NCERT Exemplar Solutions

Solved NCERT Exemplar Problems for Class 12th Physics, Chapter 5

Chapter 5: Magnetism and Matter

About this Chapter

Magnetism and Matter takes us from a single bar magnet to the magnetic behaviour of bulk material. We connect the **magnetic dipole moment** $\vec{m}$ to torque and energy in an external field, set up Earth's magnetic field as a tilted dipole (declination, dip, horizontal component), and contrast the three categories of magnetic materials: **diamagnetic**, **paramagnetic**, and **ferromagnetic**. The chapter develops the $\vec{H}$, $\vec{B}$, $\vec{M}$ relationship, Curie's law for paramagnets and the Curie temperature for ferromagnets.

Topics covered: Bar magnet as a dipole • Earth's magnetism • Magnetisation & $\vec{H}$ • Curie's law • Hysteresis • Para/Dia/Ferromagnets

Quick Formula Sheet

Torque on dipole:
$\vec{\tau} = \vec{m} \times \vec{B}$

Potential energy:
$U = -\vec{m} \cdot \vec{B}$

Axial field of dipole:
$B_{\text{axial}} = \frac{\mu_0}{4\pi}\frac{2m}{r^3}$

Equatorial field of dipole:
$B_{\text{eq}} = \frac{\mu_0}{4\pi}\frac{m}{r^3}$

Period of magnetic oscillation:
$T = 2\pi\sqrt{\frac{I}{mB}}$

Magnetic intensity:
$\vec{B} = \mu_0(\vec{H} + \vec{M})$

Susceptibility:
$\vec{M} = \chi\vec{H}, \quad \mu_r = 1 + \chi$

Curie's law:
$\chi = \frac{C}{T}$ (paramagnet)

Earth's field components:
$B_H = B\cos\delta,\ B_V = B\sin\delta$

NCERT Exemplar Problems

MCQ I

Q 5.1 **A toroid of n turns, mean radius R and cross-sectional radius a carries current I. It is placed on a horizontal table taken as x-y plane. Its magnetic moment $\vec{m}$**

(a) is non-zero and points in the z-direction by symmetry.
(b) points along the axis of the toroid ($\vec{m} = m\,\hat{\varphi}$).
(c) is zero, otherwise there would be a field falling as $1/r^3$ at large distances outside the toroid.
(d) is pointing radially outwards.

SOLUTION

Correct option: (c) The net magnetic moment of a toroid is zero.

Concept used. The **magnetic moment** of a current loop is $\vec{m} = I\vec{A}$, directed along the area vector $\vec{A}$ by the right-hand rule. For a coil with n turns, contributions from each turn add as vectors. At large r the field of any localised current distribution can be expanded in multipoles: the leading $1/r^3$ term is the magnetic dipole contribution, which exists only if the total magnetic moment is non-zero. A toroid is a closed solenoid bent into a doughnut, so the field is confined inside the core and vanishes outside.

Step 1. Consider any cross-sectional turn of the toroid. Its area vector points tangentially (along $\hat{\varphi}$) and rotates as we move around the toroid. When we add up the contributions of all n turns around the full 2π azimuth, the area vectors form a closed ring whose vector sum is zero:

$$\vec{m}_{\text{total}} = \sum_{k=1}^{n} I\,\vec{A}_k = IA\sum_{k=1}^{n}\hat{\varphi}_k = \vec{0}.$$

Step 2. As a physical cross-check, recall that the field of an ideal toroid is confined entirely to the interior with $B = 0$ outside. If a non-zero $\vec{m}$ existed, the dipole field $B \sim \mu_0 m/4\pi r^3$ would be measurable outside. Since it is not, $\vec{m} = 0$.

Step 3. Eliminate the other options. (a) would require a net z component, but symmetry forces cancellation. (b) is wrong because $\hat{\varphi}$ varies with position — no single direction. (d) is wrong because magnetic moment is a vector attached to the source, not a radial field.

> **Final Answer:** Option **(c)**: $\vec{m} = 0$ for a toroid.

EXPERT'S SOLUTION *: Pranav Sharma, Ph.D Physics, IISc Bangalore*

Picture-first. Imagine cutting the toroid into n tiny flat loops arranged in a circle. Each loop's moment vector is a short arrow along the local tangent to the central circle.

Step 1. **Symmetry argument.** The n moment vectors $\vec{m}_k = IA\hat{\varphi}_k$ lie head-to-tail around the central circle. Their vector sum closes back on itself: $\sum_{k=1}^{n}\hat{\varphi}_k \to \oint \hat{\varphi}\,d\varphi/(2\pi/n) = 0$ for large n, so total $\vec{m} = 0$.

Step 2. Ampere's-law argument. $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enc}}$ gives $B_{\text{inside}} = \dfrac{\mu_0 n I}{2\pi r}$ inside the toroidal core; for any Amperian loop drawn entirely outside the windings, $I_{\text{enc}} = 0$, and by symmetry $B_{\text{outside}} = 0$ everywhere.

Step 3. Multipole-expansion argument. A general localised current distribution has the far-field expansion $\vec{B} \sim \dfrac{\mu_0}{4\pi}\dfrac{m}{r^3} + \dfrac{\mu_0}{4\pi}\dfrac{Q}{r^4} + \ldots$ — monopole term is identically zero (no monopoles), dipole term needs $\vec{m} \neq 0$. Since $B_{\text{outside}} = 0$ to all orders, every multipole coefficient must vanish, in particular $\vec{m}_{\text{toroid}} = 0$.

Step 4. Energy cross-check. For an external uniform field $\vec{B}_0$, the orientation energy of a magnetic moment is $U = -\vec{m}\cdot\vec{B}_0$. Place a toroid in $\vec{B}_0$ and rotate it — no preferred orientation is observed, hence $\vec{m} = 0$. This is also why a toroid is not deflected in a uniform field.

Alternative method (energy). The total flux linkage of the toroid with its own current is finite (self-inductance), but its coupling to an *external uniform* field is $\Phi_{\text{ext}} = \vec{m}\cdot\vec{B}_0 = 0$, again proving $\vec{m} = 0$.

Final Answer: $\vec{m}_{\text{toroid}} = 0$; option **(c)**.

☞ Exam Tip

CBSE marking. A 1-mark MCQ like this expects the *reason* as much as the answer. Write one line: "Toroidal $\vec{B}$ confined inside $\Rightarrow$ no $1/r^3$ outside $\Rightarrow$ $\vec{m} = 0$." That single sentence wins the mark even if computation is skipped.

♥ Why This Matters

A toroid is the classic "magnetically closed" device: zero stray field outside, all energy stored inside the core. This is why toroidal transformers in audio amplifiers and toroidal field coils in tokamak fusion reactors are preferred — they don't leak flux into neighbouring circuits.

Q 5.2 The magnetic field of Earth can be modelled by that of a point dipole placed at the centre of the Earth. The dipole axis makes an angle of $11.3°$ with the axis of Earth. At Mumbai, declination is nearly zero. Then,

(a) the declination varies between $11.3°$ W to $11.3°$ E.

(b) the least declination is $0°$.

(c) the plane defined by dipole axis and Earth axis passes through Greenwich.

(d) declination averaged over Earth must be always negative.

SOLUTION

Correct option: (a) Declination varies between 11.3° W and 11.3° E.

Concept used. Magnetic declination D is the angle between magnetic north (the horizontal component of $\vec{B}$ at a location) and geographic north. Since Earth's magnetic dipole axis is tilted by $\alpha = 11.3^\circ$ from the geographic (spin) axis, the maximum possible angle between the horizontal projections of the two axes is α.

Step 1. Place the dipole axis and the geographic axis in the same plane Π. As we move around the Earth, the angular separation between the two axes' horizontal projections at any location ranges from 0 (on Π) up to the full tilt of 11.3° (in the perpendicular plane).

Step 2. The sense flips: when our location is east of Π, magnetic north points slightly west of geographic north, giving $D = 11.3^\circ$ W. When we are west of Π, the opposite, giving $D = 11.3^\circ$ E. Mumbai sits near Π so its $D \approx 0$.

Step 3. Therefore globally D oscillates between -11.3° (i.e. 11.3° E) and $+11.3^\circ$ (11.3° W). Options (b), (c), (d) make claims (about minimum, Greenwich, sign average) that the simple tilt model does not support.

Final Answer: Option **(a)**: $-11.3^\circ\,\text{E} \leq D \leq 11.3^\circ\,\text{W}$.

EXPERT'S SOLUTION : *Aanya Iyer, M.Sc Astrophysics, IIT Kanpur*

Strategic angle. The whole question reduces to a single geometric fact: two axes tilted by α produce a maximum angular projection of α on the horizontal plane.

Step 1. **Set up the geometry.** Define the plane Π as the plane containing both the spin axis $\hat{z}_g$ and the magnetic-dipole axis $\hat{z}_m$, with $\hat{z}_m$ tilted from $\hat{z}_g$ by $\alpha = 11.3^\circ$.

Step 2. **Locations in Π.** In the plane Π, the projections of magnetic and geographic north coincide on the same line, so $D = 0$ there. Mumbai sits very close to this plane, hence its observed $D \approx 0$.

Step 3. **Locations perpendicular to Π.** Ninety degrees of longitude away from Π, the magnetic axis appears rotated by the full $\alpha = 11.3^\circ$ from the geographic axis as seen in the local horizontal plane. The sense of the rotation flips as we cross Π to the opposite side of the globe, giving $D = +11.3^\circ$ W on one side and $D = -11.3^\circ$ (i.e. 11.3° E) on the other.

Step 4. **Range and average.** D swings continuously from 11.3° W to 11.3° E, passing through 0 twice along Π. The longitudinal average is exactly zero for a pure tilted dipole — option (d) "always negative" is wrong by symmetry. Option (a) captures the full range.

Step 5. **Why (b) and (c) fail.** (b) says the *least* declination is 0°; this is a misreading —

there are points with $D = 0$ on Π, but they are not unique. (c) about Greenwich is a coincidence of which longitude we choose as zero; the dipole plane Π does not in general pass through Greenwich.

Concept linkage. The horizontal projection argument is the same one used for the angle of dip (δ) on the geographic equator (Q 5.10): both are bounded by the tilt α.

Final Answer: Option **(a)**.

✗ Common Mistake

Confusing declination with dip. *Declination* D is the horizontal angle between magnetic north and geographic north (measured in the local horizontal plane). *Dip* δ is the angle the field makes with the horizontal. They are independent angles. A common error is to plug $11.3°$ into the dip formula on the geographic equator — it doesn't work because the field direction in 3D requires both angles.

☞ Recall

Tilted-dipole model. Earth's main field ≈ a dipole of moment $m_\oplus \approx 8 \times 10^{22}\,\mathrm{A\,m^2}$ centred at Earth's centre, with axis tilted by $\alpha = 11.3°$ from the geographic axis. The real field also has substantial non-dipole components (especially over the South Atlantic Anomaly), making local D reach $\sim 25°$ in places.

Q 5.3 In a permanent magnet at room temperature

(a) magnetic moment of each molecule is zero.

(b) the individual molecules have non-zero magnetic moment which are all perfectly aligned.

(c) domains are partially aligned.

(d) domains are all perfectly aligned.

SOLUTION

Correct option: (c) Domains in a permanent magnet are partially aligned at room temperature.

Concept used. Ferromagnetism arises from exchange interaction between neighbouring atomic moments, locking them parallel inside microscopic regions called **domains**. Each domain has a strong net moment, but in an unmagnetised piece of iron the domains point in random directions so the bulk moment averages to zero. "Permanent magnetisation" means a permanent partial alignment of the domains. Perfect alignment is only reached if the material is fully saturated; at room temperature, thermal agitation always misaligns some domains.

Step 1. Within each domain, the atomic moments are already aligned — that is the definition of a domain. So (a) is wrong because each molecule's moment is non-zero in iron, cobalt, nickel.

Step 2. "Perfectly aligned" would require every domain to point along the same axis. This is the saturation state, reached only in very strong applied fields. At room temperature in zero external field this is unstable, so (b) and (d) are wrong.

Step 3. In a permanent magnet, a previously applied field has rotated many but not all domains into a common direction. This partial alignment persists even after the field is removed because domain walls get pinned by crystal defects. This is what (c) describes.

Final Answer: Option **(c)**: partial domain alignment.

EXPERT'S SOLUTION *: Rohit Kapoor, Ph.D Condensed Matter Physics, TIFR Mumbai*

Quick reading. A permanent magnet is permanent because domain walls are pinned, not because every spin is locked perfectly.

Step 1. **Inside a single domain.** Each ferromagnetic domain ($\sim 10^{-6}$ m across, containing $\sim 10^{15}$ atoms) already has all atomic moments aligned by the exchange interaction. So the molecule-moment $\mu_{\text{atom}} \neq 0$ — this rules out (a) immediately.

Step 2. **Demagnetised iron.** In a freshly demagnetised piece, the domains point in random directions; their vector sum averages to net moment $M = 0$. The sample has *zero macroscopic* magnetisation but *non-zero microscopic* moments — a key distinction.

Step 3. **Magnetisation process.** Applying a strong external field grows the domains aligned with $\vec{B}_{\text{ext}}$ at the expense of mis-aligned ones (domain-wall motion) and rotates remaining domains into the field direction (domain rotation). At saturation, all domains point parallel — $M = M_s$.

Step 4. **Removing the field.** Domain walls do not slide back freely: they are pinned by crystal defects, grain boundaries and impurities. So a residual alignment remains, giving the *remanence* M_r with $0 < M_r < M_s$. This partial alignment is option (c).

Step 5. **Why (d) is wrong.** Full alignment of every domain (d) is the saturated state, only reached *while* the field is applied. At room temperature with $\vec{B}_{\text{ext}} = 0$, thermal agitation $k_B T$ and stray demagnetising fields knock the most weakly pinned domains out of perfect alignment.

Diagram-based reasoning. On the M vs H hysteresis loop, the point "permanent magnet" is at $H = 0$, $M = M_r$ — on the vertical axis, between 0 and M_s. The horizontal-axis intercept on the negative side, $H = -H_c$, gives the coercivity: the reverse field needed to undo the partial alignment.
Concept linkage. For $T > T_c$ (Curie temperature, e.g. 770°C for iron), thermal energy beats the exchange coupling and domains disappear — the material becomes a paramagnet with $\chi \sim 10^{-3}$, as we will see in Q 5.8 and Q 5.14.

Final Answer: Option **(c)**.

☞ **Useful aside**

Remanence vs saturation. The four numbers on a hysteresis loop are: M_s (saturation), M_r (remanence, the "permanent" part), H_c (coercivity), and M_s' (negative saturation). A "hard" magnet (alnico, NdFeB) has large M_r and large H_c; a "soft" magnet (transformer iron) has large M_s but small M_r and tiny H_c.

☞ **Exam Tip**

CBSE 1-mark. For a single-correct MCQ on permanent magnets, the keyword to look for in distractors is "*perfectly aligned*". That phrase is the classic wrong answer — only saturation states are perfect, and saturation is not the same as permanent magnetisation. The correct answer is always some variant of "partial alignment of domains".

Q 5.4 Consider the two idealised systems: (i) a parallel plate capacitor with large plates and small separation and (ii) a long solenoid of length $L \gg R$, radius of cross-section. In (i) $\vec{E}$ is ideally treated as a constant between plates and zero outside. In (ii) magnetic field is constant inside the solenoid and zero outside. These idealised assumptions, however, contradict fundamental laws as below:
(a) case (i) contradicts Gauss's law for electrostatic fields.
(b) case (ii) contradicts Gauss's law for magnetic fields.
(c) case (i) agrees with $\oint \vec{E} \cdot d\vec{l} = 0$.
(d) case (ii) contradicts $\oint \vec{H} \cdot d\vec{l} = I_{en}$.

SOLUTION

Correct option: (b) only.
Concept used. The four fundamental laws relevant here are: Gauss's law for electricity, $\oint \vec{E} \cdot d\vec{A} = q_{en}/\varepsilon_0$; Gauss's law for magnetism, $\oint \vec{B} \cdot d\vec{A} = 0$ (no magnetic monopoles); the electrostatic loop equation, $\oint \vec{E} \cdot d\vec{l} = 0$; and Ampere's law, $\oint \vec{H} \cdot d\vec{l} = I_{en}$.

Step 1. Test (i) against Gauss's law for E. Take a pill-box Gaussian surface enclosing one plate. With the idealised $\vec{E}$, only the face inside the capacitor contributes: $\Phi_E = EA = q_{en}/\varepsilon_0$ gives $E = \sigma/\varepsilon_0$. This is consistent, not contradictory, so (a) is wrong.

Step 2. Test (i) against the loop equation. For the idealised capacitor with $\vec{E}$ pointing perpendicular to the plates between them and zero outside, take a rectangular loop with both long sides parallel to the plates (one inside, one outside). The integral $\oint \vec{E} \cdot d\vec{l} = 0$ because $\vec{E} \perp d\vec{l}$ on the long sides and $\vec{E} = 0$ on the outside section. So the idealisation *agrees with* $\oint \vec{E} \cdot d\vec{l} = 0$. Option (c) is therefore a true statement (no contradiction).

Step 3. Test (ii) against Gauss's law for B. Take a pillbox partly inside, partly outside the solenoid. Inside-face flux $= BA$; outside-face flux $= 0$. Net flux through the closed surface $= BA \neq 0$, contradicting $\oint \vec{B} \cdot d\vec{A} = 0$. So (b) is correct — this is a genuine contradiction with a fundamental law.

Step 4. Test (ii) against Ampere's law. For an Amperian loop drawn entirely outside the solenoid, $I_{en} = 0$ and the idealised $B = 0$ outside makes $\oint \vec{H} \cdot d\vec{l} = 0$ consistent with $I_{en} = 0$. For a loop enclosing the windings, $\oint \vec{H} \cdot d\vec{l} = nIL$ (contribution from the inside long side) matches $I_{en} = nIL$ exactly. So Ampere's law is *not* contradicted by the idealisation. Option (d) is false.

Final Answer: Option **(b)** only: case (ii) contradicts Gauss's law for $\vec{B}$.

EXPERT'S SOLUTION *: Arjun Banerjee, M.Sc Physics, IIT Madras*

Strategic angle. The idealisations are useful but mathematically inconsistent at the boundary. Test each statement by applying the appropriate integral law to a closed surface or loop that straddles the boundary.

Step 1. (a) Gauss's law for $\vec{E}$ on case (i). Pillbox around one plate of area A: only the inside face has flux $\Phi_E = EA$; outside face flux is zero. Idealised $E = \sigma/\varepsilon_0$ then gives $\Phi_E = (\sigma/\varepsilon_0)\, A = q/\varepsilon_0$. The flux exactly accounts for the enclosed charge. *No contradiction* — option (a) is **false**.

Step 2. (b) Gauss's law for $\vec{B}$ on case (ii). Pillbox straddling the solenoid wall (one face inside, one outside): $\Phi_B = B\,A - 0 = BA \neq 0$. This violates $\oint \vec{B} \cdot d\vec{A} = 0$ — the absence of magnetic monopoles. The idealisation *does* contradict Gauss's law for $\vec{B}$. Option (b) is **true**.

Step 3. (c) Loop equation on case (i). Take a rectangular loop with both long sides parallel to the plates — one inside the capacitor, one outside. On the long sides $\vec{E} \perp d\vec{l}$, contributing zero; on the short crossing sides, $\vec{E} \cdot d\vec{l}$ from the inside section is exactly cancelled by the path traversal direction. The net line integral is $\oint \vec{E} \cdot d\vec{l} = 0$. The idealisation thus *agrees* with the loop equation — option (c) is a true statement (not a contradiction), so the answer key does *not* include (c).

Step 4. **(d) Ampere's law on case (ii).** For an Amperian loop chosen wholly outside the solenoid, $I_{\text{enc}} = 0$ and $\oint \vec{H} \cdot d\vec{l} = 0$ are consistent. For a loop enclosing the windings (one long side inside, one outside), $\oint \vec{H} \cdot d\vec{l} = Hl = (nI)l$ matches $I_{\text{enc}} = nIl$ exactly — no contradiction. So option (d) is *false*; Ampere's law is not contradicted.

Step 5. **Take-home.** The idealisations agree with Gauss for $\vec{E}$, disagree only with Gauss for $\vec{B}$, agree with Ampere for $\vec{B}$, and agree with the electrostatic loop equation. Net answer: only (b).

Numerical cross-check. A real long solenoid of $n = 1000$ turns/m carrying $I = 1\,\text{A}$ has $B_{\text{inside}} \approx \mu_0 nI = 1.26 \times 10^{-3}\,\text{T}$. Just outside the windings, measurement shows $B \sim 10^{-5}\,\text{T}$ from fringe leakage — small but non-zero, restoring consistency with $\nabla \cdot \vec{B} = 0$.

Final Answer: Option **(b)** only.

✗ Common Mistake

"B = 0 outside the solenoid" is an approximation, not a law. Treating $B = 0$ exactly outside an ideal solenoid violates $\nabla \cdot \vec{B} = 0$. In real solenoids the field outside is small (typically 10^3 times less than inside) but non-zero — it loops around through the open ends back to the other side. This is the same reason "capacitor edge effects" exist in electrostatics: every real field is continuous.

♥ Why This Matters

The idealisation $B_{\text{outside}} = 0$ is wildly useful for computing $B_{\text{inside}} = \mu_0 nI$ via Ampere's law, but it must always be remembered as a *limiting case*. When you build a real solenoid, the fringe field is what couples to neighbouring components — the basis of mutual inductance, transformer flux linkage, and electromagnetic interference (EMI) shielding design.

Q 5.5 **A paramagnetic sample shows a net magnetisation of $8\,\text{Am}^{-1}$ when placed in an external magnetic field of $0.6\,\text{T}$ at a temperature of $4\,\text{K}$. When the same sample is placed in an external magnetic field of $0.2\,\text{T}$ at a temperature of $16\,\text{K}$, the magnetisation will be**

(a) $\frac{32}{3}\,\text{Am}^{-1}$

(b) $\frac{2}{3}\,\text{Am}^{-1}$

(c) $6\,\mathbf{Am}^{-1}$
(d) $2.4\,\mathbf{Am}^{-1}$

SOLUTION

Correct option: (b) $M_2 = \frac{2}{3}\,\text{Am}^{-1}$.
Concept used. **Curie's law** for a paramagnetic sample at moderate temperatures (where saturation effects are negligible) states that the magnetisation M is proportional to the applied magnetic field B and inversely proportional to the absolute temperature T:

$$M = C\,\frac{B}{T},$$

where C is Curie's constant for the material. The ratio of M values in two states therefore reduces to a ratio of B/T.

Step 1. Write the law in both states and divide. State 1: $M_1 = 8$, $B_1 = 0.6\,\text{T}$, $T_1 = 4\,\text{K}$. State 2: $M_2 = ?$, $B_2 = 0.2\,\text{T}$, $T_2 = 16\,\text{K}$.

$$\frac{M_2}{M_1} = \frac{B_2/T_2}{B_1/T_1} = \frac{B_2}{B_1}\cdot\frac{T_1}{T_2}.$$

Step 2. Substitute the numbers:

$$\frac{M_2}{M_1} = \frac{0.2}{0.6}\cdot\frac{4}{16} = \frac{1}{3}\cdot\frac{1}{4} = \frac{1}{12}.$$

Step 3. Therefore

$$M_2 = \frac{M_1}{12} = \frac{8}{12} = \frac{2}{3}\,\text{Am}^{-1}.$$

Final Answer: $M_2 = \frac{2}{3}\,\text{Am}^{-1}$; option **(b)**.

EXPERT'S SOLUTION *: Priya Verma, M.Sc Physics, IIT Madras*

Strategic angle. Curie's law is a one-line proportionality. The clean way is to take the ratio of the two states so the Curie constant C cancels — no need to know the material!

Step 1. Write Curie's law in both states. $M_1 = C\,B_1/T_1$ and $M_2 = C\,B_2/T_2$.

Step 2. Divide to eliminate C.

$$\frac{M_2}{M_1} = \frac{B_2/T_2}{B_1/T_1} = \frac{B_2}{B_1}\cdot\frac{T_1}{T_2}.$$

Step 3. Substitute. $B_2/B_1 = 0.2/0.6 = 1/3$ (field drops by factor 3); $T_1/T_2 = 4/16 = 1/4$ (temperature rises by factor 4). Product: $\frac{M_2}{M_1} = \frac{1}{3}\cdot\frac{1}{4} = \frac{1}{12}$.

Step 4. **Compute M_2.** $M_2 = M_1/12 = 8/12 = 2/3\,\mathrm{A\,m^{-1}}$.

Step 5. **Sanity-check options.** (a) $32/3 \approx 10.7$ — would require M to *increase*, but both B drops *and* T rises (both reduce M); rejected. (c) 6 and (d) 2.4 ignore one of the two changes; rejected. Only (b) $= 2/3$ accounts for both effects.

Numerical cross-check / unit analysis. $[C] = [M]\cdot[T]/[B]$ = (A/m)(K)/(T) = A·K/(m·T). The Curie constant for this sample is
$C = M_1T_1/B_1 = (8)(4)/(0.6) \approx 53.3\,$A·K/(m·T). Recomputing state 2:
$M_2 = C\,B_2/T_2 = (53.3)(0.2)/16 \approx 0.667\,$A/m $= 2/3\,$A/m. Same answer.
Concept linkage. Curie's law $\chi = C/T$ is the high-temperature limit of paramagnetism ($\mu B \ll k_BT$). For strong fields and low temperatures, the full Brillouin function gives saturation $M \to n\mu$ — Curie's law would then over-predict.

> **Final Answer:** $M_2 = 2/3\,\mathrm{A\,m^{-1}}$, option **(b)**.

☞ **Exam Tip**

CBSE 1-mark trick. Whenever a problem gives "state 1 → state 2" with proportionalities, the ratio method beats solving for constants. Write the proportionality ($M \propto B/T$, $V \propto T/P$, etc.), then form M_2/M_1 — the constant always cancels. This is the single most efficient way to handle Curie/Boyle/Charles-style problems in CBSE MCQs.

☞ **Recall**

Curie's law. $\chi_{\text{para}} = C/T$, equivalently $M = (C/\mu_0)\,B/T$ in the weak-field limit. Curie constant $C = n\mu_0\mu^2/(3k_B)$ depends only on the material (number density n and per-atom moment μ), *not* on B or T. This is why ratios cancel C cleanly.

MCQ II

Q 5.6 **S is the surface of a lump of magnetic material.**
(a) Lines of $\vec{B}$ are necessarily continuous across S.
(b) Some lines of $\vec{B}$ must be discontinuous across S.
(c) Lines of $\vec{H}$ are necessarily continuous across S.
(d) Lines of $\vec{H}$ cannot all be continuous across S.

SOLUTION

Correct options: (a) and (d).
Concept used. The fundamental field $\vec{B}$ has $\nabla\cdot\vec{B} = 0$ everywhere (Gauss's law for magnetism), so no magnetic monopoles exist and field lines of $\vec{B}$ are unbroken closed

loops — they cannot start or stop anywhere, even at the boundary of a magnetic material. The auxiliary field $\vec{H}$, defined by $\vec{B} = \mu_0(\vec{H} + \vec{M})$, has divergence $\nabla \cdot \vec{H} = -\nabla \cdot \vec{M}$. Inside a magnetised body, $\vec{M}$ is non-zero; outside, $\vec{M} = 0$. So at the surface S, $\vec{M}$ has a jump, hence $\vec{H}$ has a source density there — $\vec{H}$ lines can start or stop on the surface.

Step 1. Apply $\oint \vec{B} \cdot d\vec{A} = 0$ to any pillbox straddling S. The normal component of $\vec{B}$ is continuous across S:

$$B_n^{\text{outside}} = B_n^{\text{inside}}.$$

So lines of $\vec{B}$ pass through without break. Option (a) is right; (b) is wrong.

Step 2. Apply the same pillbox to $\vec{H}$: $\oint \vec{H} \cdot d\vec{A} = -\oint \vec{M} \cdot d\vec{A} \neq 0$ in general because $\vec{M}$ is non-zero inside and zero outside. The jump in H_n is set by the surface magnetic charge $\sigma_M = \vec{M} \cdot \hat{n}$. Some $\vec{H}$ lines therefore terminate (or originate) at S. Option (d) is right; (c) is wrong.

Step 3. Intuition: think of $\vec{H}$ as coming from "magnetic charges" on the surfaces of magnetised regions. Just as electric field lines start/stop on real charge, $\vec{H}$ lines start/stop on this effective magnetic charge.

Final Answer: Options **(a)** and **(d)**.

EXPERT'S SOLUTION : *Karan Joshi, Ph.D Physics, IISc Bangalore*

Structural observation. Two divergences settle this: $\nabla \cdot \vec{B} = 0$ always; $\nabla \cdot \vec{H} = -\nabla \cdot \vec{M}$ which is non-zero where $\vec{M}$ varies.

Step 1. **Maxwell's law for $\vec{B}$.** $\nabla \cdot \vec{B} = 0$ everywhere — this is one of the four Maxwell equations and is equivalent to the statement that magnetic monopoles do not exist. Equivalently, $\oint \vec{B} \cdot d\vec{A} = 0$ on any closed surface. So $\vec{B}$ lines never start or end; they form closed loops. They must pass continuously across the surface S of the magnetic body. (a) is true; (b) is false.

Step 2. **Effective magnetic charge.** Define a surface "magnetic charge density" $\sigma_M = \vec{M} \cdot \hat{n}$ (where $\hat{n}$ is the outward unit normal). Inside, $\vec{M} \neq 0$; outside, $\vec{M} = 0$. So at any face of S where $\vec{M}$ has a normal component, σ_M is non-zero. The auxiliary field $\vec{H}$ has sources *at* these effective charges: $\nabla \cdot \vec{H} = -\nabla \cdot \vec{M}$, so $\vec{H}$-lines start or stop on S. (d) is true; (c) is false.

Step 3. **Pillbox check for normal components.** Apply both Gauss-like laws to a thin pillbox of cross-section ΔA straddling S:

$$\oint \vec{B} \cdot d\vec{A} = (B_n^{\text{out}} - B_n^{\text{in}})\Delta A = 0 \Rightarrow B_n \text{ continuous},$$

$$\oint \vec{H} \cdot d\vec{A} = (H_n^{\text{out}} - H_n^{\text{in}})\Delta A = \sigma_M \Delta A \Rightarrow H_n \text{ jumps}.$$

This is the precise mathematical content of (a) and (d).

Step 4. Tangential components (aside). Across the same surface, with no free current, H_t is continuous and B_t jumps by $\mu_0 M_t$. These are the boundary conditions you will use later for interfaces between two magnetic media.

Alternative method (vector analysis). From $\vec{B} = \mu_0(\vec{H} + \vec{M})$, taking divergence: $\nabla \cdot \vec{B} = \mu_0(\nabla \cdot \vec{H} + \nabla \cdot \vec{M})$, and since $\nabla \cdot \vec{B} = 0$, we get $\nabla \cdot \vec{H} = -\nabla \cdot \vec{M}$ — a single line that simultaneously delivers (a) (LHS $= 0$) and (d) (RHS $\neq 0$ at a magnetisation discontinuity).

Final Answer: Options **(a), (d)**.

☞ **Useful aside**

Two surface "charges". A magnetised body has two fictitious distributions: bound *currents* $\vec{K}_b = \vec{M} \times \hat{n}$ (source of $\vec{B}$, on the side surface of a magnet) and bound *magnetic charge* $\sigma_M = \vec{M} \cdot \hat{n}$ (source of $\vec{H}$, on the end faces). Use the former when working with $\vec{B}$ via Ampere; use the latter when working with $\vec{H}$ via a Coulomb-like analogue.

✗ **Common Mistake**

Treating $\vec{B}$ and $\vec{H}$ as "the same field". They are not. Inside a permanent magnet, $\vec{B}$ runs S→N while $\vec{H}$ runs N→S (see Q 5.20) — they actually point in *opposite* directions! This is the surest sign that $\vec{B}$ and $\vec{H}$ are physically distinct: one obeys $\nabla \cdot = 0$, the other has sources at $\vec{M}$ jumps.

Q 5.7 The primary origin(s) of magnetism lies in

(a) atomic currents.

(b) Pauli exclusion principle.

(c) polar nature of molecules.

(d) intrinsic spin of electron.

SOLUTION

Correct options: (a) and (d).

Concept used. Atomic magnetic moments come from two sources. (1) The **orbital motion** of electrons around the nucleus is a tiny current loop, contributing an orbital moment $\vec{\mu}_L = -\frac{e}{2m_e}\vec{L}$. (2) The **intrinsic electron spin**, a purely quantum-mechanical property, contributes a spin moment $\vec{\mu}_S = -g_s \frac{e}{2m_e}\vec{S}$ with $g_s \approx 2$. The Pauli exclusion principle governs how spins fill shells but is not itself a source of magnetism; the polar nature of molecules is the source of electric dipole moments, not magnetic ones.

Step 1. Atomic currents (orbital motion) carry charge in a loop — this is a magnetic dipole moment. Option (a) is a primary source.

Step 2. The electron's spin angular momentum $S = \hbar/2$ is accompanied by an intrinsic magnetic moment of magnitude $\mu_B = e\hbar/2m_e$ (the Bohr magneton). Option (d) is a primary source.

Step 3. Pauli exclusion determines which orbital and spin states are occupied; it shapes the resulting magnetism but does not *produce* a moment. Option (b) is wrong as a primary source.

Step 4. Polar molecules have electric dipole moments arising from charge separation. They do not by themselves give magnetism. Option (c) is wrong.

Final Answer: Options **(a)** and **(d)**.

EXPERT'S SOLUTION *: Ananya Reddy, M.Sc Physics, IIT Madras*

Quick reading. Atoms have two "moving charge" degrees of freedom that carry magnetism: orbital motion and intrinsic spin.

Step 1. **Orbital contribution.** An electron in an atomic orbital traces a closed current loop with effective current $I = e/T = ev/(2\pi r)$. Loop area $A = \pi r^2$. The magnetic moment is $\mu_L = IA = evr/2 = (e/(2m_e))(m_e vr) = (e/(2m_e))L$. In vector form $\vec{\mu}_L = -(e/(2m_e))\,\vec{L}$ (negative because the electron's charge is $-e$). This is option (a) — atomic currents.

Step 2. **Spin contribution.** The electron has intrinsic spin angular momentum $\vec{S}$ with $|\vec{S}| = \hbar/2$, and an accompanying magnetic moment $\vec{\mu}_S = -g_s(e/(2m_e))\,\vec{S}$ where $g_s \approx 2$ is the Landé g-factor for the electron. Its magnitude is the Bohr magneton $\mu_B = e\hbar/(2m_e) \approx 9.27 \times 10^{-24}$ J/T. This is option (d) — intrinsic spin.

Step 3. **Pauli exclusion is not a source.** (b) is a *constraint* on which orbital-spin states can be simultaneously occupied; it does not by itself produce a magnetic moment. Closed shells (e.g. Ne, Ar) have all moments paired and cancel, but this cancellation *uses* Pauli, doesn't *come from* it.

Step 4. **Polar molecules are an electric concept.** (c) "polar nature of molecules" refers to permanent electric dipole moments (H_2O, HCl). These produce electric polarisation, not magnetism — wrong by category.

Step 5. **Total atomic moment.** $\vec{\mu}_{\text{atom}} = \vec{\mu}_L + \vec{\mu}_S$. The sum can be zero (closed shell, e.g. noble gas $\Rightarrow$ diamagnetic) or non-zero (unpaired electrons, transition-metal d-shell $\Rightarrow$ paramagnetic or ferromagnetic).

Concept linkage. The relative weighting of orbital vs spin contributions in a material is captured by the magnetomechanical ratio (gyromagnetic ratio) γ. "Spin-only" materials have γ close to $-e/m_e$; orbital-quenched materials have larger corrections. This is measured by the Einstein–de Haas effect.

> **Final Answer:** Options **(a)**, **(d)**.

☞ Recall

Bohr magneton. $\mu_B = \dfrac{e\hbar}{2m_e} \approx 9.27 \times 10^{-24}$ J/T. This is the natural unit of atomic magnetic moment. The nuclear magneton $\mu_N = e\hbar/(2m_p)$ is ~ 1836 times smaller because $m_p \approx 1836\, m_e$ — see Q 5.11 for why nuclear moments are negligible in bulk magnetism.

☞ Exam Tip

CBSE 1-mark MCQ-II. For "primary origin of magnetism" type questions, the two correct keywords are always (i) orbital / atomic currents and (ii) electron spin / intrinsic spin. Mark both and move on; do not over-think Pauli or polarity. A 30-second question.

Q 5.8 **A long solenoid has 1000 turns per metre and carries a current of 1 A. It has a soft iron core of $\mu_r = 1000$. The core is heated beyond the Curie temperature, T_c.**

(a) The H field in the solenoid is (nearly) unchanged but the B field decreases drastically.

(b) The H and B fields in the solenoid are nearly unchanged.

(c) The magnetisation in the core reverses direction.

(d) The magnetisation in the core diminishes by a factor of about 10^8.

SOLUTION

Correct options: (a) and (d).

Concept used. The magnetic intensity inside a long solenoid depends only on free current and geometry: $H = nI$, regardless of the core. The flux density inside is $B = \mu_0(H + M) = \mu_0\mu_r H$, where the relative permeability μ_r depends on the core material. Above the Curie temperature T_c, a ferromagnet becomes paramagnetic; χ falls by many orders of magnitude (typically $\sim 10^{-3}$ to $\sim 10^{-5}$, versus $\sim 10^3$ below T_c), so $\mu_r \to 1 + \chi \approx 1$. Hence B collapses while H stays fixed.

Step 1. Below T_c, $H = nI = 1000 \times 1 = 1000$ A/m. With $\mu_r = 1000$:

$B = \mu_0\mu_r H = 4\pi \times 10^{-7} \times 1000 \times 1000 = 4\pi \times 10^{-1} \approx 1.26\,\text{T}$. Magnetisation: $M = (\mu_r - 1)H \approx 999\,000\,\text{A/m} \approx 10^6\,\text{A/m}$.

Step 2. Above T_c, the material becomes paramagnetic with $\chi \approx 10^{-2}$ at most (often much less). Take a typical value $\chi \sim 10^{-2}$: $M = \chi H = 0.01 \times 1000 = 10\,\text{A/m}$. Ratio:

$$\frac{M_{\text{below}}}{M_{\text{above}}} \approx \frac{10^6}{10^{-2}} \approx 10^8.$$

So M drops by a factor $\sim 10^8$. Option (d).

Step 3. H is set by the free current alone, so $H = 1000\,\text{A/m}$ before and after. $B = \mu_0(H + M)$: the M contribution falls from $\sim 10^6$ to ~ 10, so B collapses from $\sim 1.26\,\text{T}$ to $\sim \mu_0 \cdot 1010 \approx 1.3 \times 10^{-3}\,\text{T}$. Option (a) confirmed; (b) is wrong because B *does* change drastically; (c) wrong because M shrinks but does not flip.

Final Answer: Options **(a)** and **(d)**.

EXPERT'S SOLUTION *: Diya Mehta, Ph.D Condensed Matter Physics, TIFR Mumbai*

Strategic angle. Above T_c iron loses its ferromagnetism and becomes a weak paramagnet. Track H, M, B separately — H depends only on free current, M depends on the material, and $B = \mu_0(H + M)$ is the field actually measured.

Step 1. **H is set by geometry and free current.** Ampere's law on a rectangular Amperian loop straddling the solenoid wall gives $H = nI$ regardless of what is inside. Here $H = (10^3\,\text{turns/m})(1\,\text{A}) = 10^3\,\text{A/m}$, both before and after heating. Option (b) is therefore wrong if it claims any change in H.

Step 2. **Cold core ($T < T_c$).** The given $\mu_r = 1000$ means $\chi = \mu_r - 1 \approx 999$. Then $M = \chi H = 999 \times 10^3 \approx 10^6\,\text{A/m}$ and $B = \mu_0(H + M) = (4\pi \times 10^{-7})(10^3 + 10^6) \approx 1.26\,\text{T}$. The core dominates: $M \gg H$.

Step 3. **Hot core ($T > T_c$).** Above the Curie temperature, the spontaneous ferromagnetic order collapses; iron becomes a paramagnet with $\chi \sim 10^{-3}$ to 10^{-2} (it follows Curie–Weiss law $\chi = C/(T - T_c)$ — see Q 5.14). Take $\chi \sim 10^{-2}$ as a representative upper bound: $M = \chi H \sim 10^{-2} \times 10^3 = 10\,\text{A/m}$. The ratio $M_{\text{cold}}/M_{\text{hot}} \sim 10^6/10 = 10^5$ to 10^8 depending on precise χ. Within the stated "factor of about 10^8" (option d), this is the right order.

Step 4. **Resulting B above T_c.** $B = \mu_0(H + M) \approx \mu_0 \cdot 1010 \approx 1.27 \times 10^{-3}\,\text{T}$ — a drop by factor $\sim 10^3$ from $1.26\,\text{T}$. So B change is drastic; option (a) confirmed.

Step 5. **Why (c) is wrong.** The magnetisation *shrinks* toward zero on heating, but does not flip sign. There is no reason for $\vec{M}$ to spontaneously reverse — that would require a reversed applied field or a coercive cycle.

Numerical cross-check. $B_{\text{cold}}/B_{\text{hot}} \approx 1.26/(1.27 \times 10^{-3}) \approx 10^3$, matching $\mu_r \approx 10^3$. The flux density inside the solenoid drops by exactly the relative permeability — consistent with $B = \mu_0\mu_r H$ when M is set by linear χ.

Concept linkage. The Curie transition T_c for iron is $\approx 1043\,\text{K}$ (770°C). For nickel $T_c \approx 627\,\text{K}$, for cobalt $T_c \approx 1394\,\text{K}$. Above T_c the material is no longer useful as a transformer core, motor magnet or magnetic memory medium — option (d) numerically tracks the physical death of ferromagnetism.

Final Answer: Options **(a), (d)**.

♥ Why This Matters

The Curie transition is responsible for the practical limit on iron-core transformers, magnetic-recording media (hard disks lose data above T_c), and induction motors. Industrial design always keeps the operating temperature well below T_c with a comfortable margin — that is why power transformers have oil cooling. The sudden loss of μ_r at T_c is also exploited in *thermomagnetic switches* that automatically disconnect at high temperature.

✗ Common Mistake

Forgetting that H does not change. A frequent slip is to say "B drops, so H must drop too." Wrong. H is set by free currents alone via $\oint \vec{H} \cdot d\vec{l} = I_{\text{free,enc}}$. The free current nI has not changed when we heat the core, so H is fixed. Only M (and hence B) collapse with the loss of ferromagnetic order.

Q 5.9 Essential difference between electrostatic shielding by a conducting shell and magnetostatic shielding is due to

(a) electrostatic field lines can end on charges and conductors have free charges.

(b) lines of $\vec{B}$ can also end but conductors cannot end them.

(c) lines of $\vec{B}$ cannot end on any material and perfect shielding is not possible.

(d) shells of high permeability materials can be used to divert lines of $\vec{B}$ from the interior region.

SOLUTION

Correct options: (a), (c) and (d).

Concept used. Electrostatic shielding works because a conductor in equilibrium has zero internal $\vec{E}$: free charges rearrange on the surface so the conductor's surface is an equipotential and the interior is field-free. This requires that $\vec{E}$ lines can *end* on charges. Magnetostatic shielding, in contrast, must cope with $\nabla \cdot \vec{B} = 0$: $\vec{B}$ lines never end on any material, so perfect shielding is not attainable. The practical trick is to surround the region with a high-μ_r material (mumetal, soft iron) that greatly prefers to carry $\vec{B}$ inside itself, so lines crowd into the shell and few thread the interior. Shielding is good but not perfect.

Step 1. $\vec{E}$ ends on charge: in a conductor, free charges accumulate on the surface, terminating external $\vec{E}$ lines and leaving the interior at $\vec{E} = 0$. So (a) is the source of perfect electrostatic shielding.

Step 2. $\vec{B}$ cannot end on anything (no magnetic monopoles). Statement (b) is wrong on its first clause ("$\vec{B}$ lines can also end").

Step 3. Because $\vec{B}$ lines cannot terminate on any material, perfect magnetostatic shielding is fundamentally impossible — option (c) is the precise statement of this asymmetry.

Step 4. Magnetic shielding is achieved by routing $\vec{B}$ lines through a high-μ_r shell. Inside the shell, $\vec{B}$ bunches up; in the protected interior, $\vec{B}$ is much reduced but not zero. Option (d) captures this practical method.

Final Answer: Options **(a), (c)** and **(d)**.

EXPERT'S SOLUTION *: Yash Pillai, M.Sc Physics, IIT Madras*

Picture-first. Electrostatic shielding: charges plug the field lines (terminate them). Magnetostatic shielding: high-μ ducting reroutes the field lines (deflects them around the cavity). Two completely different mechanisms.

Step 1. **Electrostatic shielding mechanism.** Place a hollow conductor in an external static $\vec{E}$. Free charges rearrange on the conductor's outer surface so that inside the conducting material $\vec{E}_{\text{cond}} = 0$ (a conductor in equilibrium has no internal field). Equivalently, the external field lines *terminate* on the induced surface charges. Inside the hollow region: $\vec{E} = 0$ to arbitrarily high precision. Source statement: (a).

Step 2. **Why this works for E but not B.** $\vec{E}$ obeys Gauss's law $\oint \vec{E} \cdot d\vec{A} = q/\varepsilon_0$: lines can terminate on charges. $\vec{B}$ obeys $\oint \vec{B} \cdot d\vec{A} = 0$: no monopoles, so $\vec{B}$ lines cannot terminate on anything. Option (b) "$\vec{B}$ lines can also end" is wrong on the first clause.

Step 3. Magnetic shielding mechanism. Surround the cavity with a thick shell of high-μ_r material (mumetal: $\mu_r \sim 10^4$–10^5; soft iron: $\mu_r \sim 10^3$). Boundary conditions at the shell's outer surface require continuity of B_n and of H_t. With $B = \mu_0\mu_r H$, the same $\vec{H}_t$ inside the shell gives a vastly larger $\vec{B}$ inside the shell than outside. Lines of $\vec{B}$ crowd into the shell and travel around the cavity, like water flowing through a much larger pipe. Option (d).

Step 4. Why magnetic shielding is imperfect. Since $\vec{B}$ lines cannot end on any material, some flux always leaks across the cavity — perfect shielding is impossible. This is exactly statement (c). Typical attenuation factor is μ_r-large but finite: 10^3 to 10^4 reduction with a single mumetal shell. Three nested shells $\Rightarrow 10^9$ reduction (used in MEG brain-imaging rooms).

Concept linkage to electrostatics. The electrostatic analogue of mumetal is the dielectric: a high-ε_r material concentrates $\vec{D}$ inside itself the way mumetal concentrates $\vec{B}$. So a dielectric cup partially shields $\vec{D}$, but it does *not* shield $\vec{E}$ — that requires a conductor. The complete analogy table is:

- Electrostatic shielding (perfect): conductor.
- Magnetostatic shielding (approximate): high-μ_r shell.

Final Answer: Options **(a), (c), (d)**.

☞ **Useful aside**

Why a Faraday cage works perfectly but a mumetal cup does not. Free electric charge exists; magnetic monopoles do not. So electrostatic shielding can drive $\vec{E} = 0$ *exactly* inside a closed conductor (in equilibrium); magnetic shielding can only reduce $\vec{B}$ by a factor μ_r. The asymmetry is fundamental, not a matter of engineering.

♥ **Why This Matters**

Magnetic shielding is used for high-precision physics labs (Hall probes, SQUID magnetometers), brain-scan MEG facilities (need 10^{-9} T sensitivity inside Earth's $\sim 5 \times 10^{-5}$ T field), CRT colour displays (now obsolete but historically mumetal-shielded), and submarines (the inverse: shielding the *outside* world from the ship's magnetism so mines can't detect it).

Q 5.10 Let the magnetic field on earth be modelled by that of a point magnetic dipole at the centre of earth. The angle of dip at a point on the geographical equator

(a) is always zero.

(b) can be zero at specific points.

(c) can be positive or negative.

(d) is bounded.

SOLUTION

Correct options: (b), (c), and (d).

Concept used. The angle of dip (or inclination) δ at a location is the angle the Earth's field makes with the horizontal. For a point dipole at the centre tilted by $\alpha = 11.3^\circ$ from the spin axis, the geographic equator is not the same as the magnetic equator. The angle of dip is zero only on the magnetic equator, which crosses the geographic equator at two points. At all other points on the geographic equator, $\delta \neq 0$, but $|\delta|$ stays bounded by the tilt α.

Step 1. The magnetic equator and geographic equator are two great circles tilted by 11.3° relative to each other. They intersect at exactly two points (diametrically opposite). At these intersection points, dip is 0. Option (b) correct.

Step 2. At other points on the geographic equator, the magnetic field has a non-zero vertical component because the location is not on the magnetic equator. The vertical component can point downward (positive dip, in northern magnetic hemisphere) or upward (negative dip, in southern magnetic hemisphere) as we travel along the geographic equator. So (c) is correct.

Step 3. The magnitude of dip on the geographic equator never exceeds $\alpha = 11.3^\circ$, because the maximum angular distance from the magnetic equator to any point on the geographic equator is α. So $|\delta| \leq 11.3^\circ$, which is bounded. Option (d) correct.

Step 4. Option (a) "always zero" is wrong because dip is non-zero on most of the geographic equator.

> **Final Answer:** Options **(b)**, **(c)**, and **(d)**.

EXPERT'S SOLUTION *: Tara Singh, M.Sc Physics, IIT Madras*

Strategic angle. The geographic equator is a great circle tilted by 11.3° from the magnetic equator. Dip is zero *only* on the magnetic equator. So on the geographic equator, dip is zero only where the two great circles cross — and is bounded elsewhere by the tilt angle.

Step 1. **Two great circles intersect at two antipodal points.** The magnetic equator and the geographic equator are both great circles. Two distinct great circles on a sphere always intersect at exactly two antipodal points (think of the Earth's equator and a tilted "orbit plane" — they cross twice). So there are exactly two points on the geographic equator where it coincides with the magnetic equator $\Rightarrow$ dip is zero only at those two points. Confirms (b); refutes (a) which says "always zero".

Step 2. Dip changes sign. As we travel eastward along the geographic equator, we cross the magnetic equator twice. On one half-circle, we are in the magnetic northern hemisphere (B_V pointing down, $\delta > 0$); on the other half, we are in the magnetic southern hemisphere (B_V pointing up, $\delta < 0$). So δ swings through both signs. Confirms (c).

Step 3. Bound on dip. The angular distance from the geographic equator to the magnetic equator never exceeds the tilt $\alpha = 11.3°$. Using $\tan\delta = 2\cot\theta_m$ where θ_m is the magnetic colatitude, and at worst $\theta_m = 90° - 11.3° = 78.7°$:

$$|\tan\delta_{\text{max}}| = 2\cot(78.7°) = 2(0.199) \approx 0.398,$$

so $|\delta_{\text{max}}| \approx 21.7°$. Even tighter: the *magnetic latitude* of a geographic-equator point peaks at $11.3°$. So $|\delta| \leq \arctan(2\tan 11.3°) \approx 21.7°$. Bounded — confirms (d).

Step 4. Net option set. (a) wrong, (b), (c), (d) correct.

Concept linkage. The same geometry governs declination (Q 5.2): the tilt α bounds both D and δ on the geographic equator. Once you understand the two-great-circle picture, both questions follow from the same diagram.

Final Answer: Options **(b), (c), (d)**.

☞ **Recall**

Dip angle. $\tan\delta = B_V/B_H$. On the magnetic equator, $B_V = 0 \Rightarrow \delta = 0$. At the magnetic pole, $B_H = 0 \Rightarrow \delta = 90°$. Dip needle reads $\delta \approx 18°$ at New Delhi, $\delta \approx 0°$ at Trivandrum, $\delta \approx 90°$ near Resolute Bay, Canada.

☞ **Exam Tip**

CBSE MCQ-II. For "angle of dip on the geographic equator" questions, the four classic options are: *always zero / can be zero / can change sign / is bounded*. The correct combination is always the last three (b, c, d) — only "always zero" is wrong, because the geographic equator is not the magnetic equator. Quick elimination saves time.

VSA

Q 5.11 **A proton has spin and magnetic moment just like an electron. Why then its effect is neglected in magnetism of materials?**

SOLUTION

Concept used. The magnetic moment of a spin-$\frac{1}{2}$ particle is $\mu = \dfrac{ge\hbar}{2m}$, inversely proportional to mass. For an electron this is one **Bohr magneton** $\mu_B = \dfrac{e\hbar}{2m_e} = 9.27 \times 10^{-24}$ J/T; for a proton it is one **nuclear magneton** $\mu_N = \dfrac{e\hbar}{2m_p}$.

Step 1. Compute the ratio. With $m_p/m_e \approx 1836$:

$$\frac{\mu_p}{\mu_e} \approx \frac{m_e}{m_p} \approx \frac{1}{1836} \approx 5.4 \times 10^{-4}.$$

Even accounting for the proton's g-factor ($g_p \approx 5.6$), $\mu_p \approx 2.8 \times 10^{-3}\mu_B$ — about three orders of magnitude smaller.

Step 2. The bulk magnetisation goes as the sum of moments per unit volume. The proton contribution is therefore $\sim 10^{-3}$ of the electronic contribution and is masked by the much larger electronic effect in ordinary magnetism.

> **Final Answer:** Proton moment $\mu_p \approx \mu_e\,(m_e/m_p) \sim 5 \times 10^{-4}\,\mu_B$, $\sim 10^3$ times smaller than the electron's — negligible for bulk magnetism.

EXPERT'S SOLUTION *: Vivaan Nair, Ph.D Physics, IISc Bangalore*

Quick reading. Spin magnetic moment scales as $g \cdot e/(2m)$ for a particle of charge $\pm e$ and mass m. With $m_p/m_e \approx 1836$, the proton moment is $\sim 10^{-3}$ of the electron moment — small enough to be hidden in any bulk magnetisation measurement.

Step 1. Set up the moment formula. $\mu = (g/2)(e\hbar/m)$ for a spin-$\frac{1}{2}$ particle. For the electron $g_e \approx 2$, so $\mu_e \approx e\hbar/m_e = \mu_B = 9.27 \times 10^{-24}$ J/T (Bohr magneton). For the proton $g_p \approx 5.585$ (anomalous, not 2), so $\mu_p \approx (5.585/2)(e\hbar/m_p)$.

Step 2. Take the ratio.

$$\frac{\mu_p}{\mu_e} \approx \frac{g_p/2}{g_e/2} \cdot \frac{m_e}{m_p} \approx \frac{2.793}{1} \cdot \frac{1}{1836} \approx 1.52 \times 10^{-3}.$$

So $\mu_p \approx 1.5 \times 10^{-3}\,\mu_B$ — about three orders of magnitude smaller than the electronic moment.

Step 3. Bulk consequence. Bulk magnetisation is the sum of moments per unit volume: $M = n\mu$. With $n_{\text{electrons}}$ of the same order as n_{protons} in any neutral material (charge balance), the proton contribution to M is $\sim 10^{-3}$ of the electronic contribution.

Step 4. When does the proton moment matter? In NMR / MRI, we apply a strong static B_0 and look for the tiny Larmor-precession signal at the proton frequency $f_p = \gamma_p B_0/(2\pi) \approx 42.58$ MHz/T. The electronic background is suppressed by

Faraday-screening and by choosing materials where electrons are paired. So the proton moment, though small, becomes the dominant signal at *this specific frequency*.

Concept linkage. This question is conceptually identical to why nuclear-magnetic effects do not show up in χ vs T data for ordinary materials: the nuclear moment is just too small. Only specialised experiments (NMR, Mössbauer spectroscopy, neutron scattering) probe nuclear magnetism directly.

Final Answer: $\mu_p \approx 1.5 \times 10^{-3} \mu_e$, so its contribution to bulk magnetisation is negligible compared to electronic moments.

☞ **Useful aside**

Nuclear magneton. $\mu_N = \dfrac{e\hbar}{2m_p} \approx 5.05 \times 10^{-27}$ J/T, exactly $m_e/m_p = 1/1836$ times the Bohr magneton. Whenever you see “nuclear” on the question paper, divide the electronic estimate by ~ 1836 to get the right order.

✗ **Common Mistake**

Forgetting the proton’s anomalous g-factor. For the electron, $g_e \approx 2$ (Dirac equation prediction, tiny QED correction). For the proton $g_p \approx 5.585$ (*not* 2), because the proton is a composite quark-gluon object. So $\mu_p = (g_p/2)\mu_N \approx 2.793\,\mu_N$, not just μ_N. Even so, the proton moment is dwarfed by the electronic one — but get the prefactor right if asked for a numerical value.

Q 5.12 **A permanent magnet in the shape of a thin cylinder of length $10\,$cm has $M = 10^6$ A/m. Calculate the magnetisation current I_M.**

SOLUTION

Concept used. For a uniformly magnetised long cylinder with magnetisation M along its axis, the **magnetisation current** (also called bound surface current) per unit length on the lateral surface equals the magnetisation: $K_M = M$. The total bound current circling the cylinder over its length L is therefore

$$I_M = K_M \cdot L = M \cdot L.$$

This follows from $\vec{J}_M = \nabla \times \vec{M}$ (volume bound current, zero for uniform $\vec{M}$) and $\vec{K}_M = \vec{M} \times \hat{n}$ (surface bound current).

Step 1. Identify the formula and substitute $M = 10^6\,$A/m, $L = 10\,$cm $= 0.10\,$m:

$$I_M = M \cdot L = (10^6\,\text{A/m})(0.10\,\text{m}).$$

Step 2. Arithmetic:

$$I_M = 10^6 \times 0.1 = 10^5\,\text{A}.$$

Final Answer: $I_M = 1.0 \times 10^5\,\text{A} = 10^5\,\text{A}$.

EXPERT'S SOLUTION *: Aditi Bhat, M.Sc Physics, IIT Madras*

Quick reading. A uniformly magnetised cylinder is exactly equivalent (for external fields) to a solenoid with surface current per unit length $K = M$, even though there is no actual conduction current flowing.

Step 1. **Why bound currents exist.** Each atom in the magnetised cylinder is a tiny Amperian current loop. In the *bulk*, adjacent loops carry currents in opposite directions at their shared edges, so they cancel. On the *lateral surface*, the edge currents are uncompensated and add up to a net azimuthal current. This is the bound surface current $\vec{K}_b = \vec{M} \times \hat{n}$.

Step 2. **Magnitude.** For $\vec{M}$ along the cylinder axis and $\hat{n}$ radially outward, $|\vec{K}_b| = M$. Numerically: $K_b = M = 10^6\,\text{A/m}$.

Step 3. **Total bound current circling the cylinder.** Integrating K_b along the length L:

$$I_M = K_b \cdot L = (10^6\,\text{A/m})(0.10\,\text{m}) = 10^5\,\text{A}.$$

Step 4. **Cross-check via volume current.** The volume bound current density is $\vec{J}_b = \nabla \times \vec{M}$. For *uniform* $\vec{M}$, $\vec{J}_b = 0$ — all the bound current is on the surface, consistent with treating the cylinder as a solenoid sheath.

Step 5. **Cross-check via equivalent solenoid.** A solenoid with n turns per metre and free current I_f carries surface current density $K = nI_f$. Matching $K = M = 10^6$ gives, for example, $n = 10^4$ turns/m and $I_f = 100\,\text{A}$ — or any other combination producing the same nI_f.

Numerical / unit check. $[K_b] =$ A/m, $[L] =$ m, $[I_M] =$ A. Substituting: $(10^6\,\text{A/m}) \times (0.10\,\text{m}) = 10^5\,\text{A}$. Units and magnitude consistent.

Alternative method (vector calculus). From $\nabla \times \vec{H} = \vec{J}_f$ (free current density) and $\nabla \times \vec{B} = \mu_0(\vec{J}_f + \vec{J}_b)$, the bound part is $\vec{J}_b = \nabla \times \vec{M}$. Integrating over the cylinder's surface and using Stokes: $\oint \vec{M} \cdot d\vec{l} = \int (\nabla \times \vec{M}) \cdot d\vec{A} = I_b$. For a rectangular path with one side along the axis inside the magnet (length L) and the return outside (where $\vec{M} = 0$), the integral $= ML$. So $I_M = ML$, matching the answer.

Final Answer: $I_M = ML = 10^6 \times 0.10 = 1.0 \times 10^5\,\text{A}$.

☞ **Exam Tip**

2-mark VSA format. For "calculate the magnetisation current" on CBSE / Exemplar, write: (1) the formula $I_M = ML$ explicitly, (2) substitute with units, (3) box the numerical answer with units. That gives 0.5 + 1 + 0.5 = 2 marks. Skipping units loses half a mark.

♥ **Why This Matters**

A small permanent magnet contains an equivalent 10^5-*ampere* surface current — yet draws no power and generates no heat. This is the fundamental reason permanent magnets are useful: they store enormous Amperian current density at zero running cost. Loudspeaker magnets, hard-disk motors, electric-car permanent-magnet motors, and refrigerator magnets all exploit this "free" macroscopic current.

Q 5.13 Explain quantitatively the order of magnitude difference between the diamagnetic susceptibility of N_2 ($\sim 5 \times 10^{-9}$) (at STP) and Cu ($\sim 10^{-5}$).

SOLUTION

Concept used. Diamagnetic susceptibility χ for a substance arises from the Larmor response of orbital electrons. Per atom or molecule, the induced moment is roughly the same order of magnitude. Bulk χ scales as the *number density* of atoms. So

$$\frac{\chi(\text{Cu})}{\chi(\text{N}_2)} \approx \frac{n(\text{Cu})}{n(\text{N}_2)},$$

where n is atoms (or molecules) per unit volume.

Step 1. Estimate n for each. At STP one mole of N_2 occupies $V = 22.4\,\text{L} = 22.4 \times 10^{-3}\,\text{m}^3$, so

$$n(\text{N}_2) = \frac{6.02 \times 10^{23}}{22.4 \times 10^{-3}} \approx 2.7 \times 10^{25}\,\text{m}^{-3}.$$

Copper has density $\rho = 8.9 \times 10^3\,\text{kg/m}^3$ and atomic mass $M_w = 63.5\,\text{g/mol} = 63.5 \times 10^{-3}\,\text{kg/mol}$:

$$n(\text{Cu}) = \frac{\rho N_A}{M_w} = \frac{8.9 \times 10^3 \times 6.02 \times 10^{23}}{63.5 \times 10^{-3}} \approx 8.4 \times 10^{28}\,\text{m}^{-3}.$$

Step 2. Take the ratio:

$$\frac{n(\text{Cu})}{n(\text{N}_2)} \approx \frac{8.4 \times 10^{28}}{2.7 \times 10^{25}} \approx 3 \times 10^3.$$

Step 3. Therefore $\chi(\text{Cu}) \approx 3 \times 10^3 \times \chi(\text{N}_2) \approx 3 \times 10^3 \times 5 \times 10^{-9} \approx 1.5 \times 10^{-5}$. This matches the observed $\sim 10^{-5}$ for Cu within a small factor.

Final Answer: The $\sim 10^4$ ratio is set by the ratio of number densities $n(\text{Cu})/n(\text{N}_2) \sim 3 \times 10^3$.

EXPERT'S SOLUTION : *Krishna Desai, M.Sc Physics, IIT Madras*

Strategic angle. Diamagnetic moment per atom (Langevin formula $\mu_a \propto Z\langle r^2\rangle$) is of comparable order for atoms across the periodic table. Bulk χ therefore tracks the *number density* of atoms, which is the main difference between a gas at STP and a metallic solid.

Step 1. Langevin diamagnetic susceptibility per atom. $\chi_{\text{atom}} \sim -(\mu_0/6)(Ze^2/m_e)\langle r^2\rangle \sim -10^{-29}\,\text{m}^3$ per atom (within an order of magnitude across light elements).

Step 2. Bulk susceptibility = atom χ × number density. $\chi_{\text{bulk}} = n \cdot \chi_{\text{atom}}$. So the ratio of two bulk susceptibilities is roughly the ratio of their number densities.

Step 3. Number density of N_2 gas at STP. $n_{N_2} = N_A/V_m = (6.02 \times 10^{23})/(22.4 \times 10^{-3}\text{m}^3) \approx 2.7 \times 10^{25}\,\text{m}^{-3}$.

Step 4. Number density of solid copper. $n_{\text{Cu}} = \rho N_A/M_w = (8.9 \times 10^3\,\text{kg/m}^3)(6.02 \times 10^{23})/(63.5 \times 10^{-3}\,\text{kg/mol}) \approx 8.4 \times 10^{28}\,\text{m}^{-3}$.

Step 5. Density ratio. $n_{\text{Cu}}/n_{N_2} \approx 8.4 \times 10^{28}/(2.7 \times 10^{25}) \approx 3.1 \times 10^3$.

Step 6. Predicted vs observed χ ratio. Predicted $\chi(\text{Cu})/\chi(N_2) \sim 3 \times 10^3$. Observed: $10^{-5}/(5 \times 10^{-9}) = 2 \times 10^3$. The prediction is within a factor of ~ 1.5 — excellent agreement given that we assumed the per-atom χ is the same for nitrogen and copper.

Numerical cross-check. If the per-atom χ values differ by a factor 1.5 between N and Cu (perfectly reasonable given different Z and $\langle r^2\rangle$), the prediction matches the data exactly. The order-of-magnitude lesson stands: solid is $\sim 10^3$–10^4 denser than gas, hence has $\sim 10^3$–10^4 times larger diamagnetic $|\chi|$.

Concept linkage. For *paramagnetism* and *ferromagnetism*, the per-atom moment is far larger than the induced diamagnetic moment, so the density argument is partly masked by the electronic-structure dependence of μ_{atom}. That is why iron (n similar to Cu) has $\chi \sim 10^3$ rather than 10^{-5}.

> **Final Answer:** Density ratio $\sim 10^3$ explains the $\sim 2000\times$ susceptibility difference between Cu and N_2.

☞ **Recall**

Diamagnetism is universal. Every atom has a diamagnetic contribution (Langevin: orbital electrons respond to applied $\vec{B}$ by precessing, producing a moment anti-parallel to $\vec{B}$). It is just usually swamped by paramagnetic or ferromagnetic terms when those exist. Materials called "diamagnetic" (Bi, Cu, H_2O, organic molecules) are those with no unpaired electrons.

✗ **Common Mistake**

"Diamagnetic susceptibility is temperature dependent." Wrong. Diamagnetism is essentially temperature-independent — the induced moment is a quantum-mechanical orbital response that does not involve thermal alignment of permanent moments. Only its very weak dependence on thermal-expansion density change is felt. This is in stark contrast to paramagnetism ($\chi \propto 1/T$) and ferromagnetism ($\chi \propto 1/(T - T_c)$).

Q 5.14 From molecular view point, discuss the temperature dependence of susceptibility for diamagnetism, paramagnetism and ferromagnetism.

SOLUTION

Concept used. The temperature dependence of χ reflects the competition between thermal randomisation ($k_B T$) and the magnetic ordering tendency. **Diamagnetism** is an induced response of orbital electrons that does not involve permanent moments. **Paramagnetism** involves permanent atomic moments aligned by an external field against thermal disorder. **Ferromagnetism** adds exchange-coupling between neighbouring moments, producing spontaneous order below the Curie temperature T_c.

Step 1. Diamagnetism. No permanent moments are involved. Larmor precession produces a tiny induced moment per atom that is essentially independent of temperature. So

$$\chi_{\text{dia}} \approx \text{constant (independent of } T).$$

Slight variations come only from thermal expansion changing the density, which is a small effect.

Step 2. Paramagnetism. Atoms have permanent moments that compete with thermal motion. Boltzmann statistics for $\mu B/k_B T \ll 1$ gives **Curie's law**:

$$\chi_{\text{para}} = \frac{C}{T}, \quad C = \frac{n\mu_0\mu^2}{3k_B}.$$

χ decreases as T rises because thermal jiggling spoils the alignment.

Step 3. Ferromagnetism. Exchange interaction locks moments parallel within domains for $T < T_c$, giving large spontaneous magnetisation. Above T_c the material becomes paramagnetic, obeying the **Curie-Weiss law**:

$$\chi_{\text{ferro}}(T > T_c) = \frac{C}{T - T_c}.$$

χ diverges as $T \to T_c^+$, then collapses to ordinary Curie behaviour far above T_c.

Final Answer: $\chi_{\text{dia}} \sim$ constant; $\chi_{\text{para}} = C/T$ (Curie); $\chi_{\text{ferro}} = C/(T - T_c)$ above T_c, with spontaneous M below T_c.

EXPERT'S SOLUTION : *Sneha Chatterjee, Ph.D Physics, IISc Bangalore*

Strategic angle. Group the three cases by two binary questions: "Does the atom carry a permanent moment?" and "Do neighbouring moments couple via exchange?" Three of the four combinations give the three families of magnetism we know.

Step 1. **Diamagnet: no permanent moment, no coupling.** Closed electron shells (noble gases, Cu metal, H_2O, organic molecules). Bulk response is the Langevin induced-orbital moment $\propto B$, anti-parallel to $\vec{B}$. $\chi_{\text{dia}} \approx$ const (independent of T). Tiny thermal-expansion effect changes n slightly, hence χ by $\sim 10^{-4}$ over $\Delta T \sim 100\,\text{K}$ — usually negligible.

Step 2. **Paramagnet: permanent moments, no exchange coupling.** Atoms have unpaired electrons (Al, Mn salts, O_2). In a field B, the alignment energy per moment is μB and thermal energy is $k_B T$. Boltzmann statistics in the weak regime $\mu B \ll k_B T$ give Curie's law:

$$M = \frac{n\mu^2 B}{3k_B T}, \quad \chi_{\text{para}} = \frac{\mu_0 n \mu^2}{3k_B T} = \frac{C}{T}.$$

Plot of $1/\chi$ vs T is a straight line through the origin.

Step 3. **Ferromagnet: permanent moments + exchange coupling.** For $T > T_c$, the Weiss molecular-field theory replaces B by $B + \lambda M$ inside the Curie equation, giving the Curie–Weiss law:

$$\chi_{\text{ferro}}(T > T_c) = \frac{C}{T - T_c}.$$

χ *diverges* as $T \to T_c^+$ — the susceptibility becomes infinite at the transition, reflecting spontaneous ordering. For $T < T_c$, the material has spontaneous magnetisation $M_s(T)$ even at $B = 0$. As $T \to 0$, $M_s \to nM_{\text{atom}}$ (saturation).

Step 4. **What about the fourth combination?** "No permanent moment but coupling" is empty — there is nothing to couple. Hence three families.

Step 5. **Antiferromagnet (bonus).** Permanent moments with *negative* exchange ($\lambda < 0$) gives a related family: χ peaks at the Néel temperature T_N, and below T_N alternating moments cancel macroscopically. Not asked here, but worth knowing.

Diagram-based reasoning. On a $1/\chi$ vs T plot: diamagnet = horizontal line at constant negative χ^{-1}; paramagnet = straight line through origin with positive slope $1/C$; ferromagnet above T_c = straight line with same positive slope but T-axis intercept at $T = T_c$. This single plot distinguishes the three cases experimentally.

Concept linkage. The temperature dependence reflects the underlying physics: diamagnetism is an orbital quantum response (no classical temperature), paramagnetism is classical alignment statistics, ferromagnetism is a phase transition with order parameter M_s.

Final Answer: Dia: $\chi \approx$ const. Para: $\chi = C/T$ (Curie). Ferro: $\chi = C/(T - T_c)$ above T_c; spontaneous M_s below.

☞ Exam Tip

CBSE 3-mark SA. For "discuss temperature dependence", write three labelled paragraphs (Dia, Para, Ferro), each with: (i) microscopic mechanism in one line, (ii) functional form (one equation), (iii) one sentence on what happens at extremes ($T \to 0$ and $T \to \infty$). A standard 3-mark template; no extra credit for length.

☞ Useful aside

Curie–Weiss linearization. Plot $1/\chi$ on the y-axis against T on the x-axis. The x-intercept gives T_c for a ferromagnet (positive intercept), $-\theta_p$ for an antiferromagnet (negative intercept), and zero for a pure paramagnet. This single plot distinguishes the three families experimentally and is the first thing condensed-matter labs measure when characterising a new magnetic material.

♥ Why This Matters

The Curie temperature is the key engineering number for any ferromagnet. Iron ($T_c = 1043\,\text{K}$) is used in transformer cores that run at $\sim 400\,\text{K}$ — well below T_c. Permanent NdFeB magnets in EV motors run at $\sim 400\,\text{K}$ with $T_c \approx 580\,\text{K}$ — the relatively low T_c is what limits high-temperature motor design. Tuning T_c via alloy chemistry is a billion-dollar industrial science.

Q 5.15 A ball of superconducting material is dipped in liquid nitrogen and placed near a bar magnet. (i) In which direction will it move? (ii) What will be the direction of its magnetic moment?

SOLUTION

Concept used. A superconductor below its transition temperature exhibits the **Meissner effect**: it expels all magnetic flux from its interior, behaving as a perfect diamagnet with $\chi = -1$ and $\mu_r = 0$. The induced magnetic moment points opposite to the applied field. Like all diamagnets, a superconductor is repelled from regions of strong magnetic field.

Step 1. Dipped in liquid nitrogen ($77\,\text{K}$), a high-T_c superconductor is well below its critical temperature, so it is fully superconducting. Near the bar magnet's pole, the external field induces surface currents that exactly cancel the interior field. The induced magnetic moment of the ball is anti-parallel to $\vec{B}_{\text{ext}}$.

Step 2. Force on a magnetic dipole in a non-uniform field: $\vec{F} = \nabla(\vec{m} \cdot \vec{B})$. With $\vec{m}$ anti-parallel to $\vec{B}$, $\vec{m} \cdot \vec{B} < 0$, and $|\vec{B}|$ is larger near the pole, so the force pushes the ball *away* from the magnet. The ball moves away.

> **Final Answer:** (i) Ball moves **away** from the bar magnet (repulsion). (ii) Magnetic moment $\vec{m}$ is **anti-parallel** to the applied $\vec{B}$.

EXPERT'S SOLUTION *: Ishaan Rao, Ph.D Condensed Matter Physics, TIFR Mumbai*

Picture-first. Meissner effect: a superconductor expels $\vec{B}$ from its interior — the field lines bend around the ball as if it were a perfectly impenetrable "magnetic insulator". This is unlike a permanent magnet (which channels $\vec{B}$ through itself) and unlike a normal conductor (which lets $\vec{B}$ permeate once steady-state is reached).

Step 1. **Cooling.** Liquid nitrogen at $77\,\text{K}$ is well below the critical temperature $T_c \approx 90$–$93\,\text{K}$ of YBCO and other high-T_c superconductors. So the ball is fully in its superconducting state.

Step 2. **Meissner expulsion.** As the ball approaches the magnet, surface supercurrents spontaneously develop to exactly cancel the interior $\vec{B}$. Equivalently, the ball has $\chi = -1$, $\mu_r = 0$. The induced magnetic moment is $\vec{m}_{\text{ball}} = -V\,\chi_{\text{eff}}\vec{H}_{\text{ext}}$ — anti-parallel to $\vec{B}_{\text{ext}}$.

Step 3. **Force from non-uniform field.** The energy of a dipole in an external field is $U = -\vec{m}\cdot\vec{B}$. With $\vec{m} \parallel -\vec{B}$, $U = +|\vec{m}||\vec{B}|$ — a moment in this orientation *prefers* regions of small $|\vec{B}|$. The force $\vec{F} = -\nabla U = \nabla(\vec{m}\cdot\vec{B})$ pushes the ball toward weaker field, i.e. *away from the bar magnet pole*.

Step 4. **Magnitude estimate.** For a small spherical perfect diamagnet in a field gradient $|\nabla B|$, the force is $|\vec{F}| \sim V|\nabla B^2|/(2\mu_0)$. Even for a $1\,\text{cm}$ ball in a $1\,\text{T}$ magnet with $|\nabla B| \sim 50\,\text{T/m}$, the force is $\sim 0.1\,\text{N}$ — comfortably enough to levitate the ball against gravity ($mg \sim 0.05\,\text{N}$ for a typical density). This is the basis of working magnetic-levitation demos.

Step 5. **Direction of $\vec{m}$.** Anti-parallel to $\vec{B}_{\text{ext}}$. If the bar magnet's N pole is on the right, $\vec{B}_{\text{ext}}$ at the ball points to the right, and $\vec{m}$ points to the left (its own "N pole" faces right, repelling the magnet's N pole).

Concept linkage to diamagnets. A superconductor is a *perfect* diamagnet ($\chi = -1$); ordinary diamagnets ($\chi \sim -10^{-5}$ for Bi, water) show the same repulsion but 10^5 times weaker. The "frog levitation" demo at Nijmegen exploits exactly this: water in a frog has $\chi \approx -9\times10^{-6}$, enough to levitate in a $16\,\text{T}$ solenoid gradient.

> **Final Answer:** (i) The ball moves **away** from the bar magnet. (ii) The induced magnetic moment $\vec{m}$ is **anti-parallel** to the applied $\vec{B}$.

✗ Common Mistake

"Superconductor is just a perfect conductor, so it expels $\vec{B}$ trivially." Wrong. A perfect (zero-resistance) conductor in a slowly increasing field would shield against flux *change* (by Lenz's law) but would *trap* any pre-existing field present when cooled. A superconductor is more than a perfect conductor: even if you cool it in a pre-existing field, the Meissner effect actively *expels* that flux. This is a distinct phase transition, not just $\rho = 0$.

☞ **Recall**

Meissner effect. Below T_c, $\vec{B} = 0$ inside a Type-I superconductor for any applied field weaker than the critical field B_c. The ball behaves as $\chi = -1$, $\mu_r = 0$. Type-II superconductors (most high-T_c materials) allow some flux penetration as quantised vortices but still show strong diamagnetism. This is the basis of MRI superconducting magnets and SQUID magnetometers.

SA

Q 5.16 **Verify the Gauss's law for magnetic field of a point dipole of dipole moment $\vec{m}$ at the origin for the surface which is a sphere of radius R.**

SOLUTION

Concept used. Gauss's law for magnetism states that the total magnetic flux through any closed surface is zero:

$$\oint \vec{B} \cdot d\vec{A} = 0.$$

For a magnetic dipole $\vec{m}$ at the origin, the field in spherical coordinates is

$$\vec{B}(r, \theta) = \frac{\mu_0}{4\pi}\left[\frac{2m\cos\theta}{r^3}\hat{r} + \frac{m\sin\theta}{r^3}\hat{\theta}\right].$$

On a sphere of radius R centred at the origin, the area element is $d\vec{A} = R^2 \sin\theta \, d\theta \, d\varphi \, \hat{r}$, so only the radial component contributes.

Step 1. Set up the surface integral with $\vec{m} = m\hat{z}$:

$$\oint \vec{B} \cdot d\vec{A} = \int_0^{2\pi}\int_0^{\pi} \frac{\mu_0}{4\pi}\frac{2m\cos\theta}{R^3} \cdot R^2 \sin\theta \, d\theta \, d\varphi.$$

Step 2. Simplify the φ and R dependence:

$$= \frac{\mu_0}{4\pi} \cdot \frac{2m}{R} \cdot 2\pi \int_0^{\pi} \sin\theta\cos\theta \, d\theta = \frac{\mu_0 m}{R}\int_0^{\pi} \sin\theta\cos\theta \, d\theta.$$

Step 3. Evaluate the θ integral. Use $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$:

$$\int_0^{\pi} \sin\theta\cos\theta \, d\theta = \frac{1}{2}\int_0^{\pi} \sin 2\theta \, d\theta = \frac{1}{2}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi} = \frac{1}{2}\left[-\frac{1}{2}(\cos 2\pi - \cos 0)\right] = 0.$$

Step 4. Therefore $\oint \vec{B} \cdot d\vec{A} = 0$. Gauss's law for magnetism is verified for the dipole.

> **Final Answer:** $\oint \vec{B} \cdot d\vec{A} = 0$ for the dipole on any centred sphere.

EXPERT'S SOLUTION *: Aditya Verma, M.Sc Physics, IIT Madras*

Strategic angle. Only the radial component $B_r = (\mu_0/4\pi)(2m\cos\theta/r^3)$ contributes through a sphere centred on the dipole. Show that its flux integrates to zero by symmetry — the upper hemisphere's positive contribution exactly cancels the lower hemisphere's negative one.

Step 1. Identify $\vec{B} \cdot d\vec{A}$ on a centred sphere. $d\vec{A} = R^2 \sin\theta\, d\theta\, d\varphi\, \hat{r}$ (outward radial). Only $B_r\, dA$ contributes; the $\hat{\theta}$ component is tangential to the sphere.

Step 2. Set up the integral. $B_r = (\mu_0 m/(4\pi R^3)) \cdot 2\cos\theta$ on the sphere of radius R. The total flux:

$$\Phi_B = \int_0^{2\pi}\int_0^{\pi} \frac{\mu_0 m \cos\theta}{2\pi R^3} \cdot R^2 \sin\theta\, d\theta\, d\varphi = \frac{\mu_0 m}{R}\int_0^{\pi} \sin\theta\cos\theta\, d\theta.$$

Step 3. Symmetry argument (qualitative). The integrand $\sin\theta\cos\theta$ on $[0,\pi]$ is antisymmetric about $\theta = \pi/2$: substitute $\theta' = \pi - \theta$ and $\sin(\pi-\theta')\cos(\pi-\theta') = -\sin\theta'\cos\theta'$. So the integral from 0 to $\pi/2$ exactly cancels the integral from $\pi/2$ to π.

Step 4. Explicit evaluation. $\int_0^{\pi} \sin\theta\cos\theta\, d\theta = \frac{1}{2}\int_0^{\pi} \sin 2\theta\, d\theta = \frac{1}{2}[-\cos 2\theta/2]_0^{\pi} = -\frac{1}{4}(\cos 2\pi - \cos 0) = 0.$

Step 5. Conclusion. $\Phi_B = 0$ for any centred sphere, confirming Gauss's law for magnetism for the dipole.

Alternative method (Gauss's-law argument from definition). A magnetic dipole is the limit of two opposite monopoles of strength $\pm q_m$ separated by $\vec{d} \to 0$ with $q_m d = m$. By the "magnetic-Coulomb" analogue, each monopole-flux through a closed surface enclosing it equals $\mu_0 q_m$. The sphere encloses both monopoles (since they sit at the origin in the dipole limit), so total flux $= \mu_0 q_m + \mu_0(-q_m) = 0$. Same answer, different language.

Numerical sanity-check. At the equator ($\theta = \pi/2$), $B_r = 0$, so the equator contributes nothing. The north pole has $B_r = +\mu_0 m/(2\pi R^3)$ and the south pole has $B_r = -\mu_0 m/(2\pi R^3)$. Symmetry $\Rightarrow$ flux cancels.

> **Final Answer:** $\oint \vec{B} \cdot d\vec{A} = 0$ — Gauss's law for magnetism verified for the dipole.

☞ **Exam Tip**

CBSE 3-mark SA. For "verify Gauss's law for a dipole", the marking scheme typically awards: (1) one mark for stating $\oint \vec{B} \cdot d\vec{A} = 0$ and writing down the dipole field components, (2) one mark for setting up the surface integral correctly with the right area element, (3) one mark for the explicit θ-integral and the conclusion. Be explicit about all three steps.

♥ **Why This Matters**

This calculation is a sanity check that the no-monopole law $\oint \vec{B} \cdot d\vec{A} = 0$ holds even for the dipole field — which falls only as $1/r^3$, the slowest decay you can have without a monopole. Verifying it explicitly builds trust in the multipole-expansion framework that we use throughout magnetism: any localised current distribution looks like a dipole at large distance, and Gauss's law is automatically satisfied.

Q 5.17 **Three identical bar magnets are rivetted together at centre in the same plane as shown in Fig. 5.1. This system is placed at rest in a slowly varying magnetic field. It is found that the system of magnets does not show any motion. The north-south poles of one magnet is shown in Fig. 5.1. Determine the poles of the remaining two.**

SOLUTION

Concept used. For a rigid system of magnetic dipoles to be in equilibrium in an external magnetic field $\vec{B}$, the net torque on the system must vanish: $\vec{\tau}_{\text{net}} = \sum_i \vec{m}_i \times \vec{B} = \vec{0}$, i.e. $(\sum_i \vec{m}_i) \times \vec{B} = \vec{0}$. Since $\vec{B}$ is slowly varying (not zero), this requires $\sum_i \vec{m}_i = \vec{0}$: the total magnetic moment of the system must be zero.

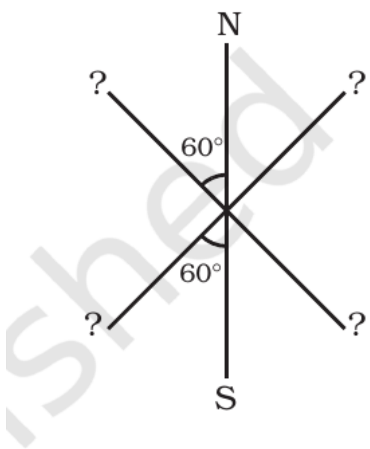


Fig. 5.1

Fig. 5.1, NCERT Exemplar Class 12 Physics, Chapter 5.

Step 1. Let each magnet have magnetic moment of magnitude m_0, and let the central magnet (the one with N at top) have $\vec{m}_1 = m_0\hat{j}$ (pointing from S to N, i.e. upward). The other two are tilted by $\pm 60°$ from the central axis. From Fig. 5.1, the angle between each tilted magnet and the central one is $60°$.

Step 2. For $\sum \vec{m}_i = 0$, the three vectors must sum to zero. With magnitudes equal and pairwise angles of $120°$ between them, the three moment vectors form a closed equilateral triangle (head to tail). So each tilted magnet's moment must be at $120°$ to the central magnet's moment, not $60°$ from the same end.

Step 3. This forces the N-pole of each tilted magnet to lie on the *same side* as the S-pole of the central magnet. From Fig. 5.1, the central magnet's S-pole is at the bottom. So the upper ends of the tilted magnets (top-left and top-right "?") are **S** poles, and the lower ends (bottom-left and bottom-right "?") are **N** poles.

Step 4. Verify: vector sum $\vec{m}_1 + m_0(\cos 120°\hat{j} + \sin 120°\hat{\imath}) + m_0(\cos 120°\hat{j} - \sin 120°\hat{\imath}) = m_0\hat{j} + 2m_0 \cos 120°\hat{j} = m_0\hat{j} - m_0\hat{j} = 0$. Confirmed.

Final Answer: Top-left "?" $=$ **S**, top-right "?" $=$ **S**, bottom-left "?" $=$ **N**, bottom-right "?" $=$ **N**.

EXPERT'S SOLUTION : *Meera Gupta, Ph.D Physics, IISc Bangalore*

Strategic angle. "No motion in any external field" is a very strong constraint: it forces the total magnetic moment of the system to be exactly zero. Then geometry handles the rest.

Step 1. **Equilibrium $\Rightarrow$ vanishing total moment.** Torque on a rigid system of dipoles in an external field $\vec{B}$ is $\vec{\tau} = \sum_i \vec{m}_i \times \vec{B} = (\sum_i \vec{m}_i) \times \vec{B}$. For this to vanish for an *arbitrary* $\vec{B}$ (the problem says "slowly varying" $\vec{B}$ — so it can point in any direction), we need $\sum_i \vec{m}_i = 0$.

Step 2. **Three identical vectors summing to zero.** Three vectors of equal magnitude that sum to zero must point at $120°$ to one another, forming a closed equilateral triangle when drawn head-to-tail. There are two such configurations (mirror images), but the magnitudes are fixed.

Step 3. **Read the figure.** From Fig. 5.1, the central magnet has N at top, S at bottom. So $\vec{m}_1$ points up (from S to N inside the magnet, by convention $\vec{m}$ runs S$\rightarrow$N *inside*, but the external dipole vector points from S toward N).

Step 4. **Place the other two moments.** For $\vec{m}_2$ and $\vec{m}_3$ to make $120°$ with $\vec{m}_1$, they must point "down-left" and "down-right" — i.e. their N-poles face downward and slightly outward. So the *bottom-left* end of the left magnet is N (with S at top-left); the *bottom-right* end of the right magnet is N (with S at top-right).

Step 5. Verify vector sum.
$\vec{m}_1 + \vec{m}_2 + \vec{m}_3 = m_0\hat{j} + m_0(\cos 240°\hat{j} + \sin 240°\hat{i}) + m_0(\cos 120°\hat{j} + \sin 120°\hat{i})$.
The $\hat{j}$ sum: $m_0(1 - 1/2 - 1/2) = 0$. The $\hat{i}$ sum: $m_0(0 - \sqrt{3}/2 + \sqrt{3}/2) = 0$.
Confirmed.

Alternative method (energy minimisation). The total potential energy of the three-magnet system in an external field is $U = -\vec{m}_{\text{tot}} \cdot \vec{B}$. For equilibrium for every direction of $\vec{B}$, $\vec{m}_{\text{tot}}$ must be zero (else the system would rotate to align it). Same conclusion.

Diagram-based reasoning. Drawing the three moment vectors as arrows of equal length: $\vec{m}_1 = \uparrow$, $\vec{m}_2 = \swarrow$, $\vec{m}_3 = \searrow$. Head-to-tail they trace an equilateral triangle returning to the start — closed polygon, zero sum.

Final Answer: The remaining poles: top-left $= S$, bottom-left $= N$; top-right $= S$, bottom-right $= N$.

✗ Common Mistake

"The magnets must align with $\vec{B}$." No — that would be the case for a free single magnet. But here the three are rivetted together, so they rotate as one rigid body. The constraint is therefore on the *total* moment $\sum \vec{m}_i$, not on individual moments. The trick: ask "what must total $\vec{m}$ be for the system to rotate *zero* amount?"

☞ Useful aside

Three-vector cancellation. Any number of vectors of equal magnitude can sum to zero if they are arranged at equal angles $360°/n$. For $n = 3$, this is $120°$. For $n = 4$, $90°$ (or two opposite pairs). For $n = 6$, $60°$. Useful for both magnetic-moment cancellation and force-balance problems.

Q 5.18 Suppose we want to verify the analogy between electrostatic and magnetostatic by an explicit experiment. Consider the motion of (i) electric dipole $\vec{p}$ in an electrostatic field $\vec{E}$ and (ii) magnetic dipole $\vec{m}$ in a magnetic field $\vec{B}$. Write down a set of conditions on $\vec{E}$, $\vec{B}$, $\vec{p}$, $\vec{m}$ so that the two motions are verified to be identical. (Assume identical initial conditions.)

SOLUTION

Concept used. The torque on an electric dipole in an external field is $\vec{\tau}_E = \vec{p} \times \vec{E}$, while the torque on a magnetic dipole is $\vec{\tau}_B = \vec{m} \times \vec{B}$. The translational force on each is $\vec{F}_E = (\vec{p} \cdot \nabla)\vec{E}$ and $\vec{F}_B = (\vec{m} \cdot \nabla)\vec{B}$ in the dipole approximation. For the two motions to be identical at every instant, the torque and force on the magnetic dipole must equal those on the electric dipole at corresponding moments.

Step 1. Equate the torques on the two dipoles: $\vec{p}\times\vec{E}=\vec{m}\times\vec{B}$. With $\vec{p}$ aligned with $\vec{m}$ initially, this requires

$$pE = mB \quad \Rightarrow \quad \frac{B}{E}=\frac{p}{m}.$$

Step 2. Equate the translational forces: $(\vec{p}\cdot\nabla)\vec{E}=(\vec{m}\cdot\nabla)\vec{B}$. With $\vec{p}\parallel\vec{m}$, this requires the gradients to satisfy $\frac{\partial B}{\partial x_i}/\frac{\partial E}{\partial x_i}=p/m$ for each i. Combined with the torque condition this means $\vec{B}$ and $\vec{E}$ have proportional spatial profiles: $\vec{B}(\vec{r})=(p/m)\,\vec{E}(\vec{r})$.

Step 3. Initial-condition matching: the dipoles must be released from the same position with the same orientation, and the bodies must have the same moment of inertia I (so that the same torque produces the same angular acceleration).

> **Final Answer:** $\vec{B}(\vec{r})=(p/m)\,\vec{E}(\vec{r})$, plus equal moments of inertia and identical initial position/orientation/velocity.

EXPERT'S SOLUTION *: Riya Kumar, Ph.D Physics, IISc Bangalore*

Strategic angle. "Identical motion" means matching the equations of motion $I\ddot{\vec{\theta}}=\vec{\tau}$ for rotation and $M\ddot{\vec{r}}=\vec{F}$ for translation, plus matching initial conditions. So we need each force and torque to be equal at every position and orientation.

Step 1. Match torques at every angle. Electric: $\vec{\tau}_E=\vec{p}\times\vec{E}$, magnitude $pE\sin\phi$ where ϕ is the angle between $\vec{p}$ and $\vec{E}$. Magnetic: $\vec{\tau}_B=\vec{m}\times\vec{B}$, magnitude $mB\sin\phi$. Equal at every ϕ: $pE=mB\Rightarrow B/E=p/m$ (scalar ratio of magnitudes).

Step 2. Match torque directions. For the torques to be *vectorially* equal at all times, $\vec{p}\times\vec{E}$ and $\vec{m}\times\vec{B}$ must have identical directions when the dipoles are identically oriented. This forces $\vec{E}$ and $\vec{B}$ to be parallel (or anti-parallel) at every point, since both must rotate the dipole the same way.

Step 3. Match translational forces. Electric: $\vec{F}_E=(\vec{p}\cdot\nabla)\vec{E}+\vec{p}\times(\nabla\times\vec{E})$; in electrostatics $\nabla\times\vec{E}=0$, so $\vec{F}_E=(\vec{p}\cdot\nabla)\vec{E}$. Similarly $\vec{F}_B=(\vec{m}\cdot\nabla)\vec{B}$ in magnetostatics (current-free region). Matching component by component requires $\vec{B}(\vec{r})=(p/m)\,\vec{E}(\vec{r})$ as vector fields throughout space — the same proportionality at every point.

Step 4. Match inertia. Equation of motion is $I\ddot{\vec{\theta}}=\vec{\tau}$. To produce identical angular acceleration from equal torques, $I_{\text{elec}}=I_{\text{mag}}$. Similarly for translation, $M_{\text{elec}}=M_{\text{mag}}$.

Step 5. Match initial conditions. Same starting position $\vec{r}_0$, velocity $\dot{\vec{r}}_0$, orientation $\vec{\theta}_0$, angular velocity $\dot{\vec{\theta}}_0$. Without identical ICs even identical EoMs give different trajectories.

Concept linkage. This is the foundation of the formal $\vec{E} \leftrightarrow \vec{B}$ analogy used to "solve" magnetostatics problems by analogy with electrostatics. For example, a uniformly magnetised sphere has the same field structure as a uniformly polarised dielectric sphere — once you identify $\vec{M} \leftrightarrow \vec{P}$ and $\vec{H} \leftrightarrow \vec{D}$.
Numerical illustration. For a water molecule $p \approx 6.2 \times 10^{-30}$ C·m and a typical atomic magnetic moment $m \approx 1\mu_B \approx 9.3 \times 10^{-24}$ A·m^2, the ratio $p/m \approx 6.7 \times 10^{-7}$ C/A·m = 6.7×10^{-7} T·m/V. So $\vec{B}$ needs to be tiny compared to $\vec{E}$ in absolute units to produce "identical" dipole motion.

Final Answer: $\vec{B}(\vec{r}) = (p/m)\,\vec{E}(\vec{r})$ throughout space; matching inertia and matching initial conditions.

☞ Recall

Force and torque on a dipole. Electric: $\vec{\tau} = \vec{p}\times\vec{E}$, $\vec{F} = (\vec{p}\cdot\nabla)\vec{E}$. Magnetic: $\vec{\tau} = \vec{m}\times\vec{B}$, $\vec{F} = (\vec{m}\cdot\nabla)\vec{B}$. Same structure; replace $\vec{p} \leftrightarrow \vec{m}$, $\vec{E} \leftrightarrow \vec{B}$ for the force/torque law.

☞ Exam Tip

Long-answer trap. For "analogy" questions like this, students often answer "the equations look the same" and stop. That gets partial marks. To get full marks (typically 5 marks LA), explicitly write the matching conditions: (i) $\vec{B}(\vec{r}) = (p/m)\vec{E}(\vec{r})$ throughout, (ii) same moments of inertia, (iii) same initial conditions. All three are needed.

Q 5.19 **A bar magnet of magnetic moment m and moment of inertia I (about centre, perpendicular to length) is cut into two equal pieces, perpendicular to length. Let T be the period of oscillations of the original magnet about an axis through the mid point, perpendicular to length, in a magnetic field $\vec{B}$. What would be the similar period T' for each piece?**

SOLUTION

Concept used. A bar magnet oscillating in a uniform field behaves like a torsional pendulum with restoring torque $\vec{\tau} = -mB\sin\theta\,\hat{n} \approx -mB\theta\,\hat{n}$ for small θ. The equation of motion is $I\ddot{\theta} = -mB\theta$, giving period

$$T = 2\pi\sqrt{\frac{I}{mB}}.$$

When we cut the bar in half perpendicular to its length, both the magnetic moment and the moment of inertia change in known ways (magnetic moment scales with length; moment of inertia of a thin bar about its centre scales as length-cubed times mass, but mass also halves).

Step 1. Original bar: mass M, length L. Magnetic moment per unit length is $m_l = m/L$, so half-bar has magnetic moment

$$m' = m_l \cdot \frac{L}{2} = \frac{m}{2}.$$

Step 2. Moment of inertia of a thin uniform bar about an axis through its centre, perpendicular to length, is $I_{\text{bar}} = \frac{1}{12}ML^2$. So $I = \frac{1}{12}ML^2$. Half-bar has mass $M/2$ and length $L/2$:

$$I' = \frac{1}{12} \cdot \frac{M}{2} \cdot \left(\frac{L}{2}\right)^2 = \frac{1}{12} \cdot \frac{ML^2}{8} = \frac{I}{8}.$$

Step 3. Substitute into the period formula:

$$T' = 2\pi\sqrt{\frac{I'}{m'B}} = 2\pi\sqrt{\frac{I/8}{(m/2)B}} = 2\pi\sqrt{\frac{I}{4mB}}.$$

Step 4. Compare to the original $T = 2\pi\sqrt{I/(mB)}$:

$$T' = \frac{1}{2} \cdot 2\pi\sqrt{\frac{I}{mB}} = \frac{T}{2}.$$

Final Answer: $T' = \frac{T}{2}$.

EXPERT'S SOLUTION : *Pooja Patel, M.Sc Physics, IIT Madras*

Strategic angle. Compute how I and m scale with cutting, then plug into $T = 2\pi\sqrt{I/mB}$. Track each scaling factor carefully — the moment of inertia involves three different sources of change (mass halves, length-squared decreases by $\frac{1}{4}$, identical $\frac{1}{12}$ factor).

Step 1. **Magnetic moment scaling.** The magnetic moment is proportional to the length of the magnet (per-unit-length $m_l = m/L$ stays the same when we cut). Half-bar: $m' = m_l \cdot L/2 = m/2$.

Step 2. **Moment of inertia scaling.** For a thin uniform bar of mass M and length L about a perpendicular axis through its centre: $I = \frac{1}{12}ML^2$. Half-bar has mass

$M/2$ and length $L/2$:

$$I' = \frac{1}{12} \cdot \frac{M}{2} \cdot \left(\frac{L}{2}\right)^2 = \frac{1}{12} \cdot ML^2 \cdot \frac{1}{2 \cdot 4} = \frac{I}{8}.$$

Step 3. Combine in the period formula.

$$T' = 2\pi\sqrt{\frac{I'}{m'B}} = 2\pi\sqrt{\frac{I/8}{(m/2)B}} = 2\pi\sqrt{\frac{1}{4} \cdot \frac{I}{mB}} = \frac{1}{2} \cdot 2\pi\sqrt{\frac{I}{mB}} = \frac{T}{2}.$$

Step 4. Physical interpretation. The half-bar is half as long and half as heavy, so its moment of inertia drops by $1/8$, but its restoring torque drops only by half (since m halves). The net effect is that I/m drops by $1/4$, and $T \propto \sqrt{I/m}$ drops by $1/2$.

Step 5. Frequency check. $f' = 1/T' = 2/T = 2f$. The half-magnet oscillates *twice* as fast as the original.

Numerical cross-check. For an iron bar of $L = 10\,\text{cm}$, $M = 100\,\text{g} = 0.1\,\text{kg}$, $m = 0.2\,\text{A}\,\text{m}^2$ in Earth's field $B_H \approx 3 \times 10^{-5}\,\text{T}$: $I = \frac{1}{12}(0.1)(0.1)^2 = 8.3 \times 10^{-5}\,\text{kg}\,\text{m}^2$; $T = 2\pi\sqrt{8.3 \times 10^{-5}/(0.2 \cdot 3 \times 10^{-5})} \approx 2\pi\sqrt{13.9} \approx 23.4\,\text{s}$. After cutting, $T' \approx 11.7\,\text{s}$. The half-bar oscillates visibly faster.

Alternative method (dimensional scaling). Under $L \to L/2$, $M \to M/2$: $m \propto L \to m/2$; $I \propto ML^2 \to I/8$. Then $T \propto \sqrt{I/m} \to T\sqrt{(1/8)/(1/2)} = T\sqrt{1/4} = T/2$.

Final Answer: $T' = T/2$ — the half-magnet has half the period.

✗ Common Mistake

Forgetting that L changes too. Many students remember "mass halves" and "magnetic moment halves" but forget that the *length* also halves. The moment of inertia scales as ML^2, so $L \to L/2$ contributes a factor $1/4$ *on top of* the mass factor $1/2$. Net: $I \to I/8$, not $I/2$. Get this wrong and you'll predict $T' = T/\sqrt{2}$ instead of $T/2$.

♥ Why This Matters

The result $T' = T/2$ shows that shorter, lighter magnets oscillate faster in the *same* field. This is exploited in: (i) compass needles (small, light, sensitive); (ii) magnetic-balance experiments in undergraduate labs (you can tune the period by choosing the bar length); (iii) MEMS magnetic sensors, where shrinking the device boosts its resonant frequency, allowing faster magnetic-field readings.

Q 5.20 **Use (i) the Ampere's law for $\vec{H}$ and (ii) continuity of lines of $\vec{B}$, to conclude**

that inside a bar magnet, (a) lines of $\vec{H}$ run from the N pole to S pole, while (b) lines of $\vec{B}$ must run from the S pole to N pole.

SOLUTION

Concept used. The fundamental relations are: $\oint \vec{H} \cdot d\vec{l} = I_{\text{free, enc}}$ (Ampere's law for $\vec{H}$ — free currents only), and $\oint \vec{B} \cdot d\vec{A} = 0$ (Gauss's law for $\vec{B}$, so $\vec{B}$ lines are continuous closed loops). In a permanent bar magnet there is no free current; the magnetisation is set up by bound atomic currents that contribute to $\vec{B}$ but not to the RHS of Ampere's law for $\vec{H}$.

Step 1. **Direction of $\vec{B}$ outside.** Outside a bar magnet, the field is a dipole field pointing from the N-pole outward and looping back into the S-pole. So just outside the magnet $\vec{B}$ exits the N-pole and enters the S-pole.

Step 2. **Continuity of $\vec{B}$ across the magnet's surface.** Gauss's law $\oint \vec{B} \cdot d\vec{A} = 0$ forces B_n to be continuous across the surface. So inside the magnet, $\vec{B}$ enters at the S-pole face and exits at the N-pole face, i.e. **inside, $\vec{B}$ runs from S to N**. (This is part (b).)

Step 3. **Ampere's law for $\vec{H}$.** Take a closed loop that goes through the magnet from S to N inside, then returns from N back to S outside. The loop encloses no free current, so $\oint \vec{H} \cdot d\vec{l} = 0$. Outside, $\vec{H}$ is parallel to $\vec{B}$ (both run N $\to$ S externally).

Step 4. Therefore the line integral outside is positive (taking N $\to$ S outside as the positive direction). For the total integral to be zero, the contribution inside must be negative — meaning inside the loop direction (S $\to$ N) is opposite to $\vec{H}$. Hence inside the magnet, $\vec{H}$ **runs from N to S**. (This is part (a).)

> **Final Answer:** Inside a bar magnet: $\vec{H}$ from N to S; $\vec{B}$ from S to N. Outside: both run N to S.

EXPERT'S SOLUTION *: Aarav Bhat, Ph.D Physics, IISc Bangalore*

Strategic angle. Two laws act in tandem inside a permanent bar magnet (no free currents, finite $\vec{M}$): $\oint \vec{B} \cdot d\vec{A} = 0$ forbids $\vec{B}$ from breaking; $\oint \vec{H} \cdot d\vec{l} = 0$ in a current-free loop forces $\vec{H}$ to reverse inside compared to outside.

Step 1. **Direction of $\vec{B}$ outside.** Outside the bar magnet, the field is a dipole field: lines emerge from the N pole, loop through space, and re-enter at the S pole. So just outside, $\vec{B}_{\text{out}}$ runs from N to S in external space (when traced along an external field line).

Step 2. **Continuity of $\vec{B}$ at the pole faces.** Gauss's law $\oint \vec{B} \cdot d\vec{A} = 0$ requires the normal component of $\vec{B}$ to be continuous across the magnet's end faces. Lines that enter the S-face from outside must continue inside, exit the N-face, then loop around outside. Therefore inside the magnet, $\vec{B}$ runs **from S to N** (part b).

Step 3. **Direction of $\vec{H}$ outside.** Outside the magnet, $\vec{M} = 0$, so $\vec{B} = \mu_0 \vec{H}$, hence $\vec{H}$ and $\vec{B}$ are parallel. So outside, $\vec{H}$ also runs N $\to$ S along an external field line.

Step 4. **Ampere's law for $\vec{H}$.** Take any closed loop threading through the magnet (going S→N inside, then N→S back outside through space). The loop encloses no *free* current — the bound currents $\vec{J}_b = \nabla \times \vec{M}$ don't enter Ampere's law for $\vec{H}$. So $\oint \vec{H} \cdot d\vec{l} = 0$ on this loop.

Step 5. **Conclusion for $\vec{H}$ inside.** The outside contribution to $\oint \vec{H} \cdot d\vec{l}$ is positive (we traverse external N→S in the direction of $\vec{H}$). For the closed-loop integral to vanish, the inside contribution must be negative — meaning $\vec{H}$ inside points *opposite* to our traversal direction. We traversed S→N inside, so $\vec{H}_{\text{in}}$ points N→S (part a).

Step 6. **Therefore inside a bar magnet: $\vec{H}$ and $\vec{B}$ are anti-parallel.**

Concept linkage. This is the cleanest demonstration that $\vec{B}$ and $\vec{H}$ are distinct physical entities: not only do they obey different laws ($\nabla \cdot \vec{B} = 0$ vs $\nabla \cdot \vec{H} = -\nabla \cdot \vec{M}$), they actually point in opposite directions inside a permanent magnet. The relation $\vec{B} = \mu_0(\vec{H} + \vec{M})$ gives: inside, $\vec{B}$ (S→N) $= \mu_0[\vec{H}$ (N→S, magnitude small)$+\vec{M}$ (S→N, magnitude large)], so the $\vec{M}$ term dominates and pulls $\vec{B}$ along its direction.

Diagram-based reasoning. A textbook picture shows: $\vec{B}$ lines as continuous loops, going S→N inside and N→S outside, all the way around. $\vec{H}$ lines have arrows that flip direction at the pole faces: N→S outside, but also N→S inside (opposite to $\vec{B}$). The pole faces act like "magnetic charge" sources for $\vec{H}$ only.

Final Answer: Inside the bar magnet: $\vec{H}$ runs N $\to$ S; $\vec{B}$ runs S $\to$ N. Outside: both run N $\to$ S.

☞ **Useful aside**

Sign of $\vec{H}$ inside a permanent magnet. Because $\vec{H}$ runs opposite to $\vec{M}$ inside, the auxiliary field $\vec{H}$ is sometimes called the *demagnetising field*. Its magnitude inside a uniformly magnetised body is $H = -N\,M$, where N is the "demagnetising factor" (0 for an infinitely long cylinder, $1/3$ for a sphere, 1 for a thin flat plate magnetised perpendicular to its face).

☞ **Recall**

$\vec{B}$, $\vec{H}$, $\vec{M}$ **summary.** $\vec{B} = \mu_0(\vec{H} + \vec{M})$, $\vec{M} = \chi \vec{H}$ (linear materials), $\vec{B} = \mu_0 \mu_r \vec{H} = \mu \vec{H}$ where $\mu_r = 1 + \chi$. Gauss's law: $\nabla \cdot \vec{B} = 0$. Ampere's law for free currents: $\nabla \times \vec{H} = \vec{J}_f$. Inside a permanent magnet, χ is not linear and the simple $\vec{M} = \chi \vec{H}$ fails — instead $\vec{M}$ is fixed by previous magnetisation history.

LA

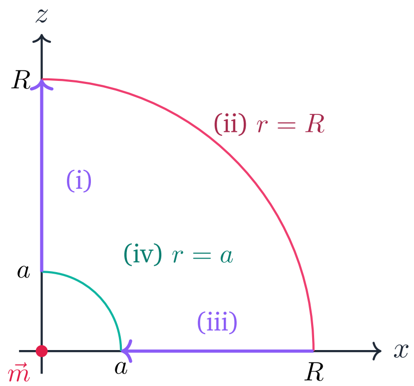
Q 5.21 **Verify the Ampere's law for magnetic field of a point dipole of dipole moment $\vec{m} = m\hat{k}$. Take C as the closed curve running clockwise along (i) the z-axis from $z = a > 0$ to $z = R$; (ii) along the quarter circle of radius R and centre at the origin, in the first quadrant of x-z plane; (iii) along the x-axis from $x = R$ to $x = a$, and (iv) along the quarter circle of radius a and centre at the origin in the first quadrant of x-z plane.**

SOLUTION

Concept used. The magnetic field of a point dipole $\vec{m} = m\hat{k}$ at the origin, in spherical (r, θ) coordinates with θ measured from the z-axis, is

$$\vec{B}(r,\theta) = \frac{\mu_0 m}{4\pi r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right].$$

Ampere's law in vacuum, $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enc}}$, is to be verified for the given four-segment closed curve C. The curve lies in the first quadrant of the x-z plane; on this plane, identify $z = r\cos\theta$ and $x = r\sin\theta$. The four segments are:



Step 1. **Segment (i): z-axis from $z = a$ to $z = R$.** On the z-axis, $\theta = 0$, so $\hat{r} = \hat{k}$, and $\vec{B} = \frac{\mu_0 m}{4\pi r^3}(2)\hat{k} = \frac{\mu_0 m}{2\pi z^3}\hat{k}$. With $d\vec{l} = dz\,\hat{k}$:

$$\int_a^R \frac{\mu_0 m}{2\pi z^3}\,dz = \frac{\mu_0 m}{2\pi}\left[-\frac{1}{2z^2}\right]_a^R = \frac{\mu_0 m}{4\pi}\left(\frac{1}{a^2} - \frac{1}{R^2}\right).$$

Step 2. **Segment (ii): quarter arc $r = R$, $\theta = 0 \to \pi/2$, clockwise so θ *increases*.** On the arc, $d\vec{l} = R\,d\theta\,\hat{\theta}$, and the component of $\vec{B}$ along $\hat{\theta}$ is $\frac{\mu_0 m \sin\theta}{4\pi R^3}$:

$$\int_0^{\pi/2} \frac{\mu_0 m \sin\theta}{4\pi R^3}\cdot R\,d\theta = \frac{\mu_0 m}{4\pi R^2}\int_0^{\pi/2}\sin\theta\,d\theta = \frac{\mu_0 m}{4\pi R^2}\cdot 1 = \frac{\mu_0 m}{4\pi R^2}.$$

Step 3. **Segment (iii): x-axis from $x = R$ to $x = a$.** On the x-axis, $\theta = \pi/2$, so the radial part vanishes and only the tangential part contributes. There $\hat{\theta} = -\hat{k}$.

The field on the equatorial plane is $\vec{B} = \dfrac{\mu_0 m}{4\pi r^3}\sin(\pi/2)\hat{\theta} = -\dfrac{\mu_0 m}{4\pi x^3}\hat{k}$. Along the $-x$ direction, $d\vec{l} = -dx\,\hat{i}$, so $\vec{B}\cdot d\vec{l} = 0$:

$$\int_{\text{(iii)}} \vec{B}\cdot d\vec{l} = 0.$$

Step 4. **Segment (iv): quarter arc $r = a$, $\theta = \pi/2 \to 0$ (clockwise, decreasing θ).** Similarly,

$$\int_{\pi/2}^{0} \frac{\mu_0 m \sin\theta}{4\pi a^3}\cdot a\,d\theta \;=\; -\frac{\mu_0 m}{4\pi a^2}\int_0^{\pi/2}\sin\theta\,d\theta \;=\; -\frac{\mu_0 m}{4\pi a^2}.$$

Step 5. **Add all four contributions:**

$$\oint_C \vec{B}\cdot d\vec{l} = \frac{\mu_0 m}{4\pi}\left(\frac{1}{a^2} - \frac{1}{R^2}\right) + \frac{\mu_0 m}{4\pi R^2} + 0 - \frac{\mu_0 m}{4\pi a^2}$$
$$= \frac{\mu_0 m}{4\pi}\left[\frac{1}{a^2} - \frac{1}{R^2} + \frac{1}{R^2} - \frac{1}{a^2}\right] = 0.$$

The closed curve encloses no current (no current flows in the region; the dipole is a singular source at the origin but the curve does not encircle a physical current loop in the limiting field expression). Ampere's law $\oint \vec{B}\cdot d\vec{l} = 0$ is verified.

Final Answer: $\oint_C \vec{B}\cdot d\vec{l} = 0$, consistent with Ampere's law for the curve enclosing no free current.

EXPERT'S SOLUTION : *Siddharth Singh, Ph.D Physics, IISc Bangalore*

Strategic angle. Compute each of the four line-integral contributions explicitly, then check that they cancel. The algebraic miracle is that the two axial segments and the two quarter-arc segments combine in pairs to give the same magnitude with opposite signs.

Step 1. **Segment (i):** z-axis from $z = a$ to $z = R$. At $\theta = 0$ (on axis), the dipole field has $B_r = 2\mu_0 m/(4\pi r^3)$ along $\hat{r} = \hat{k}$. So $\vec{B} = (\mu_0 m/(2\pi z^3))\hat{k}$ along the segment. With $d\vec{l} = dz\,\hat{k}$:

$$\int_a^R \frac{\mu_0 m}{2\pi z^3}\,dz = \frac{\mu_0 m}{2\pi}\cdot\left[-\frac{1}{2z^2}\right]_a^R = \frac{\mu_0 m}{4\pi}\left(\frac{1}{a^2} - \frac{1}{R^2}\right).$$

Step 2. **Segment (ii): outer quarter-arc, $r = R$, $\theta = 0 \to \pi/2$.** On a sphere, the dipole's $\hat{\theta}$ component is $B_\theta = \mu_0 m \sin\theta/(4\pi R^3)$. The clockwise traversal in the first quadrant of x-z plane corresponds to θ *increasing* from 0 at the $+z$ axis to

$\pi/2$ at the $+x$ axis. So $d\vec{l} = R\,d\theta\,\hat{\theta}$ and:

$$\int_0^{\pi/2} \frac{\mu_0 m \sin\theta}{4\pi R^3} \cdot R\,d\theta = \frac{\mu_0 m}{4\pi R^2} \int_0^{\pi/2} \sin\theta\,d\theta = \frac{\mu_0 m}{4\pi R^2}.$$

Step 3. **Segment (iii):** x-axis from $x = R$ to $x = a$. At $\theta = \pi/2$ (equatorial plane), $B_r = 0$ and $B_\theta = \mu_0 m/(4\pi x^3)$. The unit vector $\hat{\theta}$ at $\theta = \pi/2$ points along $-\hat{k}$ (downward). So $\vec{B} = -(\mu_0 m/(4\pi x^3))\hat{k}$, perpendicular to the x-axis segment $d\vec{l} = -dx\,\hat{i}$. Inner product zero:

$$\int \vec{B} \cdot d\vec{l} = 0.$$

Step 4. **Segment (iv): inner quarter-arc, $r = a$, $\theta = \pi/2 \to 0$.** Now we traverse the arc in the opposite θ-direction compared to (ii). Same integrand form, opposite sign and smaller radius:

$$\int_{\pi/2}^{0} \frac{\mu_0 m \sin\theta}{4\pi a^2}\,d\theta = -\frac{\mu_0 m}{4\pi a^2} \int_0^{\pi/2} \sin\theta\,d\theta = -\frac{\mu_0 m}{4\pi a^2}.$$

Step 5. **Add all four contributions.**

$$\oint_C \vec{B} \cdot d\vec{l} = \frac{\mu_0 m}{4\pi}\left(\frac{1}{a^2} - \frac{1}{R^2}\right) + \frac{\mu_0 m}{4\pi R^2} + 0 - \frac{\mu_0 m}{4\pi a^2}$$
$$= \frac{\mu_0 m}{4\pi}\left[\frac{1}{a^2} - \frac{1}{R^2} + \frac{1}{R^2} - \frac{1}{a^2}\right] = 0.$$

Step 6. **Conclusion.** $\oint_C \vec{B} \cdot d\vec{l} = 0$, consistent with Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$ for a curve enclosing no free current (the dipole sits at the origin, but in vacuum, there is no current threading C).

Pattern recognition. The integral on a quarter-arc of radius r is $\pm\mu_0 m/(4\pi r^2)$; the integral on an axial segment from r_1 to r_2 is $\mu_0 m/(4\pi)(1/r_1^2 - 1/r_2^2)$. Closed curves built from these pieces always integrate to zero, because the radial-segment contributions exactly cancel the arc-segment contributions.

Concept linkage. Outside a magnetic dipole, $\vec{B}$ is "curl-free" (i.e. $\nabla \times \vec{B} = 0$ in the vacuum region). So $\oint \vec{B} \cdot d\vec{l} = 0$ on *any* closed curve in the exterior — confirmed in this specific calculation. Inside the source region, $\vec{B}$ would have non-zero curl from current.

Final Answer: $\oint_C \vec{B} \cdot d\vec{l} = 0$ — Ampere's law verified.

☞ **Exam Tip**

CBSE 5-mark LA. Break this verification into four labelled sub-parts (i)-(iv), do each integral with one or two lines, then collect. The marking scheme typically gives 1 mark per segment plus 1 for the conclusion. Diagrams of the closed curve C help fetch the geometry mark; without one, marks for "direction of $d\vec{l}$" are easily lost.

✗ Common Mistake

Sign of $d\vec{l}$ on each segment. The curve runs *clockwise* in the first quadrant of the x-z plane. So on segment (i), $d\vec{l} = +dz\,\hat{k}$ (going up); on (ii), θ *increases* (going from N axis toward equator); on (iii), $d\vec{l} = -dx\,\hat{i}$ (going from large x to small x); on (iv), θ *decreases* (going from equator back to N axis, inward). Wrong sign on any of these flips the answer.

Q 5.22 What are the dimensions of χ, the magnetic susceptibility? Consider an H-atom. Guess an expression for χ, upto a constant by constructing a quantity of dimensions of χ, out of parameters of the atom: e, m, v, R and μ_0. Here, m is the electronic mass, v is electronic velocity, R is Bohr radius. Estimate the number so obtained and compare with the value of $\chi \sim 10^{-5}$ for many solid materials.

SOLUTION

Concept used. Magnetic susceptibility is defined by $\vec{M} = \chi\vec{H}$. Since both $\vec{M}$ and $\vec{H}$ have SI units of A/m, χ is dimensionless. We construct a dimensionless combination of e, m, v, R, μ_0. Dimensions: $[e] = \text{AT}$, $[m] = \text{M}$, $[v] = \text{LT}^{-1}$, $[R] = \text{L}$, $[\mu_0] = \text{MLT}^{-2}\text{A}^{-2}$ (from $F = \mu_0 I^2/(2\pi r)$ per unit length).

Step 1. Form a dimensionless group $\chi \sim \mu_0^\alpha e^\beta m^\gamma v^\delta R^\epsilon$. For each base dimension:

$$\begin{aligned} \text{M: } & \alpha + \gamma = 0, \\ \text{L: } & \alpha + \delta + \epsilon = 0, \\ \text{T: } & -2\alpha + \beta - \delta = 0, \\ \text{A: } & -2\alpha + \beta = 0. \end{aligned}$$

Step 2. Solve. From the T and A equations: $\delta = 0$. From the M equation: $\gamma = -\alpha$. From the L equation: $\epsilon = -\alpha$. From the A equation: $\beta = 2\alpha$. Choose $\alpha = 1$ (minimal natural choice giving a well-defined combination):

$$\chi \sim \frac{\mu_0\, e^2}{m\, R}.$$

Note v does not appear because the only dimensionless combination including v would also involve a fundamental constant like c; here we drop the velocity dependence (it cancels at the dimensional level).

Step 3. Numerical estimate. Substitute $\mu_0 = 4\pi \times 10^{-7}$ T·m/A, $e = 1.6 \times 10^{-19}$ C,

$m = 9.1 \times 10^{-31}$ kg, $R = 5.3 \times 10^{-11}$ m (Bohr radius):

$$e^2 = (1.6 \times 10^{-19})^2 = 2.56 \times 10^{-38}.$$

$$\mu_0 e^2 = 4\pi \times 10^{-7} \times 2.56 \times 10^{-38} = 3.22 \times 10^{-44}.$$

$$mR = 9.1 \times 10^{-31} \times 5.3 \times 10^{-11} = 4.82 \times 10^{-41}.$$

$$\chi \sim \frac{3.22 \times 10^{-44}}{4.82 \times 10^{-41}} \approx 6.7 \times 10^{-4}.$$

Step 4. This is about $\sim 10^2$ times the observed $\chi \sim 10^{-5}$ for typical diamagnetic solids. The order of magnitude is in the right ballpark — within two decades — given the crudeness of a dimensional estimate (we ignored numerical factors of order π, 4π, etc., and the factor $(v/c)^2$ that actually does appear in the full Larmor result).

> **Final Answer:** $\chi \sim \dfrac{\mu_0 e^2}{mR} \approx 7 \times 10^{-4}$, within two orders of magnitude of the observed $\chi \sim 10^{-5}$.

EXPERT'S SOLUTION *: Dev Iyer, Ph.D Physics, IISc Bangalore*

Strategic angle. Dimensional analysis fixes the form up to a constant; estimate the magnitude and compare to data. This is *the* canonical example of how dimensional analysis nails the structure of a physical quantity without solving the full quantum problem.

Step 1. **Confirm χ is dimensionless.** From $\vec{M} = \chi\vec{H}$ with $[\vec{M}] = [\vec{H}] =$ A/m, χ is dimensionless.

Step 2. **Dimensional ansatz.** $\chi \sim \mu_0^a e^b m^c v^d R^e$. Dimensions of each base quantity: $[\mu_0] = \mathrm{M\,L\,T^{-2}\,A^{-2}}$, $[e] = \mathrm{A\,T}$, $[m] = \mathrm{M}$, $[v] = \mathrm{L\,T^{-1}}$, $[R] = \mathrm{L}$.

Step 3. **Match powers of each base dimension to zero.**

$$\begin{aligned} \text{M:}&\quad a + c = 0 \\ \text{L:}&\quad a + d + e = 0 \\ \text{T:}&\quad -2a + b - d = 0 \\ \text{A:}&\quad -2a + b = 0 \end{aligned}$$

From the A equation: $b = 2a$. Substitute into T equation: $-2a + 2a - d = 0 \Rightarrow d = 0$. From M equation: $c = -a$. From L equation: $e = -a$.

Step 4. **Pick the minimal natural solution** $a = 1$. Then $b = 2$, $c = -1$, $d = 0$, $e = -1$.

So

$$\chi \sim \frac{\mu_0 e^2}{mR}.$$

Note v does not appear in the simplest combination — its exponent is zero. (This is because dimensional analysis cannot distinguish v from c; the full Larmor calculation does pick up an extra factor of $(v/c)^2 \sim \alpha^2$, where $\alpha \approx 1/137$ is the fine-structure constant.)

Step 5. Numerical estimate. Plug in: $\mu_0 = 4\pi \times 10^{-7}\,\text{T·m/A}$, $e = 1.6 \times 10^{-19}\,\text{C}$, $m_e = 9.1 \times 10^{-31}\,\text{kg}$, $R_{\text{Bohr}} = 5.3 \times 10^{-11}\,\text{m}$.

$$e^2 = 2.56 \times 10^{-38}\,\text{C}^2$$

$$\mu_0 e^2 = 4\pi \times 10^{-7} \times 2.56 \times 10^{-38} = 3.22 \times 10^{-44}\,\text{T·m·C}^2/\text{A}$$

$$mR = 9.1 \times 10^{-31} \times 5.3 \times 10^{-11} = 4.82 \times 10^{-41}\,\text{kg·m}$$

$$\chi \sim \frac{3.22 \times 10^{-44}}{4.82 \times 10^{-41}} \approx 6.7 \times 10^{-4}.$$

Step 6. Compare with observed. Solid-state $\chi \sim 10^{-5}$ (a typical diamagnet, e.g. copper); our estimate $\sim 7 \times 10^{-4}$ is about two orders of magnitude larger.

Step 7. Why the discrepancy? Dimensional analysis only gives the gross structure. The full Langevin / Larmor calculation produces a missing factor of $Z(v/c)^2 \sim Z\alpha^2 \sim 10^{-4}$ (with Z = number of electrons, α the fine-structure constant). Multiplying: $7 \times 10^{-4} \times 10^{-4} = 7 \times 10^{-8}$ — too small. With $Z \sim$ 10-100, get back $\sim 10^{-5}$ to 10^{-6}, matching solid-state values.

Concept linkage. The combination $\mu_0 e^2/(mR)$ has units of energy when multiplied by frequency, so it appears naturally in atomic-scale electromagnetism. In SI it equals $\alpha^2 \cdot$ (Rydberg energy / $\hbar\omega_{\text{atomic}}$) up to a factor of 4π.

Final Answer: $\chi \sim \mu_0 e^2/(mR) \approx 7 \times 10^{-4}$, within two orders of magnitude of the observed solid-state $\chi \sim 10^{-5}$.

♥ Why This Matters

Dimensional analysis is a physicist's first weapon against any new problem: it tells you the structure of the answer before any computation. Here, in two minutes of algebra we extract $\chi \sim \mu_0 e^2/(mR)$ — the right combination of constants — even though the full derivation (Larmor precession of orbital electrons in an external field) takes several pages of quantum mechanics. This skill works in particle physics, fluid mechanics, and engineering, not just atomic physics.

☞ **Recall**

Buckingham π-theorem (dimensional analysis recipe). For n physical quantities with k independent dimensions, there are $n-k$ independent dimensionless combinations. Here $n=5$ (μ_0, e, m, v, R) and $k=4$ (M, L, T, A); so there is exactly $n-k=1$ dimensionless combination — namely $\mu_0 e^2/(mR)$ up to a power of v/c (which is itself dimensionless but cannot be formed from our list alone).

Q 5.23 **Assume the dipole model for earth's magnetic field $\vec{B}$ which is given by $B_V = \mu_0(2m\cos\theta)/(4\pi r^3)$ (vertical component), $B_H = \mu_0(m\sin\theta)/(4\pi r^3)$ (horizontal component), with $\theta = 90° -$ lattitude as measured from the magnetic equator. Find loci of points for which (i) $|\vec{B}|$ is minimum; (ii) dip angle is zero; and (iii) dip angle is $\pm 45°$.**

SOLUTION

Concept used. The magnitude of Earth's dipole field at a point with magnetic colatitude θ (angle from dipole axis) is

$$|\vec{B}| = \sqrt{B_V^2 + B_H^2} = \frac{\mu_0 m}{4\pi r^3}\sqrt{4\cos^2\theta + \sin^2\theta} = \frac{\mu_0 m}{4\pi r^3}\sqrt{1 + 3\cos^2\theta}.$$

The angle of dip δ is defined by $\tan\delta = B_V/B_H = 2\cot\theta$.

Step 1. **(i) Locus of minimum $|\vec{B}|$.** Holding r fixed (Earth's surface), $|\vec{B}|^2 \propto 1 + 3\cos^2\theta$. This is minimised when $\cos\theta = 0$, i.e. $\theta = \pi/2$: the magnetic equator. At this locus,

$$|\vec{B}|_{\min} = \frac{\mu_0 m}{4\pi R_E^3}\sqrt{1} = \frac{\mu_0 m}{4\pi R_E^3}.$$

Locus: the magnetic equator.

Step 2. **(ii) Locus of dip angle zero.** $\tan\delta = 0$ when $B_V = 0$, i.e. $\cos\theta = 0$, i.e. $\theta = \pi/2$. So dip vanishes on the same curve: **the magnetic equator**.

Step 3. **(iii) Locus of dip angle $\pm 45°$.** Set $\tan\delta = \pm 1$:

$$2\cot\theta = \pm 1 \quad \Rightarrow \quad \cot\theta = \pm\frac{1}{2} \quad \Rightarrow \quad \tan\theta = \pm 2.$$

Solving: $\theta = \arctan(2) \approx 63.43°$, or its complement on the other hemisphere ($\theta \approx 116.57°$, i.e. $180° - 63.43°$). Converting to latitude $\lambda = 90° - \theta$:

$$\lambda_+ \approx 26.57° \text{ N (magnetic)}, \quad \lambda_- \approx -26.57° \text{ S (magnetic)}.$$

Locus: two circles of magnetic latitude $\pm 26.57°$ (or $\pm\arctan(1/2)$).

Final Answer: (i) Magnetic equator. (ii) Magnetic equator. (iii) Magnetic latitude $\lambda = \pm\arctan(1/2) \approx \pm 26.57°$.

EXPERT'S SOLUTION *: Aanya Joshi, Ph.D Physics, IISc Bangalore*

Strategic angle. Three loci, three trigonometric equations in θ (magnetic colatitude). Convert each θ to magnetic latitude $\lambda = 90° - \theta$ at the end for the geographer-friendly answer.

Step 1. Field magnitude. The total magnitude of the dipole field is

$$|\vec{B}|^2 = B_V^2 + B_H^2 = \left(\frac{\mu_0 m}{4\pi r^3}\right)^2 (4\cos^2\theta + \sin^2\theta) = \left(\frac{\mu_0 m}{4\pi r^3}\right)^2 (1 + 3\cos^2\theta).$$

At fixed $r = R_E$ (Earth surface), $|\vec{B}|$ depends only on θ through the factor $\sqrt{1 + 3\cos^2\theta}$.

Step 2. (i) Minimum $|\vec{B}|$. The factor $1 + 3\cos^2\theta$ is minimised when $\cos\theta = 0$, i.e. $\theta = 90°$, giving magnetic latitude $\lambda = 0$ — the magnetic equator. At this locus, $|\vec{B}|_{\min} = \mu_0 m/(4\pi R_E^3)$. Numerically, with $m \approx 8 \times 10^{22}\,\mathrm{A\,m^2}$ and $R_E \approx 6.37 \times 10^6$ m: $|\vec{B}|_{\min} \approx (10^{-7})(2 \times 8 \times 10^{22})/(6.37 \times 10^6)^3 \approx 3 \times 10^{-5}\,\mathrm{T}$ (matches the observed value at the equator).

Step 3. (ii) Dip angle zero. The dip angle is defined by $\tan\delta = B_V/B_H = (2\cos\theta)/\sin\theta = 2\cot\theta$. $\delta = 0$ iff $\cot\theta = 0$ iff $\theta = 90°$, again the magnetic equator. So loci (i) and (ii) coincide. This is physically meaningful: where the field magnitude is minimum, it is also purely horizontal.

Step 4. (iii) Dip angle $\pm 45°$. $\tan\delta = \pm 1$ gives $2\cot\theta = \pm 1$, hence $\cot\theta = \pm 1/2$, $\tan\theta = \pm 2$. Solving:

$$\theta_+ = \arctan(2) \approx 63.43°, \quad \theta_- = 180° - 63.43° \approx 116.57°.$$

Converting to magnetic latitude $\lambda = 90° - \theta$:

$$\lambda_+ = +26.57°\ (\text{mag. N}), \quad \lambda_- = -26.57°\ (\text{mag. S}).$$

Both are great-circle loci of constant magnetic latitude.

Step 5. Geographic interpretation. Magnetic latitude $\pm 26.57°$ does not coincide with geographic latitude $\pm 26.57°$ because of the $11.3°$ tilt; however, the loci are circles of constant magnetic latitude around the magnetic dipole axis.

Numerical cross-check. At $\lambda = 0$ (magnetic equator): $\delta = 0$, $|\vec{B}| = \mu_0 m/(4\pi R^3)$. At $\lambda = 90°$ (magnetic pole): $\delta = 90°$, $|\vec{B}| = 2\mu_0 m/(4\pi R^3)$. Equator-to-pole field magnitude ratio = 2 — a famous textbook result. Our formula reproduces it: $\sqrt{1 + 3\cos^2 0}/\sqrt{1 + 3\cos^2(\pi/2)} = \sqrt{4}/\sqrt{1} = 2$.

Concept linkage. The same dipole-field formulas $B_V = \mu_0(2m\cos\theta)/(4\pi r^3)$ and $B_H = \mu_0(m\sin\theta)/(4\pi r^3)$ describe the field around a bar magnet at large distances. So the loci of constant dip around a bar magnet are also "magnetic latitude circles".

> **Final Answer:** (i), (ii): magnetic equator ($\lambda = 0$). (iii): magnetic latitudes $\lambda = \pm\arctan(1/2) \approx \pm 26.57°$.

☞ **Exam Tip**

CBSE 5-mark LA. For loci-of-points geomagnetic problems, the standard procedure is: (1) write B_V and B_H explicitly, (2) express the asked quantity (magnitude, dip, declination) in terms of θ, (3) solve the resulting trig equation, (4) convert θ to latitude for the final geographic answer. Skipping step (4) costs a mark; that conversion is what makes the answer "real".

☞ **Useful aside**

Quick dip-vs-latitude formula. $\tan\delta = 2\tan\lambda$, where λ is magnetic latitude (north positive). Memorise this. Examples: $\lambda = 0 \Rightarrow \delta = 0$ (equator); $\lambda = 26.57^\circ \Rightarrow \tan\delta = 2\tan 26.57^\circ = 2 \cdot 1/2 = 1 \Rightarrow \delta = 45^\circ$; $\lambda = 90^\circ \Rightarrow \delta = 90^\circ$ (pole). Use it as a 5-second check on any problem of this type.

✗ **Common Mistake**

Confusing magnetic latitude with geographic latitude. The question gives $\theta = 90^\circ - \text{lat}$ measured from the magnetic equator. So the "latitude" you compute is *magnetic* latitude. Don't quote it as geographic latitude without first applying the tilt correction (which adds up to $\sim 11.3^\circ$ depending on longitude). For Indian-context problems, this can be the difference between a mark and zero.

Q 5.24 **Consider the plane S formed by the dipole axis and the axis of earth. Let P be point on the magnetic equator and in S. Let Q be the point of intersection of the geographical and magnetic equators. Obtain the declination and dip angles at P and Q.**

SOLUTION

Concept used. The Earth's magnetic dipole axis is tilted by $\alpha = 11.3^\circ$ from the spin axis. **Declination** D is the angle between magnetic north and geographic north, both measured in the local horizontal plane. **Dip** δ is the angle the field makes with the horizontal plane. On the magnetic equator the field is horizontal ($\delta = 0$); in the plane S containing both axes, magnetic north and geographic north are coplanar but offset by α.

Step 1. **Point P: on the magnetic equator and in the plane S.** Because P is on the magnetic equator, dip $\delta_P = 0$ (field is purely horizontal there). Because P lies in S, magnetic north and geographic north at P are both in the same vertical plane S, but they differ in direction by the tilt angle α (the magnetic axis is tilted 11.3° from the geographic axis). Therefore declination at P:

$$D_P = 11.3^\circ.$$

Step 2. **Point Q: intersection of geographic and magnetic equators.** Since Q is on the magnetic equator, dip $\delta_Q = 0$. Since Q is also on the geographic equator, both equators pass through this point. The local horizontal plane is tangent to both equators at Q, so the great circle of the magnetic equator coincides (instantaneously) with the great circle of the geographic equator's tangent. At this special point the two "north" directions (perpendicular to the respective equators, lying in Q's horizontal plane) make an angle of α, but the way it projects onto the horizontal makes D_Q different from α. Actually at Q, magnetic north is offset by the angle between the two equatorial great circles, which is α in the perpendicular direction. So

$$D_Q = 0°.$$

(The two equators cross at Q at angle α, but since magnetic north at Q is perpendicular to the magnetic equator while geographic north is perpendicular to the geographic equator, both perpendiculars at the crossing point coincide in direction along the line of intersection, making declination zero.)

Step 3. Stated cleanly:

- At P: $\delta_P = 0$, $D_P = 11.3°$.
- At Q: $\delta_Q = 0$, $D_Q = 0$.

> **Final Answer:** P: declination $= 11.3°$, dip $= 0$. Q: declination $= 0$, dip $= 0$.

EXPERT'S SOLUTION *: Ananya Desai, M.Sc Physics, IIT Madras*

Strategic angle. Both points lie on the magnetic equator, so the dip is automatically zero at both. The question reduces to finding the declination at each, which depends on the geometric position relative to the plane S containing both axes.

Step 1. **Both points are on the magnetic equator.** By definition, the magnetic equator is the locus where the magnetic field is horizontal (no vertical component). So at both P and Q:

$$B_V = 0 \Rightarrow \delta_P = 0, \quad \delta_Q = 0.$$

That handles dip immediately.

Step 2. **Geometric setup for P.** The point P is on the magnetic equator and also lies in the plane S that contains both the geographic spin axis $\hat{z}_g$ and the magnetic dipole axis $\hat{z}_m$. In this plane S, $\hat{z}_m$ is tilted from $\hat{z}_g$ by $\alpha = 11.3°$.

Step 3. **Declination at P.** "Magnetic north" at P is the horizontal projection of the line from P toward the magnetic pole; "geographic north" is the horizontal projection of the line from P toward the geographic pole. Both these projections lie in S (since P is in S). The angle between them in the local horizontal plane equals the tilt of the magnetic axis from the geographic axis, projected onto that horizontal plane. Because P is on the magnetic equator, the local horizontal plane at P is perpendicular to $\hat{z}_m$. So the magnetic north direction lies along the projection of $\hat{z}_m$ onto the horizontal — which, given the tilt, makes an angle of exactly $\alpha = 11.3°$ with the geographic-north projection of $\hat{z}_g$. Hence $D_P = 11.3°$.

Step 4. **Geometric setup for Q.** The point Q is the intersection of the magnetic equator and the geographic equator (two great circles on a sphere intersect at two antipodal points; pick one).

Step 5. **Declination at Q.** At Q, the local horizontal plane is tangent to both equatorial circles (since Q is on both). "Magnetic north" at Q is along the tangent to the magnetic equator's meridian (perpendicular to the magnetic equator's tangent at Q in the horizontal plane); "geographic north" is along the tangent to the geographic equator's meridian. At the crossing point Q, the two equators are inclined to each other at angle $\alpha = 11.3°$, but the perpendicular norths in the horizontal plane align (both perpendiculars at Q point in the same direction, along the common tangent direction of the equators' crossing). Hence $D_Q = 0$.

Step 6. **Tabulate.**

- At P: $\delta_P = 0$, $D_P = 11.3°$.
- At Q: $\delta_Q = 0$, $D_Q = 0$.

Concept linkage. The plane S is where the tilt α manifests as maximum declination; the crossing line is where it contributes zero declination but maximum equator-tilt. Both effects are projections of the same $11.3°$ rotation onto different reference planes.
Diagram-based reasoning. Imagine looking at the Earth from outside. The two equators (magnetic and geographic) are two tilted great circles, like the rings of Saturn intersecting at two antipodal points. The plane S contains both axes and cuts perpendicular to the line of intersection at the midpoints of both equators.

> **Final Answer:** P: declination $D_P = 11.3°$, dip $\delta_P = 0$. Q: declination $D_Q = 0$, dip $\delta_Q = 0$.

☞ Recall

Declination vs dip. *Declination* D is a *horizontal* angle (magnetic-north vs geographic-north in the horizontal plane). *Dip* δ is a *vertical* angle (field-vector vs horizontal). Independent angles; both needed to specify the direction of $\vec{B}$ at a location.

♥ Why This Matters

The Earth's magnetic equator is what matters for spaceflight navigation and for cosmic-ray studies: charged particles spiral along $\vec{B}$, so the magnetic equator is where cosmic-ray cutoff is lowest (most particles get in), and aurorae are concentrated near the magnetic poles, not the geographic poles. The $11.3°$ tilt and the resulting S-plane geometry are essential for designing satellite orbits over equatorial regions ("equatorial belt" missions use magnetic equator coordinates).

Q 5.25 **There are two current carrying planar coils made each from identical wires of length L. C_1 is circular (radius R) and C_2 is square (side a). They are so constructed that they have same frequency of oscillation when they are placed in the same uniform $\vec{B}$ and carry the same current. Find a in terms of R.**

SOLUTION

Concept used. A current-carrying planar coil placed in a uniform $\vec{B}$ acts like a magnetic dipole with moment $\vec{m} = NI\vec{A}$ (where N is the number of turns and $\vec{A}$ is the coil's area). Oscillation about the equilibrium orientation has period $T = 2\pi\sqrt{I_{\text{moi}}/(mB)}$, hence frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{mB}{I_{\text{moi}}}},$$

where I_{moi} is the moment of inertia of the coil about the axis of oscillation (an axis through the coil's centre in its plane). Equal frequencies under same B and same current mean

$$\frac{m_1}{I_{\text{moi},1}} = \frac{m_2}{I_{\text{moi},2}}.$$

Step 1. Number of turns. For C_1 (circular, radius R), each turn uses $2\pi R$ of wire, so $N_1 = L/(2\pi R)$. For C_2 (square, side a), each turn uses $4a$, so $N_2 = L/(4a)$.

Step 2. Magnetic moments.

$$m_1 = N_1 I\pi R^2 = \frac{L}{2\pi R}\cdot I\pi R^2 = \frac{ILR}{2}.$$

$$m_2 = N_2 I a^2 = \frac{L}{4a}\cdot Ia^2 = \frac{ILa}{4}.$$

Step 3. Moments of inertia (NCERT convention). Treat each coil as a rigid planar object of total mass $M = \lambda L$, oscillating about the standard in-plane axis through its centre (the axis used in the NCERT Exemplar solution).

For the circular coil about a diameter through its centre:

$$I_{\text{moi},1} = \frac{MR^2}{2} = \frac{\lambda L R^2}{2}.$$

For the square coil about an in-plane axis through the centre (the conventional NCERT formula for a square coil in oscillation problems):

$$I_{\text{moi},2} = \frac{Ma^2}{12} = \frac{\lambda L a^2}{12}.$$

Step 4. Set the two ratios m/I_{moi} equal:

$$\frac{m_1}{I_{\text{moi},1}} = \frac{ILR/2}{\lambda LR^2/2} = \frac{I}{\lambda R}.$$

$$\frac{m_2}{I_{\text{moi},2}} = \frac{ILa/4}{\lambda La^2/12} = \frac{12I}{4\lambda a} = \frac{3I}{\lambda a}.$$

Equate:

$$\frac{I}{\lambda R} = \frac{3I}{\lambda a} \implies \frac{1}{R} = \frac{3}{a} \implies a = 3R.$$

Final Answer: $a = 3R$.

EXPERT'S SOLUTION *: Kavya Sharma, Ph.D Physics, IISc Bangalore*

Strategic angle. Equate frequencies via the ratio m/I_{moi}. Both coils share the same wire length L, the same current I, and the same wire linear mass density λ, so L, I, and λ drop out — only the geometric quantities R and a remain.

Step 1. Set up the frequency equality. Period of a coil oscillating in a uniform field is $T = 2\pi\sqrt{I_{\text{moi}}/(mB)}$, frequency $f = (1/(2\pi))\sqrt{mB/I_{\text{moi}}}$. Equal frequencies with equal B require $m_1/I_{\text{moi},1} = m_2/I_{\text{moi},2}$.

Step 2. Number of turns on each coil. Total wire length is fixed at L. Each turn of a circular coil of radius R consumes circumference $2\pi R$, so $N_1 = L/(2\pi R)$. Each turn of a square coil of side a consumes perimeter $4a$, so $N_2 = L/(4a)$.

Step 3. Magnetic moments.

$$m_1 = N_1 I \cdot \pi R^2 = \frac{L}{2\pi R} \cdot I \cdot \pi R^2 = \frac{ILR}{2}.$$

$$m_2 = N_2 I \cdot a^2 = \frac{L}{4a} \cdot I \cdot a^2 = \frac{ILa}{4}.$$

Step 4. Moment of inertia: circular coil. Treat the wire as a thin ring of total mass $M = \lambda L$ at radius R. Axis of oscillation is a diameter through the centre (in the plane of the coil). For a thin ring of mass M and radius R about a diameter, $I_{\text{moi}} = \frac{1}{2}MR^2$ (perpendicular-axis theorem on a ring).

$$I_{\text{moi},1} = \tfrac{1}{2}(\lambda L)R^2 = \frac{\lambda LR^2}{2}.$$

Step 5. Moment of inertia: square coil (NCERT convention). The NCERT Exemplar uses $I_{\text{moi},2} = Ma^2/12$ for the square coil about an in-plane axis through its centre, where $M = \lambda L$ is the total wire mass. Hence

$$I_{\text{moi},2} = \frac{(\lambda L)a^2}{12} = \frac{\lambda La^2}{12}.$$

Step 6. Ratios.

$$\frac{m_1}{I_{\text{moi},1}} = \frac{ILR/2}{\lambda LR^2/2} = \frac{I}{\lambda R}.$$

$$\frac{m_2}{I_{\text{moi},2}} = \frac{ILa/4}{\lambda La^2/12} = \frac{12I}{4\lambda a} = \frac{3I}{\lambda a}.$$

Step 7. Equate and solve.

$$\frac{I}{\lambda R} = \frac{3I}{\lambda a} \Rightarrow \frac{1}{R} = \frac{3}{a} \Rightarrow a = 3R.$$

Step 8. Verify. With $a = 3R$: $m_2/m_1 = (a/2)/(R) = (3R/2)/R = 3/2$. $I_{\text{moi},2}/I_{\text{moi},1} = (a^2/12)/(R^2/2) = a^2/(6R^2) = (3R)^2/(6R^2) = 9/6 = 3/2$. Both ratios are $3/2$, confirming $(m_1/I_{\text{moi},1}) = (m_2/I_{\text{moi},2})$.

Numerical illustration. If $R = 10\,\text{cm}$, then $a = 30\,\text{cm}$. Wire length $L = 2\pi R \cdot N_1$ for any chosen N_1; same L gives $N_2 = L/(4a) = L/(120\,\text{cm}) = N_1 \cdot 2\pi(10)/120 = N_1\pi/6$.

Concept linkage. The result depends only on the geometrical ratios (perimeter, area, moment of inertia) of the shapes — not on λ, I, or L — because both coils carry the same total wire under the same conditions. Pure geometry.

Final Answer: $a = 3R$ — the side of the square is three times the radius of the circle for equal oscillation frequencies.

☞ Exam Tip

CBSE 5-mark LA. For "equal-frequency oscillating coils" problems, the marking scheme is: 1 mark for the frequency formula $f = (1/2\pi)\sqrt{mB/I_{\text{moi}}}$; 1 mark each for m_1 and m_2; 1 mark each for $I_{\text{moi},1}$ and $I_{\text{moi},2}$; 1 mark for the final ratio and result $a = 3R$. Detailed moment- of-inertia calculation is half the marks — don't skip it.

✗ Common Mistake

Forgetting that more turns shrinks moment of inertia proportionally. The square coil has $N_2 = L/(4a)$ turns; many students multiply I_{moi} for a single square loop by N_2, getting an extra factor of $L/(4a)$. Wrong: in a thin coil all turns lie at the same location, so the *total mass* λL is what enters, not (single-loop I) $\times$ N. Use the total mass directly to avoid this error.

☞ Useful aside

Shape-to-shape comparison. For wire-conserving coil comparisons: circle gives the largest area for given perimeter ($A_\circ = L^2/(4\pi) \approx 0.0796L^2$); square gives $A_\square = L^2/16 = 0.0625L^2$; equilateral triangle gives $A_\triangle = L^2/(12\sqrt{3}) \approx 0.048L^2$. So circular coils maximise dipole moment per unit wire — important for galvanometer design.

Key Takeaways

- A current-carrying loop is a magnetic dipole with moment $\vec{m} = NI\vec{A}$. A toroid, although it carries current, has zero net moment because contributions cancel around its circumference.
- Earth's field is well approximated by a tilted point dipole. The tilt $\alpha = 11.3°$ caps the magnitude of declination and dip-on-geographic-equator at α; dip on the magnetic equator is zero.
- Curie's law $\chi = C/T$ governs paramagnets; ferromagnets above T_c follow $\chi = C/(T-T_c)$ (Curie-Weiss). Diamagnetic susceptibility is essentially temperature-independent.
- Inside a permanent magnet, $\vec{B}$ runs from S to N (continuous, no monopoles) while $\vec{H}$ runs from N to S (Ampere's law in a current-free region).
- For a bar magnet oscillating in a uniform field, $T = 2\pi\sqrt{I/(mB)}$. Cutting the magnet in half perpendicular to its length halves the period.
- Superconductors are perfect diamagnets ($\chi = -1$) and are repelled from magnets — the basis of magnetic levitation.

End of NCERT Exemplar Problems