Collegedunia NCERT Notes

The Ultimate NCERT Revision Guide for Class 12 Physics

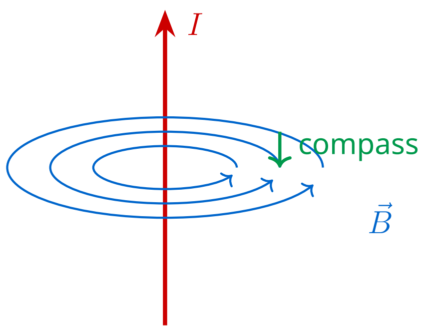
Chapter 4: Moving Charges and Magnetism

1 Magnetic Field and the Lorentz Force

In the previous chapters we treated charges as static and let them push and pull each other through electric fields. The moment a charge starts moving, something new happens: it produces, and is influenced by, a magnetic field. The bridge between electricity and magnetism was discovered by Hans Christian Oersted in 1820, when he noticed that a compass needle deflected near a current-carrying wire. That single observation changed physics — electricity and magnetism turned out to be two faces of the same phenomenon.

1.1 Oersted's experiment and the magnetic field

When a current flows through a straight wire, a compass placed near it does not point north any more. It aligns tangentially to a circle drawn around the wire. Reverse the current and the compass flips through $180°$. The conclusion is direct: **a steady electric current produces a magnetic field around it**, with field lines forming closed circles concentric with the wire.



Magnetic field around a current-carrying wire

The direction of the field is given by the **right-hand thumb rule**: point your right thumb in the direction of the conventional current; your curled fingers give the direction of $\vec{B}$ along the field line.

Two new symbols

A field or current pointing *out of the page* is shown as a dot ($\odot$), like the tip of an arrow coming toward you. A field or current pointing *into the page* is shown as a cross ($\otimes$), like the tail feathers of an arrow going away. These symbols are everywhere in this chapter — learn them on day one.

1.2 The Lorentz force

Once the magnetic field exists, a moving charge in it experiences a force. If the charge q moves with velocity $\vec{v}$ in a region with electric field $\vec{E}$ and magnetic field $\vec{B}$, the total electromagnetic force on it is:

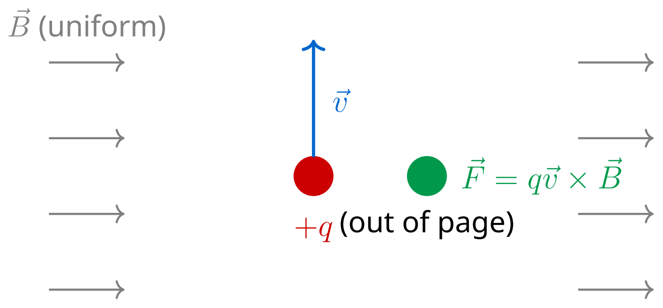
Lorentz Force

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$

The first piece is the familiar electric force; the second, $q(\vec{v} \times \vec{B})$, is the magnetic force. **SI unit:** newton (N).

The magnetic part is what makes this chapter different from the last. Three features of $\vec{F}_{\text{mag}} = q(\vec{v} \times \vec{B})$ deserve attention:

- **It depends on velocity.** A charge at rest feels no magnetic force, no matter how strong $\vec{B}$ is. This is the simplest distinction from the electrostatic force.
- **It is perpendicular to both $\vec{v}$ and $\vec{B}$.** The cross product guarantees $\vec{F} \perp \vec{v}$, which has a remarkable consequence: *the magnetic force does no work*. Hence the speed of a charged particle in a pure magnetic field never changes — only its direction.
- **Its magnitude is $F = qvB\sin\theta$**, where θ is the angle between $\vec{v}$ and $\vec{B}$. The force is maximum when $\vec{v} \perp \vec{B}$ and zero when $\vec{v} \parallel \vec{B}$ (the charge sails through unaffected).



1.3 Direction by the right-hand rule

The cross product $\vec{v}\times\vec{B}$ has a direction; the right-hand rule pins it down. Stretch the fingers of your right hand in the direction of $\vec{v}$ and curl them toward $\vec{B}$ through the smaller angle. Your thumb points in the direction of $\vec{v}\times\vec{B}$. For a positive charge, this is also the direction of $\vec{F}$. For a negative charge (like an electron), the force points the opposite way.

Memory Aid

"FBI" hand rule. For positive charges, point your right hand: **F**irst finger = **B** (field), **M**iddle finger = current/velocity, **T**humb = **F**orce. The three are mutually perpendicular. Check the geometry on every problem before plugging numbers — the sign of the answer depends on it.

1.4 The unit of magnetic field

From the Lorentz formula, when $\vec{v}\perp\vec{B}$:

$$B = \frac{F}{qv}$$

The SI unit of B is the **tesla (T)**: $1\text{ T} = 1\text{ N}/(\text{A}\cdot\text{m}) = 1\text{ kg}/(\text{A}\cdot\text{s}^2)$.

A non-SI unit still common in textbooks is the **gauss (G)**: $1\text{ G} = 10^{-4}\text{ T}$. Earth's magnetic field at the surface is about $0.5\text{ G} = 5\times10^{-5}\text{ T}$, while a strong neodymium magnet can produce ~ 1 T at its surface, and an MRI machine ~ 1.5–3 T.

Real-World Application

The 1.5 T field inside an MRI machine is roughly $30{,}000$ times stronger than Earth's natural field. The same Lorentz physics that we use for electron beams underpins the imaging — protons in body tissue precess in this strong field, and their relaxation signals build up the medical image.

Quick Tip

If the question asks for the speed of a charged particle to remain unchanged in a region, check whether only $\vec{B}$ is present (no $\vec{E}$). If so, $\vec{B}$ alone can never change the particle's kinetic energy; it can only steer it.

2 Motion of a Charged Particle in a Magnetic Field

Because the magnetic force is always perpendicular to the velocity, charged particles in magnetic fields trace very specific shapes: circles when the velocity is fully transverse to the field, and helices when it has a component along the field too. These two cases cover almost every particle-trajectory problem in the chapter.

2.1 Circular motion ($\vec{v} \perp \vec{B}$)

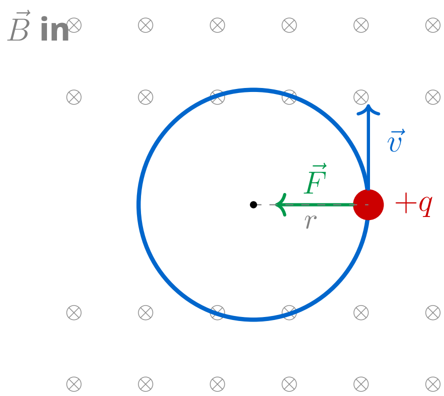
Consider a particle of charge q and mass m entering a uniform field $\vec{B}$ with velocity $\vec{v}$ exactly perpendicular to $\vec{B}$. The magnetic force qvB acts perpendicular to $\vec{v}$ at every instant. A force always perpendicular to motion is precisely the centripetal condition — the particle moves in a circle of radius r:

$$qvB = \frac{mv^2}{r} \implies r = \frac{mv}{qB}$$

Radius, period, and frequency of circular motion

$$r = \frac{mv}{qB}, \qquad T = \frac{2\pi m}{qB}, \qquad f = \frac{qB}{2\pi m}$$

The frequency f is called the **cyclotron frequency**. Notice that T and f depend only on m/q and B — *not on the speed* v.



Why is the period independent of speed? A faster particle has more momentum but follows a proportionally larger circle, so the time per revolution is unchanged. This curious property is what makes accelerator design (like the cyclotron) possible in principle.

Cyclotron frequency ω_c

The angular frequency of the circular motion is

$$\omega_c = \frac{qB}{m}$$

A typical exam question gives you B and the particle (e, proton, α) and asks for ω_c or T. Memorise the form, not the numbers.

2.2 Helical motion ($\vec{v}$ has both $\parallel$ and $\perp$ components)

What if the velocity makes an angle θ with $\vec{B}$ that is neither $0°$ nor $90°$? Decompose $\vec{v}$ into a component $v_{\parallel} = v\cos\theta$ along $\vec{B}$ and $v_{\perp} = v\sin\theta$ perpendicular to $\vec{B}$.

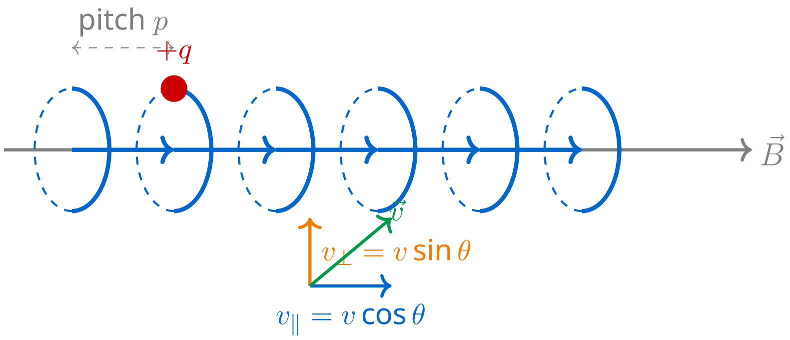
- The parallel component feels no magnetic force — it carries the particle steadily along the field direction.
- The perpendicular component drives circular motion in the plane perpendicular to $\vec{B}$.

The combined motion is a **helix**: a circle wrapped around an axis parallel to $\vec{B}$, drifting steadily along that axis.

Pitch and radius of the helix

$$r = \frac{mv_\perp}{qB} = \frac{mv\sin\theta}{qB}, \qquad p = v_\parallel \cdot T = \frac{2\pi m\, v\cos\theta}{qB}$$

The **pitch** p is the axial distance covered in one full revolution.



Helical motion: circular loop drifting along $\vec{B}$

Real-World Application

Charged particles from the solar wind spiral down Earth's magnetic field lines toward the magnetic poles, where they collide with the upper atmosphere and produce auroras — the aurora borealis in the north and aurora australis in the south. The helical-motion math we just wrote is exactly what governs their descent.

Common Mistake

For helical motion, students often substitute the full speed v when computing r instead of $v_\perp = v\sin\theta$. Always project the velocity onto the directions $\parallel$ and $\perp$ to $\vec{B}$ first — only $v_\perp$ produces the circular part.

3 Magnetic Force on a Current-Carrying Conductor

A current is just a stream of moving charges, so a current-carrying wire placed in a magnetic field must feel a force — the Lorentz forces on each individual charge add up to a force on the wire as a whole.

3.1 Force on a straight conductor

Consider a straight wire of length ℓ carrying current I in a uniform field $\vec{B}$. Each free electron drifts with average velocity $\vec{v}_d$. Summing $\vec{F} = q(\vec{v}_d \times \vec{B})$ over all carriers in the wire gives a clean result in terms of macroscopic quantities:

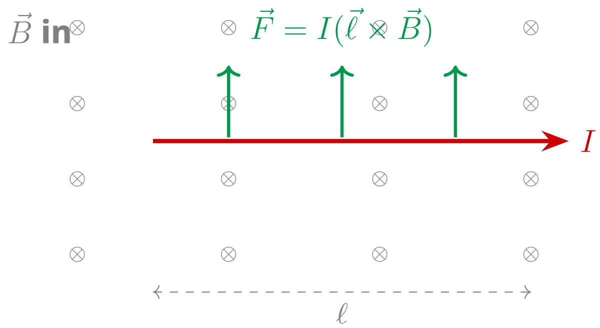
Force on a current-carrying conductor

$$\vec{F} = I(\vec{\ell} \times \vec{B}), \qquad |F| = BI\ell \sin\theta$$

where $\vec{\ell}$ is a vector with magnitude equal to the wire's length and direction along the conventional current. θ is the angle between $\vec{\ell}$ and $\vec{B}$.

The direction of $\vec{F}$ is given by Fleming's left-hand rule (for conventional current direction): align the forefinger with $\vec{B}$, the middle finger with $\vec{I}$ — the thumb shows $\vec{F}$. Special cases:

- $\theta = 90°$: maximum force $F = BI\ell$.
- $\theta = 0°$ or $180°$: zero force — a wire carrying current parallel to the field feels nothing.



Brief derivation. If the wire has cross-section A and free-electron density n, each electron contributes $\vec{F}_1 = -e(\vec{v}_d \times \vec{B})$. The total number of carriers in length ℓ is $nA\ell$, so

$$\vec{F}_{\text{total}} = nA\ell \cdot (-e)(\vec{v}_d \times \vec{B}) = (nAev_d)(\hat{\ell}) \times \vec{B} \cdot \ell = I(\vec{\ell} \times \vec{B})$$

using $I = nAev_d$. Notice how the microscopic ingredients (n, v_d, e) reorganise into the single macroscopic quantity I.

Force is what current carries to the field

A wire with no current feels no magnetic force (even in a strong field). Switch the current on, and the same wire suddenly experiences a sideways push. This is the principle behind every electric motor — the wire's translational push gets converted into rotational motion via clever geometry.

Quick Tip

For a wire bent into an arbitrary shape in a *uniform* magnetic field, the net

> force depends only on the straight-line vector from one end to the other — not on the actual path taken. Replace the curve with a straight wire from start to end and compute $I\vec{\ell} \times \vec{B}$. This trick saves a lot of integration.

4 Biot–Savart Law and Its Applications

So far we have computed forces *due to* a given magnetic field. Now the inverse: given a current, what field does it produce? The Biot–Savart law is the magnetic analogue of Coulomb's law — it tells you the contribution to $\vec{B}$ from each tiny element of a current.

4.1 Statement of the Biot–Savart law

For a small element $d\vec{\ell}$ of a wire carrying current I, the magnetic field it produces at a point P at position vector $\vec{r}$ from the element is

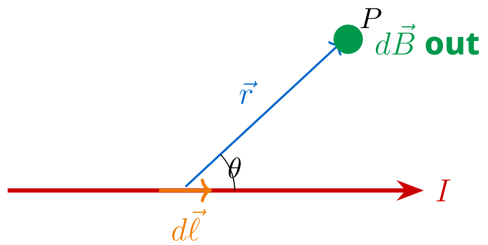
Biot–Savart Law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\ell} \times \hat{r}}{r^2}$$

where $\mu_0 = 4\pi \times 10^{-7}$ T m/A is the **permeability of free space**, and $\hat{r}$ is the unit vector from the element to P. **Magnitude:** $dB = \frac{\mu_0}{4\pi}\frac{I\,d\ell\sin\theta}{r^2}$.

The total field at P is found by integrating $d\vec{B}$ over the entire wire. Three features parallel Coulomb's law nicely:

- It is an **inverse-square law in** r.
- It obeys **superposition**: contributions from different elements add as vectors.
- It involves a **cross product** (unlike Coulomb's law) — so $d\vec{B}$ is perpendicular to the plane containing $d\vec{\ell}$ and $\vec{r}$.



4.2 Field due to a long straight wire

For an infinitely long straight wire carrying current I, integrating $d\vec{B}$ from each element along the wire gives a field at perpendicular distance a:

Field of a long straight wire

$$B = \frac{\mu_0 I}{2\pi a}$$

The field forms concentric circles around the wire. **Direction:** given by the right-hand thumb rule.

The result $B \propto 1/a$ shows that the field falls off more slowly than the $1/r^2$ field of a point charge — the same dimensional pattern we saw for the electric field of an infinite line of charge.

4.3 Field at the centre of a circular loop

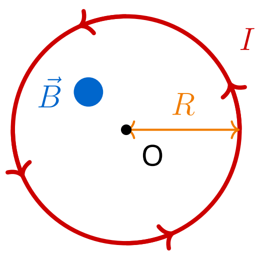
Consider a circular loop of radius R carrying current I. By symmetry, every element $d\vec{\ell}$ is perpendicular to its position vector $\hat{r}$ from the centre, so $d\vec{\ell} \times \hat{r} = d\ell$ in magnitude. The contributions from all elements point in the same direction (along the axis through the centre), so they simply add:

Field at the centre of a circular loop

$$B_{\text{centre}} = \frac{\mu_0 I}{2R} \quad \text{(single turn)}, \qquad B = \frac{\mu_0 N I}{2R} \quad (N \text{ turns})$$

Direction: along the axis of the loop, given by the right-hand curl rule.

Right-hand curl rule for a loop: curl the fingers of your right hand in the direction of the conventional current; your thumb points in the direction of $\vec{B}$ on the axis at the centre.



Top-down view: $\vec{B}$ at centre is out of the page

4.4 Field on the axis of a circular loop [JEE/NEET extension]

For a point on the axis of the loop at a distance x from the centre, the symmetry argument generalises. The components perpendicular to the axis cancel in pairs; only the axial components add:

Field on the axis of a circular loop

$$B_{\text{axis}}(x) = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

At $x = 0$, this reduces to $B = \mu_0 I/(2R)$. For $x \gg R$, $B \approx \mu_0 I R^2/(2x^3)$ — a $1/x^3$ falloff characteristic of a magnetic dipole.

Quick Tip

The far-field behaviour $B \propto 1/x^3$ identifies a circular loop as a **magnetic dipole** with moment $m = IA = I\pi R^2$ (covered in detail in Chapter 5). This is one of the deepest connections in classical electromagnetism: a tiny current loop is, magnetically, what a pair of $\pm q$ charges is electrically.

A common useful sub-result

For a **semicircular** arc of radius R at its centre, $B = \mu_0 I/(4R)$ (half of the full-loop value). For a **quarter** circle, $B = \mu_0 I/(8R)$, and so on. For an arc subtending angle ϕ (in radians) at the centre, $B = \mu_0 I\phi/(4\pi R)$.

Common Mistake

A frequent slip: students take the loop's centre-field formula $B = \mu_0 I/(2R)$ and use it for a *straight* wire by writing $B = \mu_0 I/(2\pi a)$ — forgetting the factor of π in the denominator. Memorise the two distinctly: *loop centre $\to$ no π; straight wire $\to$ π in denominator.*

5 Ampere's Circuital Law and Its Applications

The Biot–Savart law works in principle for any geometry, but for high-symmetry problems the integrals can be cumbersome. Ampere's law plays the same role in magnetostatics that Gauss's law plays in electrostatics: a powerful symmetry-based shortcut.

5.1 Statement of Ampere's law

Ampere's Circuital Law

The line integral of $\vec{B}$ around any closed loop in vacuum equals μ_0 times the net current enclosed:

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$$

The closed loop C is called an **Amperian loop**.

A few points to internalise:

- It is true for *any* closed loop, but it is useful only when the symmetry lets you pull B out of the integral.
- I_{enc} is the *net* current threading the loop, with directions counted via the right-hand rule applied to the loop's orientation.
- Currents *outside* the loop contribute zero to the line integral (they may contribute to $\vec{B}$ at points along C, but the integral closes back to zero).
- It is equivalent in content to the Biot–Savart law for steady currents, just as Gauss's law is equivalent to Coulomb's.

5.2 Field of a long straight wire (using Ampere's law)

By symmetry, $\vec{B}$ around a long straight wire is tangent to circles concentric with the wire and has the same magnitude on each circle. Pick a circular Amperian loop of radius a:

$$\oint \vec{B} \cdot d\vec{\ell} = B(2\pi a) = \mu_0 I \implies B = \frac{\mu_0 I}{2\pi a}$$

This recovers the Biot–Savart result with one line of work — a strong illustration of the law's power.

5.3 Field inside a long solenoid (qualitative + quantitative)

A solenoid is a tightly wound helical coil of wire. For a solenoid that is long compared to its diameter, the field inside is essentially **uniform along the axis** and very nearly zero outside. Pick a rectangular Amperian loop with one side L inside the solenoid (parallel to the axis) and the parallel side outside.

- Along the inside side: $\vec{B} \cdot d\vec{\ell} = BL$.
- Along the outside side: $\vec{B} \approx 0$, contribution is zero.
- Along the two short transverse sides: $\vec{B} \perp d\vec{\ell}$, contribution is zero.

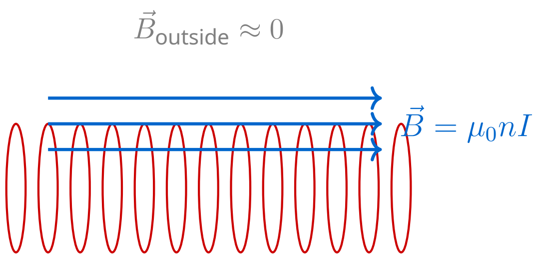
If n is the number of turns per unit length, the enclosed current is $I_{\text{enc}} = nLI$. So

$$BL = \mu_0 (nLI) \implies B = \mu_0 n I$$

Field inside a long solenoid

$$B = \mu_0 n I$$

where n = turns per unit length, I = current. The field inside is **uniform** and depends only on n and I — not on the solenoid's radius or its length (as long as it is ”long”).



Long solenoid: uniform field inside, near-zero outside

Real-World Application

The MRI scanner mentioned earlier is essentially a giant superconducting solenoid. Tens of kilometres of niobium-titanium wire are wound into a tube the size of a person, and a current of hundreds of amperes flows through it without resistance, producing the ~ 1.5 T uniform field that makes medical imaging possible.

Quick Tip

For exam problems with a long solenoid, B inside is uniform — so the force on a charge or wire placed inside the solenoid uses just one constant value of B, regardless of where in the solenoid the object sits. Don't try to integrate.

5.4 Biot–Savart vs Ampere — when to use which

Both laws give $\vec{B}$ from a current. They are not different physics; they are different tools.

Biot–Savart law	Ampere's circuital law
Differential form: gives $d\vec{B}$ from each current element.	Integral form: relates $\oint \vec{B} \cdot d\vec{\ell}$ to I_{enc}.
Always works, but may need messy integration.	Works easily only when the field has high symmetry.
Best for: **finite-length** wires, arcs, partial loops, off-axis fields.	Best for: **infinite or symmetric** wires, solenoids, toroids.

The two laws are equivalent for steady currents — they are related the way Coulomb's law and Gauss's law are in electrostatics. Choose Biot–Savart when the geometry is awkward; choose Ampere when symmetry lets you pull B outside the integral.

5.5 Electrostatics vs Magnetostatics — the parallels

The chapter sits inside a larger pattern that runs through all of electromagnetism. Side by side:

Aspect	Electrostatics	Magnetostatics
Source of field	Static charges	Steady currents (moving charges)
Force on a test charge	$\vec{F} = q\vec{E}$	$\vec{F} = q\vec{v} \times \vec{B}$
Falloff with distance (point/wire)	$1/r^2$ point, $1/r$ line	$1/r^2$ none, $1/r$ wire
Integral law	Gauss: $\oint \vec{E} \cdot d\vec{A} = q/\varepsilon_0$	Ampere: $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$
Field lines	Open: start on $+$, end on $-$	Closed loops, no start/end
Work done on a charge	$\vec{F} \cdot d\vec{r} \neq 0$ in general	Magnetic force does **zero work**

The deep symmetry

Electric field lines never close on themselves; magnetic field lines always do. This is the geometric statement of ”no isolated magnetic poles” — there are no magnetic monopoles to terminate the lines on. Maxwell’s equations encode this as $\nabla \cdot \vec{B} = 0$.

6 Forces Between Parallel Currents and Torque on Loops

Now we combine the two previous strands. A current creates a field; a second current placed in that field feels a force. This is the cleanest demonstration that a moving charge can exert a magnetic force on another moving charge — and it gives the modern definition of the ampere.

6.1 Force between two long parallel wires

Consider two long, straight, parallel wires separated by a distance d, carrying currents I_1 and I_2. Wire 1 produces, at the location of wire 2, a field

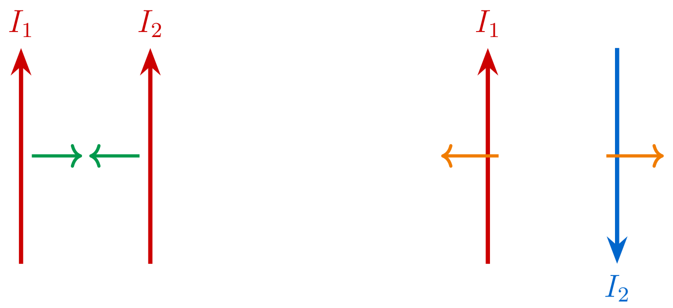
$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

The force on a length ℓ of wire 2 placed in this field is $F_2 = B_1 I_2 \ell$. Hence the force per unit length:

Force per unit length between parallel wires

$$\frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

Direction: *Parallel currents attract; anti-parallel currents repel.*



Parallel: attractive **Anti-parallel: repulsive**

Direction logic: If both currents go up (parallel), the field of wire 1 at wire 2's location points into the page, and $\vec{F}_2 = I_2(\vec{\ell} \times \vec{B}_1)$ comes out toward wire 1 — so they pull together. Reverse one current, the field reverses, and the force becomes repulsive.

6.2 Definition of the ampere

The historical SI definition of one ampere came directly from this formula. *One ampere is the steady current that, when flowing through two infinitely long parallel wires placed* 1 *m apart in vacuum, produces a force of* 2×10^{-7} *N per metre of length on each wire.* (Modern SI now defines the ampere via a fixed value of the elementary charge, but this older operational definition still anchors the physical intuition.)

Why the strange numerical value?

Substitute $I_1 = I_2 = 1$ A, $d = 1$ m, $\mu_0 = 4\pi \times 10^{-7}$ in the force formula:

$$\frac{F}{\ell} = \frac{(4\pi \times 10^{-7})(1)(1)}{2\pi(1)} = 2 \times 10^{-7} \text{ N/m}$$

The "ugly" 2×10^{-7} falls out naturally from $\mu_0/(2\pi)$.

6.3 Torque on a current loop in a uniform field

Place a rectangular loop of wire (sides a and b, so area $A = ab$) carrying current I in a uniform field $\vec{B}$. Let $\hat{n}$ be the unit vector normal to the loop's plane (right-hand curl rule with the current). The angle between $\hat{n}$ and $\vec{B}$ is θ.

- Forces on the two sides of length a (perpendicular to $\vec{B}$) are equal and opposite, but offset — they form a couple producing torque.
- Forces on the two sides of length b either cancel or stretch the loop, but contribute zero torque about the rotation axis.

The magnitude of the torque is

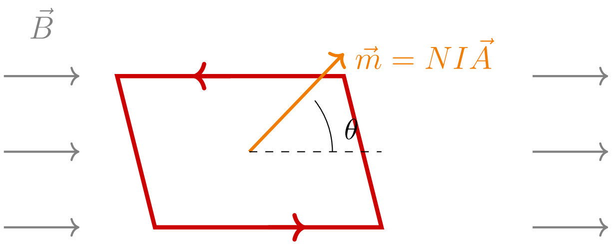
Torque on a current loop

$$\tau = NIAB\sin\theta = mB\sin\theta$$

where $\vec{m} = NI\vec{A}$ is the **magnetic dipole moment** of the loop (N turns), and the vector form is

$$\vec{\tau} = \vec{m} \times \vec{B}$$

The direction of $\hat{n}$ (and hence of $\vec{m}$) is determined by curling the right-hand fingers along the current; the thumb gives the direction of $\vec{m}$.



6.4 Stable and unstable equilibria

The torque tries to align $\vec{m}$ with $\vec{B}$. From $\vec{\tau} = \vec{m} \times \vec{B}$:

- $\theta = 0$ ($\vec{m} \parallel \vec{B}$): $\tau = 0$, **stable equilibrium**. A small perturbation generates a restoring torque.
- $\theta = \pi$ ($\vec{m}$ anti-parallel to $\vec{B}$): $\tau = 0$, but **unstable equilibrium**. A small perturbation grows.
- $\theta = \pi/2$: torque is maximum, $\tau = mB$.

Memory Aid

"Same direction = same energy minimum": just like an electric dipole tries to align $\vec{p}$ with $\vec{E}$, a current loop tries to align $\vec{m}$ with $\vec{B}$. The two stories run in perfect parallel — only the names of the dipole moment and the field change.

6.5 Comparison: electric and magnetic dipoles

Electric dipole	Magnetic dipole (current loop)
$\vec{p} = q\,\vec{d}$ (charge $\times$ separation)	$\vec{m} = NI\vec{A}$ (current $\times$ area)
Field at large distance $\propto 1/r^3$	Field at large distance $\propto 1/r^3$
$\vec{\tau} = \vec{p} \times \vec{E}$	$\vec{\tau} = \vec{m} \times \vec{B}$
Aligns with $\vec{E}$	Aligns with $\vec{B}$

Real-World Application

Every electric motor exploits the loop torque. A coil carrying current sits in a permanent magnet's field; it tries to rotate to align $\vec{m}$ with $\vec{B}$. A clever switching mechanism (a *commutator*) reverses the current at the right moment, so the coil never reaches alignment and keeps rotating. The maximum-torque condition $\theta = \pi/2$ is exactly when this kick is most useful.

7 The Moving Coil Galvanometer

The moving coil galvanometer (MCG) is a direct application of the loop-torque idea. Built right, it lets us measure tiny currents — and with two simple resistor add-ons, the same instrument becomes either an ammeter (large currents) or a voltmeter.

7.1 Construction and principle

A rectangular coil of N turns, carrying the current to be measured, is suspended in the field of a permanent magnet shaped to keep $\vec{B}$ *radial* at every position of the coil. A spring provides a restoring torque proportional to the deflection angle ϕ. A small mirror or pointer attached to the coil amplifies the deflection.

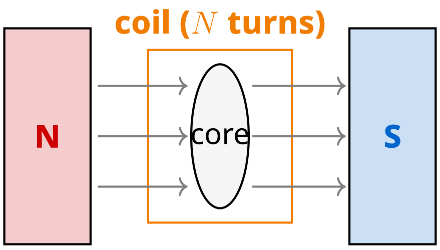
The radial-field design ensures that the coil's plane is always parallel to $\vec{B}$, so the deflecting torque is always at its maximum value:

$$\tau_{\text{def}} = NIAB$$

At equilibrium, the deflecting torque equals the restoring torque from the spring, $\tau_{\text{spring}} = k\phi$, where k is the torsional constant. So

$$NIAB = k\phi \implies \phi = \left(\frac{NAB}{k}\right) I$$

The deflection ϕ is *linear* in the current I — the most desirable feature of any measuring instrument.



Radial $\vec{B}$ keeps the coil's plane always parallel to $\vec{B}$

7.2 Sensitivities

Two figures of merit characterise a galvanometer:

Galvanometer sensitivities

Current sensitivity: $\frac{\phi}{I} = \frac{NAB}{k}$ (deflection per unit current)

Voltage sensitivity: $\frac{\phi}{V} = \frac{NAB}{kR_g}$ (deflection per unit voltage)

where R_g is the galvanometer's coil resistance.

To make a galvanometer more sensitive, one increases N, A, or B, or reduces k. There are practical limits: too many turns make R_g large (lowering voltage sensitivity), and too soft a spring makes the instrument fragile.

Common Mistake

Increasing the number of turns N increases current sensitivity *but does not necessarily increase voltage sensitivity* — R_g rises with N too. So an instrument with high current sensitivity may not have correspondingly high voltage sensitivity. A board-favourite trap.

7.3 Conversion to an ammeter (low-resistance shunt)

A galvanometer can typically take only a tiny full-scale current (microamps to a few milliamps). To measure currents much larger than this, we connect a very small resistance r_s — the **shunt** — in parallel with the galvanometer coil.

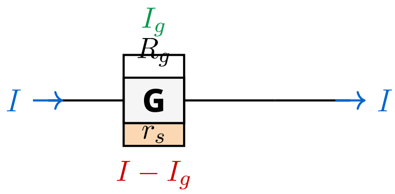
If full-scale current is I_g (through the galvanometer alone), and we want the meter to read up to total current I:

$$I_g R_g = (I - I_g)\, r_s \implies r_s = \frac{I_g R_g}{I - I_g}$$

Shunt resistance for ammeter conversion

$$r_s = \frac{I_g R_g}{I - I_g}, \qquad R_{\text{ammeter}} = \frac{R_g\, r_s}{R_g + r_s}$$

The ammeter's effective resistance is the parallel combination, which is even smaller than r_s. A good ammeter has resistance close to zero so it doesn't disturb the circuit.



Galvanometer + shunt r_s = ammeter

7.4 Conversion to a voltmeter (high-resistance series)

To measure voltage instead, we want the meter to draw very little current (so it doesn't load the circuit). We add a **large resistance** R in *series* with the galvanometer. For a voltage range V at full scale:

$$V = I_g(R_g + R) \implies R = \frac{V}{I_g} - R_g$$

Series resistance for voltmeter conversion

$$R = \frac{V}{I_g} - R_g, \qquad R_{\text{voltmeter}} = R_g + R \approx R$$

The voltmeter's effective resistance is large (close to R), which is desirable — it draws negligible current from the circuit.

	Ammeter	Voltmeter
Modification	Add small shunt r_s in parallel	Add large R in series
Effective resistance	Very low (near zero)	Very high (near R)
Connected in circuit as	In series with the load	In parallel with the load
What it measures	Current through the load	Voltage across the load

Quick Tip

Ideal ammeter has zero resistance; ideal voltmeter has infinite resistance. If a problem says "neglect meter resistance" or "use an ideal voltmeter", this is exactly what's meant. Any deviation from these ideals introduces a small systematic error in the measurement.

Real-World Application

Modern digital multimeters use the same idea but replace the moving coil with electronic comparators and an A/D converter. Underneath the digital display, the same shunt and series-resistor network sets the current/voltage range you select with the dial.

Memory Aid

"Ammeter Across with shunt" is wrong — and that's the point. **Ammeter $\rightarrow$ in series, low R. Voltmeter $\rightarrow$ in parallel, high R.** If you swap them, the ammeter (low resistance) would short-circuit the voltage you wanted to measure, and the voltmeter (high resistance) would read almost no current.

8 Problem-Solving Patterns

This chapter has roughly six recurring problem types. Recognising the type from the wording shaves real time off an exam.

8.1 Pattern 1: Charged particle in $\vec{B}$ alone

Tells: "An electron / proton / α-particle enters a uniform magnetic field with speed v..." Asks for radius, period, frequency, kinetic energy, or pitch.

Steps: (i) Identify whether $\vec{v}$ is $\perp$ or at an angle to $\vec{B}$. (ii) If $\perp$, use $r = mv/(qB)$, $T = 2\pi m/(qB)$. (iii) If at angle θ, decompose into $v_\perp = v\sin\theta$ and $v_\parallel = v\cos\theta$; use $v_\perp$ for the radius, $v_\parallel$ for the pitch contribution. (iv) Remember $\vec{B}$ does no work, so $|v|$ stays fixed.

8.2 Pattern 2: Charged particle in $\vec{E}$ and $\vec{B}$ both

Tells: "...in mutually perpendicular electric and magnetic fields..."

Steps: Write the two forces: $\vec{F}_E = q\vec{E}$ and $\vec{F}_B = q\vec{v} \times \vec{B}$. For the particle to pass undeviated, the two must cancel: $|\vec{F}_E| = |\vec{F}_B|$ gives $v = E/B$. This is the principle of a velocity selector.

8.3 Pattern 3: Field at a point due to a current geometry

Tells: "Find $\vec{B}$ at point P due to..." some specific shape (straight segment, full loop, semicircle, polygon).

Steps: (i) If the shape has high symmetry (infinite wire, solenoid, full loop), use the standard formula directly. (ii) Otherwise, break the shape into segments; compute each contribution; add as vectors. (iii) Watch for segments where $d\vec{\ell} \parallel \hat{r}$ — they contribute zero (e.g., the straight portions of a "U"-shaped loop when P is on the line of those wires).

8.4 Pattern 4: Force on a wire / between wires

Tells: "A wire of length... carries current I... in a magnetic field $\vec{B}$..." or "Two parallel wires..."

Steps: (i) For a single wire, use $\vec{F} = I\vec{\ell} \times \vec{B}$; magnitude $F = BI\ell\sin\theta$. (ii) For two parallel wires, use $F/\ell = \mu_0 I_1 I_2/(2\pi d)$; check direction (parallel attract, anti-parallel repel). (iii) For a wire in a non-uniform field, integrate $dF = I\,d\ell\,B(r)$ along the wire.

8.5 Pattern 5: Torque on a current loop

Tells: "A coil of N turns and area A carrying current I is placed in field $\vec{B}$..."

Steps: (i) Magnetic moment $m = NIA$. (ii) Torque $\tau = mB\sin\theta$ where θ is the angle between $\hat{n}$ (loop normal) and $\vec{B}$. (iii) For a coil free to rotate, the equilibrium is when $\hat{n} \parallel \vec{B}$ (stable). (iv) If asked for work to rotate from one orientation to another, use $W = mB(\cos\theta_1 - \cos\theta_2)$.

8.6 Pattern 6: Galvanometer conversion

Tells: "Convert a galvanometer with $R_g = \ldots$ and $I_g = \ldots$ to read up to $I = \ldots$ A or $V = \ldots$ V"

Steps: (i) For ammeter, find shunt: $r_s = I_g R_g/(I - I_g)$, connected in parallel. (ii) For voltmeter, find series resistor: $R = V/I_g - R_g$, connected in series. (iii) Check that the modified meter has appropriate effective resistance (low for ammeter, high for voltmeter).

Quick Tip

Always sketch the geometry first — positions of charges/wires/loops, directions of currents, orientation of $\vec{B}$. Three-dimensional vector problems are very hard to do mentally; a 2-second sketch with $\odot$ and $\otimes$ markers prevents most sign errors.

9 Quick Reference Summary

9.1 Key formulas at a glance

Quantity / Concept	Formula
Lorentz force	$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$
Magnetic force on charge	$F = qvB\sin\theta$
Radius of circular motion	$r = mv/(qB)$
Cyclotron period	$T = 2\pi m/(qB)$
Cyclotron frequency	$f = qB/(2\pi m)$, $\omega_c = qB/m$
Helical motion: pitch	$p = (2\pi m\, v\cos\theta)/(qB)$
Force on current-carrying wire	$\vec{F} = I(\vec{\ell} \times \vec{B})$, $F = BI\ell\sin\theta$
Biot–Savart law	$d\vec{B} = (\mu_0/4\pi)(I\, d\vec{\ell} \times \hat{r})/r^2$
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ T m/A
Field of long straight wire	$B = \mu_0 I/(2\pi a)$
Field at centre of circular loop	$B = \mu_0 NI/(2R)$
Field on axis of circular loop	$B = \mu_0 IR^2/[2(R^2 + x^2)^{3/2}]$
Field at centre of arc (angle ϕ rad)	$B = \mu_0 I\phi/(4\pi R)$
Ampere's circuital law	$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$
Field inside long solenoid	$B = \mu_0 nI$
Force per unit length, parallel wires	$F/\ell = \mu_0 I_1 I_2/(2\pi d)$
Magnetic moment of a loop	$\vec{m} = NI\vec{A}$

Quantity / Concept	Formula
Torque on a current loop	$\vec{\tau} = \vec{m} \times \vec{B}$, $\tau = NIAB \sin\theta$
Galvanometer current sensitivity	$\phi/I = NAB/k$
Galvanometer voltage sensitivity	$\phi/V = NAB/(kR_g)$
Shunt for ammeter	$r_s = I_g R_g/(I - I_g)$
Series resistor for voltmeter	$R = V/I_g - R_g$

9.2 Useful constants and unit conversions

- Permeability of free space: $\mu_0 = 4\pi \times 10^{-7}$ T m/A
- $\mu_0/(4\pi) = 10^{-7}$ T m/A
- Earth's magnetic field at the surface: ~ 0.5 G $= 5 \times 10^{-5}$ T
- 1 tesla (T) $= 10^4$ gauss (G)
- Mass of electron: $m_e = 9.11 \times 10^{-31}$ kg
- Mass of proton: $m_p = 1.67 \times 10^{-27}$ kg
- Charge of electron/proton: $e = 1.6 \times 10^{-19}$ C

9.3 Symbol conventions

Symbol	Meaning	Use
$\odot$ (dot)	Out of the page	Field or current emerging
$\otimes$ (cross)	Into the page	Field or current entering

9.4 Three right-hand rules at a glance

- **Thumb rule (straight wire):** thumb along I, curled fingers along $\vec{B}$.
- **Curl rule (loop):** fingers along I around the loop, thumb along $\vec{B}$ on the axis (and along $\vec{m}$).
- **Cross-product rule (force):** fingers along $\vec{v}$ (or $\vec{\ell}$), curl toward $\vec{B}$, thumb gives $\vec{F}$ direction (for positive q).

The big picture

This chapter builds the machinery in three layers: **Lorentz force** ($q\vec{v}\times\vec{B}$) is the elementary interaction; **Biot–Savart** and **Ampere's law** let you find $\vec{B}$ from a current; combining the two recovers the force on a wire and the torque on a loop. The galvanometer is the practical reward — one piece of physics that turns a coil-on-spring into a measuring instrument for both current and voltage.