Chapter 4.

Moving Charges & Magnetism

A moving charge produces a magnetic field ; any magnetic field exerts a force on another moving charge or current.

Oersted's Discovery (1820)

A compass needle placed near a current - carrying wire is deflected. So current produces a magnetic field around it.

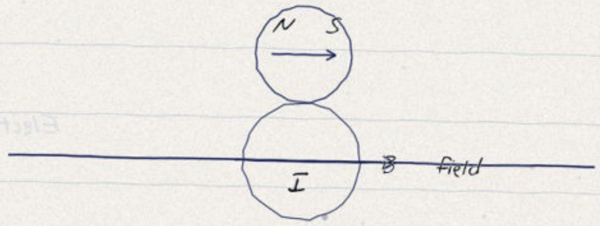


Fig. current deflects nearby compass needle

Key Ideas to come

1. Force on a moving charge : $F = q v \times B$
2. Force on a current wire : $F = I L \times B$
3. Field by a current : Biot - ~~Sevart~~ Savart , Ampere.
4. Devices : cyclotron , galvanometer.

Magnetic Force on a Moving Charge

A charge q moving with velocity v in a magnetic field B experiences a force F given by :

$$\boxed{F = q \; v \times B}$$ <- vector cross <- product

Magnitude : $F = q \, v \, B \sin \theta$

where theta = angle between v and B.

Properties

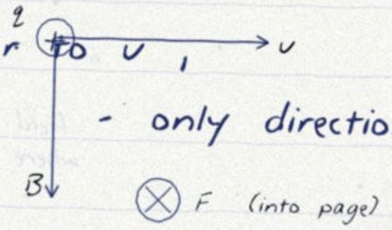
1. Maximum force when v perpendicular B :

   $F_{max} = q \, v \, B$ (theta = 90)

2. Zero force when v parallel B :

   $F = 0$ (theta = 0 or 180)

3. F is always perpendicular to v , so it cannot change v - only direction.



4. F is always perpendicular to B too.

Work done by magnetic force is always zero.
Magnetic force is a centripetal type force.

Lorentz Force & Units of B

Total electromagnetic force on a charge in both E and B fields :

$$\boxed{F = q(E + v \times B)}$$ <-Lorentz force <-(complete form)

Electric part : qE, parallel to E.

Magnetic part : $q v \times B$, perpendicular to v.

Unit of B

From $F = qvB$: $B = F/(qv)$

SI unit : $N/(C.m/s) = N/(A.m)$
$= 1$ tesla (T)

1 T = 1 weber / m 2 (very strong field).

Smaller unit : 1 gauss = 10 -4 T.

Earth's field 0.3 - 0.6 gauss 5 x 10 -5 T.

Right-hand rule for v x B

Point fingers along v ; curl them towards B.

Thumb gives $v \times B$: i.e. direction of F on +q.

If q is negative, force reverses direction.

(electron in B will feel opposite to $v \times B$.)

Force on a Current-Carrying Wire

Consider a wire of length L, area A, carrying current I (n free electrons per unit volume).

Total number of free electrons in wire $= n\,A\,L$.

Force on each electron $= e\, v_d \times B$.

Total force $= n\,A\,L \cdot e\,(v_d \times B)$

$= (n\,e\,A\,v_d) \cdot (L \times B)$

$= I\,(L \times B)$ [using $I = n\,e\,A\,v_d$]

$$\boxed{F = I\,(L \times B)}$$

<- L along I

<- (direction of I)

Magnitude : $F = B\,I\,L \sin\theta$

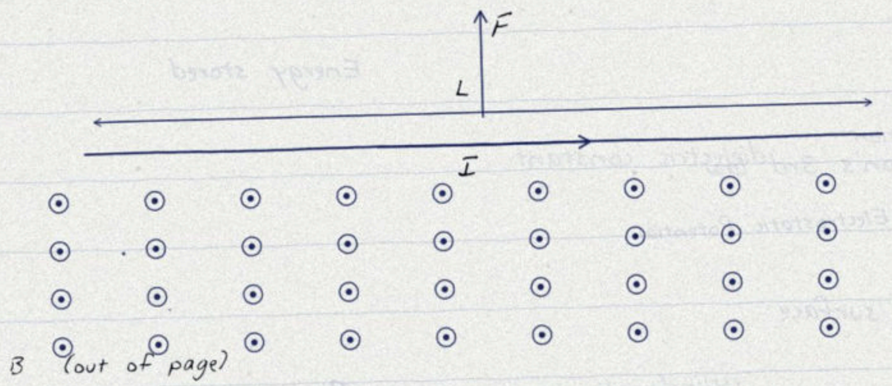


Fig. wire in B field experiences force $F = IL \times B$

Maximum F when wire perpendicular B ; zero when parallel.

Motion of a Charge in a Magnetic Field

v perpendicular B -> circular motion

Magnetic force $F = q v B$ always perpendicular to v -> acts as centripetal force.

$$q v B = m v^2 / r$$

$$\boxed{r = m v / (q B)}$$

<- radius of
<- circular path

Time period & frequency*

$$\boxed{T = 2 \pi m / (q B)}$$

<- T independent
<- of v and r !

Frequency : $f = 1 / T = q B / (2 \pi m)$

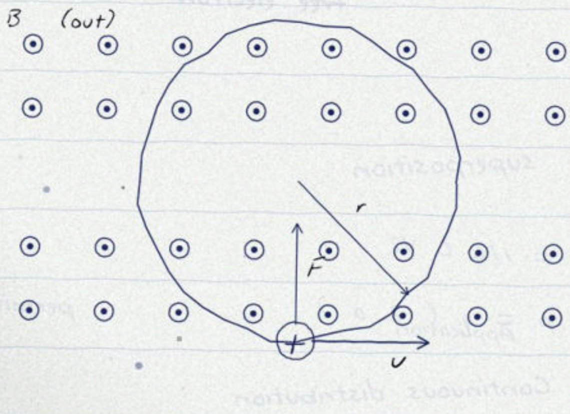


Fig. positive charge moving in B (out of page)

v at angle to B -> helical motion

Helical Motion & Velocity Selector

Helix : v not perpendicular to B

Split v into v_{para} (along B) and v_{perp} (perpendicular B). Force acts only on v_{perp}.

Result : particle spirals along B with circular radius r and forward speed v_{para}.

$$r = m\, v_{perp} / (q B)$$

$$\text{Pitch} = v_{para} \cdot T = 2 \pi m\, v_{para} / (q B)$$

Velocity Selector

Apply E and B perpendicular to each other and to v. For a charge to pass undeflected, electric and magnetic forces must cancel :

$$q E = q v B$$

$$\boxed{v = E / B}$$

<- selects one v
<- from a beam

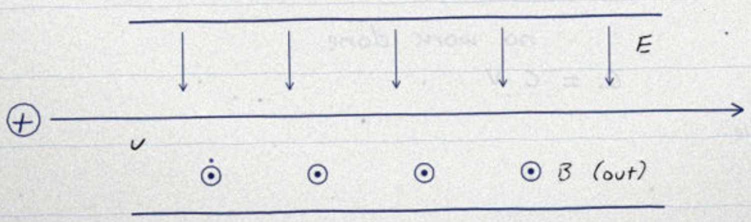


Fig. used in mass spectrometers (Thomson e/m).

Cyclotron

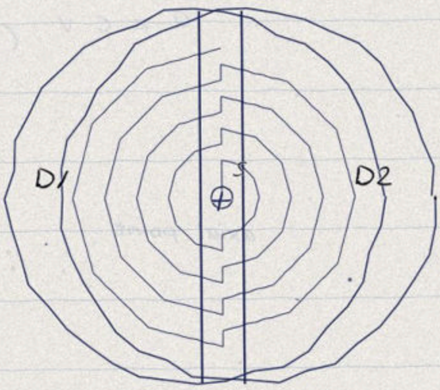
Device to accelerate positive ions to high energies using both E and B together.

Construction

(1) Two hollow D-shaped chambers (dees) placed in a strong perpendicular B.

(2) A high-frequency alternating E in the gap between the dees gives the ion a kick.

(3) Ion source S placed at the centre.



Cyclotron Frequency

$$\boxed{f_c = qB / (2 \pi m)}$$

Biot - Savart Law

Gives d B produced at a point P by a small current element I dl of a wire.

$$\boxed{d B = (\mu_0 / 4 . pi) . I \ d l \times \text{r-hat} / r^2}$$

Magnitude : $dB = (\mu_0 \ I \ dl \ \sin \theta) / (4 \ pi \ r^2)$

Direction

d B is perpendicular to the plane of d l and r ; right-hand rule decides the sense.

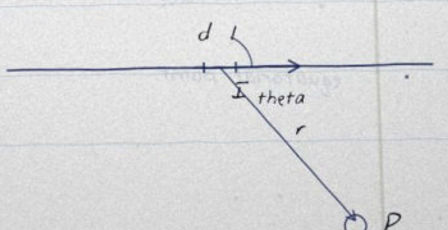
Special features

① dB = 0 if theta = 0 (along dl).

② dB max if theta = 90 (perp.).

③ Inverse square in r , like Coulomb's.

④ u_0 = 4 pi x 10 -7 T m / A.

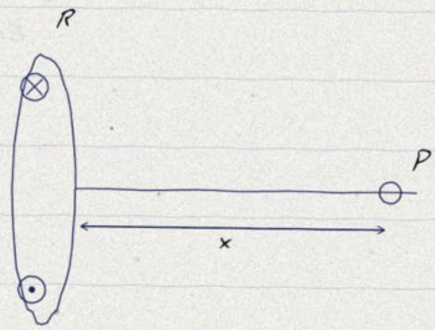


B on the Axis of a Circular Loop

Setup

Circular loop of radius R, current I.

Find B at point P on axis, distance x from 0.



Derivation (sketch)

Each element dl is perpendicular to r.

$$dB = (\mu_0 / 4 \pi)(I\, dl) / r^2 \quad \text{(with theta = 90)}$$

By symmetry only axial components add up :

$$dB_{axial} = dB \cos\theta = dB \cdot R / r$$

Integrate around full loop (length $2 \pi R$) :

$$\boxed{B = \mu_0 I R^2 / [\, 2 (R^2 + x^2)^{3/2} \,]}$$ <- axial

Direction along axis ; sense by right-hand rule
(curl fingers along I, thumb $= B$).

B at the Centre of a Loop

Setting $x = 0$ in the axial formula :

$$\boxed{B = \mu_0 I / (2R)}$$

<- centre of
<- circular loop

For N turns close together

$$\boxed{B = N \mu_0 I / (2R)}$$

<- coil with
<- N tight turns

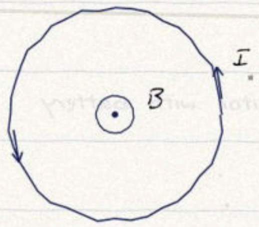


Fig. field at centre is perpendicular to the loop

B at the centre of an arc

Arc subtending angle phi at the centre :

$$\boxed{B = \mu_0 I \phi / (4 \pi R)}$$

<- phi in radians

Ampere's Circuital Law

Statement

Line integral of B around any closed loop equals μ_0 times the net current threading the loop.

$$\boxed{\int B \cdot dl = \mu_0 I_{enc}}$$

<- closed path
<- (amperian loo

Sign convention

Curl right-hand fingers along dl. Currents in the direction of thumb -> positive ; opposite -> negative.

When is it useful ?

1. Symmetric situations where B is constant along the chosen path.

2. B is parallel (or perp.) to dl along path.

Examples : long straight wire , solenoid , toroid , planar current sheets.

(In static cases Biot - Savart & Ampere agree.)

B due to a Long Straight Wire

Take a wire carrying current I along z - axis.
Find B at distance r from the wire.

Apply Ampere's Law

Choose circular amperian loop of radius r.
By symmetry B is constant on it, and B is along d l everywhere :

$$B \cdot (2 \pi r) = \mu_0 I$$

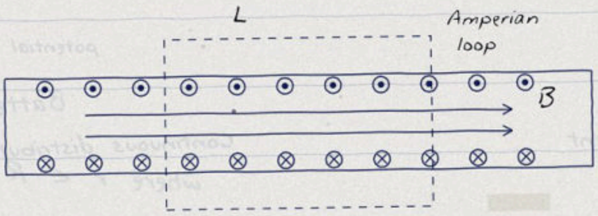
$$\boxed{B = \mu_0 I / (2 \pi r)}$$

<- B falls as 1/r
<- (inverse first power)

Direction : right-hand grip, thumb along I ;
fingers curl in sense of B.
For an ideal infinite straight wire.

Magnetic Field due to a Solenoid

Solenoid : long , tightly wound cylindrical coil with n turns per unit length.



Derivation

Outside solenoid (long , ideal) : $B = 0$.

Inside : B uniform and along axis.

Ampere's law on the rectangular loop :

$$B \cdot L + 0 + 0 + 0 = \mu_0 (n L) I$$

$$\boxed{B = \mu_0 \, n \, I}$$

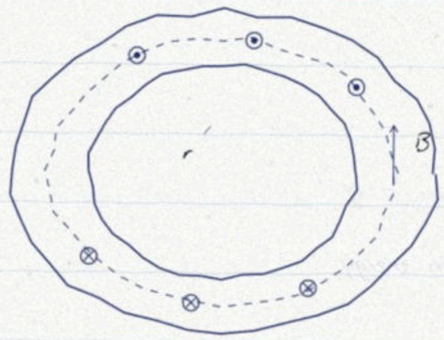
<- uniform B
<- inside solenoid

At one end of a long solenoid : B = u_0 n I / 2 .

(Half of the inside value - field lines diverge.)

Magnetic Field due to a Toroid

Toroid : a solenoid bent into a circle (a doughnut). Total turns = N, mean radius r.



Apply Ampere

On the amperian circle of radius r :

$$B \cdot (2 \pi r) = \mu_0 N I$$

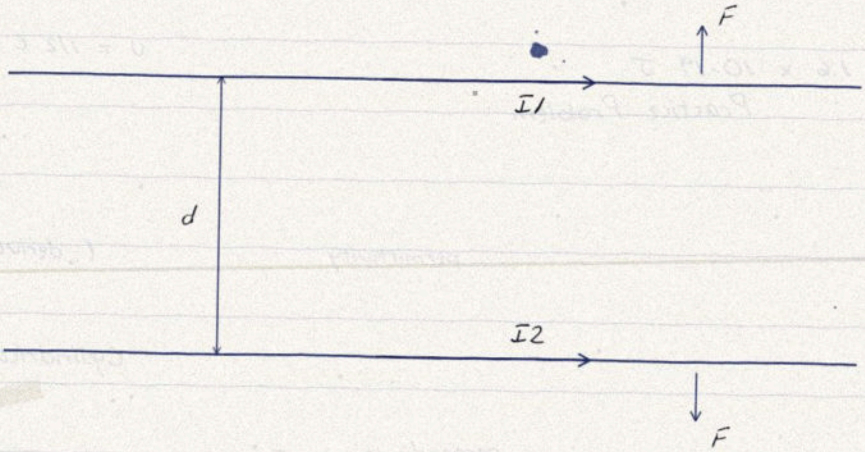
$$\boxed{B = \mu_0 N I / (2 \pi r)}$$ <- field inside <- the core only

B is zero outside (both inner & outer) ;
field lines are circles inside the core.

If $n = N / (2 \pi r)$, the formula matches the solenoid.

Force between Parallel Currents

Two long parallel wires carry currents I1 , I2 ;
they are separated by distance d.



Force per unit length

Wire 2 sits in field of wire 1 : B = u_O I1 / 2 pi d

Force on length L of wire 2 : F = I2 L B

$$\boxed{F / L = \mu_0 I_1 I_2 / (2 \text{ pi } d)}$$

<- attract if
<- I1, I2 sam

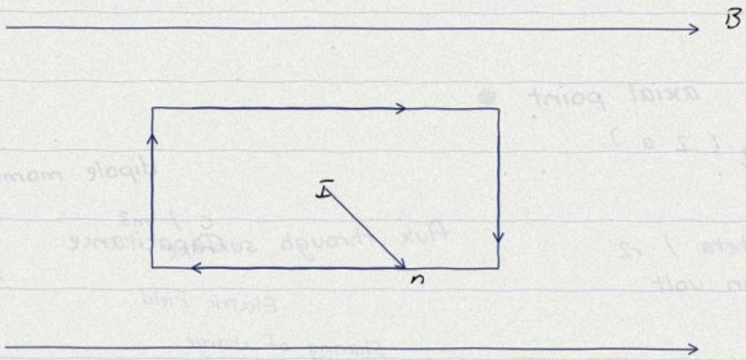
Parallel currents -> attract ;

Anti-parallel currents -> repel.

1 A = current that makes F/L = 2×10^{-7} N/m @ d = 1 m.

Torque on a Current Loop in B

Rectangular loop of sides a (along B) and b, current I, placed in uniform B with normal n making angle theta with B.



Torque

Forces on sides parallel to B cancel ; forces on perpendicular sides form a couple :

$$\boxed{T = N I A B \sin \theta}$$

<- T = torque
<- A = area , N = turns

Vector form : $T = N I A \times B = m \times B$

Stable equilibrium : theta = 0 . (normal · B).

Magnetic Dipole Moment

A current loop behaves like a magnetic dipole with moment :

$$\boxed{m = N I A}$$

<- magnetic dipole
<- moment (A m 2)

Direction : along the normal n , by right-hand rule (curl fingers along I , thumb = m).

Compact formulas

Torque : $T = m \times B$

PE : $U = - m : B$

(Identical to electric dipole formulas with m -> p, B -> E.)

Analogy with electric dipole

Electric	Magnetic
p (= $q \cdot 2a$)	m (= $N I A$)
$T = p \times E$	$T = m \times B$
$U = - p \cdot E$	$U = - m \cdot B$
$B_{axial} = (\mu_0 / 4 \text{ pi}) \, 2m / r^3$	point dipole

Use : a small current loop = magnetic dipole.
(very useful approximation far from the loop.)

Electron's Magnetic Moment

Orbiting electron model

Electron orbits nucleus with speed v in radius r.

Period $T = 2 \pi r / v$.

Current $I = e / T = e v / (2 \pi r)$

Area $A = \pi r^2$; $m = I A = e v r / 2$

$$\boxed{m = e v r / 2}$$

<- moment of
<- orbiting electron

Bohr Magneton

For the smallest orbit allowed by Bohr's quantisation, $m v r = h / (2 \pi)$:

$$\boxed{\mu_B = e \cdot h / (4 \pi m_e)}$$

<- Bohr magneton
<- 9.27×10^{-24} A m

Gyromagnetic ratio

Ratio of magnetic moment to orbital angular momentum :

$$m / L = e / (2 m_e) \quad \text{(orbital)}$$

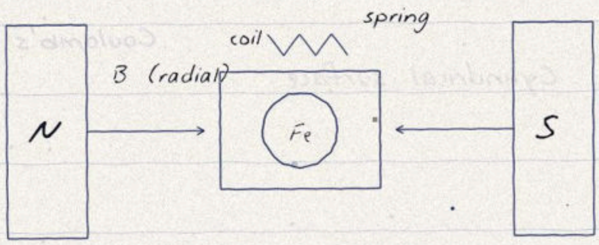
For spin, an extra factor of 2 (anomalous) appears.

Quantum-mechanical origin, no classical analogue.

Moving Coil Galvanometer

Principle

A current loop in a magnetic field experiences a torque - which a spring resists. Deflection of the coil measures the current.



Working

When current I flows : torque = N I A B.

Spring exerts restoring torque κ phi.

$$N I A B = \kappa \phi$$

$$\boxed{\phi = (N A B / \kappa) I}$$

<- phi I -> linear
<- scale on dial

Radial B (curved pole faces) -> always sin theta = 1 ;
ensures linear deflection.

Galvanometer Sensitivity

Current sensitivity

Deflection per unit current :

$$\boxed{\phi / I = N A B / \kappa}$$

<- units : rad / A
<- or div / A

Voltage sensitivity

Deflection per unit voltage :

$$\boxed{\phi / V = \phi / (I R_g)}$$

<- units : rad / V
<- R_g = coil resistance

$$\boxed{\phi / V = N A B / (\kappa R_g)}$$

How to make sensitive ?

1. Increase N , A , B ;

2. Decrease κ (use a soft spring) ;

3. For voltage sensitivity ,
also keep R_g small.

Note : S_i vs S_v

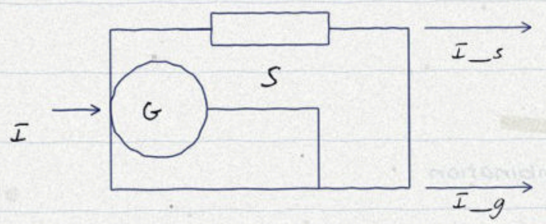
If you increase N , R_g rises too. So a more current-sensitive G may be less voltage-sensitive.

Galvanometer -> Ammeter

An ammeter measures current ; placed in series with circuit. It must have very LOW resistance (so it doesn't disturb the circuit).

Method : shunt in parallel

Connect a small resistance S in parallel with the galvanometer (G has resistance R_g).



Derivation

Same V across G and S :

$$I_g R_g = (I - I_g) S$$

$$\boxed{S = I_g R_g / (I - I_g)}$$

<- shunt to give
<- full-scale at I

Effective ammeter resistance : R_eq = R_g S / (R_g + S)

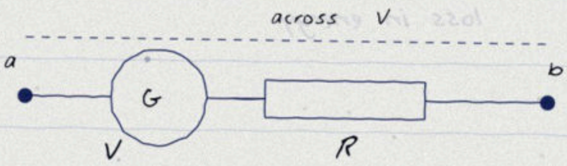
Since S << R_g , R_eq is very small (good).

Galvanometer -> Voltmeter

A voltmeter measures voltage ; placed in parallel with the element. It must have very HIGH resistance (so it draws negligible I).

Method : high R in series

Connect a large resistance R in series with G.



Derivation

Full-scale deflection current = I_g ;
max voltage to be measured = V :

$$V = I_g (R_g + R)$$

$$\boxed{R = V / I_g - R_g}$$

<- series R for
<- given range V

Ideal voltmeter : R = infinity (draws no I).

Practical : R chosen so I_g << I (less than 1 %).

Summary - Key Formulae

Forces

$F = q\, v \times B$; $F = q\,(E + v \times B)$

$F = I\,(L \times B)$; $F = B\, I\, L \sin\theta$

Motion in B

$r = m\, v / (q\, B)$; $T = 2\, pi\, m / (q\, B)$

$\text{Pitch} = v_{para} \cdot T$

Fields

Wire : $B = \mu_0\, I / (2\, pi\, r)$

Loop centre : $B = \mu_0\, I / (2\, R)$

Loop axis : $B = \mu_0\, I\, R^2 / [2(R^2 + x^2)^{3/2}]$

Solenoid : $B = \mu_0\, n\, I$

Toroid : $B = \mu_0\, N\, I / (2\, pi\, r)$

Parallel wires

$F / L = \mu_0\, I_1\, I_2 / (2\, pi\, d)$

Torque & dipole

$T = N\, I\, A\, B \sin\theta$; $m = N\, I\, A$

$T = m \times B$; $U = - m \cdot B$

Galvanometer

Ammeter : $S = I_g\, R_g / (I - I_g)$. Voltmeter : $R = V / I_g - R_g$