Magnetism & Matter

The Bar Magnet (intro)

A natural magnet (lodestone) or man-made magnet has two poles : North (N) and South (S) , inseparable. from each other.

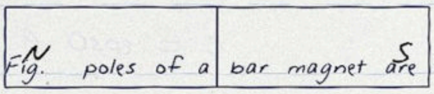


Fig. poles of a bar magnet are inseparable

Basic Properties

1. Attracts iron , nickel , cobalt.

2. Like poles repel ; unlike attract.

3. Freely suspended magnet -> N - S axis (aligns with earth's magnetic field).

4. Monopole does not exist : even half of a bar magnet has both N and S.

This is unlike electric ~~charges~~ poles - which CAN be isolated (+q alone exists).

Magnetic Field Lines

Properties

① Closed continuous loops ; emerge from N pole , re-enter S , close inside.

② Tangent at any point gives the direction of B at that point.

③ Density of lines = magnitude of B.

④ Two lines never intersect.

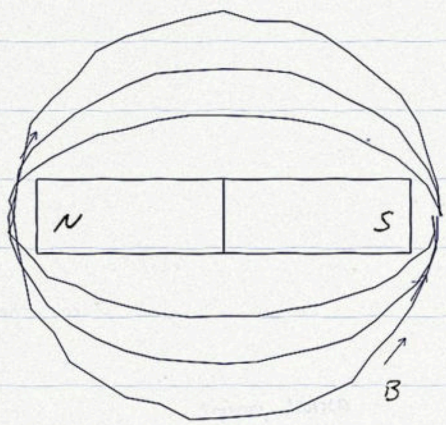


Fig. field lines emerge from N , return to S

Inside the magnet : lines go from S to N.

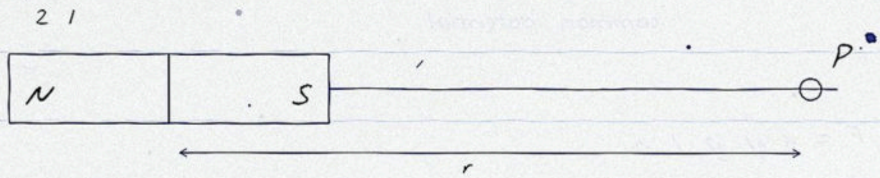
Bar Magnet - Axial Field

Magnetic Dipole Moment

Pole strength = m ; separation = 2 l :

$$\boxed{M = m \cdot (2 l)}$$

<- SI : A m 2
<- direction : S -> N



Derivation

$$B_{N\ at\ P} = \mu_0\, m / [4\ pi\ (r - l)^2]$$
$$B_{S\ at\ P} = \mu_0\, m / [4\ pi\ (r + l)^2]$$

Net B = B_N - B_S , along axis (away from N):

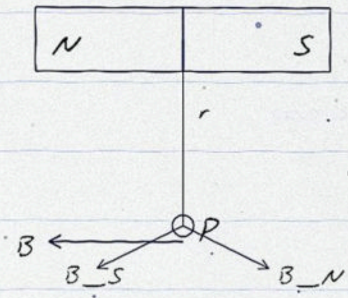
$$B = \mu_0 \cdot 2 M r / [\, 4\ pi\ (r^2 - l^2)^2 \,]$$

For $r \gg l$:

$$\boxed{B_{ax} = \mu_0 \cdot 2 M / (4\ pi\ r^3)}$$

Bar Magnet - Equatorial Field

Point P on perpendicular bisector of the magnet, at distance r from centre O.



Result

Each pole at distance sqrt(r^2 + l^2) ;
only components along M survive (anti-M) :

$$B = \mu_0 M / [4 \pi (r^2 + l^2)^{3/2}]$$

For r >> l :

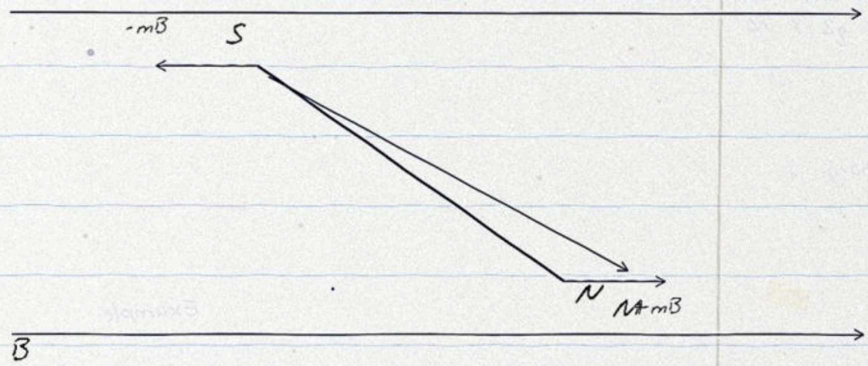
$$\boxed{B_{eq} = \mu_0 M / (4 \pi r^3)}$$

<- anti - parallel
<- to M

Ratio : B_ax / B_eq = 2 (same r).

Torque on a Bar Magnet in B.

Bar magnet of moment M placed in a uniform external field B at angle theta with B.



Net force vs torque

Net force = $+mB + (-mB) = 0$ (uniform B).

But there's a non-zero couple (torque) :

$$\boxed{T = MB\sin\theta}$$

<- vector : $M \times B$

Stable equilibrium : theta = 0 (M B)

Unstable equilibrium : theta = 180 (M anti - B)

Magnet rotates to align M along B.

Potential Energy of the Dipole

Work done in rotating the dipole from theta_1 to theta_2 against the magnetic torque :

$$W = M B (\cos \theta_1 - \cos \theta_2)$$

Take U(theta = 90) = 0 (perpendicular) :

$$\boxed{U = - M B \cos \theta = - M . B}$$ <- minimum at <- theta = 0

Specific values

U (theta = 0) = - M B (most stable)

U (theta = 90) = 0 (perpendicular)

U (theta = 180) = + M B (most unstable)

Oscillations of a magnet

Slightly tilt the magnet from theta = 0 :

$$T = - M B \sin \theta \qquad - M B \theta$$

$$I \, d^2\theta / dt^2 = - M B \theta$$

$$\boxed{T = 2 \text{ pi } \text{sqrt}(I / M B)}$$ <- I = moment of <- inertia of magnet

Used in vibration magnetometers.

Bar Magnet = Solenoid

A solenoid of length $2l$ and area A ; carrying n turns per unit length and current I , behaves like a bar magnet of moment :

$$\boxed{M = N I A = (n \cdot 2l) I A}$$

<- N = total turns

Fig. (b) equivalent bar magnet

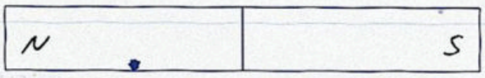
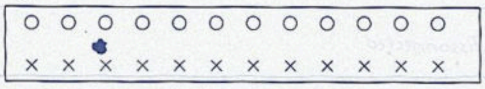


Fig. (a) solenoid



Field on the axis (sketch)

For a long solenoid , integration over the current elements gives (for $r >> l$) :

$$\boxed{B = \mu_0 \cdot 2M / (4 \pi r^3)}$$

<- matches
<- bar - magn

Confirms that magnetism = electric currents.

Electric vs Magnetic Dipole

Far - field of an electric dipole p and a magnetic dipole m have exactly the same form.

Side-by-side comparison

Electric

$p \ (= q \cdot 2a)$

$E_{ax} = 2 \kappa p / r^3$

$E_{eq} = \kappa p / r^3$

$T = p E \sin \Theta$

$U = - p \cdot E$

Magnetic

$M \ (= m \cdot 2l)$

$B_{ax} = \mu_0 2 M / (4 \text{ pi } r^3)$

$B_{eq} = \mu_0 M / (4 \text{ pi } r^3)$

$T = M B \sin \Theta$

$U = - M \cdot B$

Replacements

① p -> M (dipole moments)

② E -> B (fields)

③ 1 / (4 pi eps_0) -> u_0 / (4 pi)

Equivalently : μ_0 plays the role of 1 / eps_0.

(beautiful symmetry of electromagnetism.)

Gauss's Law for Magnetism

Statement

Net magnetic flux through any closed surface is always zero.

$$\boxed{\text{sum } B \cdot dA = 0}$$

<- no magnetic
<- monopole exists

Why ?

Magnetic field lines are closed loops. Every line leaving the surface re-enters it -> net = 0.

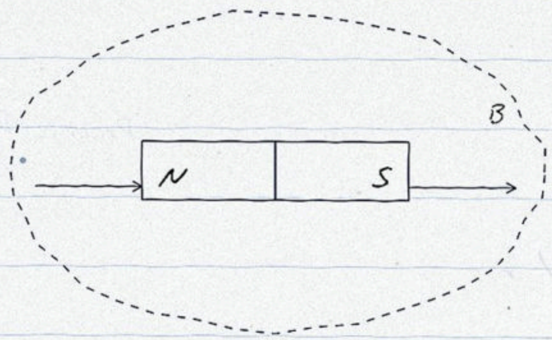


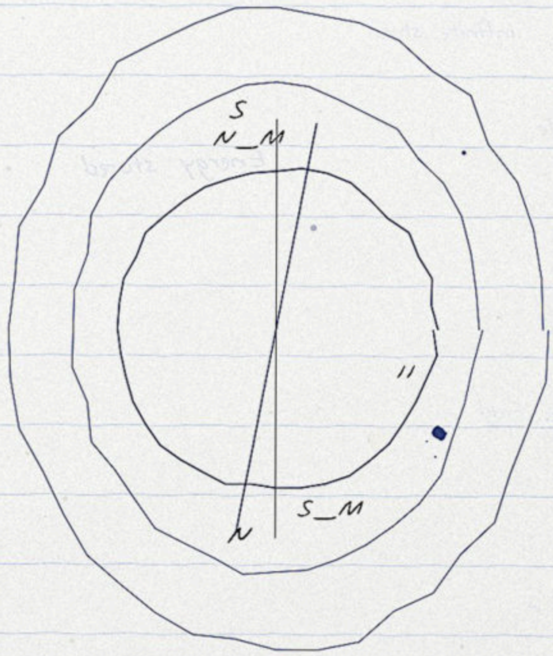
Fig. flux entering = flux leaving (no source/sink)

Compare with Gauss for E

E : flux = q_{enc} / eps_0 (monopoles exist)
B : flux = 0 (no monopoles)

Earth's Magnetism

Earth behaves like a giant bar magnet with its S - pole near geographic North and N - pole near geographic South (that's why N - end of a compass points geographically N).



Two axes : geographic vs magnetic

① Geographic axis : rotation axis ;
defines N and S geographic poles.

② Magnetic axis : tilted by 11 deg.
Geographic N is near magnetic S , and vice versa.

B_E magnitude 0.3 to 0.6 gauss = $3\text{-}6 \times 10^{-5}$ T.

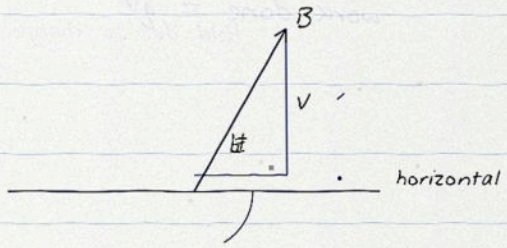
Declination & Inclination

Magnetic Declination (D)

Angle between the geographic meridian and the magnetic meridian at a given place.
(meridian = vertical plane through the place.)

Magnetic Inclination / Dip (I)

Angle that the earth's B at a place makes with the horizontal in the magnetic meridian.



Components

$$H = B \cos I \qquad \text{(horizontal)}$$

$$V = B \sin I \qquad \text{(vertical)}$$

$$\tan I = V / H$$

$$B = \text{sqrt}(H^2 + V^2)$$

At magnetic equator $I = 0$; at magnetic poles $I = 90$.

Elements of Earth's Field

Three independent elements

1. Declination D : compass needle vs true north (varies by place ; e.g. India 0).

2. Dip / Inclination I : angle of B with horizontal in magnetic meridian.

3. Horizontal component H : $H = B \cos I$. Knowing D , I , H -> full B vector.

Variation across earth

Magnetic equator : dip $I = 0$, $V = 0$, $B = H$.

Magnetic poles : dip $I = 90$, $H = 0$, $B = V$.

Earth's B is not constant in time :
secular variation (slow drift) ,
magnetic reversals (every 0.1 - 1 Myr) ,
diurnal & solar effects.

Typical numbers

Delhi : B 0.4 G , I 41 , D 0.5.

Equator : B 0.3 G , $I = 0$.

Poles : B 0.6 G , $I = 90$.

Magnetisation M & Intensity H

Magnetisation (M)

Net magnetic moment per unit volume :

$$\boxed{M = m_{net} / V}$$

<- SI : A / m
<- vector quantity

Magnetic Intensity (H)

Magnetic field produced by external sources (currents in coils), independent of medium :

$$\boxed{H = B_0 / \mu_0}$$

<- SI : A / m
<- same unit as M

Total B in a medium

$$\boxed{B = \mu_0 (H + M)}$$

<- external +
<- induced

Compare with electric : D = eps_0 E + P ;
B plays the role of D , H that of E , M that of P.

Inside a long solenoid (vacuum) : B_0 = u_0 n I
-> H = n I (independent of u_0).

Useful : H depends only on free currents.

Susceptibility & Permeability

Magnetic susceptibility (chi_m)

$$\boxed{M = \chi_m H}$$

<- chi : dimensionless
<- (material property)

Sign tells the type of material :

chi < 0 -> diamagnetic
chi > 0 (small) -> paramagnetic
chi >> 1 -> ferromagnetic

Relative permeability (u_r)

Substituting M = chi_m H into B = u_0 (H + M) :

$$B = \mu_0 (1 + \chi_m) H$$

$$\boxed{\mu_r = 1 + \chi_m}$$

<- u_r dimensionless
<- (= u / u_0)

Permeability (u)

$$\boxed{\mu = \mu_0 \mu_r}$$

<- SI : T m / A
<- = H / m

So $B = \mu_0 \mu_r H = \mu H$.

Summary Table : M , H , B

Quick reference for the three vectors :

Definitions

B : total magnetic field. (T)
H : magnetising field (A/m)
M : magnetisation (A/m)

Key relations :

$$\boxed{B = \mu_0 (H + M)}$$

$$\boxed{M = \chi_m H}$$

$$\boxed{\mu_r = 1 + \chi_m}$$

$$\boxed{B = \mu_0 \mu_r H = \mu H}$$

Classification quick view

Diamagnetic : chi small , < 0 . u_r < 1.
Paramagnetic : chi small , > 0 . u_r > 1.
Ferromagnetic : chi large , > 0 . u_r >> 1.

(Next 3 pages cover each class in detail.)

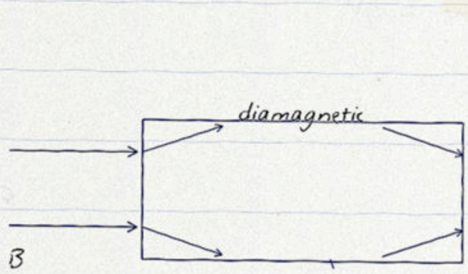
Diamagnetic Substances

Atoms have no permanent magnetic moment (paired electrons, net $m = 0$).

In an external B, Lenz - like induction sets up a weak moment opposite to B.

Properties

1. chi_m small & negative.
2. u_r slightly less than 1. *
3. Expelled from regions of strong B.
4. *Field inside is less than outside.



Examples

Bismuth, copper, water, gold, silver, Hg, N_2.

Paramagnetic Substances

Atoms have permanent magnetic moments (un - paired electrons) but randomly oriented, so the net $M = 0$ in the absence of B.

In an external B, moments partially align along B -> weak net M parallel to B.

Curie's Law

$$\boxed{\chi_m = C / T}$$

<- C = Curie const
<- T = absolute T (K)

At higher T, thermal jitter wins -> less align; M / B = chi_m / u_0 falls as 1 / T.

Properties

1. chi_m small & positive.

2. u_r slightly greater than 1.

3. Attracted weakly toward strong B.

Examples : Al, Pt, Cr, Mn, Na, O2 (liquid).

Ferromagnetic Substances

Strong spontaneous alignment of atomic moments within domains - clusters of 10 17 atoms all pointing the same way.

In an external B :

1. Domains in B grow at expense of those opposed to B (domain - wall motion).

2. Remaining domains rotate to align.

Curie - Weiss Law

$$\boxed{\chi_m = C / (T - T_c)}$$

<- T_C : Curie
<- temperature

Above T_C , ferromagnet -> paramagnet.
Curie temp : Fe 1043 K , Ni 631 K.

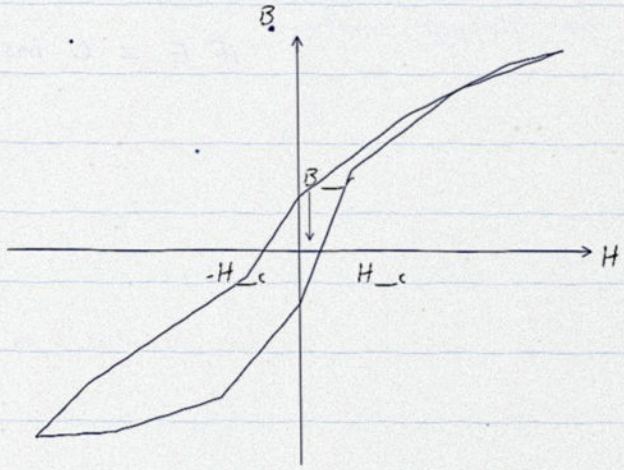
Properties

1. chi_m very large , positive (10 3 - 10 5).

2. u_r very high -> field amplification.

Hysteresis Loop

Plot of B (y - axis) vs H (x - axis) as H is increased , reversed , and cycled back.



Key features

① Retentivity (B_r) : B left when H = 0.

② Coercivity (H_c) : reverse H needed to bring B back to zero.

Loop area energy lost per cycle as heat.

Soft vs Hard Ferromagnets

Soft ferromagnets

1. Low retentivity, low coercivity.
2. Narrow hysteresis loop; small energy loss per cycle (low heating).
3. Easily magnetised and demagnetised.
4. Examples : soft iron, silicon steel, mu-metal (Ni-Fe alloy).
5. Used in transformer cores, relays, electromagnets, inductor cores.

Hard ferromagnets

1. High retentivity, high coercivity.
2. Fat hysteresis loop; hold field strongly.
3. Examples : steel, alnico, ferrites.
4. Used : permanent magnets, data storage.

Worked Example

Problem : A bar magnet of moment 0.5, A m 2 is placed in a uniform field of 0.2 T at angle 60 deg with the field. Find (a) torque , (b) work done to rotate from 0 to 60 , (c) potential energy at 60 .

(a) Torque

$$T = M B \sin \theta$$
$$= (0.5)(0.2) \sin 60$$
$$= 0.1 \cdot (\text{sqrt } 3 / 2)$$
$$= 0.0866 \text{ N m}$$

(b) Work done

$$W = M B (\cos 0 - \cos 60)$$
$$= (0.5)(0.2) (1 - 0.5)$$
$$= 0.1 \cdot 0.5 = 0.05 \text{ J}$$

(c) Potential energy

$$U = - M B \cos \theta$$
$$= - (0.5)(0.2)(0.5)$$
$$= \cancel{+0.05} - 0.05 \text{ J}$$

Note : U is negative since theta < 90 (still below the reference). Lower U = more stable.

Summary - Key Formulae

Bar magnet

$M = m \cdot (2l)$; m = pole strength

$B_{ax} = \mu_0 \, 2M / (4 \pi r^3)$

$B_{eq} = \mu_0 M / (4 \pi r^3)$

Torque, energy

$T = M B \sin\theta$; $T = M \times B$

$U = - M \cdot B$; $T = 2 \pi \, sqrt(I / MB)$

Gauss for magnetism

sum $B \cdot dA = 0$ (no monopoles)

Earth

$B = sqrt(H^2 + V^2)$; $\tan I = V / H$

3 elements : D , I , H.

M , H , B relations

$B = \mu_0 (H + M)$

$M = \chi_m H$

$\mu_r = 1 + \chi_m$; $\mu = \mu_0 \mu_r$

Materials

Dia : chi < 0 , u_r < 1 ; Para : chi > 0 , u_r > 1

Ferro : chi >> 1 ; Curie - Weiss : chi = C/(T - T_C)