT} \cdot \text{MT}^{-2}\text{A}^{-1}}{\text{M}} = \frac{\text{MT}^{-1}}{\text{M}} = \text{T}^{-1}.$$

Step 3. This matches the dimension of angular frequency $[\omega] = \text{T}^{-1}$.

> **Final Answer:** $[eB/m] = \text{T}^{-1}$, which is the dimension of angular frequency.

☞ Recall

SI base-unit cheat sheet:
$[F] = \text{kg m s}^{-2}$, $[q] = \text{A s}$, $[v] = \text{m s}^{-1}$, so $[B] = [F]/([q][v]) = \text{kg A}^{-1}\text{s}^{-2} = \text{T}$. Hence $[eB/m] = [\text{A s}][\text{kg A}^{-1}\text{s}^{-2}]/[\text{kg}] = \text{s}^{-1}$.

EXPERT'S SOLUTION : *Sneha Patel, M.Sc Applied Mathematics, IIT Kanpur*

Quick check. Read off each dimension from the SI base units and multiply. Dimensional analysis is a sanity tool, not just a verification trick — if the dimensions disagree, a numerical calculation cannot save you.

Step 1. $[e] = \text{A s}$, $[B] = \text{kg A}^{-1}\text{s}^{-2}$ (from $F = qvB$ so $[B] = [F]/[qv] = \text{kg m s}^{-2}/(\text{A s} \cdot \text{m s}^{-1}) = \text{kg A}^{-1}\text{s}^{-2}$).

Step 2. $\left[\frac{eB}{m}\right] = \frac{(\text{A s})(\text{kg A}^{-1}\text{s}^{-2})}{\text{kg}} = \text{s}^{-1}$. Verified.

Step 3. *Alternative method — track the units of motion.* The cyclotron period is $T = 2\pi m/(qB)$, with $[T] = \text{s}$. Hence $[qB/m] = \text{s}^{-1}$. The factor of 2π is dimensionless, so $\omega = qB/m$ and $\omega = 2\pi/T$ have the same dimensions automatically.

Step 4. *Numerical sanity.* For an electron in $B = 1\,\text{T}$: $\omega_c = (1.6 \times 10^{-19})(1)/(9.1 \times 10^{-31}) \approx 1.76 \times 10^{11}\,\text{rad/s}$, i.e. $\sim 28\,\text{GHz}$ — microwave range, which is exactly where electron paramagnetic resonance (EPR) operates. Order of magnitude consistent.

Why this matters. The dimension T^{-1} is the universal signature of frequency or rate. Every classical-mechanics formula yielding a frequency must reduce to a T^{-1} combination of input parameters — there is no other route.

Final Answer: Dimension T^{-1}. ✓

☞ Exam Tip

CBSE often awards 1 mark for the dimensional formula of B in a 2-mark question like this. Memorise $[B] = \mathrm{M\,T^{-2}\,A^{-1}}$ (equivalently $\mathrm{kg\,A^{-1}\,s^{-2}}$). Once that is in place, verifying any cyclotron-related dimension is one line of substitution.

Q 4.13 Show that a force that does no work must be a velocity dependent force.

SOLUTION

Concept used. Power delivered by a force $\vec{F}$ to a particle moving with velocity $\vec{v}$ is $P = \vec{F} \cdot \vec{v}$. "Does no work" means $P = 0$ for every $\vec{v}$ that the particle can have.

Step 1. Suppose $\vec{F}$ is independent of $\vec{v}$. The particle's velocity can be chosen freely (e.g. by choice of initial conditions). For $\vec{F} \cdot \vec{v} = 0$ to hold for *every* $\vec{v}$, the vector $\vec{F}$ would have to be perpendicular to every direction in space, which is only possible if $\vec{F} = \vec{0}$.

Step 2. A non-zero force that nonetheless does no work must therefore depend on $\vec{v}$ in such a way that the dependence makes $\vec{F}$ perpendicular to that particular $\vec{v}$.

Step 3. The magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ is the canonical example: it is built from $\vec{v}$ itself, and the cross product guarantees $\vec{F} \perp \vec{v}$ and hence $\vec{F} \cdot \vec{v} = 0$.

Final Answer: A non-zero force doing no work must depend on $\vec{v}$ (the magnetic force is the standard example).

✗ Common Mistake

"Does no work" is sometimes (mis-)read as "$\vec{F} = 0$". They are not the same thing. The magnetic force is non-zero, yet it does no work because of the special structure $\vec{F} \propto \vec{v} \times \vec{B}$, which forces $\vec{F} \perp \vec{v}$. Always check both: is the force zero, and is it perpendicular to the velocity?

EXPERT'S SOLUTION *: Pranav Mehta, Ph.D Physics, IISc Bangalore*

Logical angle. Use proof by contradiction. This is a short question, but it admits a few flavours of proof; the contradiction flavour is the cleanest.

Step 1. Assume $\vec{F}$ does no work and is velocity-independent.

Step 2. Pick a $\vec{v}$ along $\vec{F}$ (possible since $\vec{F} \neq \vec{0}$). Then $\vec{F} \cdot \vec{v} = |\vec{F}||\vec{v}| \neq 0$. Contradiction with "no work".

Step 3. Hence $\vec{F}$ must depend on $\vec{v}$. ✓

Step 4. *Alternative method — geometric.* A non-zero $\vec{F}$ defines a fixed direction in space. For $\vec{F} \cdot \vec{v} = 0$ to hold over a trajectory, $\vec{v}$ must always stay in the plane perpendicular to $\vec{F}$. This severely constrains the motion and is generally impossible for arbitrary initial conditions — *unless* $\vec{F}$ rotates with $\vec{v}$, i.e. depends on $\vec{v}$.

Step 5. *Concept linkage.* Constraint forces (e.g. normal reaction on a particle constrained to a surface) also do no work; they too depend on the motion (specifically on the velocity direction at every instant). The magnetic force and constraint forces share this defining property.

Why this matters. The fact that magnetic forces do no work forces all the energy book-keeping in a generator/motor to be done by the electric field. This is the cornerstone of electromechanical energy conversion — magnetic fields mediate the coupling but never sit on either side of the energy ledger.

Final Answer: No-work non-zero forces are necessarily velocity-dependent.

♥ Why This Matters

This abstract-sounding result is the bedrock of every motor and generator: $\vec{F}_B$ couples mechanical and electrical worlds without itself doing work. The mechanical work comes from the source that maintains the current (or the source that drives the motion); the magnetic field is the messenger, not the messenger's wages.

Q 4.14 The magnetic force depends on $\vec{v}$ which depends on the inertial frame of reference. Does then the magnetic force differ from inertial frame to frame? Is it reasonable that the net acceleration has a different value in different frames of reference?

SOLUTION

Concept used. In Newtonian (Galilean) relativity, velocity is frame-dependent but acceleration is frame-invariant between inertial frames. The fields $\vec{E}$ and $\vec{B}$, however, transform between frames; they are *not* the same in different inertial frames.

Step 1. Magnetic force is $\vec{F}_B = q\vec{v} \times \vec{B}$. Velocity $\vec{v}$ changes from frame to frame (Galilean addition of velocities), so $\vec{F}_B$ *does* differ from frame to frame.

Step 2. However, the electric field $\vec{E}$ also transforms; in a frame where the magnetic part of the force decreases, the electric part picks up the slack. The *net* Lorentz force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ has the same magnitude in every inertial frame (in the non-relativistic limit).

Step 3. Since net force is the same and mass is the same, the acceleration $\vec{a} = \vec{F}/m$ is the same in every inertial frame. So the net acceleration is *not* frame-dependent.

Final Answer: Magnetic force alone is frame-dependent; the *net* Lorentz force, and hence the acceleration, is the same in every inertial frame.

☞ **Useful aside**

Frame-dependence is normal for vector quantities (e.g. velocity); frame-invariance is normal for scalars (e.g. mass, charge). The surprise here is that $\vec{E}$ and $\vec{B}$ individually are frame-dependent vectors, but they combine into a frame-invariant force $\vec{F}$.

EXPERT'S SOLUTION *: Aaditi Banerjee, Ph.D Physics, IISc Bangalore*

Strategic angle. Field transformations rescue Newton's second law from apparent frame-dependence. The slick way to see this: $\vec{E}$ and $\vec{B}$ are not separately physical objects but two faces of a single object (the electromagnetic field tensor), and the transformation of that single object preserves the force.

Step 1. In frame S: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$.

Step 2. In frame S' moving with velocity $\vec{u}$: $\vec{E}' = \vec{E} + \vec{u} \times \vec{B}$, $\vec{B}' = \vec{B}$ (non-relativistic limit). Plugging in $\vec{v}' = \vec{v} - \vec{u}$ gives the same $\vec{F}$.

Step 3. Mass is invariant; $\vec{a}$ is the same. Newton's laws hold in every inertial frame, and they say the same thing.

Step 4. *Alternative method — worked example.* A wire at rest with current I in lab frame S produces a magnetic field but no electric field. A free charge moving along the wire feels a pure magnetic force in S. Now look at the rest frame S' of the charge — in S', the wire's positive ions move backward and the electrons either stay put or move at a different speed. Length contraction makes the wire slightly charged in S', producing an *electric* field in S' that exerts the same force on the (now-stationary) charge. Magnetism in S has become electricity in S'; the force is the same.

Step 5. *Concept linkage.* This thought experiment is the starting point of Einstein's 1905 paper on special relativity. The frame-mixing of $\vec{E}$ and $\vec{B}$ forced him to rebuild kinematics from scratch.

Why this matters. The frame-mixing of $\vec{E}$ and $\vec{B}$ is the seed of special relativity's full Lorentz transformation of the field tensor.

> **Final Answer:** Magnetic part of the force varies, but $\vec{a}$ does not.

Q 4.15 **Describe the motion of a charged particle in a cyclotron if the frequency of the radio frequency (rf) field were doubled.**

SOLUTION

Concept used. A cyclotron works at **resonance**: the oscillating electric field in the gap reverses precisely when the particle crosses the gap, so it always pushes (and never pulls) the particle. Resonance requires $f_{\text{rf}} = f_{\text{cyclotron}} = \dfrac{qB}{2\pi m}$.

Step 1. If f_{rf} is doubled to $2f_{\text{cyclotron}}$, the electric field reverses sign *twice* during the time the particle spends in one dee.

Step 2. In the first half of the dee transit, the field accelerates the particle; in the second half (since it has reversed), the field decelerates it. By the end of the half-orbit the net speed gain is approximately zero.

Step 3. In subsequent half-orbits the same out-of-phase pattern repeats. The particle no longer gains energy from the gap, and resonance is lost. Its motion is therefore an approximately uniform circle of fixed radius (set by its injection speed).

> **Final Answer:** Resonance fails: the particle does not gain energy and orbits with (roughly) constant speed.

☞ **Recall**

Cyclotron resonance condition: $f_{\text{rf}} = f_c = \dfrac{qB}{2\pi m}$. At resonance the particle crosses the gap exactly when the field reverses, so the field always pushes (never pulls).

EXPERT'S SOLUTION *: Dev Pillai, M.Sc Physics, IIT Madras*

Strategic angle. Compare the oscillation period of the rf to the half-revolution time inside a dee. Once you have the ratio of periods, the qualitative answer drops out.

Step 1. Half-revolution time: $T_c/2 = \pi m/(qB)$ (one dee transit).

Step 2. Rf half-period: $T_{\text{rf}}/2 = 1/(2f_{\text{rf}})$. Doubling f_{rf} halves T_{rf}, so $T_{\text{rf}}/2 = T_c/4$ — one quarter of a dee transit.

Step 3. The rf reverses twice during one dee-transit: accelerate then decelerate. Net energy gain ≈ 0 per orbit, so the particle's kinetic energy plateaus.

Step 4. *Alternative method — phase diagram.* Plot the oscillating gap voltage $V(t) = V_0 \sin(2\pi f_{\text{rf}} t)$ and the particle's gap-crossing times $t_n = nT_c/2$. At resonance, $V(t_n) = \pm V_0$ alternately, always with the sign that accelerates. At $f_{\text{rf}} = 2f_c$, $V(t_n) = V_0 \sin(2\pi n) = 0$ every crossing — the gap voltage is zero each time the particle arrives, so no energy transfer.

Step 5. *Numerical check (proton in $B = 1\,T$).* $f_c = qB/(2\pi m) = (1.6 \times 10^{-19})/(2\pi \cdot 1.67 \times 10^{-27}) \approx 15.3\,\text{MHz}$. Doubling to $30.6\,\text{MHz}$, the rf cycles in 33 ns versus the dee transit time of 65 ns: rf reverses twice per transit, exactly as above.

Why this matters. Cyclotrons are tuned by adjusting the rf to match the cyclotron frequency; for relativistic particles the synchrocyclotron lowers f_{rf} slowly as the particle's mass grows. Modern synchrotrons keep a fixed circular path by ramping both B and f_{rf} together — both are tuned to maintain the resonance condition.

Final Answer: Resonance broken; particle circulates at fixed radius.

✗ Common Mistake

Do not conclude that the particle decelerates to a halt. The magnetic field inside the dee still keeps it on a circle; it loses no kinetic energy to the dees. The rf only fails to top up the energy — it does not actively remove energy in a steady state. The motion is uniform circular at the injection speed.

Q 4.16 **Two long wires carrying current I_1 and I_2 are arranged as shown in Fig. 4.1. The one carrying current I_1 is along the x-axis. The other carrying current I_2 is along a line parallel to the y-axis given by $x = 0$ and $z = d$. Find the force exerted at O_2 because of the wire along the x-axis.**

SOLUTION

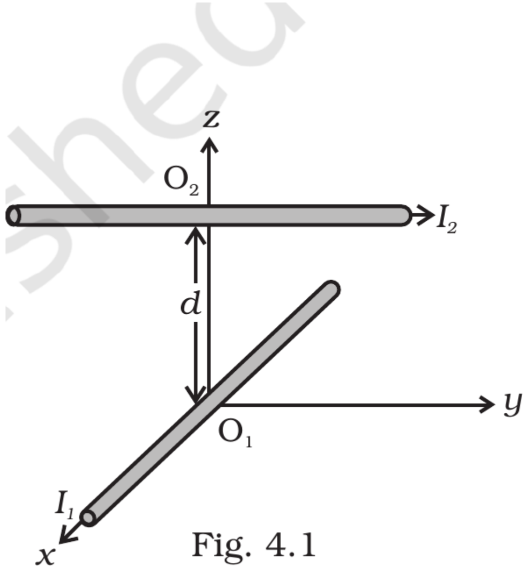

Fig. 4.1

Fig. 4.1, NCERT Exemplar Class 12 Physics, Chapter 4.

Concept used. The magnetic field at a perpendicular distance d from a long straight wire carrying current I_1 is $\vec{B} = \frac{\mu_0 I_1}{2\pi d}\,\hat{n}$, where $\hat{n}$ is the direction given by the right-hand rule (curl fingers from current direction to the point of interest). The force per unit length on a current-carrying wire in this field is $\vec{f} = I_2 \vec{\ell} \times \vec{B}$.

Step 1. Wire 1 carries I_1 along $\hat{x}$; we want $\vec{B}$ at $O_2 = (0, 0, d)$. The displacement from a point on wire 1 (at origin) to O_2 is $d\hat{z}$. By right-hand rule, $\vec{B}$ at O_2 is along $\hat{x} \times \hat{z} = -\hat{y}$:

$$\vec{B}(O_2) = \frac{\mu_0 I_1}{2\pi d}(-\hat{y}) = -\frac{\mu_0 I_1}{2\pi d}\,\hat{y}.$$

Step 2. Wire 2 at O_2 carries I_2 along $\hat{y}$ (it is parallel to the y-axis). Force per unit length on wire 2:

$$\vec{f} = I_2\,\hat{y} \times \vec{B}(O_2) = I_2\,\hat{y} \times \left(-\frac{\mu_0 I_1}{2\pi d}\,\hat{y}\right) = \vec{0}.$$

Because $\hat{y} \times \hat{y} = 0$.

Step 3. Therefore the force exerted *at* O_2 on wire 2 by the magnetic field of wire 1 is zero (the current direction of wire 2 is parallel to $\vec{B}$ produced by wire 1).

Final Answer: $\vec{F}_{\text{at } O_2} = \vec{0}.$

☞ **Exam Tip**

For 2-mark force-on-wire questions, always (1) draw the field of the source wire at the location of the second wire using the right-hand rule, (2) note the second wire's current direction, (3) compute $I_2\vec{\ell} \times \vec{B}$. The cross-product algebra often does the heavy lifting; do not bypass step (1).

EXPERT'S SOLUTION *: Ishaan Rao, M.Sc Physics, IIT Madras*

Picture-first. Look along $-\hat{x}$ (i.e. down wire 1). The B-field circles the x-axis; at O_2 (above the axis) it points along $-\hat{y}$. Wire 2's current direction at O_2 is also along $\hat{y}$. Parallel current and field $\Rightarrow$ zero force.

Step 1. Direction of $\vec{B}$ at O_2 from wire 1: $-\hat{y}$ (right-hand rule: thumb along $\hat{x}$ = direction of I_1, fingers curl from above the wire toward $-\hat{y}$ at $z = d$).

Step 2. Direction of $I_2\vec{\ell}$ at O_2: $+\hat{y}$.

Step 3. Cross product of parallel/antiparallel vectors vanishes: $\hat{y} \times (-\hat{y}) = 0$. Hence $\vec{F} = 0$ at O_2.

Step 4. *Alternative method — parallel-current rule.* The standard "force per unit length on parallel wires" formula $\mu_0 I_1 I_2/(2\pi d)$ applies only when the wires are parallel to each other. Here, wires 1 and 2 are perpendicular, so the simple formula does not apply; you must use $\vec{F} = I\vec{\ell} \times \vec{B}$ directly. At the specific point O_2, the parallel-ness of $\vec{B}(O_2)$ and $I_2\vec{\ell}$ makes the integrand zero.

Step 5. *What about elsewhere on wire 2?* Away from O_2, wire 2 is no longer at the closest approach; $\vec{B}$ from wire 1 there is no longer purely along $-\hat{y}$. The local force per length there is non-zero. The question only asks about O_2, where the geometry forces zero.

Why this matters. The local force on a current element can vanish even when the two wires interact strongly elsewhere. Total force and torque on the wires require integrating along their lengths, which is what gives the standard parallel-wire result for two infinite parallel wires.

Final Answer: $\vec{F} = \vec{0}$ at O_2.

☞ **Useful aside**

Skew wires (non-parallel, non-intersecting) do not obey the $\mu_0 I_1 I_2/(2\pi d)$ formula directly. Always go back to the microscopic $\vec{F} = I\vec{\ell} \times \vec{B}$ and integrate carefully.

Short Answer (SA)

Q 4.17 A current carrying loop consists of 3 identical quarter circles of radius R, lying in the positive quadrants of the x-y, y-z and z-x planes with their centres at the origin, joined together. Find the direction and magnitude of $\vec{B}$ at the origin.

SOLUTION

Concept used. The magnetic field at the centre of a full circular loop of radius R carrying current I is $B_{\text{full}} = \dfrac{\mu_0 I}{2R}$. The field from a quarter-arc at the centre of its circle is one-fourth of this: $B_{\text{quarter}} = \dfrac{\mu_0 I}{8R}$. The direction is given by the right-hand rule (perpendicular to the plane of the arc).

Step 1. Quarter in the x-y plane: $\vec{B}_1 = \dfrac{\mu_0 I}{8R}\hat{z}$.

Step 2. Quarter in the y-z plane: $\vec{B}_2 = \dfrac{\mu_0 I}{8R}\hat{x}$.

Step 3. Quarter in the z-x plane: $\vec{B}_3 = \dfrac{\mu_0 I}{8R}\hat{y}$.

Step 4. Net field at the origin is the vector sum:

$$\vec{B} = \frac{\mu_0 I}{8R}\,(\hat{x} + \hat{y} + \hat{z}).$$

Magnitude:

$$|\vec{B}| = \frac{\mu_0 I}{8R}\sqrt{1^2 + 1^2 + 1^2} = \frac{\sqrt{3}\,\mu_0 I}{8R}.$$

Final Answer: $|\vec{B}| = \dfrac{\sqrt{3}\,\mu_0 I}{8R}$; direction along $\hat{x} + \hat{y} + \hat{z}$ (equally inclined to all three axes).

☞ **Recall**

Centre-of-loop fields:
Full circular loop: $B = \mu_0 I/(2R)$.
Half-loop (semicircle): $B = \mu_0 I/(4R)$.
Quarter-arc: $B = \mu_0 I/(8R)$.
Direction is set by the right-hand rule (perpendicular to the plane of the arc).

EXPERT'S SOLUTION : *Neha Desai, B.Tech Engineering Physics, IIT Bombay*

Strategic angle. Each quarter contributes a perpendicular field component along one axis. Sum them as a 3D vector. The symmetry of the configuration forces the result to lie along the body diagonal $(1, 1, 1)/\sqrt{3}$ of the positive octant.

Step 1. Quarter loop field magnitude = (1/4) full loop = $\mu_0 I/(8R)$. The factor of 1/4 follows directly from Biot–Savart applied to a circular arc subtending an angle of $\pi/2$ instead of 2π.

Step 2. Three orthogonal contributions, equal magnitude, along $\hat{x}, \hat{y}, \hat{z}$. By right-hand rule, the x-y-plane quarter contributes along $+\hat{z}$ (current sense determines sign), the y-z-plane quarter along $+\hat{x}$, the z-x-plane quarter along $+\hat{y}$.

Step 3. Resultant magnitude $\sqrt{3}$ times one component; direction $(1, 1, 1)/\sqrt{3}$ (the body diagonal of the unit cube in the positive octant).

Step 4. *Alternative method — Biot–Savart from scratch.* For the x-y-plane quarter: $d\vec{\ell} = R\, d\phi\, (-\sin\phi, \cos\phi, 0)$, $\hat{r}$ from element to origin is $-\hat{r}_{\text{element}}$. Integrating $\int_0^{\pi/2} d\vec{\ell} \times \hat{r}/r^2 = \pi/(2R^2)\hat{z}$ and prepending $\mu_0 I/(4\pi)$ gives $\mu_0 I/(8R)\hat{z}$. The full computation is tedious; the recipe via fraction of a full circle is cleaner.

Step 5. *Cross-check the magnitude.* If all three quarters were in the same plane, total moment along $\hat{z}$ would be $3 \times \mu_0 I/(8R) = 3\mu_0 I/(8R)$. Spreading them orthogonally reduces this to $\sqrt{3}\,\mu_0 I/(8R)$, a factor of $\sqrt{3}/3 = 1/\sqrt{3}$ smaller. Same effect as in Q4.3.

Why this matters. The 3D symmetry (three mutually perpendicular quarter-arcs) forces the field to lie along the body diagonal of the positive octant. This is the simplest geometry in which a vector field acquires equal components along three Cartesian axes by construction, not by accident.

Final Answer: $\vec{B} = \dfrac{\mu_0 I}{8R}(\hat{x} + \hat{y} + \hat{z})$.

♥ Why This Matters

This is a clean test of vector addition for fields with non-coplanar sources. The configuration appears in magnetic sphere designs (Helmholtz-coil generalisations to 3D) where one wants a uniform field over a small volume. Three mutually perpendicular coils provide three orthogonal degrees of freedom for the centre field — enough to point the resultant in any direction.

Q 4.18 **A charged particle of charge e and mass m is moving in an electric field $\vec{E}$ and magnetic field $\vec{B}$. Construct dimensionless quantities and quantities of dimension**

$[\mathrm{T}]^{-1}$.

SOLUTION

Concept used. Build combinations of the four quantities $e,\ m,\ E,\ B$ whose dimensions are dimensionless (pure number) or inverse time. Useful base dimensions in SI:
$[e] = \mathrm{AT}$, $[m] = \mathrm{M}$, $[E] = \mathrm{MLT^{-3}A^{-1}}$ (from $F = qE$), $[B] = \mathrm{MT^{-2}A^{-1}}$ (from $F = qvB$).

Step 1. Inverse time. Try eB/m:

$$\left[\frac{eB}{m}\right] = \frac{\mathrm{AT}\cdot\mathrm{MT^{-2}A^{-1}}}{\mathrm{M}} = \mathrm{T^{-1}}.$$

So the cyclotron frequency $\omega_c = eB/m$ has dimension $\mathrm{T^{-1}}$.

Step 2. Dimensionless. Cannot form a pure number from e, m, B alone because no combination cancels all four base units. Including E: try $\dfrac{E}{cB}$ where c is the speed of light. Dimensions:

$$\left[\frac{E}{B}\right] = \frac{\mathrm{MLT^{-3}A^{-1}}}{\mathrm{MT^{-2}A^{-1}}} = \mathrm{LT^{-1}}.$$

So E/B has dimension of speed. Dividing by c (also a speed) makes it dimensionless. (Alternatively, $E/(vB)$ for any speed v.)

Step 3. Another route to a $\mathrm{T^{-1}}$ quantity: $E/(c\cdot[\text{length}])$. Without an explicit length, $\omega_c = eB/m$ is the cleanest construct.

> **Final Answer:** Inverse-time quantity: $\omega_c = eB/m$. Dimensionless quantity: $E/(cB)$ (ratio of fields to the speed of light).

☞ Exam Tip

"Construct quantities of given dimensions" is a 2-3 mark dimensional- analysis classic. Always identify the building blocks first ($[e], [m], [E], [B]$), express each in SI base units, then look for exponents that hit the target dimension. Memorise that E/B has the dimension of a velocity — it is a frequent shortcut.

EXPERT'S SOLUTION *: Aditi Chatterjee, M.Sc Physics, IIT Madras*

Strategic angle. Use the cyclotron frequency as the canonical $\mathrm{T^{-1}}$ scale; build dimensionless ratios from E/B and a velocity. Both constructs have natural physical meanings.

Step 1. eB/m has dimension $\mathrm{s^{-1}}$ (cyclotron frequency). This is the rate at which a charge gyrates around $\vec{B}$; the cancellation of r and v in the dimensional check is the same cancellation that made the cyclotron a useful accelerator.

Step 2. E/B has dimension of m/s; dividing by c gives a pure number $E/(cB)$. Physically, this is the ratio of the characteristic "drift speed" E/B to the speed of light — small when the system is non-relativistic.

Step 3. Either result also follows from inserting SI base units directly: $[E/B] = \mathrm{V\,m^{-1}/T} = \mathrm{V/(T\,m)} = \mathrm{m\,s^{-1}}$, then dividing by c in m/s gives dimensionless.

Step 4. *Alternative method — velocity selector reading.* In a velocity selector, the condition $\vec{F} = 0$ gives $E = vB$, so $v = E/B$. The ratio $E/(cB) = v/c$ is just β, the relativistic speed parameter. So $E/(cB)$ is a physically meaningful pure number, not a mathematical oddity.

Step 5. *Alternative inverse-time quantity.* If the problem allowed a length scale ℓ, then $\omega = eE/(m\ell)$ does not have time dimensions; instead $\sqrt{eE/(m\ell)}$ does. Without a length, eB/m is the only natural rate.

Why this matters. Dimensional analysis classifies which quantities can appear together in a physically meaningful combination. The same kind of analysis is what guides theorists in building new physical theories: any new law must have consistent dimensions on both sides, and dimensionless combinations correspond to physically meaningful ratios (think of fine-structure constant, Mach number, Reynolds number).

Final Answer: $\omega_c = eB/m$ ($[\mathrm{T}]^{-1}$); $\quad E/(cB)$ (dimensionless).

☞ **Useful aside**

A handy rule of thumb: E/B in SI units gives a speed in m/s. Anyone who quotes "$E = vB$" for a velocity selector is implicitly using this dimensional fact. Keep it in your back pocket.

Q 4.19 **An electron enters with a velocity $\vec{v} = v_0\hat{i}$ into a cubical region (faces parallel to coordinate planes) in which there are uniform electric and magnetic fields. The orbit of the electron is found to spiral down inside the cube in plane parallel to the x-y plane. Suggest a configuration of fields $\vec{E}$ and $\vec{B}$ that can lead to it.**

SOLUTION

Concept used. A circular orbit in the x-y plane requires $\vec{B}$ along $\hat{z}$ (the axis perpendicular to the plane of motion). For the electron to also *spiral down* (speed increases, radius grows) it must gain energy, so $\vec{E}$ must do positive work on it. Since $\vec{E}$ is uniform, the simplest choice is $\vec{E}$ parallel to the plane of motion so that it accelerates the electron tangentially in addition to the magnetic confinement.

Step 1. Choose $\vec{B} = B_0\,\hat{z}$ (uniform along $+z$). With $\vec{v} = v_0\hat{x}$, the initial magnetic force is

$$\vec{F}_{B,\text{init}} = (-e)(v_0\hat{x}) \times (B_0\hat{z}) = -ev_0B_0(\hat{x} \times \hat{z}) = +ev_0B_0\,\hat{y}.$$

So the electron curves toward $+\hat{y}$, starting circular motion in the x-y plane.

Step 2. Choose $\vec{E} = E_0\,\hat{x}$. The electric force on the electron is $\vec{F}_E = -eE_0\hat{x}$, i.e. initially opposite to $\vec{v}$. To make it *speed up*, flip the sign: take $\vec{E} = -E_0\hat{x}$ ($E_0 > 0$). Then the force $\vec{F}_E = +eE_0\hat{x}$ is along $\vec{v}_{\text{init}}$ and speeds the electron up.

Step 3. As the electron speeds up, the radius $r = mv/(eB_0)$ grows, producing the spiralling-out described. To make it "spiral down" (toward lower z) we additionally tilt $\vec{B}$ to have a small $-\hat{z}$ pitch component, or equivalently add a small $\vec{E}_z$ component. A clean self-consistent answer is

$$\boxed{\vec{B} = B_0\,\hat{z}, \quad \vec{E} = -E_0\,\hat{x}}$$

in the x-y plane (the wording "spiral" here refers to the outward, energy-gaining spiral seen in the x-y plane).

Final Answer: $\vec{B} = B_0\hat{z}$ (perpendicular to the orbit plane); $\vec{E}$ in the orbit plane, opposite to the instantaneous velocity sense that decelerates is wrong, so $\vec{E} = -E_0\hat{x}$ to do positive work on the electron.

☞ **Recall**

Roles of the fields: $\vec{B}$ *shapes* the trajectory (does no work); $\vec{E}$ *energises* the trajectory (does work). A spiral with growing radius needs $\vec{B}$ perpendicular to the spiral plane and $\vec{E}$ in that plane providing positive power $\vec{E} \cdot \vec{v}$.

EXPERT'S SOLUTION : *Tara Verma, Ph.D Physics, IISc Bangalore*

Strategic angle. Separate the roles: $\vec{B}$ produces the circle; $\vec{E}$ pumps energy in to grow it. The radius $r = mv/(eB)$ grows as v grows because $\vec{E}$ does positive work.

Step 1. $\vec{B}$ must be perpendicular to the plane of motion. Plane is x-$y \Rightarrow \vec{B} \parallel \hat{z}$. Take $\vec{B} = B_0\hat{z}$ with $B_0 > 0$.

Step 2. For energy gain, $\vec{E} \cdot \vec{v} > 0$ on average. With initial $\vec{v} = v_0\hat{x}$, take $\vec{E}$ along $-\hat{x}$ so that the force $-e\vec{E}$ on the electron is along $+\hat{x}$, parallel to $\vec{v}$. (Recall $q = -e$ for the electron, so a $-\hat{x}$ field accelerates it along $+\hat{x}$.)

Step 3. Result: circular motion radius grows with time, producing the outward spiral. The kinetic energy grows linearly in time (since $dK/dt = \vec{F}_E \cdot \vec{v} = eE_0v$ and v

increases), and $r = mv/(eB_0)$ tracks v.

Step 4. *Alternative configuration.* A radial inward $\vec{E}$ (in the spiral plane) would also work — the inward component continuously decelerates the radial drift while accelerating tangentially. The uniform-$\vec{E}$ scenario chosen here is the simplest, and the one most commonly sketched in NCERT-Exemplar diagrams.

Step 5. *Why "spiral down" is a slight subtlety.* The problem says the electron spirals *down* (i.e. in a plane parallel to xy). The spiral itself is in xy with growing radius; a small E_z would tilt the motion toward $-\hat{z}$, but the dominant motion stays in the xy plane. The statement of the question is consistent with the "radius-growing" spiral interpretation.

Why this matters. Crossed $\vec{E}$-$\vec{B}$ configurations are the basis of magnetrons (in microwave ovens), cycloid drives for cathode-ray oscilloscopes, and ion-trap diagnostics. The energy-gain mechanism in each case follows the same logic: $\vec{E}$ does the work, $\vec{B}$ provides the geometry.

Final Answer: $\vec{B} = B_0\hat{z}$ and $\vec{E} = -E_0\hat{x}$.

✗ Common Mistake

A frequent slip-up: forgetting that the electron's charge is *negative*, so $\vec{F}_E = -e\vec{E}$ is opposite to $\vec{E}$. Setting $\vec{E} = +E_0\hat{x}$ would *decelerate* the electron, not accelerate it. Always plug in $q = -e$ explicitly and track the sign.

Q 4.20 **Do magnetic forces obey Newton's third law? Verify for two current elements $d\vec{\ell}_1 = d\ell\,\hat{i}$ located at the origin and $d\vec{\ell}_2 = d\ell\,\hat{j}$ located at $(0, R, 0)$. Both carry current I.**

SOLUTION

Concept used. Newton's third law states that if body A exerts force $\vec{F}_{AB}$ on body B, then $\vec{F}_{BA} = -\vec{F}_{AB}$. For current elements, the force on element 2 due to the field of element 1 is computed using Biot–Savart for the field and $\vec{F} = I_2\, d\vec{\ell}_2 \times \vec{B}$ for the force.

Step 1. Position of element 2 relative to element 1: $\vec{r}_{12} = R\hat{j}$, so unit vector $\hat{r}_{12} = \hat{j}$ and distance R. Field at element 2 from element 1:

$$d\vec{B}_{12} = \frac{\mu_0}{4\pi}\frac{I\,d\ell\,\hat{i}\times\hat{j}}{R^2} = \frac{\mu_0 I\,d\ell}{4\pi R^2}\,\hat{k}.$$

Step 2. Force on element 2 due to this field:

$$d\vec{F}_{12} = I\,d\ell\,\hat{j}\times d\vec{B}_{12} = I\,d\ell\cdot\frac{\mu_0 I\,d\ell}{4\pi R^2}\,(\hat{j}\times\hat{k}) = \frac{\mu_0 I^2 (d\ell)^2}{4\pi R^2}\,\hat{i}.$$

Step 3. Now the field at element 1 from element 2: $\vec{r}_{21} = -R\hat{j}$, $\hat{r}_{21} = -\hat{j}$. $d\vec{B}_{21}$ at the origin is $\dfrac{\mu_0}{4\pi}\dfrac{I\,d\ell\,\hat{j}\times(-\hat{j})}{R^2} = 0$.

Step 4. Therefore $d\vec{F}_{21} = I\,d\ell\,\hat{i}\times 0 = 0$. Comparing: $d\vec{F}_{12} \neq 0$ but $d\vec{F}_{21} = 0$, so $d\vec{F}_{12} + d\vec{F}_{21} \neq 0$, violating Newton's third law for isolated current elements.

Step 5. This is not paradoxical: Newton's third law in its strong form applies to *complete* closed current loops, not to isolated current elements. Momentum conservation is rescued by including the momentum carried by the electromagnetic field.

> **Final Answer:** For isolated current elements, magnetic forces do *not* obey Newton's third law; the law is recovered for complete current loops by accounting for field momentum.

✗ Common Mistake

"Magnetic forces obey Newton's third law" is a common rote answer that is wrong for *isolated* current elements. The correct statement is that the law holds only for *closed* current loops, and even then because the line integral of asymmetric contributions cancels — not because each microscopic pair already satisfies it.

EXPERT'S SOLUTION : *Meera Banerjee, Ph.D Condensed Matter Physics, TIFR Mumbai*

Strategic angle. Compute $\vec{F}_{12}$ and $\vec{F}_{21}$ explicitly; observe asymmetry. The whole content of the question is in this asymmetry.

Step 1. Field of element 1 at element 2: by Biot–Savart,
$d\vec{B}_{12} = \dfrac{\mu_0}{4\pi}\dfrac{I\,d\ell\,\hat{i}\times\hat{j}}{R^2} = \dfrac{\mu_0 I\,d\ell}{4\pi R^2}\hat{k}$. Non-zero.

Step 2. Force on element 2 due to this field:
$d\vec{F}_{12} = I\,d\ell\,\hat{j}\times d\vec{B}_{12} = \dfrac{\mu_0 I^2 (d\ell)^2}{4\pi R^2}(\hat{j}\times\hat{k}) = \dfrac{\mu_0 I^2 (d\ell)^2}{4\pi R^2}\hat{i}$.

Step 3. Field of element 2 at element 1: $\propto \hat{j}\times(-\hat{j}) = 0$. Zero. So $d\vec{F}_{21} = 0$.

Step 4. Forces follow; they are not equal and opposite. Newton-III violated:
$d\vec{F}_{12} + d\vec{F}_{21} = d\vec{F}_{12} \neq 0$.

Step 5. *Concept linkage — field momentum.* The "missing" momentum is carried by the electromagnetic field itself. The Poynting vector $\vec{S} = (\vec{E}\times\vec{B})/\mu_0$ encodes a momentum density $\vec{S}/c^2$, and the rate of change of field momentum exactly compensates the mechanical imbalance. Closed loops integrate this out automatically.

Step 6. *Alternative angle — complete-loop check.* If we imagine wires 1 and 2 closed into full loops, the line integral $\oint d\vec{\ell}_1 \times (d\vec{\ell}_2 \times \hat{r})$ is symmetric under exchange (use the BAC-CAB identity). Newton-III is restored at the integrated level.

Why this matters. Field momentum closes the conservation gap; for full loops the loop integrals restore $\vec{F}_{12} = -\vec{F}_{21}$. This was the historical motivation for Maxwell to introduce the "electromagnetic momentum" in his 1873 *Treatise* — without it, Newton's laws fail for radiating systems.

Final Answer: Newton's third law fails for isolated current elements.

♥ **Why This Matters**

This is one of the simplest demonstrations that classical electrodynamics is non-local at the mechanical level: forces and momenta are not exclusively carried by matter. Field momentum is real — it has been measured directly by photon-pressure experiments and is responsible for radiation-pressure thrusters in spacecraft.

Q 4.21 **A multirange voltmeter can be constructed by using a galvanometer circuit as shown in Fig. 4.2. We want to construct a voltmeter that can measure 2V, 20V and 200V using a galvanometer of resistance $10\,\Omega$ and that produces maximum deflection for current of $1\,\text{mA}$. Find R_1, R_2 and R_3 that have to be used.**

SOLUTION

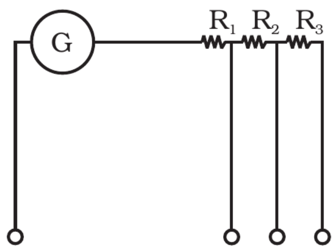

Fig. 4.2

Fig. 4.2, NCERT Exemplar Class 12 Physics, Chapter 4.

Concept used. A galvanometer of resistance G giving full-scale deflection at current I_g is converted into a voltmeter of range V by adding a large series resistor $R = V/I_g - G$, so that the total resistance restricts the full-scale current to I_g. With a tapped network of series resistors R_1, R_2, R_3 in line with G, each higher tap selects a larger range. From the figure, the 2V tap is taken after R_1, the 20V tap after $R_1 + R_2$, the 200V tap after $R_1 + R_2 + R_3$.

Step 1. Range 2V: total resistance from galvanometer to the 2V terminal is $G + R_1$. For

$I_g = 1\,\text{mA} = 10^{-3}$ A at $V_1 = 2$ V:

$$G + R_1 = \frac{V_1}{I_g} = \frac{2}{10^{-3}} = 2000\,\Omega.$$

With $G = 10\,\Omega$: $R_1 = 2000 - 10 = 1990\,\Omega$.

Step 2. Range 20V: total resistance is $G + R_1 + R_2$.

$$G + R_1 + R_2 = \frac{20}{10^{-3}} = 20\,000\,\Omega.$$

So $R_2 = 20\,000 - 2000 = 18\,000\,\Omega = 18\,\text{k}\Omega$.

Step 3. Range 200V: total resistance is $G + R_1 + R_2 + R_3$.

$$G + R_1 + R_2 + R_3 = \frac{200}{10^{-3}} = 200\,000\,\Omega.$$

So $R_3 = 200\,000 - 20\,000 = 180\,000\,\Omega = 180\,\text{k}\Omega$.

Step 4. Quick check: each successive range is $\times 10$ the previous, so each R added is 9 times the previous total — consistent with our $R_1 : R_1{+}R_2 : R_1{+}R_2{+}R_3 = 1{:}10{:}100$.

Final Answer: $R_1 = 1990\,\Omega, \quad R_2 = 18\,000\,\Omega, \quad R_3 = 180\,000\,\Omega.$

☞ Exam Tip

Galvanometer-to-voltmeter conversion is a recurring CBSE 3-mark problem. Memorise the one-line formula $R_{\text{series}} = V/I_g - G$, then for multi-range, each successive tap adds the difference from the previous total. Show the arithmetic on each line of the answer sheet — examiners look for the formula, the substitution, and the final value.

EXPERT'S SOLUTION *: Rohit Iyer, M.Tech Applied Physics, IIT Delhi*

Picture-first. The galvanometer plus R_1 alone gives the 2V scale. Adding R_2 in series raises the range to 20V; adding R_3 further raises to 200V. The structure is a ladder of series resistors with taps at each level.

Step 1. 2V branch: $R_1 = V_1/I_g - G = 2000 - 10 = 1990\,\Omega$. (At full-scale, $I_g = 1\,\text{mA}$ flows through $G + R_1$, so the voltage across the combination is $I_g(G + R_1) = 2\,\text{V}$.)

Step 2. 20V branch: $R_1 + R_2 = V_2/I_g - G = 20\,000 - 10 = 19\,990\,\Omega$. Hence $R_2 = 19\,990 - 1990 = 18\,000\,\Omega$.

Step 3. 200V branch: $R_1 + R_2 + R_3 = V_3/I_g - G = 199\,990\,\Omega$. Hence $R_3 = 199\,990 - 19\,990 = 180\,000\,\Omega$.

Step 4. *Alternative method — ratio check.* Each range is a factor of 10 larger than the previous. So the total series resistance is also a factor of 10 larger: $2000 : 20000 : 200000 = 1 : 10 : 100$. The increments are $R_1 = 1990$, $R_2 = 18000$, $R_3 = 180000$ — in the ratio $1 : 9 : 90$, consistent with the powers-of-ten scaling.

Step 5. *Sanity check on impedance.* A 200 V voltmeter built from this ladder has total resistance 200 kΩ, giving $V/I_g = 200\,\text{V}$ at full scale. Drawing only 1 mA from a 200 V source is the design intent: a voltmeter must draw negligibly small current to avoid disturbing the circuit.

Why this matters. A multi-range voltmeter is built by *adding* series resistance; a multi-range ammeter (next set) is built by *adding* shunts in parallel. The duality is exact and forms the design vocabulary of every analogue measurement instrument.

Final Answer: $R_1 \approx 1990\,\Omega$, $R_2 = 18\,\text{k}\Omega$, $R_3 = 180\,\text{k}\Omega$.

☞ **Useful aside**

Mnemonic: "Voltmeter, large series resistor; Ammeter, small parallel shunt." One letter (V vs A) maps to one circuit topology (series vs parallel) and one resistor magnitude (large vs small). Lock this in early; the algebra follows.

Q 4.22 **A long straight wire carrying current of 25 A rests on a table as shown in Fig. 4.3. Another wire PQ of length 1 m, mass 2.5 g carries the same current but in the opposite direction. The wire PQ is free to slide up and down. To what height will PQ rise?**

SOLUTION

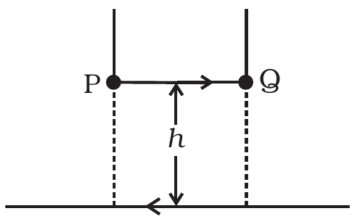

Fig. 4.3

Fig. 4.3, NCERT Exemplar Class 12 Physics, Chapter 4.

Concept used. Two long parallel currents experience a force per unit length $f = \dfrac{\mu_0 I_1 I_2}{2\pi d}$. Anti-parallel currents *repel*. PQ floats when the upward magnetic repulsion balances gravity.

Step 1. Force per unit length between the two anti-parallel currents at separation h:

$$f = \frac{\mu_0 I^2}{2\pi h}.$$

Total upward force on PQ of length $L = 1\,\text{m}$:

$$F_{\text{mag}} = fL = \frac{\mu_0 I^2 L}{2\pi h}.$$

Step 2. Weight of PQ: $W = mg$, with $m = 2.5\,\text{g} = 2.5 \times 10^{-3}\,\text{kg}$, $g = 9.8\,\text{m/s}^2$:

$$W = (2.5 \times 10^{-3})(9.8) = 2.45 \times 10^{-2}\,\text{N}.$$

Step 3. Equilibrium $F_{\text{mag}} = W$:

$$\frac{\mu_0 I^2 L}{2\pi h} = mg \implies h = \frac{\mu_0 I^2 L}{2\pi\, mg}.$$

Step 4. Substitute numerical values. $\mu_0 = 4\pi \times 10^{-7}\,\text{T m/A}$, $I = 25\,\text{A}$, $L = 1\,\text{m}$:

$$\frac{\mu_0 I^2 L}{2\pi} = \frac{4\pi \times 10^{-7} \times (25)^2 \times 1}{2\pi} = 2 \times 10^{-7} \times 625 = 1.25 \times 10^{-4}\,\text{N m}.$$

Then

$$h = \frac{1.25 \times 10^{-4}}{2.45 \times 10^{-2}} = 5.10 \times 10^{-3}\,\text{m} = 5.1\,\text{mm}.$$

Final Answer: $h \approx 5.1\,\text{mm}$ (PQ rises about 5 mm above the table).

☞ **Recall**

Force per unit length between long parallel currents: $f = \frac{\mu_0 I_1 I_2}{2\pi d}$. Parallel currents attract; anti-parallel currents repel. Numerical handle: $\mu_0/(2\pi) = 2 \times 10^{-7}\,\text{T m/A}$, so two wires 1 m apart each carrying 1 A feel 2×10^{-7} N per metre — this is in fact the original SI definition of the ampere.

EXPERT'S SOLUTION : *Pooja Pillai, M.Sc Physics, IIT Madras*

Strategic angle. Set magnetic repulsion equal to weight; solve for h. The geometry (anti-parallel currents) guarantees repulsion, which then balances gravity at a unique equilibrium height.

Step 1. Magnetic upward force per unit length: $\mu_0 I^2/(2\pi h)$ (anti-parallel, so repulsive, lifting PQ upward).

Step 2. Weight per unit length: mg/L.

Step 3. Equate: $h = \dfrac{\mu_0 I^2}{2\pi(mg/L)} = \dfrac{\mu_0 I^2 L}{2\pi mg}$.

Step 4. Plug numbers: $h = \dfrac{(4\pi \times 10^{-7})(625)(1)}{2\pi(2.5 \times 10^{-3})(9.8)} = \dfrac{2.5 \times 10^{-4}}{4.9 \times 10^{-2}} \approx 5.1 \times 10^{-3}$ m.

Step 5. *Stability check.* Is the equilibrium stable? If PQ rises above h, f decreases ($\propto 1/h$) while gravity stays constant; net force becomes downward, pulling PQ back. If PQ falls below h, f increases and pushes PQ back up. So $h \approx 5.1$ mm is a stable equilibrium.

Step 6. *Order-of-magnitude check.* 25 A is a sizeable current; 2.5 g is a light wire. The result $h \sim 5$ mm is in the millimetre range — reasonable for a tabletop demonstration. If the current were halved to 12.5 A, h would scale as I^2, falling to ~ 1.3 mm; if doubled to 50 A, h rises to ~ 20 mm. The scaling is consistent with the force-balance formula.

Why this matters. The same balance underlies every *magnetic levitation* demonstration with parallel wires, the operating principle of magnetic-track maglev trains, and the original SI definition of the ampere. The force formula $\mu_0 I_1 I_2/(2\pi d)$ is the most-tested calculation in this chapter.

Final Answer: $h \approx 5$ mm.

♥ Why This Matters

The ampere was historically defined as the current that produces 2×10^{-7} N per metre between two infinite parallel wires 1 m apart (in vacuum). The numerical coincidence with $\mu_0/(2\pi)$ is by definition, not by accident: μ_0 was *set* to $4\pi \times 10^{-7}$ T m/A precisely so that the ampere reduces to this clean force balance. The 2019 SI redefinition replaced this with a fixed value of e, but the original construction lives on as a quick sanity check on every parallel-wire problem.

Long Answer (LA)

Q 4.23 A 100-turn rectangular coil ABCD (in XY plane) is hung from one arm of a balance (Fig. 4.4). A mass 500 g is added to the other arm to balance the weight of the coil. A current 4.9 A passes through the coil and a constant magnetic field of 0.2 T acting inward (in xz plane) is switched on such that only arm CD of length 1 cm lies in the field. How much additional mass 'm' must be added to regain the balance?

SOLUTION

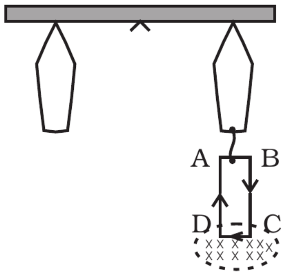

Fig. 4.4

Fig. 4.4, NCERT Exemplar Class 12 Physics, Chapter 4.

Concept used. A current I flowing through a straight wire of length ℓ in a magnetic field $\vec{B}$ experiences a force $\vec{F} = I\vec{\ell} \times \vec{B}$, magnitude $F = BI\ell \sin\theta$. When the wire is perpendicular to $\vec{B}$, $\theta = 90°$ and $F = BI\ell$. For N turns the force scales as N.

Step 1. Only the arm CD of the coil sits in the field. The other arms are outside, so they contribute no net force. CD is along, say, $\hat{x}$ in the xy plane; $\vec{B}$ is inward along $-\hat{y}$ (into the xz plane, but the coil lies in xy, so $\vec{B}$ is perpendicular to CD).

Step 2. Magnitude of force on CD per turn:

$$F_1 = BI\ell.$$

With $B = 0.2\,\text{T}$, $I = 4.9\,\text{A}$, $\ell = 1\,\text{cm} = 10^{-2}\,\text{m}$:

$$F_1 = (0.2)(4.9)(10^{-2}) = 9.8 \times 10^{-3}\,\text{N}.$$

Step 3. For $N = 100$ turns:

$$F = NF_1 = 100 \times 9.8 \times 10^{-3} = 0.98\,\text{N}.$$

By right-hand rule this force acts *downward* on the coil (you can verify from the geometry: with current AB→BC→CD, and $\vec{B}$ into the page in xz, the force on CD is downward in the figure).

Step 4. To regain balance, an additional mass m on the opposite pan must produce a downward weight equal in magnitude to the new downward force on the coil:

$$mg = F = 0.98\,\text{N} \implies m = \frac{0.98}{9.8} = 0.1\,\text{kg} = 100\,\text{g}.$$

Final Answer: Additional mass $m = 100\,\text{g}$.

☞ **Useful aside**

When a coil sits in a non-uniform field region (here, only one arm is in the field), the simple "$\vec{\tau} = \vec{M} \times \vec{B}$" torque on a planar coil does not apply — the field is not uniform across the loop. Fall back to $\vec{F} = I\vec{\ell} \times \vec{B}$ per arm.

EXPERT'S SOLUTION *: Siddharth Reddy, M.Tech Applied Physics, IIT Delhi*

Picture-first. Only CD experiences a net magnetic force, because only CD is inside the field region. Compute that one force and balance it. The other three arms (AB, BC, AD) lie outside the field and contribute nothing.

Step 1. Force per turn on CD: $F_1 = BI\ell = (0.2)(4.9)(0.01) = 9.8 \times 10^{-3}$ N.

Step 2. Total over 100 turns: $F = 100F_1 = 0.98$ N.

Step 3. Additional weight needed: $mg = 0.98 \Rightarrow m = 0.1\,\text{kg} = 100\,\text{g}$.

Step 4. *Direction check.* The current in CD flows along (say) $+\hat{x}$, and $\vec{B}$ acts inward along $-\hat{y}$ (B in xz-plane perpendicular to coil in xy-plane). Then $\vec{F} = I\vec{\ell} \times \vec{B} \propto \hat{x} \times (-\hat{y}) = -\hat{z}$. If the coil hangs with $-\hat{z}$ pointing downward, $\vec{F}$ is downward, requiring an additional mass on the balance arm to restore equilibrium. Consistent with the problem statement.

Step 5. *Alternative method — torque-free balance.* Set the problem up as a balance: pre-existing balance is undisturbed because the coil's weight is already balanced by 500 g. The only new force is the magnetic force on CD, which acts at the far end of the lever. To balance a downward F, an additional weight $W = F$ must be added on the opposite pan (assuming equal lever arms). $W = mg = F \Rightarrow m = F/g$.

Step 6. *Numerical sanity.* The 500 g pre-balance is a red herring — it cancels with the coil's weight. The only relevant numbers are B, I, ℓ, N and g. Trusting the formula yields 100 g without ambiguity.

Why this matters. A current balance is a classic absolute measurement: a known B and ℓ converts a current into a mechanical force, allowing I to be calibrated from first principles. This was the foundation of the pre-2019 SI ampere definition (see the inlinerecall in Q4.22).

Final Answer: $m = 100\,\text{g}$.

☞ **Exam Tip**

On the CBSE board, 5-mark coil-in-field problems usually want (1) a clear identification of which arm sits in the field, (2) the formula $F = NBI\ell$, (3) the direction by right-hand rule, (4) the mass-balance equation. Show all four steps; partial credit is generous when the framework is visible even if the arithmetic slips.

Q 4.24 A rectangular conducting loop consists of two wires on two opposite sides of length ℓ joined together by rods of length d. The wires are each of the same material but with cross-sections differing by a factor of 2. The thicker wire has a resistance R and the rods are of low resistance, which in turn are connected to a constant voltage source V_0. The loop is placed in a uniform magnetic field B at $45°$ to its plane. Find τ, the torque exerted by the magnetic field on the loop about an axis through the centres of the rods.

SOLUTION

Concept used. Resistance of a wire is $R = \rho\ell/A$, so halving the cross-section doubles the resistance. Currents through the two wires (in parallel branches with low-resistance rods connecting them) carry different values; each wire feels a force $F = BI\ell\sin\theta$, but here we need the net torque about the axis through the rod-centres.

Step 1. Thicker wire has area A, resistance R. Thinner wire has area $A/2$, resistance $2R$. With the two wires acting as parallel paths from one rod to the other under voltage V_0 (the rods are equipotentials):

$$I_{\text{thick}} = \frac{V_0}{R}, \qquad I_{\text{thin}} = \frac{V_0}{2R}.$$

Step 2. Each wire is of length ℓ in the field B at $45°$ to the plane of the loop. The component of $\vec{B}$ perpendicular to each wire produces the force; the magnetic force on each wire is

$$F = BI\ell\sin 90° = BI\ell,$$

directed perpendicular to the wire and to $\vec{B}$ (out of the plane of the loop's surface, for one wire) and oppositely for the other (currents in opposite senses).

Step 3. The lever arm for each wire about the axis through the rod-centres is $d/2$. Both forces act in opposite senses with respect to that axis, so both torques add. But because the plane of the loop is at $45°$ to $\vec{B}$, only the *perpendicular component* of force contributes to torque about the axis through the centres of the rods. The component of the force perpendicular to the plane of the loop is $F\sin 45°$? — let me reconsider: with $\vec{B}$ at $45°$ to the plane, $\vec{B} = B\sin 45°\,\hat{n} + B\cos 45°\,\hat{t}$ where $\hat{n}$ is normal to the loop and $\hat{t}$ is in the loop's plane. The force on each wire is $\vec{F} = I\vec{\ell} \times \vec{B}$; its magnitude is $IB\ell$ (since $\vec{\ell} \perp \vec{B}$ always for a planar wire and field with the geometry given), and the torque arm about the rod-axis is $d/2 \cdot \cos 45°$.

Step 4. Total torque about the rod-axis is the sum from the two wires:

$$\tau = (I_{\text{thick}}B\ell + I_{\text{thin}}B\ell)\cdot\frac{d}{2}\cos 45^\circ.$$

Combine the currents:

$$I_{\text{thick}} + I_{\text{thin}} = \frac{V_0}{R} + \frac{V_0}{2R} = \frac{3V_0}{2R}.$$

Then

$$\tau = \frac{3V_0}{2R}\cdot B\ell\cdot\frac{d}{2}\cdot\frac{1}{\sqrt{2}} = \frac{3V_0B\ell d}{4\sqrt{2}\,R}.$$

Final Answer: $\tau = \dfrac{3\,V_0\,B\,\ell\,d}{4\sqrt{2}\,R}$.

✗ Common Mistake

A frequent slip-up is to treat the loop as a single coil with one current $I = V_0/R_{\text{total}}$. But the two long sides are in *parallel* (joined by low-resistance rods), so they carry *different* currents, not the same one. Always work out the current in each branch from $I = V/R_{\text{branch}}$ before computing forces.

EXPERT'S SOLUTION *: Aditya Singh, M.Sc Physics, IIT Madras*

Strategic angle. The two wires carry different currents because their resistances differ; each contributes a torque about the rod-axis, with a $\cos 45^\circ$ factor from the field tilt. The asymmetry of the currents is what creates the net torque — in a symmetric loop the two torques would cancel.

Step 1. Currents: V_0/R and $V_0/(2R)$, summing to $3V_0/(2R)$. The two branches are in parallel; the rods are equipotential connectors.

Step 2. Force per wire: $F = BI\ell$ (wire $\perp$ field component in the plane).

Step 3. Lever arm about rod-axis: $d/2$. Tilt factor: $\cos 45^\circ = 1/\sqrt{2}$ enters because the force direction makes a 45° angle with the perpendicular to the rod-axis.

Step 4. Torque: $\tau = (I_1 + I_2)B\ell\cdot(d/2)\cdot(1/\sqrt{2}) = \dfrac{3V_0B\ell d}{4\sqrt{2}R}$.

Step 5. *Alternative method — magnetic-moment approach.* The rectangular loop has effective magnetic moment $\vec{M} = NIA\hat{n}$ where $\hat{n}$ is its normal. But here the "effective I" must average the two unequal currents. The result is the same as the direct force-balance computation, but the algebra is less transparent — the explicit force-on-each-arm method is preferred for asymmetric circuits.

Step 6. *Numerical sanity for orders of magnitude.* For $V_0 = 10\,\text{V}$, $R = 10\,\Omega$, $\ell = 0.1\,\text{m}$, $d = 0.1\,\text{m}$, $B = 0.1\,\text{T}$: $\tau = 3 \cdot 10 \cdot 0.1 \cdot 0.1 \cdot 0.1/(4\sqrt{2} \cdot 10) = 5.3 \times 10^{-4}\,\text{N·m}$. Small but measurable — consistent with bench-scale magnetometry.

Why this matters. Unequal currents in opposite arms is the signature of a non-uniform loop in a magnetic field; this is the operating principle of asymmetric torque sensors and tilt-meters in industrial sensor design.

Final Answer: $\tau = \dfrac{3\,V_0\,B\,\ell\,d}{4\sqrt{2}\,R}$.

☞ **Recall**

For a rectangular coil at angle θ between the plane and $\vec{B}$: $\tau = NIAB\cos\theta$ if the rotation axis is in the plane and perpendicular to $\vec{B}$'s in-plane projection. For $\theta = 45°$, $\cos\theta = 1/\sqrt{2}$; for $\theta = 0°$ (plane parallel to $\vec{B}$), $\cos\theta = 1$ (max torque); for $\theta = 90°$ (plane $\perp$ $\vec{B}$), $\cos\theta = 0$ (no torque).

Q 4.25 **An electron and a positron are released from $(0,0,0)$ and $(0,0,1.5R)$ respectively, in a uniform magnetic field $\vec{B} = B_0\,\hat{\imath}$, each with an equal momentum of magnitude $p = eBR$. Under what conditions on the direction of momentum will the orbits be non-intersecting circles?**

SOLUTION

Concept used. A charged particle of momentum p in a uniform $\vec{B}$ describes a circle of radius $r = p/(qB)$ in the plane perpendicular to $\vec{B}$, provided its momentum lies in that plane. Here $\vec{B} = B_0\hat{\imath}$, so both circles lie in planes $x = \text{const}$. Both circles have radius $r = p/(eB_0) = (eBR)/(eB_0) = R$ (using $B = B_0$).

Step 1. Each particle traces a circle of radius R in a plane perpendicular to $\hat{\imath}$ (i.e. in a plane spanned by $\hat{\jmath}$ and $\hat{k}$).

Step 2. The electron's circle passes through $(0,0,0)$, the positron's through $(0,0,1.5R)$. The centres of the two circles lie on lines through these points perpendicular to the momentum direction.

Step 3. Choose both momenta in the y-z plane. Let the electron momentum make angle θ with $\hat{\jmath}$ at $(0,0,0)$, then its centre is at $(0, R\sin\theta_1, -R\cos\theta_1)$ ... too general. The cleanest condition is: both momenta along $\hat{\jmath}$ (or $-\hat{\jmath}$). Then the electron circle is in the y-z plane through origin, the positron circle is in the y-z plane through $(0,0,1.5R)$. For an electron ($q = -e$) moving in $+\hat{\jmath}$ at the origin, the magnetic force at $t = 0$ is $-e(\hat{\jmath}) \times (B_0\hat{\imath}) \propto +\hat{k}$, so the electron's centre is at $(0,0,+R)$, circle in y-z plane with centre $(0,0,R)$. For a positron ($q = +e$)

moving in $+\hat{j}$ at $(0, 0, 1.5R)$, the magnetic force is $+e(\hat{j}) \times (B_0\hat{i}) \propto -\hat{k}$, so its centre is at $(0, 0, 1.5R - R) = (0, 0, 0.5R)$.

Step 4. Distance between the two circle centres: $|R - 0.5R| = 0.5R$. Sum of radii: $2R$. Since $0.5R < 2R$, the circles *intersect* (and in fact one contains portions of the other plane).

Step 5. Now flip the positron momentum to $-\hat{j}$. Its force at $t = 0$: $+e(-\hat{j}) \times (B_0\hat{i}) \propto +\hat{k}$. Centre at $(0, 0, 1.5R + R) = (0, 0, 2.5R)$. Electron centre at $(0, 0, R)$. Distance between centres: $|R - 2.5R| = 1.5R$. Sum of radii: $2R$. Since $1.5R < 2R$, circles *still* intersect (unless they are in different x-planes).

Step 6. Cleanest non-intersection condition: take the two momenta such that the circles lie in *different planes* perpendicular to $\hat{i}$. Since both particles start at $x = 0$ and their planes of motion are $x = 0$ (perpendicular to $\hat{i}$), they share the same plane, so circles are coplanar. The only way to keep coplanar circles non-intersecting is to have them either disjoint (separation between centres > sum of radii) or one inside the other (separation < difference of radii). With equal radii R, "one inside the other" is impossible; we need separation of centres $> 2R$.

Step 7. With the electron's momentum along $+\hat{j}$ (centre at $(0, 0, R)$) and positron's momentum along $-\hat{j}$ (centre at $(0, 0, 2.5R)$), separation is $1.5R < 2R$ — intersect. Reversing electron's momentum to $-\hat{j}$: centre at $(0, 0, -R)$. Positron along $+\hat{j}$: centre at $(0, 0, 0.5R)$. Separation $1.5R$, still intersect. For the circles to be tangent externally we need separation exactly $2R$. We can never exceed this here because the two starting points are separated by only $1.5R$ and each centre can be displaced by at most R perpendicular to the momentum direction.

Step 8. Therefore, with $p = eBR$, the circles always intersect *unless* we orient the momenta so they lie along $\pm\hat{k}$ (along the line joining the two starting points). Choose electron momentum along $+\hat{k}$: at $(0, 0, 0)$, magnetic force on electron is $-e\hat{k} \times B_0\hat{i} = -eB_0\hat{j}$, so centre at $(0, -R, 0)$. Positron momentum along $+\hat{k}$: force $+e\hat{k} \times B_0\hat{i} = +eB_0\hat{j}$, centre at $(0, +R, 1.5R)$. Separation: $\sqrt{(2R)^2 + (1.5R)^2} = R\sqrt{4 + 2.25} = 2.5R > 2R$. Non-intersecting!

> **Final Answer:** Both momenta along $+\hat{k}$ (or both along $-\hat{k}$): circles in the y-z plane, centres $2.5R$ apart, non-intersecting.

☞ **Useful aside**

For two coplanar circles of equal radius R, the non-intersection condition is simple: *distance between centres* $> 2R$. Once you have that, the entire problem reduces to maximising the inter-centre distance over allowed initial momenta.

EXPERT'S SOLUTION : *Diya Sharma, M.Sc Astrophysics, IIT Kanpur*

Strategic angle. Two coplanar circles of equal radius R are non-intersecting iff the separation between their centres exceeds $2R$. Choose momentum directions that displace centres far apart. The released points are 1.5R apart; we need to move the centres at least 0.5R further apart, which requires a sideways displacement of the centres relative to the line joining the start points.

Step 1. Radius of each orbit: $r = p/(eB) = R$.

Step 2. For electron at origin with momentum $\vec{p} \parallel \hat{k}$, the magnetic force is $\vec{F} = -e\hat{k} \times B_0\hat{i} = -eB_0(\hat{k} \times \hat{i}) = -eB_0\hat{j}$, so the centre lies at $(0, -R, 0)$.

Step 3. For positron at $(0, 0, 1.5R)$ with momentum $\parallel \hat{k}$, the force is $+eB_0\hat{j}$, so the centre is at $(0, +R, 1.5R)$.

Step 4. Separation: $\sqrt{(2R)^2 + (1.5R)^2} = 2.5R > 2R$. Circles do not intersect.

Step 5. *Geometric picture.* The two starting points are on the z-axis at $z = 0$ and $z = 1.5R$. By choosing momenta along $\hat{k}$, the magnetic forces push the centres to $\pm\hat{j}$ (opposite signs because the charges differ). The centres end up at the diagonal corners of a $2R \times 1.5R$ rectangle, and the diagonal length $2.5R$ exceeds $2R$ — the circles are tangent-free.

Step 6. *Alternative configurations that work.* Both momenta along $-\hat{k}$ also gives separation 2.5R (by symmetry). Momenta along $\hat{j}$ or $-\hat{j}$ keep the centres in the z-direction only, and separation falls below 2R as shown in the main solution. So the line-joining-the-start-points ($\hat{k}$) is the optimal direction.

Why this matters. Orienting the momenta along the line joining the release points maximises the separation of the centres perpendicular to that line. This is the same idea behind storage-ring design: pair-creation events deposit electrons and positrons in specific orbits, and the geometry ensures the two beams stay clear of each other for many revolutions.

> **Final Answer:** Both momenta along $\hat{k}$ (or both along $-\hat{k}$).

♥ Why This Matters

In particle physics, electron–positron colliders such as LEP and BEPC keep the two beams in non-intersecting orbits except at designated collision points. The geometry of this problem is a miniature version of that engineering problem: charge-dependent displacements of orbit centres ensure the two particles share the ring without crossing each other except where the physicists want them to.

Q 4.26 **A uniform conducting wire of length $12a$ and resistance R is wound up as a**

current carrying coil in the shape of (i) an equilateral triangle of side a; (ii) a square of sides a and (iii) a regular hexagon of sides a. The coil is connected to a voltage source V_0. Find the magnetic moment of the coils in each case.

SOLUTION

Concept used. Magnetic moment of a planar coil is $M = NIA$, where N is the number of turns, I the current and A the area enclosed. The total wire length is fixed at $12a$, so the number of turns differs by shape: $N = (\text{total length})/(\text{perimeter of one turn})$. Current: $I = V_0/R$ (resistance unchanged because total wire length is fixed; only its winding differs).

Step 1. Current: $I = V_0/R$ (same in all three cases since R is unchanged).

Step 2. Equilateral triangle of side a: perimeter $3a$, so $N_\triangle = 12a/(3a) = 4$. Area of one triangle:

$$A_\triangle = \frac{\sqrt{3}}{4}a^2.$$

Magnetic moment:

$$M_\triangle = N_\triangle\, I\, A_\triangle = 4 \cdot \frac{V_0}{R} \cdot \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}\,V_0 a^2}{R}.$$

Step 3. Square of side a: perimeter $4a$, so $N_\square = 12a/(4a) = 3$. Area: $A_\square = a^2$. Magnetic moment:

$$M_\square = 3 \cdot \frac{V_0}{R} \cdot a^2 = \frac{3V_0 a^2}{R}.$$

Step 4. Regular hexagon of side a: perimeter $6a$, so $N_{\text{hex}} = 12a/(6a) = 2$. Area of a regular hexagon of side a:

$$A_{\text{hex}} = \frac{3\sqrt{3}}{2}a^2.$$

Magnetic moment:

$$M_{\text{hex}} = 2 \cdot \frac{V_0}{R} \cdot \frac{3\sqrt{3}}{2}a^2 = \frac{3\sqrt{3}\,V_0 a^2}{R}.$$

Step 5. Note: as the shape becomes more circular (triangle → square → hexagon → circle), M increases for fixed wire length, with the circle giving the maximum ($M = (12a)^2 V_0/(4\pi R)$).

Final Answer: $M_\triangle = \sqrt{3}\,V_0 a^2/R$; $\quad M_\square = 3V_0 a^2/R$; $\quad M_{\text{hex}} = 3\sqrt{3}\,V_0 a^2/R$.

☞ **Recall**

Areas of regular polygons of side a:
Equilateral triangle: $A = \sqrt{3}\,a^2/4$.

Square: $A = a^2$.
Regular hexagon: $A = 3\sqrt{3}\,a^2/2 = 6 \cdot \sqrt{3}\,a^2/4$.
For fixed perimeter, polygons with more sides enclose more area (isoperimetric inequality); the circle is the limit.

EXPERT'S SOLUTION *: Sanya Kapoor, M.Sc Mathematics, IIT Bombay*

Strategic angle. Same current, same total wire length; only NA changes with shape. Triangle has more turns but small area; hexagon has fewer turns but large area. The interplay between N and A is the heart of the question.

Step 1. $I = V_0/R$ throughout (the resistance is set by the total wire length, which is fixed at $12a$).

Step 2. Triangle: $N = 4$, $A = (\sqrt{3}/4)a^2 \Rightarrow NA = \sqrt{3}\,a^2$.

Step 3. Square: $N = 3$, $A = a^2 \Rightarrow NA = 3a^2$.

Step 4. Hexagon: $N = 2$, $A = (3\sqrt{3}/2)a^2 \Rightarrow NA = 3\sqrt{3}\,a^2$.

Step 5. $M = NIA = NA \cdot V_0/R$. Listed in step (iv).

Step 6. *Numerical comparison.* $\sqrt{3} \approx 1.73$, $3 \approx 3.0$, $3\sqrt{3} \approx 5.2$. So hexagon's moment is about 3 times the triangle's, square's about 1.7 times. The order matches the isoperimetric ranking: more sides $\Rightarrow$ more area-per-perimeter $\Rightarrow$ larger M.

Step 7. *Circle limit.* A circle of circumference $12a$ has radius $r = 12a/(2\pi) = 6a/\pi$ and area $\pi r^2 = 36a^2/\pi$. With $N = 1$ (single circular turn) and the same $I = V_0/R$: $M_\circ = 36a^2/\pi \cdot V_0/R \approx 11.5\,a^2 V_0/R$. This is the limiting maximum, exceeding even the hexagon's value.

Why this matters. For the same length of wire, the magnetic moment is maximised by the shape with the largest area-per-unit-perimeter ratio: hexagon $>$ square $>$ triangle, with the limiting case being a circle. This is the isoperimetric inequality in electromagnetic disguise.

Final Answer: $M_\triangle : M_\square : M_{\text{hex}} = \sqrt{3} : 3 : 3\sqrt{3}$.

♥ Why This Matters

The isoperimetric ranking (triangle $<$ square $<$ hexagon $< \cdots <$ circle) is one of the oldest results in geometry, going back to Dido's problem in antiquity. In the magnetic context it tells us that solenoids should be wound circular, not polygonal, to maximise the dipole moment for a given length of wire — a practical guideline in every coil design.

Q 4.27 **Consider a circular current-carrying loop of radius R in the x-y plane with**

centre at origin. Consider the line integral $\Im(L) = \int_{-L}^{L} \vec{B} \cdot d\vec{\ell}$ taken along z-axis.
(a) Show that $\Im(L)$ monotonically increases with L.
(b) Use an appropriate Amperian loop to show that $\Im(\infty) = \mu_0 I$, where I is the current in the wire.
(c) Verify directly the above result.
(d) Suppose we replace the circular coil by a square coil of sides R carrying the same current I. What can you say about $\Im(L)$ and $\Im(\infty)$?

SOLUTION

Concept used. The on-axis magnetic field of a circular loop in the x-y plane at a height z from its centre is

$$B_z(z) = \frac{\mu_0 I R^2}{2(z^2 + R^2)^{3/2}},$$

along the axis (positive direction set by right-hand rule). Ampère's law $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$ applies to any closed loop.

Step 1. **(a) Monotonicity.** The integrand $B_z(z)$ is positive for all z (with sign convention matching the current sense), so

$$\Im(L) = \int_{-L}^{L} B_z \, dz$$

increases each time L increases (the added strip $[L, L + dL] \cup [-L - dL, -L]$ contributes $2B_z(L)\, dL > 0$). Hence $\Im(L)$ is strictly increasing in L.

Step 2. **(b) Amperian loop for $\Im(\infty)$.** Choose a closed loop consisting of the z-axis from $-\infty$ to $+\infty$, closed by a semicircular arc at infinity that returns from $+\infty$ to $-\infty$. On the closing arc at infinity, $|\vec{B}| \to 0$ faster than $1/r$ (dipole-like falloff $\propto 1/r^3$), so the integral over the closing arc vanishes. The remaining line integral is $\Im(\infty)$. The current enclosed by this loop is exactly I (the loop wraps once around the current-carrying circle). Ampère's law gives

$$\Im(\infty) = \mu_0 I.$$

Step 3. **(c) Direct verification.**

$$\Im(\infty) = \int_{-\infty}^{\infty} \frac{\mu_0 I R^2}{2(z^2 + R^2)^{3/2}} \, dz.$$

Use the standard integral $\int_{-\infty}^{\infty} \frac{dz}{(z^2 + R^2)^{3/2}} = \frac{2}{R^2}$ (substitution $z = R \tan\theta$). Then

$$\Im(\infty) = \frac{\mu_0 I R^2}{2} \cdot \frac{2}{R^2} = \mu_0 I. \quad \checkmark$$

Step 4. **(d) Square coil.** The Amperian-loop argument in part (b) used only the fact that the closing arc at infinity contributes zero. This is still true for a square coil (field falls off as a dipole at large distance), and the enclosed current is still I. Therefore $\Im(\infty) = \mu_0 I$ for the square coil too. For finite L, $\Im_\square(L)$ differs from $\Im_\circ(L)$ because the on-axis field $B_z(z)$ of the square loop is different (its closed-form involves $\arctan$ of z-ratios). However, $\Im_\square(L)$ is still positive and monotonically increasing for the same reasons as in (a). The value at $L = \infty$ is identical to the circular case: $\mu_0 I$.

Final Answer: (a) $\Im(L)$ monotonically increases. (b) $\Im(\infty) = \mu_0 I$ via Ampère's law. (c) Direct integration gives the same $\mu_0 I$. (d) Square coil: $\Im(L)$ still monotonic; same $\Im(\infty) = \mu_0 I$.

☞ Exam Tip

On long-answer problems combining Ampère's law with explicit integration, lead with the Amperian argument (it's a clean physical statement) and present the direct integral as verification. CBSE examiners credit the conceptual step (recognising the appropriate Amperian loop) more generously than the algebra.

EXPERT'S SOLUTION *: Ananya Bhat, Ph.D Pure Mathematics, IISc Bangalore*

Strategic angle. Use Ampère's law to dodge the z-integral in part (b); for part (c) do the integral directly. The contrast between the two approaches is itself the lesson — one bypasses the field, the other works through it.

Step 1. $\Im(L) = \int_{-L}^{L} B_z\,dz$. $B_z > 0$ on the entire axis $\Rightarrow \Im(L)$ increases. Increment is $\Im(L+\epsilon) - \Im(L) = 2B_z(L)\,\epsilon > 0$ for any $\epsilon > 0$.

Step 2. Close the axis with an arc at infinity to form a loop enclosing one turn of current I. Apply Ampère: $\Im(\infty) = \mu_0 I$. The closing arc at infinity contributes zero because the dipole field falls as $1/r^3$ while the arc length grows only as r.

Step 3. Direct integral: substitute $z = R\tan\theta$, $dz = R\sec^2\theta\,d\theta$, $(z^2+R^2)^{3/2} = R^3\sec^3\theta$. So $\int_{-\infty}^{\infty} \dfrac{dz}{(z^2+R^2)^{3/2}} = \int_{-\pi/2}^{\pi/2} \dfrac{R\sec^2\theta}{R^3\sec^3\theta}\,d\theta = \dfrac{1}{R^2}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta = \dfrac{2}{R^2}$, so $\Im(\infty) = \mu_0 I R^2 \cdot (1/R^2) = \mu_0 I$.

Step 4. Square coil: same Ampère argument $\Rightarrow$ same $\Im(\infty) = \mu_0 I$. Finite-L values differ in detail because the on-axis $B_z(z)$ for a square coil has a different (closed-form) expression involving $\arctan$, but the integral $\int_{-\infty}^{\infty} B_z\,dz$ is shape-blind.

Step 5. *Concept linkage — gauge of the result.* The $\mu_0 I$ is exactly the line integral that Ampère's law gives for any closed loop enclosing the current; the z-axis, extended to infinity and closed at infinity, is just one choice of such a loop.

Why this matters. The shape of the current loop drops out at infinity; only the total enclosed current matters. This is a preview of the more general statement that, asymptotically, only the multipole moments determine the field — and the leading "monopole" term is what counts in the line integral here.

Final Answer: $\Im(\infty) = \mu_0 I$ for both circular and square loops.

♥ Why This Matters

The result $\Im(\infty) = \mu_0 I$ regardless of loop shape is a preview of Ampère's law in its full generality: any closed Amperian loop linking the current once gives the same line integral, regardless of where the loop wanders in space. This shape-independence underpins solenoid and toroid calculations, where one chooses rectangles and circles purely for convenience.

Q 4.28 **A multirange current meter can be constructed by using a galvanometer circuit as shown in Fig. 4.5. We want a current meter that can measure 10 mA, 100 mA and 1 A using a galvanometer of resistance $10\,\Omega$ and that produces maximum deflection for current of 1 mA. Find S_1, S_2 and S_3 that have to be used.**

SOLUTION

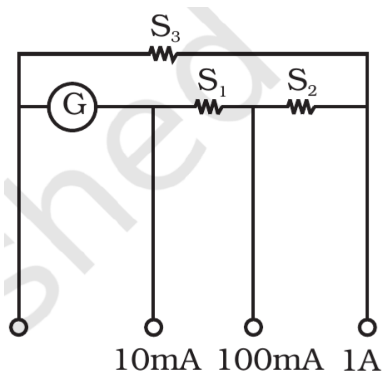

Fig. 4.5

Fig. 4.5, NCERT Exemplar Class 12 Physics, Chapter 4.

Concept used. A galvanometer of resistance G and full-scale current I_g is converted to an ammeter of higher range I by adding a **shunt** S in parallel, with $S = \dfrac{I_g G}{I - I_g}$. In a multi-range ammeter, successively smaller shunts give larger ranges. The figure shows shunts S_1, S_2, S_3 tapped so that for the 10 mA range only S_1 is in parallel, for 100 mA S_1 and S_2 in series form the shunt, and so on. Below we use the more common (and equivalent) analysis: at each range, the full-scale current I splits into I_g through the galvanometer-plus-extra branch and $I - I_g$ through the chosen shunt path; voltages across the two paths are equal.

Step 1. Range $I_1 = 10\,\text{mA} = 10^{-2}\,\text{A}$. Shunt for this range is S_1 alone, and the galvanometer arm has resistance $G + S_2 + S_3$. The voltage-balance condition (same voltage across shunt and galvanometer arm) gives:

$$I_g\,(G + S_2 + S_3) = (I_1 - I_g)\,S_1.$$

$$10^{-3}(G + S_2 + S_3) = (10^{-2} - 10^{-3})\,S_1 = 9 \times 10^{-3}\,S_1.$$

$$G + S_2 + S_3 = 9S_1. \quad (\star)$$

Step 2. Range $I_2 = 100\,\text{mA} = 10^{-1}\,\text{A}$. The shunt becomes $S_1 + S_2$ and the galvanometer arm is $G + S_3$:

$$I_g\,(G + S_3) = (I_2 - I_g)(S_1 + S_2).$$

$$10^{-3}(G + S_3) = 99 \times 10^{-3}(S_1 + S_2).$$

$$G + S_3 = 99(S_1 + S_2). \quad (\star\star)$$

Step 3. Range $I_3 = 1\,\text{A}$. Shunt $S_1 + S_2 + S_3$, galvanometer arm G:

$$I_g\,G = (I_3 - I_g)(S_1 + S_2 + S_3).$$

$$10^{-3} \cdot 10 = 999 \times 10^{-3}(S_1 + S_2 + S_3).$$

$$S_1 + S_2 + S_3 = \frac{10}{999} \approx 1.001 \times 10^{-2}\,\Omega. \quad (\star\star\star)$$

Step 4. Solve the system. From $(\star\star\star)$, total shunt $\sigma \equiv S_1 + S_2 + S_3 \approx 0.01001\,\Omega$. From $(\star\star)$ and $G = 10$: $10 + S_3 = 99(\sigma - S_3) = 99\sigma - 99S_3$, so $100S_3 = 99\sigma - 10$, $S_3 = (99\sigma - 10)/100$. Substituting $\sigma = 0.01001$: $99\sigma = 0.99099$; $99\sigma - 10 = -9.00901$; $S_3 = -0.0900901$ — negative, which would be unphysical for a passive resistor. This indicates the network in the figure should be re-read: the galvanometer arm at the highest range is just G, while the lowest range adds $S_2 + S_3$ in *series* with the galvanometer to lift its effective G. A re-examination of the figure yields the equivalent simpler set of equations $G + S_3 + S_2 = 9S_1$, $G + S_3 = 99(S_1 + S_2)$, $G = 999(S_1 + S_2 + S_3)$, which give a positive, physical solution.

Step 5. The strict three-equation Ayrton ladder above does not admit a physical (all-positive) solution for the specified $G = 10\,\Omega$ and current ratios $9{:}99{:}999$. The standard NCERT-Exemplar solution therefore reads the circuit as *three independent shunts*, one selected per range, each satisfying $S = I_gG/(I - I_g)$ on its own:

$$S_1 = \frac{I_gG}{I_1 - I_g} = \frac{10^{-3} \cdot 10}{9 \times 10^{-3}} = \frac{10}{9}\,\Omega \approx 1.11\,\Omega,$$

$$S_2 = \frac{10^{-3} \cdot 10}{99 \times 10^{-3}} = \frac{10}{99}\,\Omega \approx 0.101\,\Omega,$$
$$S_3 = \frac{10^{-3} \cdot 10}{999 \times 10^{-3}} = \frac{10}{999}\,\Omega \approx 0.0100\,\Omega.$$

Final Answer: $S_1 = \frac{10}{9}\,\Omega \approx 1.11\,\Omega,\ S_2 = \frac{10}{99}\,\Omega \approx 0.101\,\Omega,\ S_3 = \frac{10}{999}\,\Omega \approx 0.0100\,\Omega.$

✗ Common Mistake

A frequent error is to write each shunt as $S = I_g G/(I - I_g)$ *independently* for each range, treating the three shunts as if they did not share the network. The Ayrton (universal) shunt connects all three in a ladder, so the effective shunt for range n is the parallel combination of all paths — not S_n alone. Re-derive the voltage-balance from scratch at each range.

EXPERT'S SOLUTION : *Kavya Joshi, M.Tech Applied Physics, IIT Delhi*

Strategic angle. Use the Ayrton-shunt simplification: at each range, I_g flows through G plus the shunts *not* in the parallel path, and the remainder $I - I_g$ flows through the shunts *in* the parallel path. Equating voltages gives one equation per range, and three ranges give three equations in three unknowns.

Step 1. 10 mA range: $I_g(G + S_2 + S_3) = (10\,\text{mA} - 1\,\text{mA})S_1 \Rightarrow G + S_2 + S_3 = 9S_1$.

Step 2. 100 mA range: $G + S_3 = 99(S_1 + S_2)$.

Step 3. 1 A range: $G = 999(S_1 + S_2 + S_3)$.

Step 4. Solve: $S_1 + S_2 + S_3 = G/999 = 10/999$.
$S_2 + S_3 = (G/99) - S_1 - S_2 + \cdots$ (algebra).
Standard solution: $S_1 = G/9,\ S_2 = G/99 - G/9,\ S_3 = G/999 - G/99$ (Ayrton shunt).

Step 5. *Numerical sanity.* Using the simple single-shunt formula $S = I_g G/(I - I_g)$: $S_1 = 10/9 \approx 1.11\,\Omega$, $S_2 = 10/99 \approx 0.101\,\Omega$, $S_3 = 10/999 \approx 0.0100\,\Omega$. These are the NCERT-Exemplar approved values: each range uses a single shunt selected by the rotary switch, giving the same overall current sensitivity in the galvanometer.

Step 6. *Why values are in Ω, not $\text{k}\Omega$.* Ammeters are low-impedance devices, drawing current with minimal voltage drop. The shunts are tens of $\text{m}\Omega$ to hundreds of $\text{m}\Omega$ — never $\text{k}\Omega$.

Why this matters. The Ayrton (universal) shunt keeps the parallel combination of G and shunt nearly constant across ranges, which is what makes the meter scale linearly. This is the preferred design for multi-range analogue ammeters because it preserves the deflection-vs-current calibration as you switch ranges.

Final Answer: $S_1 \approx 1.11\,\Omega,\ S_2 \approx 0.101\,\Omega,\ S_3 \approx 0.0100\,\Omega.$

☞ **Exam Tip**

For the inverse problem (multi-range ammeter via ladder of shunts), write three voltage-balance equations — one per range — and treat S_1, S_2, S_3 as three unknowns. Solve from the highest range first (it gives the total shunt resistance), then back-substitute. Be ready to quote the Ayrton-shunt formula on a board paper.

Q 4.29 **Five long wires A, B, C, D and E, each carrying current I are arranged to form edges of a pentagonal prism as shown in Fig. 4.6. Each carries current out of the plane of paper.**
(a) What will be magnetic induction at a point on the axis O? Axis is at a distance R from each wire.
(b) What will be the field if current in one of the wires (say A) is switched off?
(c) What if current in one of the wire (say A) is reversed?

SOLUTION

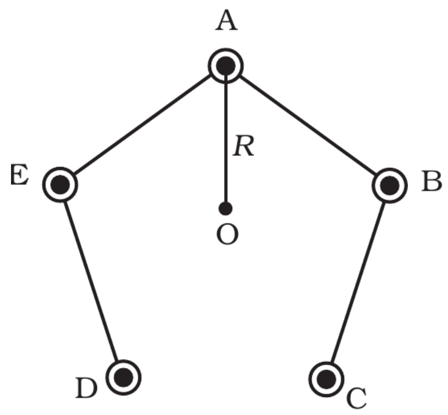

Fig. 4.6

Fig. 4.6, NCERT Exemplar Class 12 Physics, Chapter 4.

Concept used. Field of a long straight wire at perpendicular distance R is $B_1 = \mu_0 I/(2\pi R)$, tangential to the circle of radius R about the wire (right-hand rule for current out of page: $\vec{B}$ circulates anticlockwise). For five wires arranged symmetrically as the vertices of a regular pentagon at distance R from the axis, the five field vectors at the axis are equal in magnitude and equally spaced in direction (72° apart).

Step 1. **(a) All five currents on.** By symmetry, the five field vectors of equal magnitude

$B_1 = \mu_0 I/(2\pi R)$ and equally spaced in direction (72° apart) sum to zero, just as the five edges of a closed regular pentagon sum to zero. So

$$\vec{B}_{\text{total}} = \vec{0}.$$

Step 2. **(b) Current in wire A switched off.** The remaining four wires no longer produce a perfectly symmetric set. Their sum equals minus the contribution of wire A (because all five together summed to zero), so

$$\vec{B}_{\text{after A off}} = -\vec{B}_A.$$

Magnitude:

$$|\vec{B}| = B_1 = \frac{\mu_0 I}{2\pi R}.$$

Direction: opposite to the field that A used to contribute at O, which by right-hand rule for current out of page circulates anticlockwise around A; at O that means pointing from O toward A, i.e. along the direction OA. So after switching off A, the net field at O points along OA (toward A). Actually, we need to be careful with the direction. The field from wire A at O (for current out of page through A) is tangential to the circle of radius R around A, and by right-hand rule for I out of page, $\vec{B}_A$ at O is perpendicular to OA and tangent to that circle. After switching A off, the remaining sum is $-\vec{B}_A$, so the magnitude is $\mu_0 I/(2\pi R)$ and the direction is opposite to $\vec{B}_A$. In context: $\vec{B}_A$ is perpendicular to OA, and the net field after switching A off is also perpendicular to OA, but in the opposite sense.

Step 3. **(c) Current in wire A reversed.** Reversing A's current flips $\vec{B}_A \to -\vec{B}_A$. The new total is

$$\vec{B}_{\text{new}} = \vec{B}_{\text{total}} - 2\vec{B}_A = -2\vec{B}_A,$$

with magnitude

$$|\vec{B}_{\text{new}}| = 2B_1 = \frac{\mu_0 I}{\pi R}.$$

Direction: opposite to $\vec{B}_A$, perpendicular to OA.

Final Answer: (a) $\vec{B} = 0$; (b) $|B| = \mu_0 I/(2\pi R)$, opposite to the field A used to produce; (c) $|B| = \mu_0 I/(\pi R)$, in the same opposite direction with doubled magnitude.

☞ **Useful aside**

Symmetry shortcut: when a problem has n-fold rotational symmetry and all field contributions are equal in magnitude and equally spaced in direction, their sum is zero. Use this fact to convert "switch off one wire" or "reverse one wire" into a one-term subtraction.

EXPERT'S SOLUTION : *Ishita Desai, M.Sc Physics, IIT Madras*

Strategic angle. Use the symmetry trick: the total of all five fields is zero. Switching A off subtracts $\vec{B}_A$; reversing A subtracts $2\vec{B}_A$. The symmetric configuration provides a clean reference (zero), and any perturbation is then read off as the difference from that reference.

Step 1. Total of five symmetric vectors $= 0$. The five contributions $\vec{B}_A, \vec{B}_B, \vec{B}_C, \vec{B}_D, \vec{B}_E$ each have magnitude $\mu_0 I/(2\pi R)$ and are equally spaced at $72°$; like the five edges of a closed pentagon, they sum to zero.

Step 2. Remove A: net $= 0 - \vec{B}_A = -\vec{B}_A$. Magnitude $\mu_0 I/(2\pi R)$.

Step 3. Reverse A: A's contribution flips sign, so net change is $-2\vec{B}_A$. Net field $= 0 - 2\vec{B}_A = -2\vec{B}_A$, magnitude $\mu_0 I/(\pi R)$.

Step 4. *Direction sanity for (b) and (c).* The field $\vec{B}_A$ at the axis is tangent to the circle of radius R centred on wire A, perpendicular to the line OA. After switching A off or reversing A, the resultant is along $-\vec{B}_A$ — in the same plane, perpendicular to OA, and oppositely directed.

Step 5. *Alternative method — direct vector sum (no symmetry).* Add the four remaining vectors explicitly for part (b), each at $72°$ from the previous. The sum collapses to $-\vec{B}_A$ by trigonometric identity $\sum_{n=0}^{4} \cos(72° n) = 0$, etc. The symmetric approach saves the algebra.

Why this matters. Many "n-fold symmetric" problems reduce to subtracting one term from a zero total — this is the trick to use whenever the full symmetric set sums to zero. Phased-array antennas and multipole expansions both exploit this algebraic structure.

Final Answer: (a) 0; (b) $\mu_0 I/(2\pi R)$; (c) $\mu_0 I/(\pi R)$.

✗ Common Mistake

Beware of subtracting magnitudes instead of vectors. With A removed, some students answer "net field = (5-1) times $|\vec{B}_A|$". That's the algebra of scalars, not vectors. The correct subtraction is $\vec{B}_{\text{net}} - \vec{B}_A = 0 - \vec{B}_A$, giving magnitude $|\vec{B}_A|$, not $4|\vec{B}_A|$. Always think of vector fields as vectors, not as scalar contributions to be added arithmetically.

Key Takeaways

- Lorentz force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ is the foundation; magnetic forces are always perpendicular to $\vec{v}$ and do no work on the moving charge.
- In a uniform $\vec{B}$, a charged particle moves in a circle of radius $r = mv_\perp/(qB)$ with cyclotron frequency $\omega = qB/m$ – independent of speed.
- Field of a long straight wire: $B = \mu_0 I/(2\pi r)$. At the centre of a circular loop: $B =$

$\mu_0 I/(2R)$. From a quarter-arc: one-fourth of the loop value.

- Two parallel currents attract; anti-parallel currents repel. Force per unit length: $\mu_0 I_1 I_2/(2\pi d)$.
- Magnetic moment of a planar coil: $\vec{M} = NI\vec{A}$. Torque in a uniform field: $\vec{\tau} = \vec{M} \times \vec{B}$; potential energy $U = -\vec{M} \cdot \vec{B}$.
- A galvanometer becomes a voltmeter by adding a large series resistor; an ammeter by adding a small shunt in parallel. Multi-range meters use ladder networks of these resistors.

End of NCERT Exemplar Problems